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OF AUTOMATIC CONTROL

## **Iron and Steel**

### **Metallurgical Processes and Materials Handling**

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# 39



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**Metallurgical Processes**  
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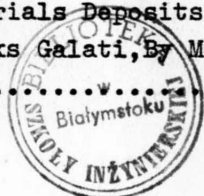


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# РАЦИОНАЛЬНЫЙ АЛГОРИТМ УПРАВЛЕНИЯ ТЕПЛОВЫМ СОСТОЯНИЕМ ДОМЕННОЙ ПЕЧИ С ИСПОЛЬЗОВАНИЕМ ИНФОРМАЦИОННО-УПРАВЛЯЮЩИХ МАШИН

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Под управлением тепловым состоянием доменной печи мы понимаем задачу по отысканию и стабилизации такого температурного поля в печи, которое соответствует оптимальному варианту ведения доменной плавки (форсированный ровный ход при минимальном расходе кокса).

Принципиальной основой рассматриваемого алгоритма управления является теория теплообмена [1], получившая широкое признание. Исходной предпосылкой этой теории служит представление о завершенности теплообмена по высоте доменной печи.

На рис.1, а показан характер температурного поля в современной доменной печи при нормальных условиях ее работы. Развитие тепло- и массообмен<sup>ных</sup> процессов между встречными потоками шихты и газа отражено ходом кривых  $t_{ш}(H)$  и  $t_{г}(H)$  (рис.1,б). Близкое совпадение этих кривых на среднем участке говорит о существовании в средней части печи области замедленного теплообмена, в пределах которой процессы теплопередачи практически завершены. Эта область разделяет верхнюю и нижнюю ступени интенсивного теплообмена со своими существенно разными закономерностями [1].

Промежуточная область замедленного теплообмена с приблизительно постоянной по высоте печи температурой шихты и газа выполняет роль своеобразного демпфера, который устраняет взаим-

ное влияние колебаний в тепловом состоянии верха и низа печи. Наличие такого демпфера создает условия для относительной автономности тепловой работы верхней и нижней ступеней теплообмена.

Автономность тепловой работы верха и низа печи подтверждается практическими данными. Нередки случаи, когда процесс доменной плавки ведется при требуемом тепловом состоянии горна печи, но с перегревом или недогревом ее шахты, то есть при условиях далеких от оптимального. Такая практика ведения технологического процесса обусловлена, в основном, недостатками некоторых существующих методов управления тепловым состоянием доменной печи в целом. Эти методы не позволяют выявлять "Тепловые перекосы" и оперативно воздействовать на температурное поле одной зоны печи, не затрагивая другую.

Из выявленной автономности тепловой работы отдельных зон следует, что тепловое состояние доменной печи принципиально неверно и невозможно оценивать в целом. Для каждой зоны должен быть свой независимый показатель, позволяющий количественно и однозначно оценивать ее тепловое состояние. Помимо этих показателей для управления тепловым режимом печи нужны также эффективные управляющие воздействия с избирательным (местным) влиянием на тепловую работу отдельных зон печи.

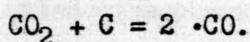
Следовательно, нами выдвигается и отстаивается идея раздельного контроля и локального управления тепловым состоянием верха и низа доменной печи.

Исходя из рассмотренных особенностей доменной печи, как теплотехнического объекта управления, предлагается рабочий объем печи делить на две зоны - верхнюю и нижнюю (рис. I, а).



За условную границу между этими зонами принимается изотермическая поверхность с температурой  $t_0$  - температурой газа в области замедленного теплообмена.

Сопоставление тепловых и физико-химических процессов доменной плавки показывает, что в пределах верхней зоны преимущественно протекают реакции непрямого восстановления железа. Нижняя зона охватывает область интенсивного развития реакций прямого восстановления, область фурменных очагов и горн печи. Температура  $t_0$ , принятая за условную границу раздела, характеризует начало заметного развития эндотермической реакции восстановления двуокиси углерода:



Можно заметить, что пространственная граница между преимущественным развитием реакций прямого и непрямого восстановления выражена менее определенно, чем предлагаемое нами деление печи на две зоны по вполне определенной изотерме. Выбор температуры  $t_0$  за границу раздела обоснован тем, что эта температура для определенных условий плавки довольно стабильна и может быть найдена заранее. Температура  $t_0$  лежит, обычно, в пределах от 850 до 900°C, в зависимости от марки выплаваемого чугуна, состава дутья и других вполне конкретных особенностей работы печи.

Количественной оценкой теплового состояния верхней зоны печи может служить температура средняя по массе  $\bar{t}_w$  того объема шихты, который заключен между уровнем засыпи и границей раздела (рассматриваемый объем шихты на рис. I, а заштрихован). Поскольку при автоматическом контроле и управлении удобнее пользоваться относительными (безразмерными) величинами, то вместо температуры  $\bar{t}_w$  предлагается другой показатель - индекс температурного

поля верха печи  $i_B = \bar{t}_w / t_o$ , который вычисляется по формуле:

$$i_B = 1 - \left[ \frac{t_k - t_{wk}}{t_k - m(t_{wk} - \Delta t_o)} \cdot \frac{1 - \exp(-A)}{A} - \frac{m \cdot \Delta t_o}{t_k - m(t_{wk} - \Delta t_o)} \right], \quad (I)$$

где  $t_k$  - и  $t_{wk}$  - температура газа и шихты на колошнике печи, °C;

$m = 0,5 \cdot (W_{wk} / W_k + 1)$  - среднее для верхней зоны отношение теплоемкостей потоков шихты и газа;

$W_{wk}$  - и  $W_k$  - теплоемкости потоков шихты и газа на уровне колошника печи, Вт/град;

$\Delta t_o = t_o - t_{wo}$  - разность температур между газом и шихтой на границе раздела между зонами, °C;

$A$  - вспомогательный коэффициент, равный

$$A = \frac{\alpha \cdot S \cdot H}{W_w} (1 - m).$$

Здесь  $\alpha$  - средний для верхней зоны объемный суммарный коэффициент теплообмена с учетом внутреннего теплового сопротивления кусков шихты, Вт/(м³.град);

$S$  - средняя площадь сечения шахты печи, м²;

$H$  - средняя высота верхней зоны печи, м;

$W_w$  - средняя для верхней зоны теплоемкость потока шихты, Вт/град.

Индекс  $i_B$  является комплексным показателем, который учитывает влияние на тепловое состояние верхней зоны печи многих переменных факторов. Однако определение этого индекса не представляет больших трудностей, так как при относительно устойчивой работе печи для вычисления индекса  $i_B$  по формуле (I) достаточ-

но текущей информации о расходе загружаемой шихты  $G_{\text{ш}}$ , температуре  $t_k$  и расходе  $G_k$  уходящего колошникового газа, то есть нужны сведения всего лишь о трех переменных величинах. От точности и надежности применяемых способов контроля этих параметров работы печи зависит достоверность значений вычисляемого индекса  $i_g$ .

Методика сбора и способы первичной обработки всей необходимой информации для определения индекса  $i_g$  рассмотрены в опубликованных ранее работах [2-7]. Там же показано влияние газораспределения на колошнике печи на величину индекса  $i_g$  и даны рекомендации по введению необходимой поправки.

Известно, что тепловое состояние нижней зоны доменной печи можно охарактеризовать затратами тепла  $Q_{\text{пл}}$  на физический и химический нагрев продуктов плавки:

$$Q_{\text{пл}} = Q_{\text{чуг}} + Q_{\text{шл}} + Q_{\text{Si, Mn, P...}} \quad \text{кДж/т чугуна,} \quad (2)$$

где  $Q_{\text{чуг}}$  - энтальпия чугуна с учетом теплоты плавления;

$Q_{\text{шл}}$  - энтальпия шлака, за вычетом тепла образования шлака;

$Q_{\text{Si, Mn, P...}}$  - тепло, затраченное на восстановление кремния, марганца, фосфора, титана и других элементов.

Относительное значение величины  $Q_{\text{пл}}$  названо нами индексом теплового состояния низа печи  $i_n = Q_{\text{пл}} / Q_{\text{опт}}$ . Оптимальные затраты тепла  $Q_{\text{опт}}$  на нагрев продуктов плавки определяется заданным составом чугуна и требуемыми свойствами шлака. Индекс  $i_n$  рекомендуется нами в качестве однозначной количественной оценки теплового состояния нижней зоны печи.

Формула (2) дает действительное значение индекса  $i_n$  за последний период работы печи, но для <sup>его</sup> расчета требуются данные,



которые могут быть получены лишь после выпуска чугуна, то есть не чаще, чем через 3–4 часа работы печи. Промежуточные (ожидаемые) значения индекса  $i_H$  предлагается находить по "мгновенному" тепловому балансу нижней зоны печи с учетом основных закономерностей теплообмена в доменных печах [1]. Для современной доменной печи, работающей на обогащенном кислородом дутье и с вдуванием природного газа, ожидаемое значение индекса  $i_H$  вычисляется по формуле:

$$i_H = \left\{ \frac{V_d \cdot \delta}{(V_{O_2})_K - (V_{O_2})_d} \left[ (c_{N_2} \cdot t_d - 1312) \cdot N_2 + (c_{O_2} \cdot t_d + 7840) \cdot O_2 + \right. \right. \\ \left. \left. + 1,24 \cdot 10^{-3} \cdot c_{H_2O} \cdot t_d \cdot \varphi_d - 11,53 (1 + 0,154 \cdot \eta_{H_2}) \cdot \varphi_d \right] + [c_a \cdot G_a + c_K \cdot K (1 - 0,01 \cdot L)] t_d - \right. \\ \left. - 6912 (1 + 0,411 \cdot \eta_{H_2}) \cdot \Gamma - 3175 \cdot Fe_o \cdot \tau_d - 104500 \frac{\alpha}{P_{сум}} - 24,6 \cdot 10^3 \right\}, \quad (3)$$

где  $V_d$  – расход дутья, м<sup>3</sup>/мин;

$\delta$  – отнимаемое у шихты количество кислорода, м<sup>3</sup>/т чугуна;

$(V_{O_2})_K$  – расход кислорода, входящего в состав CO, CO<sub>2</sub> и H<sub>2</sub>O колошникового газа, м<sup>3</sup>/мин;

$(V_{O_2})_d$  – расход кислорода, поступающего с влажным дутьем, м<sup>3</sup>/мин;

$\Gamma$  – количество инжестируемого природного газа, м<sup>3</sup>/т чугуна;

$t_d$  – температура дутья, °C;

$\varphi_d$  – влажность дутья, г/м<sup>3</sup>;

$O_2$  и  $N_2$  – содержание кислорода и азота в дутье, доли;

$\bar{c}_{O_2}$ ,  $\bar{c}_{N_2}$  и  $\bar{c}_{H_2O}$  – средние удельные теплоемкости кислорода, азота и водяных паров для интервала температур от 0

до  $t_0$ , кДж/м<sup>3</sup>·град;

$G_a$  - и  $K$  - расходы агломерата и кокса, кг/т чугуна;

$\bar{c}_a$  - и  $\bar{c}_k$  - средние удельные теплоемкости агломерата и кокса при температуре  $t_0$ , кДж/кг·град;

$\eta_{H_2}$  - степень использования водорода;

$L$  - содержание летучих в коксе, %;

$\tau_d$  - степень прямого восстановления;

$Fe_0$  - содержание железа в чугуне, за вычетом железа металлодобавок, %;

$P_{\text{сут}}$  - суточная производительность печи, 10<sup>3</sup> т;

$d$  - диаметр горна печи, м.

Для вычисления индекса  $i_H$  по формуле (3) требуется текущая информация о параметрах дутья и колошникового газа (около 15 переменных величин). Трудности сбора этой информации связаны лишь с недостаточной точностью и надежностью существующих систем автоматического контроля доменного производства. Помимо текущей информации для расчетов по формуле (3) нужны также периодические сведения об изменениях в составе шихты и некоторые справочные данные.

Достоверность ожидаемого значения индекса  $i_H$  определяется как качеством собираемой информации, так и принятым методом ее первичной обработки. При корректном решении задачи следует учитывать влияние температурного поля верха печи на изменение состава восходящих газов и на изменение физико-химических свойств шихты за время пребывания ее в верхней зоне. Предлагаемый нами способ раздельного контроля теплового состояния верха и низа печи позволяет это сделать, используя зависимости

$\Delta(CO_2)_K = \psi(i_B)$ ;  $\Delta(CO+H_2)_K = \chi(i_B)$ ,  $\Delta\tau_d = \chi(i_B)$  и другие.

Применением этих корректирующих зависимостей отличается наш метод составления "мгновенных" тепловых балансов низа печи от всех ранее известных. Другая особенность состоит в том, что трудноопределимые коэффициенты и статьи теплового баланса, такие как степень прямого восстановления  $\tau_d$  и потери тепла в окружающую среду, могут постепенно периодически уточняться путем сопоставления расчетов по формулам (2) и (3). То есть возможна самопроверка системы контроля теплового состояния низа печи и основанное на этом самообучение информационно-управляющей машины.

Особенности работы доменной печи — дискретность и цикличность загрузки сырых материалов в печь — накладывают жесткие ограничения по допустимой частоте вычисления индексов  $i_B$  и  $i_H$  по формулам (1) и (3). Значительная разница в физических свойствах и температуре отдельных порций загружаемых материалов приводит к тому, что кривые изменения мгновенных значений многих контролируемых параметров работы печи имеют сложный пульсирующий характер (рис. 2). Разная частота пульсаций и заметная разница в инерционности измерительных систем отдельных величин не позволяют использовать их мгновенные значения для вычисления индексов  $i_B$  и  $i_H$  в произвольно взятые или заранее установленные моменты времени.

Анализ особенностей доменного процесса и характера текущей информации о работе печи показал, что за оптимальный интервал между последовательными вычислениями индексов  $i_B$  и  $i_H$  следует принять время цикла загрузки — промежуток времени между 5–7 подачами (в зависимости от принятой программы загрузки). Это время не постоянно и определяется интенсивностью работы



печи, а точнее, скоростью схода подач. Например, для доменной печи объемом 1513 м<sup>3</sup> цикл загрузки длится от 20–30 мин при быстром сходе подач до 40–60 мин при замедленном ходе печи.

Следовательно, для расчетов по формулам (1) и (3) требуется информация, усредненная за время цикла загрузки, а находящиеся по этим формулам индексы  $i_B$  и  $i_H$  характеризуют тепловое состояние верха и низа печи за последний цикл загрузки, то есть, примерно, за последние полчаса работы печи.

Вспомогательные операции по усреднению мгновенных значений контролируемых величин за время цикла загрузки, которое заранее не известно, целесообразно возложить на специальные аналоговые интегрирующие устройства непрерывного действия [5–7]. В этом случае цифровая информационно-управляющая машина, выполняющая основные вычисления, может работать периодически с интервалом 15–50 мин, причем темп ее работы не устанавливается заранее, а задается интенсивностью самого технологического процесса.

Показанная принципиальная возможность раздельного теплового контроля верхней и нижней зон доменной печи позволяет ставить вопрос о реализации идеи локального управления тепловым состоянием этих зон.

У современных доменных печей средний уровень засыпи, давление на колошнике, расход дутья и программа загрузки либо стабилизированы, либо используются для управления газораспределением и ровностью схода шихтовых материалов. При таких условиях для управления тепловым режимом печи приходится пользоваться другими воздействиями, которые можно подразделить на "воздействия сверху" – изменение удельного расхода кокса или применение

специальных подач - и на "воздействия снизу" - изменение качественного состава дутья и расхода инжестируемого топлива.

Динамика влияния отдельных управляющих воздействий на тепловое состояние верхней и нижней зон печи, как объектов регулирования, дана на рис.3. Изменение индекса  $i_B$  найдено расчетом при ступенчатом увеличении количества горновых газов, вызванного применением воздействий. Небольшое первоначальное похолодание верха печи после увеличения удельного расхода кокса объясняется ростом теплоемкости потока шихты  $W_{ш}$ , поскольку удельная теплоемкость кокса больше удельной теплоемкости агломерата. Разогрев шахты печи начинается лишь тогда, когда шихта нового состава дойдет до фурм и от сгорания дополнительного кокса возрастает количество горновых газов.

Временное падение индекса  $i_H$  при подаче природного газа вызвано затратами тепла на конверсию инжестируемого топлива. В дальнейшем повышение содержания водорода в восстановительных газах снижает расход углерода кокса на прямое восстановление, в силу чего доля кокса, сгорающего на фурмах, возрастает, а это ведет к увеличению температуры и выхода горновых газов.

Из-за периодичности вычислений индексов  $i_B$  и  $i_H$  возможен только дискретный выбор локальных регулирующих воздействий с частотой, кратной циклу загрузки. При этом минимальный интервал между отдельными управляющими командами не может быть меньше того отрезка времени, за который отчетливо выявляется результат предыдущего воздействия. Поэтому (рис.3) управляющие команды можно подавать не чаще, чем через цикл загрузки при использовании "воздействий снизу", и лишь через 11-12 циклов загрузки при применении "воздействий сверху".

Чтобы не допускать длительных и глубоких переходных процессов, существенно затрудняющих контроль и процесс управления тепловым режимом печи, предлагается использовать только "воздействия снизу". Как исключение изредка допустимы изменения удельного расхода кокса, в тех случаях когда одних "воздействий снизу" недостаточно для стабилизации температурного поля печи или же дальнейшее снижение расхода инжестируемого топлива экономически не целесообразно.

На рис.4 представлены результаты расчетов [8] по изучению статических свойств всех регулирующих "воздействий снизу". Основная особенность найденных зависимостей состоит в том, что результат влияния некоторых воздействий на тепловое состояние верха и на тепловое состояние низа печи имеют разные знаки. Это позволяет подобрать такие комплексы одновременно поданных воздействий, общее влияние которых обладает избирательным (локальным) действием только на верх или только на низ печи.

Следовательно, путем правильно подобранных изменений в расходе инжестируемого топлива ( $G$ ) и температуре ( $t_g$ ) и качественном составе дутья ( $(O_2)_g, \varphi_g$ ) можно осуществить целенаправленное локальное управление тепловым состоянием как верха, так и низа печи. Составление нужного комплекса управляющих воздействий связано с трудоемкими и тщательными расчетами, так как приходится учитывать взаимосвязь отдельных воздействий через объект регулирования и динамику переходных процессов. Этими расчетами уточняются необходимые изменения расхода дутья и последовательность выполнения управляющих команд.

Все возможные варианты комплексов управляющих воздействий для конкретных условий доменного производства могут быть зара-



нее просчитанны и в закодированном виде введены в память информационно-управляющей машины. При работе в схеме автоматики эта машина по определенной логической схеме выбирает тот комплекс воздействий, который является оптимальным при фактических условиях плавки. Исходными данными являются информация об отклонениях  $\pm \Delta i_g = (i_{g, \text{опт}}) - i_g$ ,  $\pm \Delta i_n = (i_{n, \text{опт}}) - i_n$  и  $\pm \Delta t = 950 - t_g$ , сведения о резервах возможного изменения отдельных воздействий, а также заданные условия предпочтения одних воздействий другим, исходя из технологических и экономических соображений.

При отыскании оптимального значения индекса  $(i_g)_{\text{опт}}$  для сравнительно продолжительного периода работы печи можно предложить установленные нами [1,4] экстремальные зависимости между величиной индекса  $i_g$ , производительностью  $P$  и удельным расходом кокса  $K$  (рис.5). Эти зависимости для каждой печи и отдельных периодов ее работы необходимо периодически корректировать. При ориентировочных расчетах можно принять, что  $(i_g)_{\text{опт}} = 0,7 \div 0,8$ . Оптимальное значение индекса теплового состояния низа печи  $(i_n)_{\text{опт}} = 1$ , как это следует из самого определения индекса.

Таким образом, алгоритм независимого контроля и локального управления тепловым состоянием верхней и нижней зон доменной печи сводится к следующему.

По заданной марке выплавленного чугуна и требуемого состава шлака устанавливается необходимый состав шихты, удельный расход кокса,  $K$  и оптимальные затраты тепла  $Q_{\text{опт}}$  на нагрев продуктов плавки. Исходя из конкретных условий плавки и анализа работы печи за последний период уточняются все постоянные и вспомогательные коэффициенты, входящие в формулы (1) - (3), а

также корректируется зависимость  $P = \varphi(i_B)$ , по которой устанавливается  $(i_B)_{\text{опт}}$ . Уточняются условия выбора отдельных управляющих воздействий. Все эти сведения вводятся в память информационно-управляющей машины, которая по мере того, как обновляется шихта в рабочем пространстве печи, переходит на расчеты по новым коэффициентам.

С начала цикла загрузки система контроля собирает всю необходимую текущую информацию о параметрах дутья, инжестируемого топлива, колошникового газа и загружаемой шихты. По окончании цикла загрузки (через заданное число подач) определяются средние значения контролируемых параметров и выполняются все промежуточные вычисления (приведение к стандартным условиям, определение отдельных коэффициентов и т.п.), после чего по формуле (1) рассчитывается индекс  $i_B$ . Найденное значение индекса  $i_B$  уточняется по дополнительным данным о распределении газового потока на колошнике печи.

По величине индекса  $i_B$  корректируется состав колошникового газа и коэффициент  $\tau_d$ , а затем по формуле (3) определяется ожидаемое значение индекса  $i_H$ . С начала нового цикла загрузки все операции повторяются. После выпуска чугуна, то есть с интервалом в 3-4 часа, по формуле (2) находится действительное значение индекса  $i_H$  за последний период работы печи. Путем сопоставления полученных результатов уточняются отдельные коэффициенты для последующих расчетов индекса  $i_H$  по формуле (3).

Один раз за цикл загрузки, после того как будут найдены текущие значения индексов  $i_B$  и  $i_H$  определяются отклонения  $\pm \Delta i_B$ ,  $\pm \Delta i_H$  и  $\pm \Delta t_d$ , по знаку и величине которых и с учетом



дополнительных условий по заданной логической схеме выбирается оптимальный вариант комплекса управляющих воздействий. Выполнение найденного решения осуществляется путем автоматического ступенчатого или иного изменения заданий для регуляторов-стабилизаторов параметров дутья и инжестируемого топлива. При необходимости изменить удельный расход кокса система автоматики работает как "Советчик мастера". Информация об изменениях в составе шихты вводится в память машины оператором или автоматически от системы загрузки сырых материалов в печь.

Поясним сказанное простым примером. Пусть системой контроля выявлено, что  $\Delta i_g = -0,01$ ,  $\Delta i_H = 0$  и  $t_g = 950^\circ\text{C}$ . В этом случае, чтобы вернуть тепловое состояние верха печи к прежнему оптимальному значению, не затрагивая при этом теплового состояния низа печи, можно (см.рис.4) увеличить расход природного газа на  $28 \text{ м}^3/\text{т}$  чугуна и снизить температуру дутья на  $30^\circ\text{C}$ . Если же учитывать динамику переходных процессов, связанных с изменением степени прямого восстановления (см.рис.3), то целесообразно при увеличении расхода газа сначала несколько повысить температуру дутья и лишь потом постепенно ее снижать до нужного значения. Могут быть и другие варианты комплекса управляющих воздействий.

В качестве одного из вариантов реализации идеи раздельного контроля и локального управления тепловым состоянием верха и низа доменной печи предлагается блок-схема системы автоматизации ее теплового режима - система УПИ (рис.6). Помимо обычных систем контроля и стабилизации отдельных входных параметров, в системе УПИ используются аналоговые усредняющие устройства, са-

мостоятельные блоки памяти и цифровая информационно-управляющая машина, которая выполняет все основные операции предлагаемого рационального алгоритма контроля и управления тепловым состоянием доменной печи. В отличие от всех ранее опробованных алгоритмов и схем автоматизации доменного производства системой УПИ впервые отстается реальная возможность отыскания и стабилизации оптимального температурного поля во всем рабочем пространстве печи.



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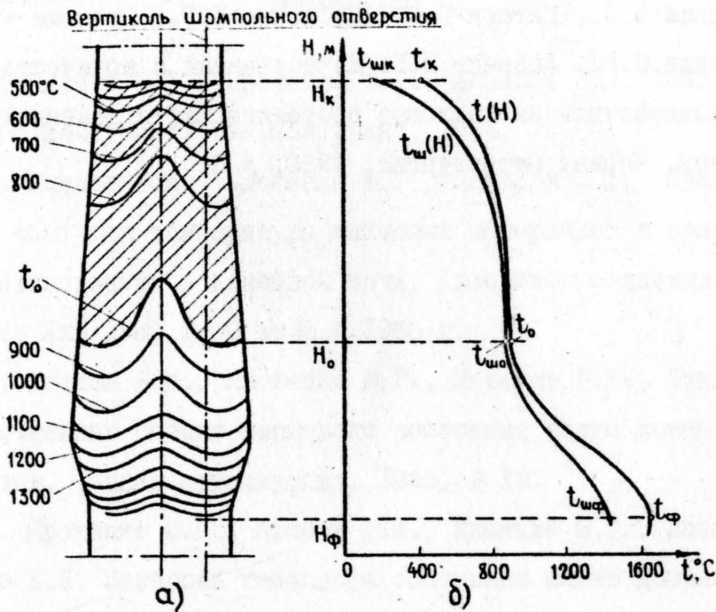
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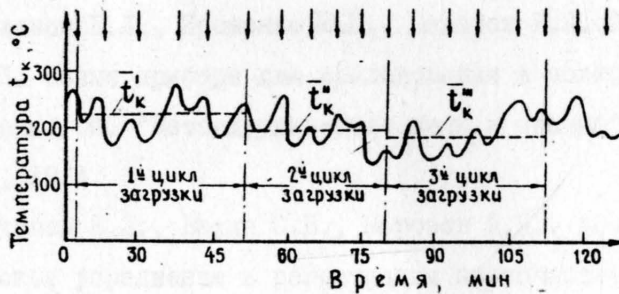
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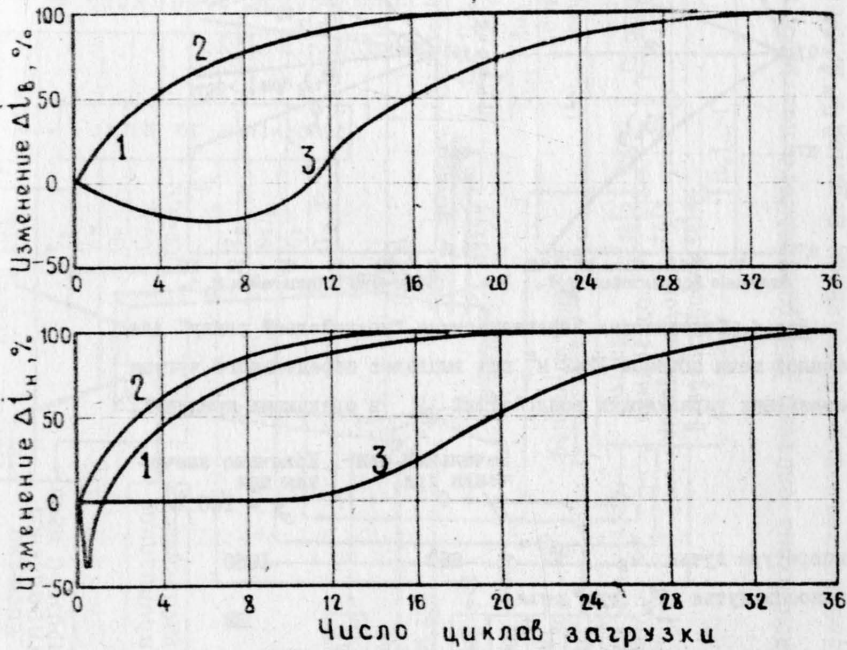
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**Рис.1.** Характер температурного поля доменной печи объемом 1242 м<sup>3</sup> при выплавке передельного чугуна и нормальных условиях работы: а - деление рабочего пространства печи на верхнюю (заштриховано) и нижнюю тепловые зоны; б - кривые изменения температуры встречных потоков шихты и газа по вертикали шомпольного отверстия.



**Рис.2.** Кривая изменения мгновенных значений температуры колошниково-го газа (стрелками указаны моменты опускания большого конуса загрузочного аппарата доменной печи):  $\bar{t}'_k$ ,  $\bar{t}''_k$  и  $\bar{t}'''_k$  - средние значения температуры колошниково-го газа за 1-й, 2-й и 3-й циклы загрузки.



**Рис.3.** Ориентировочные временные характеристики для верхней и нижней зон доменной печи как объектов регулирования:

1 - при изменении расхода инжектируемого топлива (с учетом динамики изменения степени прямого восстановления  $\tau_d$ ); 2 - при изменении параметров дутья; 3 - при изменении удельного расхода кокса.



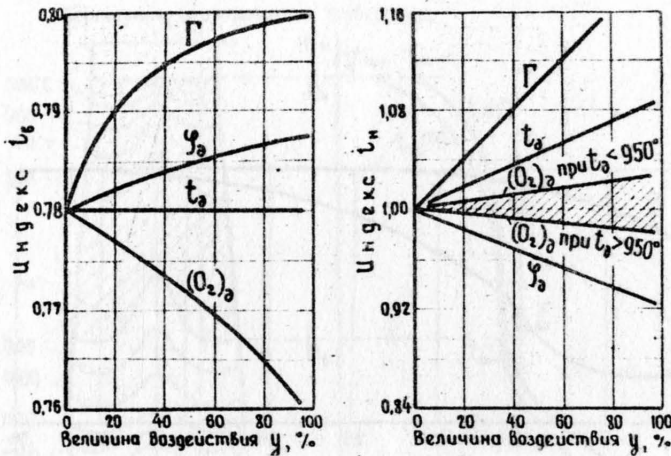


Рис.4. Статические характеристики "воздействий снизу" для доменной печи объемом 1242 м<sup>3</sup> при выплавке передельного чугуна и изменении управляющих воздействий  $\gamma$  в следующих пределах:

	Начальные значения при $\gamma = 0$	Конечные значения при $\gamma = 100\%$
Температура дутья $t_d$ , °C	850	1050
Влажность дутья $\varphi_d$ , г/м <sup>3</sup> дутья	7	32
Содержание кислорода в дутье $(O_2)_d$ , %	21	23
Расход природного газа $\Gamma$ , м <sup>3</sup> /т чугуна	0	140

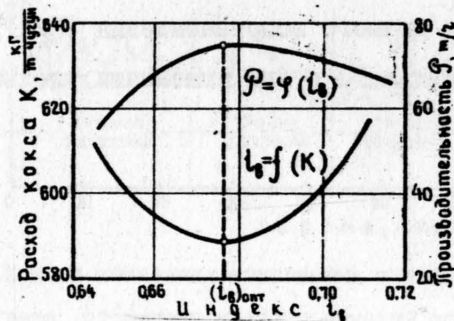


Рис.5. Зависимости  $i_s = f(K)$  и  $P = \varphi(i_s)$  для доменной печи объемом 1242 м<sup>3</sup> при выплавке передельного чугуна (по данным за октябрь-ноябрь 1966 г.)

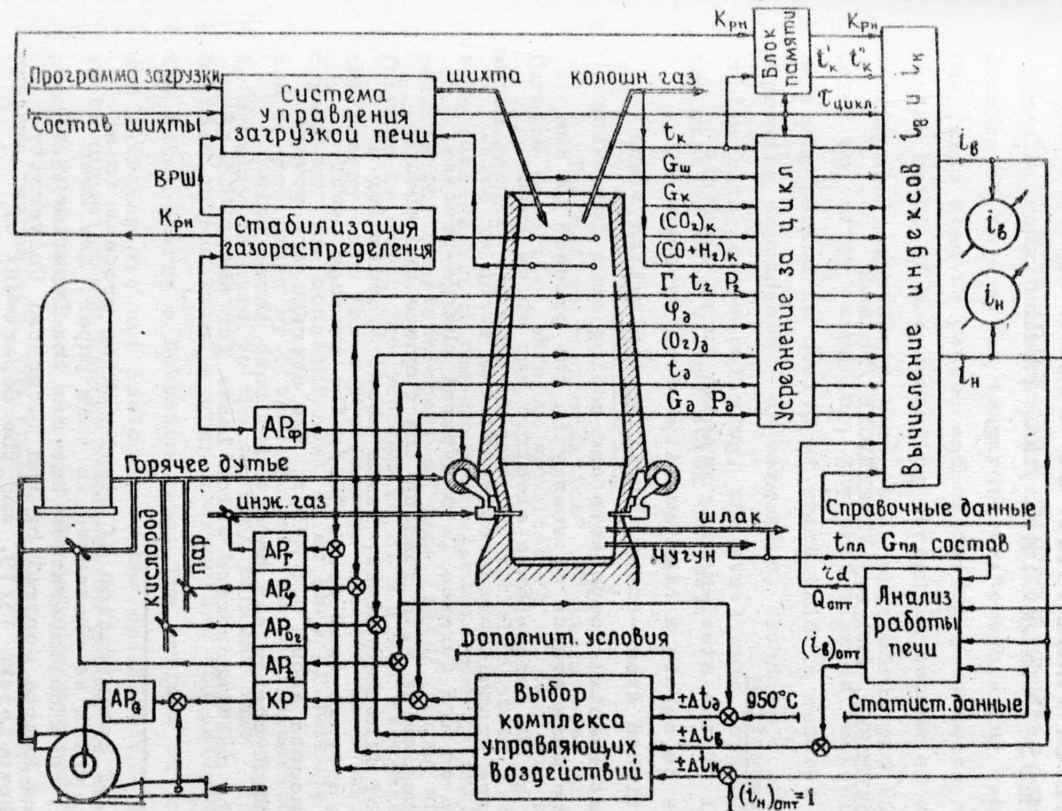


Рис.6. Блок-схема системы УПИ раздельного контроля и локального управления тепловым состоянием верхней и нижней зон доменной печи, работающей на комбинированном дутье.

# DYNAMICAL OPTIMIZATION OF STEEL-MAKING PROCESS IN ELECTRIC ARC FURNACE

by

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## 1. Introduction

The goal of the steel-making process in an electric arc furnace is to obtain a high quality steel by means of melting selectioned steel-scrap, purifying the melted steel /"bath"/ of detrimental components and adding some improving components. The main source of energy in the process is the heat supplied by an three-phase electric arc which burns in between three graphit electrodes and the scrap or bath. The process in an arc furnace is a typical charge-process; each charge, beginning with loading the furnace and ending with casting of finished steel, lasts about 6 hours for a furnace of an average load /e.g. 30t/. The average power consumption amounts about 0.6 MWh/t, the average arc power- about 3 MWh. Each charge can be divided into three main periods: 1/ the melting period which lasts for about 0.5 of

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<sup>x</sup>/The problem of optimization of steel-making process in an arc furnace, which is presented in this paper, has been investigated in a collaboration between the Department of Automatic and Remote Control of the Technical University of Warsaw, Nowowiejska 15/19, and the Department of Electrothermics of the Electrical Engineering Institute, Warsaw-Miedzylesie, as well as the Stalowa Wola Foundry by which the research has been supported. Together with the authors of the paper, Bohdan Frelek, Zygmunt Komar and Andrzej Markowski from the Department of Automatic Control, and Mirosława Sławecka, Mieczysław Solecki from the Department of Electrothermics have participated in theoretical synthesis and technical design of the optimal control system. Furthermore, M.Mazur, T.Skrzypek, P.Maj from the Department of Electrothermics and Cz.Kulak, E.Gielarek, Z.Dąbrowski and R.Hernik from the Stalowa Wola Foundry have collaborated in different aspects of this broad project.

the whole charge's time and demands about 0.7 of the whole charge's energy consumption; after melting, the bath is overheated to a given temperature /"overheating temperature"/, the first slag is casted and oxidating components are brought in the bath; 2/ the oxidating period in course of which the detrimental components - as coal, phosphorus and sulphur - are burnt up; after oxidating, the next slag is casted; 3/ the refining period in course of which oxid is reduced and the improving components - as manganium, chromium - are brought in the bath; after refining, the steel gets overheated and the casting of finished steel follows.

## 2. Setting the problem

The notion "optimization" of the process in an arc furnace describes here an application of such process control which results in minimal cost of finished steel /zł/t/, taking into consideration the cost of electric energy /zł/kWh/ as well as the constant costs of time /zł/h/ during which the furnace is used. The problem is of a great economic importance: the energy cost for a 30t furnace amounts to 10 million zł a year and the constant cost of an hour of furnace's usage can be estimated by 6 thousand zł. The optimization of a charge is usually related to the melting periode only <sup>1,2,3</sup> which decides on the total energy consumption and influences strongly the total charge's duration time. The periods of oxidating and refining are not included to the optimization because their duration and energy consumption depend on chemical processes in the bath and are strictly determined by the process technology. The optimization of loading and casting of an isolated furnace is trivial: they must be performed as intensively as it is possible.

Let us consider the simplified schema of an arc furnace with only one phase of arc circuit shown, given in Fig.1. The electrodes are supplied through the power cable TW from the furnace's transformer TP which has several taps of number  $j=1,2,\dots$  on its secondary winding; the taps are set



by by a tap-setting device that determines the nominal secondary voltage  $V_{2n}$ . The arc current  $I$  is controlled by the arc controller  $St$  /it includes the electrode servo-mechanisms  $M$  which lift or lower the electrodes/ according to the reference value  $I_0$ . Because the arc controller is acting rather quickly compared to the duration of the charge we can assume for the first approximation - apart from rapidly varying disturbances - that  $I=I_0$ . The main controls of the process are, therefore:

- $j$  - the tap number of the secondary transformer's winding;
- $I$  - the arc current /exactly speaking, the given value  $I_0$  of the arc controllers/.

It is easy to show<sup>2</sup> that the secondary voltage of the transformer should be kept as large as possible during the melting periode. As the tap number is determined in that a way, the only control during the melting period is the current  $I$ .

Theoretically, the process can be divided into two subprocesses: the electroenergetic one and the thermic one /cf.Fig.2/. The electroenergetic subprocess is practically inertialess compared to the thermic one and can be described for the optimization purposes by the two basic static relationships

$$C_m = C_m(I); \quad P_g = P_g(I) \quad /1/$$

where  $C_m$  is the momentary cost of the supplied power  $/z\dot{t}/h/$ , taking into account the active power as well as the reactive power, and  $P_g$  is the heat power of the arc. Those characteristics can be determined either analytically - by means of an equivalent schema of the electric circuit - or experimentally. Taking into consideration rapidly varying disturbances caused by arc burning, those relationships should be treated as regressions. For approximate considerations which are presented in sec.4, the relationship  $C_m(I)$  can be assumed

to be linear, the relationship  $P_g(I)$  - to be parabolic. An example of real characteristics /1/, determined experimentally, is shown on Fig.3.

Generally, a model of the thermic subprocess should be formulated as a very complicated partial differential equation with respect to the temperature  $\tau$  in the furnace's space. That equation can be approximated, however, by a system of ordinary differential equations taking into consideration three mean temperatures as the process state variables: the charge /or bath/ temperature  $x_1$ , the walls and bottom temperature  $x_2$  and the vault temperature  $x_3$ . The model of the thermic subprocess assumes then the normal form

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_2, x_3, I) = \frac{1}{W(x_1)} [P_g(I) - P_s(x_1, x_2, x_3)] \\ \dot{x}_2 &= f_2(x_1, x_2, x_3) \\ \dot{x}_3 &= f_3(x_1, x_2, x_3)\end{aligned}\quad /2/$$

where  $W(x_1)$  is the thermic capacity of the charge,  $P_s(x_1, x_2, x_3)$  - the thermic power transmitted from the charge to the walls, bottom and vault by conduction, radiation and convection. The forms of the functions  $f_1, f_2, f_3$  are rather complicated and depend on the kind of the heat exchange between walls, bottom and vault, on the furnace's geometry and physical properties. It turns out, however, that the decisive part of the thermic power  $P_s$  depends mainly on the temperature  $x_1$  and not on the temperatures  $x_2$  and  $x_3$ . For that reason for approximate considerations - cf. sec.4 - the model of the thermic subprocess can be, therefore, reduced to the first from the equations /2/ which can be further approximated by a linear equation of a simple inertial element.

For a given load, the performance of the melting period can be expressed by the cost index

$$Q = \int_0^{t_a} [C_o + C_m(I)] dt \quad /3/$$

where  $t_d$  is the melting period duration which is not given but determined by the overheating temperature  $x_1(t_d) = x_1^d$ . The constant  $C_0$  signifies the constant cost of an hour of furnace's usage /zk/h/, which results from the wages, redemption and so on.

The mean power transmitted by the furnace's transformer is constrained. Because the transmitted power is approximately proportional to the momentary cost the constraint can be expressed as

$$M = \frac{\int_0^{t_d} C_m(I) dt}{t_d} \leq M_d \quad /4/$$

where  $M_d$  is a given admissible mean value.

Now we can formulate the optimization problem: determine such a function of time  $I = I(t)$ ,  $t \in [0, t_d]$ , which minimizes the performance functional /3/ by the differential conditions /2/, the global constraints /4/, and by the final condition  $x_1(t_d) = x_1^d$ . The solution of the problem can be based on the maximum principle <sup>4,5,6</sup>.

### 3. The method of solution

Because of the existence of the special constraint /4/ let us introduce new state variables  $x_1^0$  and  $x_2^0$  involved in equations of the form

$$\begin{aligned} \dot{x}_1^0 &= 1 \\ \dot{x}_2^0 &= C_m(I) \end{aligned} \quad /5/$$

Thus, the initial problem can be reformulated as the problem with the constrained ratio of the variables  $\frac{x_2^0(t_d)}{x_1^0(t_d)}$  in the final moment  $t_d$ . It is useful, however, to reformulate the problem further by introducing new cost index with "penalty for violating the constraint" instead of the index /3/

$$Q_z = Q + K(M - M_d)^2 \cdot 1(M - M_d) \quad /6/$$

where  $K$  is a sufficiently large constant; a violation of the admissible mean value  $M_d$  is associated, therefore, with a rapidly growth of the functional /6/. It can be shown<sup>6</sup> that at  $K \rightarrow \infty$  the reformulated problem /without constraint/ is strictly equivalent to the initial problem.

The hamiltonian of the problem has the form

$$H = -(C_0 - \psi_1^0) - (1 - \psi_2^0) C_m(I) + \sum_{i=1}^3 \psi_i f_i(x_1, x_2, x_3, I) + \psi_t \quad /7/$$

where the costates  $\psi$  are determined by the equations

$$\dot{\psi}_1^0 = 0; \dot{\psi}_2^0 = 0; \dot{\psi}_i = -\frac{\partial H}{\partial x_i}; \dot{\psi}_t = 0 \quad /8/$$

The transversality conditions for  $\psi_1^0$  and  $\psi_2^0$  have the form - cf.<sup>6</sup>

$$\begin{aligned} \psi_1^0 &= -\frac{\partial Q_z}{\partial x_1^0} = K \frac{M^2}{t_d} (M - M_d) \mathbb{I}(M - M_d) \geq 0 \\ \psi_2^0 &= -\frac{\partial Q_z}{\partial x_2^0} = -\frac{K}{t_d} (M - M_d) \mathbb{I}(M - M_d) \leq 0 \end{aligned} \quad /9/$$

It follows from the form of the hamiltonian /7/ and the conditions /9/ that the existence of the global constraint /4/ leads to a suppressing of the constant cost of furnace's usage and to a growth of the cost of electric power; evidently

$$C_0 > \psi_1^0 \quad /10/$$

/if the inequality does not hold, then the furnace's transformer is wrongly designed/.

Because the process dynamic is determined mainly by the temperature  $x_1$  we can assume

$$|\psi_1 f_1| \gg |\psi_2 f_2 + \psi_3 f_3| \quad \text{where } f_1 > 0, \psi_1 > 0 \quad /11/$$

As the inequalities /10/ and /11/ hold, we can apply the principle of optimizing feedback<sup>6</sup> and reformulate the necessary condition of optimality

$$\max_I H(I) = H(\bar{I}) = 0 \quad \text{for all } t \in [0, t_d] \quad /12/$$



where  $\hat{I} = \hat{I}(t)$  is the optimal control. We introduce, therefore, the following momentary performance index

$$F = \frac{P_g(I) - P_g(x_1, x_2, x_3) + \eta(x_1, x_2, x_3, t)}{C_o^* + C_m(I)} \quad /13/$$

and then we can rewrite the necessary condition /12/ in the form

$$F(\hat{I}) = \max_I F(I) \quad \text{for all } t \in [0, t_d] \quad /14/$$

The function

$$\eta(x_1, x_2, x_3, t) = \frac{W(x_1)}{\psi_1(t)} [\psi_2(t) f_2(x_1, x_2, x_3) + \psi_3(t) f_3(x_1, x_2, x_3)] \quad /15/$$

is a correction which represents the influence of temperatures  $x_2, x_3$  on the optimal solution of the problem whereas the constant

$$C_o^* = \frac{C_o - \psi_1^0}{1 - \psi_2^0} \leq C_o \quad /16/$$

is the normalized constant cost of the time of furnace's usage considering the constraint /4/; the equality  $C_o^* = C_o$  holds if the constraint does not influence the optimal solution of the problem. It should be noted that if the influence of the temperatures  $x_2, x_3$  on the process dynamics is negligible then we can assume  $\eta = 0$ . If the influence of temperatures  $x_2, x_3$  is small, then the correction  $\eta$  is small compared to  $P_g - P_s$  and can be determined without high accuracy. Thus, determination of the dynamic optimal control  $\hat{I}(t)$  can be reduced to the peak-holding control of the momentary performance index  $F$  only on the basis of the measurements of the powers  $P_g, P_s$  and the cost  $C_m$ , and of an initial knowledge of the normalized cost  $C_o^*$  without necessity of an exact knowledge of the functions  $f_1, f_2, f_3$  and of the correction  $\eta$ . This is the essential property of the optimizing feedback.

In the previous papers referred to the control of arc furnaces, the optimization problem has been treated as a static /steady-state/ problem only and the process dynamics /2/ as well as the global constraint /4/ have not been taken into consideration. In that approach one assume that the arc current  $I$  is constant in time. This current is then determined in such a way that it minimizes the performance function

$$Q_s = t_d [C_o + C_m(I)] \quad /17/$$

which is a statical analogue to the performance functional /3/. On the basis of the thermic balance, we get

$$t_d [P_g(I) - P_{ss}] = x_1^d \bar{W}; \quad t_d = \frac{x_1^d \bar{W}}{P_g(I) - P_{ss}} \quad /18/$$

where  $P_{ss}$  is the mean, constant thermic power transmitted from the charge /the losses/. The condition of the statical optimality can be written, therefore, in the form

$$F_s = \frac{x_1^d \bar{W}}{Q_s} = \frac{P_g(I) - P_{ss}}{C_o + C_m(I)}; \quad F_s(\hat{I}_s) = \max_I F_s(I) \quad /19/$$

where  $\hat{I}_s$  is the constant, statical optimal control.

The conditions /13/, /14/ and /19/ are very similar formally but differ essentially. The difference between them is illustrated in Fig.4. The static optimal heat power  $P_g(\hat{I}_s)$  can be determined graphically /cf.Fig.4a/ by tracing a tangente to the plot  $C_m(P_g)$  from the point  $(P_{ss}, -C_o)$ . Similarly, we can determine the dynamical optimal heat power  $P_g[\hat{I}(t)]$  - cf.Fig.4b - if we assume  $P_s \gg \eta$  and trace the tangente from the point  $[P_s(x_1, x_2, x_3), -C_o^*]$ . The thermic power  $P_s(x_1, x_2, x_3)$  increases during the melting time; so does the dynamical optimal heat power  $P_g[\hat{I}(t)]$  what results clearly from Fig.4b. This property of the optimal solution has a simple physical interpretation: at the beginning of the melting period the power of thermic losses  $P_s$  is small /the charge is cold/ which implies a small heat power  $P_g$  and a high efficiency

of the electrothermic subprocess; in the course of melting period the power  $P_s$  grows and the heat power  $P_g$  must be increased in order to shorten the period of large thermic losses /or to "compensate" them/.

#### 4. Simplified analytic solution and approximate sensitivity analysis.

Choosing a sufficiently simplified model we can solve the problem analitically and get qualitative conclusions about the main features and the sensitivity of the solution. According to the mentions made in sec.2, we assume the equations of the electric circuit in the form

$$p = 1 - (1-i)^2; \quad p = \frac{P_g}{P_{gm}}; \quad i = \frac{I}{I_m}; \quad C_m = C_{mm} \cdot i \quad /20/$$

where  $P_{gm}$  denotes the maximal heat power,  $I_m$  - the arc current which results in the maximal heat power,  $C_{mm}$  - the cost of the power supplied at the maximal heat power. We assume, furthermore, the simplified equation of the dynamics

$$\frac{dx}{d\tau} = \dot{x} = p(i) - x; \quad \tau = \frac{t}{T}; \quad x = \frac{x_1}{k P_{gm}} = \frac{x_1}{x_{1m}} \quad /21/$$

where  $x$  and  $x_1$  should be treated as increments whereas the differentiating is understood as with respect to the variable  $\tau$ ;  $T$  denotes the time constant of the charge,  $k$  - the gain coefficient,  $x_{1m}$  - the maximal increment of the temperature of the charge which can be achieved by permanent applying the heat power  $P_{gm}$ . Now, the performance functional can be written as

$$q_r = \int_0^{\tau_d} (c+i) d\tau; \quad q_r = \frac{\dot{Q}}{C_{mm} T}; \quad c = \frac{C_o}{C_{mm}} \quad /22/$$

where the constant  $c$  is the relative constant cost. The constraint of the mean power supplied assumes the form

$$m = \frac{1}{\tau_d} \int_0^{\tau_d} i d\tau \leq m_d; \quad m_d = \frac{M_d}{C_{mm}} \quad /23/$$

Under the given end-point conditions

$$x(0)=0; \quad x(\tau_d)=x^d; \quad \tau_d - \text{free} \quad /24/$$

we can solve the problem of the minimization of the functional /22/ under the differential conditions /21/ and the integral constraint /23/ analytically; the solution can be based on the maximum principle. We get then the optimal control  $\hat{u} = \hat{u}(\tau)$  in the form

$$\hat{u} = 1 - \xi_0 e^{-\tau}; \quad \xi_0 = \frac{1}{1+c^* + \sqrt{2c^* + (c^*)^2}}; \quad c^* = \frac{c - \psi_1^0}{1 - \psi_2^0} \quad /25/$$

where, similarly as above,  $c^*$  can be interpreted as the relative constant cost reduced accordingly to the influence of the constraint of mean power supplied. The optimal temperature as a function of time,  $\hat{x} = \hat{x}(\tau)$ , takes a form

$$\hat{x} = (1 - \xi_0^2 e^{-\tau})(1 - e^{-\tau}) \quad /26/$$

and the value of the performance functional  $\hat{q}_n$ , the constraint functional  $\hat{m}$  and the duration of the process  $\hat{\tau}_d$  can be expressed as

$$\hat{q}_n = (1+c)\hat{\tau}_d - \xi_0(1 - e^{-\hat{\tau}_d}) \quad /27/$$

$$\hat{m} = 1 - \frac{\xi_0}{\hat{\tau}_d}(1 - e^{-\hat{\tau}_d}) \quad /28/$$

$$\hat{\tau}_d = \ln \left\{ \frac{1+c^* + \sqrt{2c^* + (c^*)^2 + x^d}}{(1-x^d)[1+c^* + \sqrt{2c^* + (c^*)^2}]} \right\} \quad /29/$$

We can assume  $c = c^*$ , if the inequality  $\hat{m}(c^*)|_{c^*=c} \leq m_d$  holds. If the inequality does not hold, we should solve the transcendental equation  $\hat{m}(c^*) = m_d$  and determine the necessary value  $c^* < c$ . However, it is simpler to assume some values of  $c^* \leq c$  and determine the values  $\hat{\tau}_d(c^*)$ ,  $\hat{m}(c^*)$  and  $\hat{q}_n(c, c^*)$  accordingly to the equations /27/, /28/, /29/. We get then not only the solution but also the sensitivity characteristics of



the solution with respect to the integral constraint as well as to the initial current  $\hat{i}_0 = 1 - \xi_0$ . /it should be stressed, that errors in determination of this initial current can be caused by a wrong estimation of a such parameters as  $C_{mm}$ ,  $P_{gm}$  or  $I_m$ . With that, it is useful to present the results achieved as the values of the sensitivity measure<sup>6</sup>

$$s_d(c, c^*) = \frac{\hat{q}_d(c, c^*) - \hat{q}_d(c, c)}{\hat{q}_d(c, c)} \cdot 100\% \quad /30/$$

In order to compare the results of dynamical and traditional statical optimization it should be assumed  $i = \text{const}$ ; then, accordingly to /21, /17/ and /18/, we get

$$x = [1 - (1-i)^2](1 - e^{-\tau}) \quad /31/$$

and

$$q_s = (c + i)\tau_{ds} \quad /32/$$

$$m_s = i \quad /33/$$

$$\tau_{ds} = \ln \frac{1 - (1-i)^2}{1 - (1-i)^2 - x^2} \quad /34/$$

The equation  $\frac{dq_s(i)}{di} = 0$  is transcendental; the simplest way to obtain the solution is to assume some values of  $i$  and to determine the values  $\tau_{ds}(i)$  and  $q_s(i)$  accordingly to /32/, /34/. It is of use to introduce the ratio

$$s_s(c, i) = \frac{q_s(c, i) - \hat{q}_d(c, c)}{\hat{q}_d(c, c)} \cdot 100\% \quad /35/$$

which shows how much worse are the results of statical optimization than those of dynamical optimization.

The results of this comparison and the sensitivity characteristics for a chosen value of  $c$  are presented in Fig.5. It is worth to mention that although the relative profits of the dynamical optimization are rather small, the sensitivity of the dynamical optimization with respect to the constraint of mean power supplied and to the choice

of initial current is much lower than the sensitivity of statical optimization. It follows from that the stronger is influence of the average power constraint the more profitable is the application of dynamical optimization.

In a similar way-cf.<sup>6</sup> - we can examine the sensitivity of the optimal solution with respect to the constant errors  $i_a$  of the determination of optimal control  $\hat{i}$  assuming

$$i = \hat{i} + i_a = 1 + i_a - \xi_a e^{-\tau} \quad /26/$$

or with respect to the amplitude  $i_a$  of rapidly varying random disturbances  $i_p$  of the arc current, assuming

$$i = \bar{i} + i_p; \quad \bar{i} = \hat{i}; \quad i_p = \begin{cases} +i_a, & P_i(+i_a) = \frac{1}{3} \\ 0, & P_i(0) = \frac{1}{3} \\ -i_a, & P_i(-i_a) = \frac{1}{3} \end{cases} \quad /37/$$

where  $\bar{i}$  is the mean value of the current,  $i_p$  is the random component, and  $P_i$  denotes probability.

It is assumed here that the temperature  $x$  depends solely on the mean value of the heat power  $p$  which is in turn determined by changes of the mean current  $\bar{i} = \hat{i}(\tau)$ . We get then, according to /20/

$$\bar{p} = p(\bar{i}) - \frac{2}{3}(i_a)^2 = 1 - (1 - \bar{i})^2 - \frac{2}{3}(i_a)^2 \quad /38/$$

Some results of this sensitivity analysis are presented in Fig.6. It should be emphasized that during the melting period the disturbances of the arc current are strong and the losses associated with them can amount to several percent of the optimal performance - cf. Fig.6 - as by dynamical as by statical optimization. It is very important, therefore, to assure a high performance of the arc controllers which counteract these disturbances.

The sensitivity analysis presented above is related to the open-loop optimal control system. An application of the optimizing feedback corresponds, accordingly to /14/,

to searching the optimal current  $\hat{i}$  by the peak-holding control of the temporary index

$$F = \frac{\rho(i) - x}{c^* + i} ; \quad \max_i F(i) = F(\hat{i}) \quad /39/$$

Thus, the optimizing feedback results in full insensitivity of the system with respect to such parameters as  $\rho_{gm}, C_{mm}, I_m, k$  and  $T$  because the values  $\rho(i), x$  and  $i$  are determined by measurements. On the other hand the sensitivity with respect to the setting the constant  $c^*$  /the estimation of the mean power constraint's influence/ and to the rapidly varying disturbances of the arc current /which cannot be suppressed by rather slowly acting optimizing feedback/ does not change.

In order to estimate the influence of the measurements in accuracy on the optimizing feedback performance further sensitivity analysis has been carried out by means of digital modelling the cases when the state  $x$  /corresponding to the power of thermic losses  $P_s$  / is measured indirectly through an output signal  $y$  which is characterized by the time constant  $\Theta$  of the measurements inertia and compensated by the forcing coefficient  $\delta$

$$\Theta \dot{y} + y = \delta \Theta \dot{x} + x \quad /40/$$

At determining the optimal control, the measured values of the signal  $y$  are put in place of the state  $x$  in the index  $F$ . The state  $x$  may be measured also with a relative error  $\lambda$

$$y = (1 + \lambda)x \quad /41/$$

Another kind of the performance losses in the optimizing feedback system may be caused by finite searching steps length  $\Delta i$  in the real peak-holding controller of the index  $F$ . The sensitivity analysis results for those three cases are presented in Fig. 7a, b, c. It follows from them that the optimizing feedback system has a very low sensitivity to the measurements inaccuracies /in a real system we can

assume  $\lambda < 0,1$ ,  $\Theta < 0,1$  which results in  $s^1 < 0,5\%$  and a reasonable sensitivity to the searching step length.

##### 5. Analysis of more accurate models and experimental verification of results achieved

By applying more accurate models of the relationships /1/ and the dynamics /2/ there several computations have been carried out; the purposes for them were the following:  
 a/ a verification of the models accuracy by computing the melting duration as well as the energy consumption when the usual arc currents are applied and by comparing the results achieved to the experimental data; b/ more accurate determination of the optimal current  $\hat{I}(t)$  on a basis of the maximum principle and the numerical algorithms related to it - cf.6;  
 c/ more accurate sensitivity analysis of various structures of the optimal control.

The results of those computations just slightly differed from the results presented above, rather quantitatively but not qualitatively.

In order to verify experimentally the results achieved the charges of 30t furnace in Stalowa Wola Foundry were run during 6 months according to the optimal /more strictly-suboptimal/ program of arc current changes determined in advance. The program was conducted by an human operator who set the reference values of arc controllers. Then it proved to be that the energy consumption for melting period decreased by about 5% /by about 3% with regard to the whole charge and 1t of steel/ and the melting duration was shortened by about 11% /by about 5% with regard to the whole charge and 1t of steel/. The improvement of the cost index was by about 8% higher than it resulted in theoretical research; it can be explained by the fact that the furnace had not previously been operated according to the static optimal conditions.



## 6. Perspectives of further research and applications.

For a technical implementation of the optimal control system of the process in arc furnace the specialized control equipment have been designed and constructed. It consists of

- a/ Modified arc current controllers and remote controlled tap-setting device
- b/ Devices for automatic measuring and transducing the temperatures and electric powers
- c/ Generator of the open-loop optimal control
- d/ Optimizer comprising of the computing device for index  $F$  and the peak-holding controller
- e/ Controller of the bath temperature that operates during oxidating and refining periods.

The simplified block diagram of that control equipment is presented in Fig.8.

An application of a digital computer for the optimal control of the steel-making process in arc furnace has also been considered. However, it has turned out that an application of a digital computer exclusively for that purpose is unprofitable. It may be profitable in a complex solution where the optimal control would be determined for the whole complex of furnaces in a foundry and associated with programming the optimal operative plans for such complex.

Optimal programs for each furnace could be determined in principle independently. However, the common constraints here occur which follow from:

- a/ admissible power supplied for the complex of furnaces
- b/ constrained transmittance of the loading devices for the complex of furnaces
- c/ constrained transmittance of the casting devices for the complex of furnaces.

In order to consider those common constraints there is a possibility to apply the multilevel optimization methods.

On a basis of those methods the numerical algorithm for determining the optimal programs of the operation of furnaces' complex has been proposed<sup>6</sup>. This algorithm is of a iterative nature: in the first computation /"iteration"/ the optimal programs for each furnace are computed independently; afterwards, "the penalties" for violating the common constraints are determined, the computations are repeated and so on. An example of the results of computations according to such algorithm are presented in Fig.9 where two furnaces are involved. In the first iteration / $i=0$ / the optimal electric powers, loadings and castings for two furnaces /numbered  $\alpha$  and  $\beta$  / are determined; the variables  $\Psi_t$  denote the co-states to the time adjoined which after a sign change can be interpreted as the cost of usage time of a furnace. After completing the first iteration and finding the violation of the common constraints, the penalty coefficients  $\pi$  are determined. These coefficients allow to repeat computations influencing with that on a nature of "the time cost" variation  $-\Psi_t$  and resulting in a speeding up of the operations for the  $\alpha$  -furnace whereas in a delay of the operations for  $\beta$  -furnace. In the subsequent iterations the operation programs for both furnaces appropriately get push apart and a violations of the common constraints are reduced to minimum.

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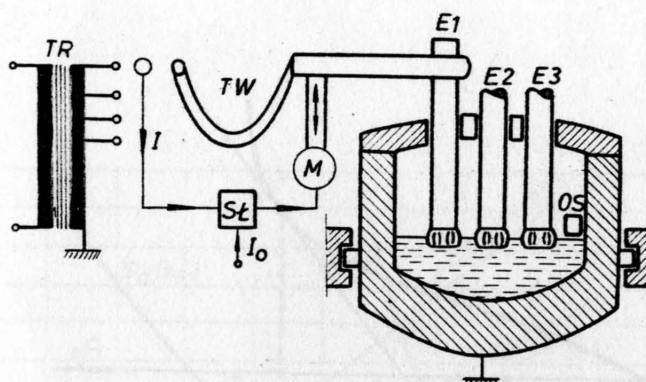


Fig.1. Scheme of an arc furnace.

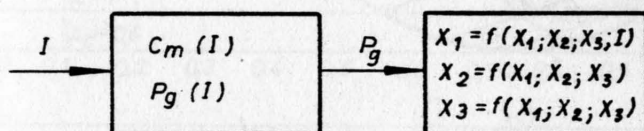
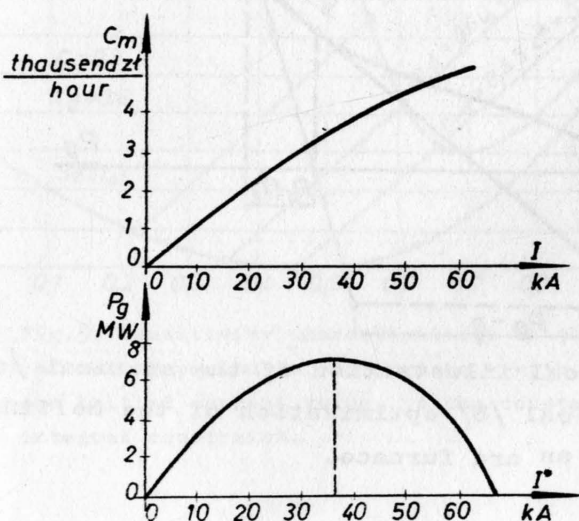


Fig.2. Dividing the steel-making process into two subprocesses: the electroenergetic subprocess and the thermic subprocess.

Fig.3. Cost of electric power  $/C_m/$  and heat power  $/P_g/$  versus arc current  $/I/$ .



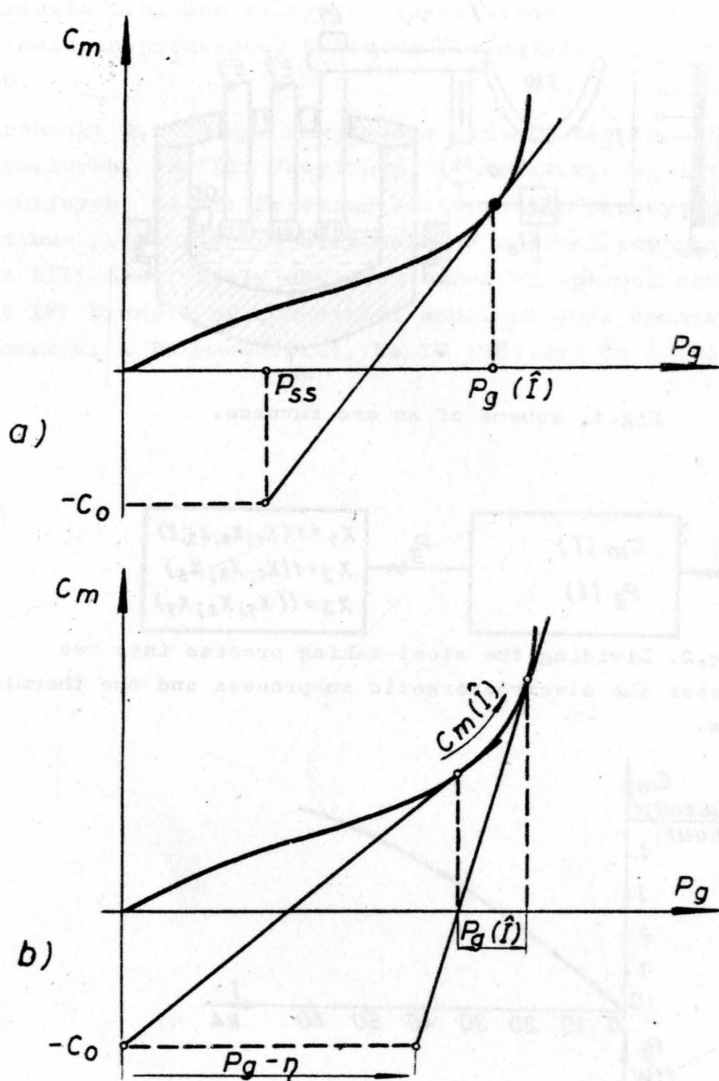


Fig.4. Graphical illustration of the statical /a/ and dynamical /b/ optimization of the melting period in an arc furnace.

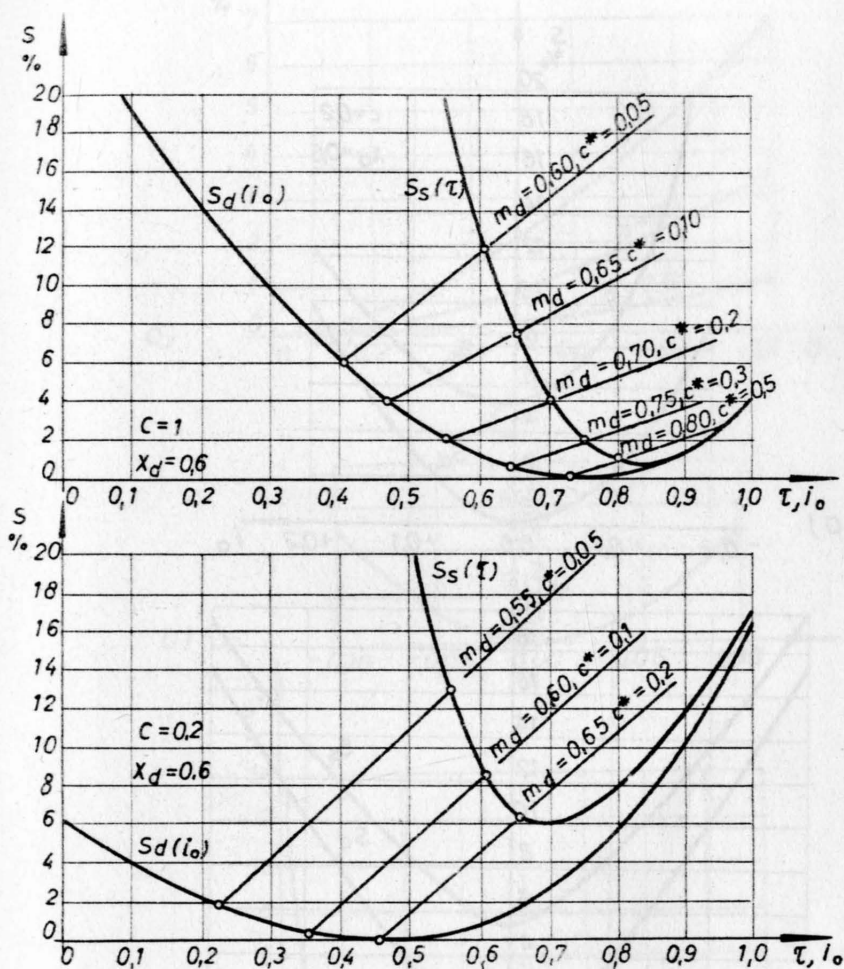


Fig.5. Sensitivity characteristics of the optimal control of melting period in arc furnace - with respect to the chosen initial current value, to the constant  $c^*$  and to the integral constraint.

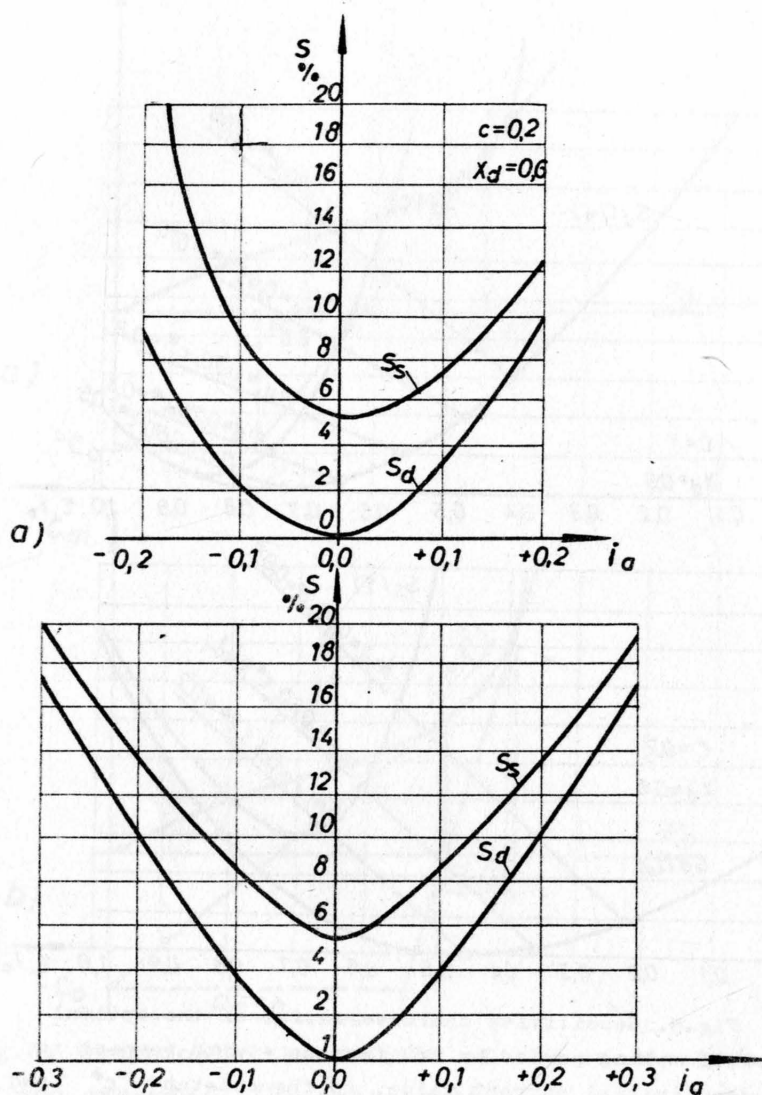


Fig.6. Sensitivity characteristics of the optimal control of melting period in arc furnace - with respect to the constant error of optimal control /a/ and to the amplitude of random disturbances /b/.

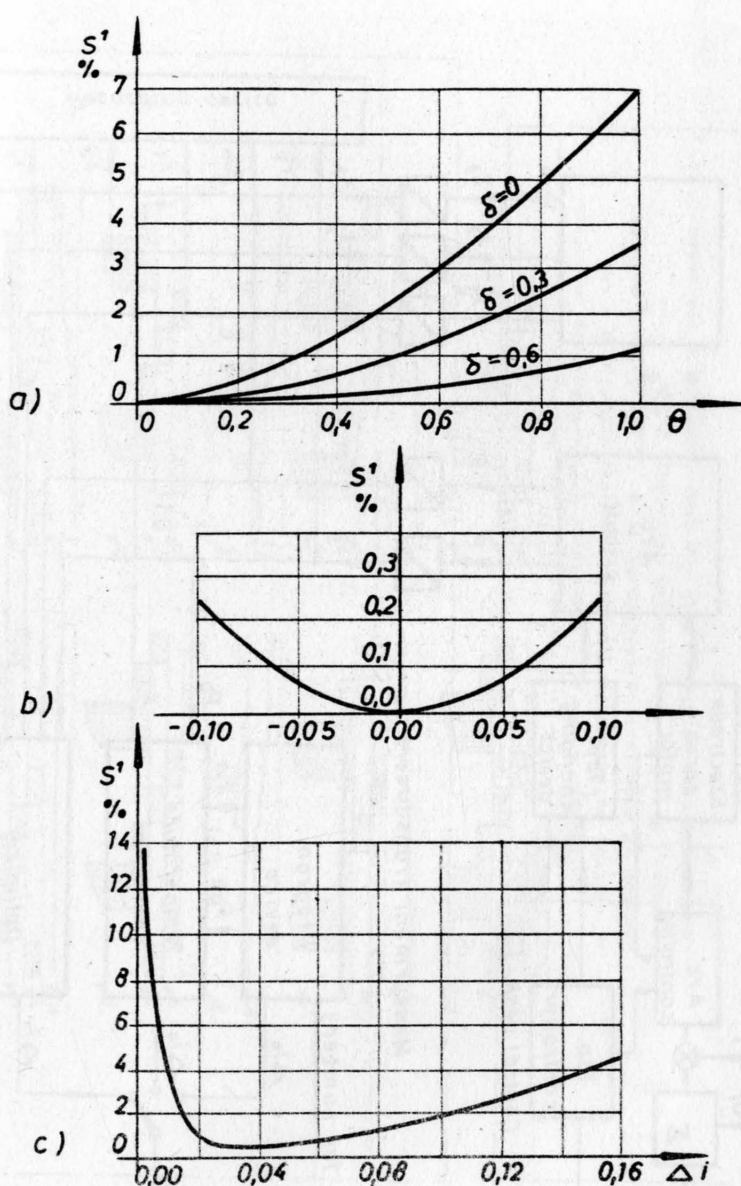


Fig.7. Sensitivity characteristics of the optimizing feedback system - with respect to the inertial measurements of the thermic losses  $P_s$  or of the state  $x$  - (a), to measurements error of the thermic losses  $P_s$  or of the state  $x$  - (b) and to the step length of the peak-holding controller - (c).



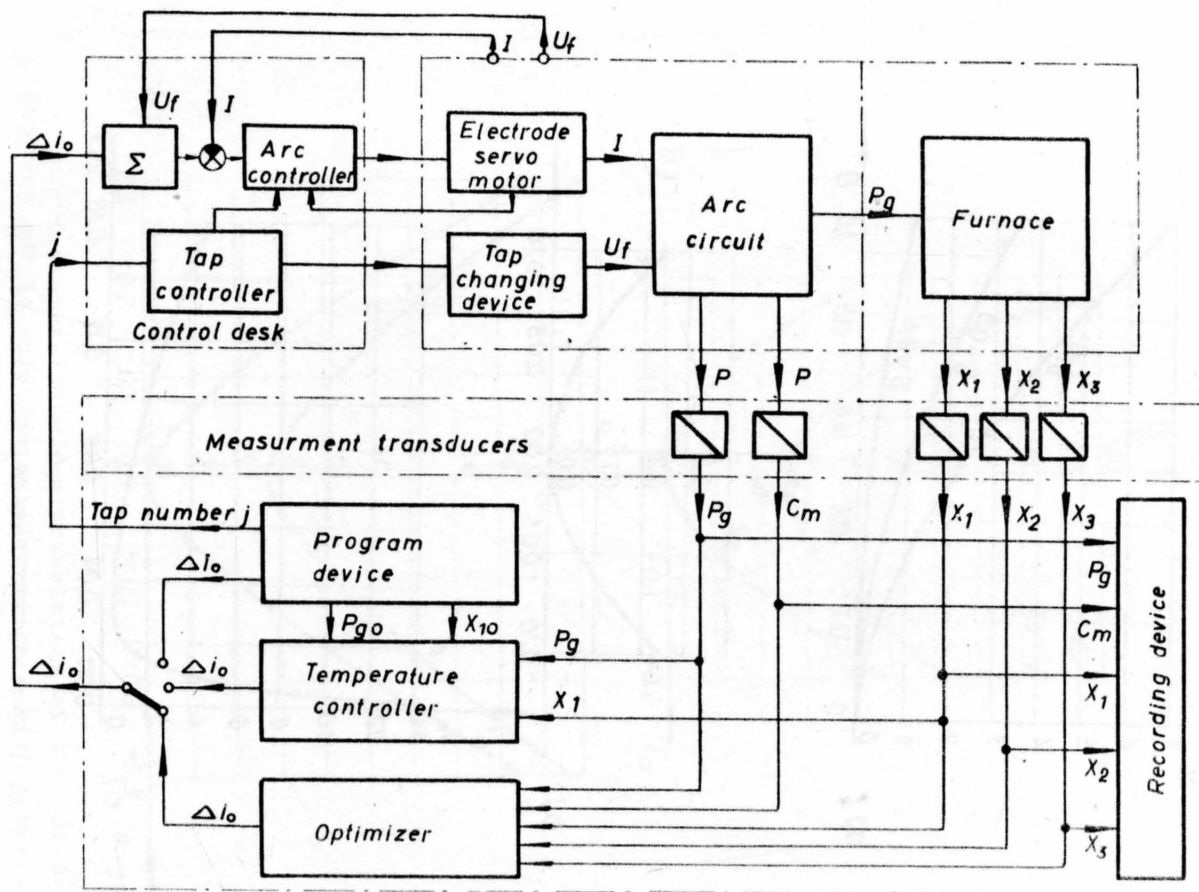


Fig.8. Block diagram of the control equipment of an arc furnace.

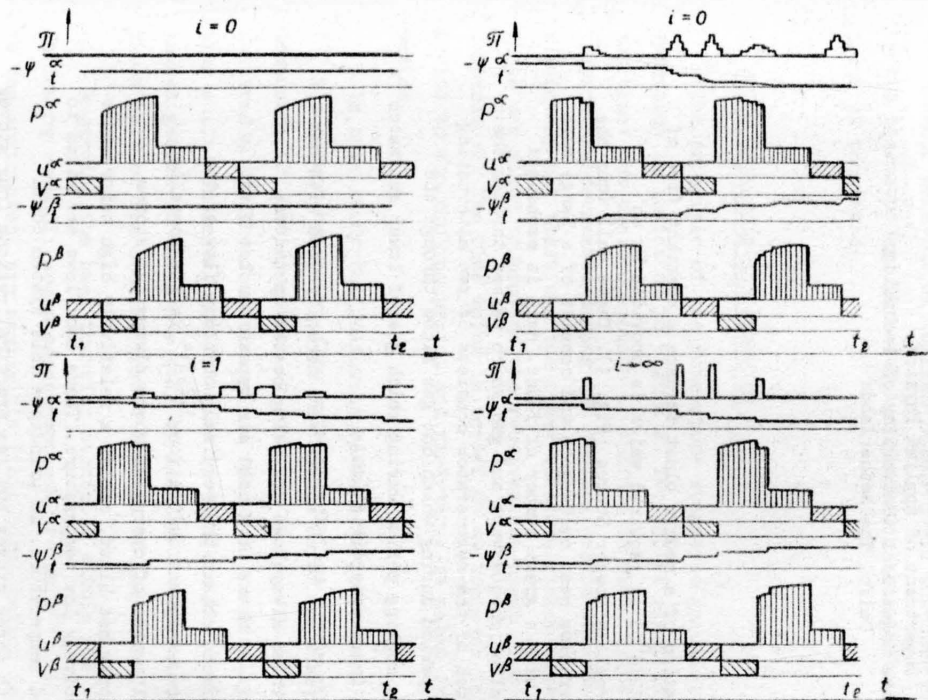


Fig.9. A simplified example of the coordinating procedure of optimal solutions for two arc furnaces. Symbols:  $i$  - the number of coordinating iteration;  $\alpha, \beta$  - the furnace's number;  $\pi$  - the coordinating variable;  $-\psi_t$  - the equivalent cost of time;  $p$  - the heat power;  $u$  - the casting;  $v$  - the loading.

# OPTIMAL OPERATION OF BLAST FURNACE STOVES

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## 1. INTRODUCTION

For the operation of a modern blast furnace a constant flow of hot blast (up to  $1250^{\circ}\text{C}$ ) is required which is provided by an arrangement of so-called Cowper Stoves (Fig. 1). These fall into the category of regenerative heat exchangers and consist of a large mass of solid material with a great number of flues. Heat is exchanged between this so-called chequerwork and gas which passes through the flues. The operation of the regenerator consists of two alternating phases: a heating period during which hot gas flows through the chequerwork, and a cooling period during which gas of lower entrance temperature flows in reverse direction.

Until a few years ago three stoves were normally used with one blast furnace of which always two are being heated up and one supplies hot blast. By means of a cold air bypass the hot blast is mixed with cold air to obtain the required hot blast flow of the desired temperature. A recent innovation <sup>1,2,3,4</sup> is the so-called staggered parallel system of operation where always two stoves simultaneously supply hot blast, one of a relatively high temperature and one of a relatively low temperature. These two flows are mixed to obtain the desired temperature. In principle this system could be implemented with three stoves but for practical reasons four stoves are employed.

It is the purpose of this paper to investigate the optimal operation of the staggered parallel system. The criterion which is used is that of maximal thermal efficiency which is motivated by the fact that fuel consumption accounts for approximately one-half of the operating costs of the stoves. Since the customary three-stove serial system of operation and the four-stove staggered parallel system are not compatible we shall compare the efficiency of the staggered parallel system with that of a four-stove serial type of operation.

The physical foundations of the theory of regenerators have been laid through the investigations of Nusselt and others (see Jakob<sup>5</sup>) and Schfield, Butterfield and Young<sup>6</sup>. They have investigated the dynamic behavior of thermal regenerators and have provided the system equations and physical insight which constitute the starting point of this study.

## 2. STOVE OPERATION AND DYNAMICS

The principle of the four-stove staggered parallel system of operation is indicated in Figure 2. Two stoves simultaneously supply hot blast but their operation is shifted in time. Since the gas exit temperature decreases during the cooling period one stove supplies relatively hot blast and the other relatively cool blast. The two flows are mixed to provide the required hot blast flow of the desired temperature. The time which is needed to switch over from the heating period to the cooling period and vice-versa is taken from the heating period.

In this study the following basic assumptions are made:

(1) A constant hot blast flow rate of constant temperature is demanded; (2) The operation is periodic (i.e., cyclic equilibrium is achieved); (3) The four stoves are operated identically apart from a time shift. These assumptions make it possible to limit the investigation to one stove only by linking the operation during the first half of the cooling period to that of the second half.

Regarding the heat transfer mechanism within the regenerator the following commonly accepted assumptions are made<sup>5,6</sup>: (a) Heat losses to the environment are negligible (this simplification is justified by the fact that the heat losses are almost independent of the operating conditions); (b) The heat capacity of the gas in the channels of the chequerwork is negligibly small relative to the heat capacity of the chequerwork; (c) Transient phenomena at the reversals can be neglected; (d) Longitudinal thermal conduction can be neglected; (e) The transversal heat conduction within the chequerwork can be accounted for by a correction of the heat transfer coefficient between gas and channel wall<sup>6</sup>.



With these assumptions the heat transfer is described by the following partial differential equations<sup>6,7</sup>:

$$\frac{\partial T_g(t,y)}{\partial y} = - \frac{ha}{c_g V(t)l} (T_g(t,y) - T_s(t,y)) \quad (1)$$

$$\frac{\partial T_s(t,y)}{\partial t} = \frac{ha}{mc_s} (T_g(t,y) - T_s(t,y)) \quad (2)$$

with  $T_g$  gas temperature  
 $T_s$  solid temperature  
 $t$  time  
 $y$  vertical distance coordinate  
 $h$  overall heat transfer coefficient  
 $a$  chequerwork heating surface area  
 $V$  mass flow rate of gas  
 $c_g$  specific heat of gas  
 $l$  height of chequerwork  
 $m$  mass of chequerwork  
 $c_s$  specific heat of chequerwork

The positive  $y$ -direction should be taken in the direction of the gas flow. The above equations hold in both the heating and the cooling period. Quantities, however, which refer to the cooling period will be primed ( $T'_s$ ,  $T'_g$ , etc.).

These equations are nonlinear because of the temperature dependence of the specific heats of chequerwork and gas and of the heat transfer coefficient  $h$ , and the dependence of  $h$  upon the flow rate of the gas. In this investigation the temperature dependences are neglected but the more important flow rate dependence of  $h$  is accounted for (see Section 4).

The boundary conditions to equations (1) and (2) and their equivalents during the cooling period are

$$T_g(t,0) = T_{gh}(t) \quad 0 \leq t \leq P \quad (3)$$

$$T'_g(t,0) = T_{gc} \quad 0 \leq t \leq P' \quad (4)$$

$T_{gh}(t)$  is the gas inlet temperature during the heating period which may vary with time, and  $T_{gc}$  is the gas inlet temperature during the cooling period which is constant.  $P$  is the duration of the heating

period and  $P'$  the duration of the cooling period (where always  $P < P'$  because of the time required for reversal).

The initial and terminal conditions are

$$T_s(P, y) = T_s'(0, l-y) \quad 0 \leq y \leq l \quad (5)$$

$$T_s'(P', l-y) = T_s'(0, y) \quad 0 \leq y \leq l \quad (6)$$

Eq. (5) expresses the continuity of the chequerwork temperature profile after reversal and Eq. (6) gives the requirements for periodicity. The distance coordinates differ because of the flow reversal from heating to cooling period.

During the cooling period always the required quantity of hot blast must be supplied which yields the condition

$$V'(t) + V'(t + \frac{1}{2}P') = V'_0, \quad 0 \leq t \leq \frac{1}{2}P' \quad (7)$$

where  $V'_0$  is the desired hot blast flow rate. To obtain the desired hot blast temperature we must impose the condition

$$\frac{V'(t)T'_g(t, l) + V'(t + \frac{1}{2}P')T'_g(t + \frac{1}{2}P', l)}{V'_0} = T_0, \quad 0 \leq t \leq \frac{1}{2}P' \quad (8)$$

where  $T_0$  is the prescribed hot blast temperature. The total heat supplied to the regenerator during the heating period is given by

$$Q_h = c_g \int_0^P V(t)(T_{gh}(t) - T_{gc}) dt \quad (9)$$

where the cold air entrance temperature during the cooling period  $T_{gc}$  is used as reference temperature.

The total amount of heat which must be supplied by one stove during the cooling period is prescribed and equal to  $Q_c = \frac{1}{2}c_g V'_0 (T_0 - T_{gc}) P'$ . The thermal efficiency of the regenerator may be defined as

$$\eta = \frac{Q_c}{Q_h} \quad (10)$$

Since  $Q_c$  is fixed, maximizing the efficiency is equivalent to minimizing  $Q_h$  as given by (9).

### 3. OPTIMIZATION METHOD

The mathematical optimization problem to find the optimal operation of the regenerator is defined by the equations of Section 2 and may be stated as follows:

Consider the distributed-parameter system described by the partial differential equations (1) and (2) and their equivalents during the cooling period, the boundary conditions (3) and (4), the initial and terminal conditions (5) and (6) and the side-conditions (7) and (8). The variables which can be manipulated are the hot gas flow rate  $V(t)$  and the gas inlet temperature  $T_{gh}(t)$  during the heating period, the blast flow rate  $V'(t)$  during the cooling period and the durations of heating period  $P$  and the cooling period  $P'$ . Determine the manipulated variables such that the total heat supplied to the stove during the heating period  $Q_h$  (given by (9)) is minimal. For technological reasons it is necessary to add the constraint:

$$T_{gh}(t) \leq T_{ghmax}, \quad 0 \leq t \leq P \quad (11)$$

where  $T_{ghmax}$  is the maximal gas inlet temperature.

In the following we shall consider the duration of the cooling period  $P'$  as fixed; its optimal value will be determined by repeating the optimization for various values of  $P'$ . It may be proved<sup>8</sup> from mathematical considerations involving the maximum-principle of Pontryagin for distributed-parameter systems<sup>9</sup> that the solution of the optimization problem has the following properties:

- (1) The gas inlet temperature during the heating period  $T_{gh}(t)$  must be chosen equal to its maximal value  $T_{ghmax}$ ;
- (2) During the heating period the optimal gas flow rate  $V(t)$  is independent of time, say  $V_0$ .
- (3) The duration of the heating period must be chosen equal to the maximal possible value  $P = P' - S$ , where  $S$  is the total time required for reversal from cooling to heating period and vice-versa.

From this point on the problem must be pursued numerically but the three conclusions considerably simplify it. The gas inlet temperature  $T_{gh}$  can be eliminated as a variable and the solution of the problem reduces to finding the minimal value of  $V_0$  for which

equations (5) through (8) are satisfied.

To make the partial differential equations (1) and (2) amenable to numerical solution they have been discretized in a manner analogous to Wilmott<sup>7</sup> (see Appendix). Thus the temperature profiles are characterized through  $M$  values along the  $y$ -direction,  $N$  values in the  $t$ -direction in the heating period and  $N'$  values along the  $t$ -direction in the cooling period. For given constant hot gas flow rate  $V_o$ , hot gas inlet temperature  $T_{ghmax}$  and cold air flow rate  $V'(t)$  during the cooling period the stationary initial and terminal temperature distribution of the chequerwork may be directly solved from (5) and (6). The details of this are given in the Appendix.

The initial temperature distribution  $T_g(0,y)$  may thus be considered as an implicit function of  $V_o$  and  $V'(t)$ ,  $0 \leq t \leq P'$ . After discretization the equations (7) and (8) constitute together  $\frac{1}{2}N' + \frac{1}{2}N = N'$  equality conditions for  $V_o$  and the  $N'$  variables  $V'[1]$ ,  $V'[2]$ , ...,  $V'[N']$  which characterize  $V'(t)$ . The solution of the problem thus amounts to the solution of a set of nonlinear equations of the form

$$f_i(V'[1], V'[2], \dots, V'[N'], V_o) = 0, \quad i=1,2,\dots,N' \quad (12)$$

with  $V_o$  minimal. This is equivalent to minimizing  $V_o$  with the  $N'$  side-conditions (12). It may be shown that this problem may be reduced to the solution of  $N'+1$  simultaneous nonlinear equations. An effective algorithm of a quasi-Newton type<sup>10</sup> has been developed for this problem and a computer program has been written in Algol for the Telefunken TR4 computer of the Technische Hogeschool at Delft. Computing times for the examples cited in Section 4 are about one to two minutes.

#### 4. NUMERICAL RESULTS

In this Section some numerical results are given of the optimization method previously discussed. The numerical data for these computations are listed in Table I. They correspond to the stoves in use with the new Blast Furnace No. 6 of the Koninklijke Nederlandse Hoogovens en Staalfabrieken N.V. in IJmuiden, The Netherlands, which is among the largest blast furnaces presently operating in the world.

$a = 43400 \text{ m}^2$	$c_s = 1118 \text{ J } ^\circ\text{C}^{-1} \text{ kg}^{-1}$
$m = 1351000 \text{ kg}$	$c_g = 1177 \text{ J } ^\circ\text{C}^{-1} \text{ kg}^{-1}$
$l = 32 \text{ m}$	$c'_g = 1110 \text{ J } ^\circ\text{C}^{-1} \text{ kg}^{-1}$
$T_{gh} = 1350 \text{ } ^\circ\text{C}$	$P'-P = 600 \text{ s}$
$T_{gc} = 100 \text{ } ^\circ\text{C}$	
$h = 9.30 + 0.320V \text{ J s}^{-1} \text{ m}^{-2} \text{ } ^\circ\text{C}^{-1} \text{ V in kg s}^{-1}$	
$h' = 6.98 + 0.309V' \text{ J s}^{-1} \text{ m}^{-2} \text{ } ^\circ\text{C}^{-1} \text{ V' in kg s}^{-1}$	
Free flow area $13.84 \text{ m}^2$	Flue diameter $0.04 \text{ m}$

TABLE I: Numerical data for optimization computations.

The specific heats  $c_s$ ,  $c_g$  and  $c'_g$  have been taken at average temperatures ( $650 \text{ } ^\circ\text{C}$ ,  $700 \text{ } ^\circ\text{C}$  and  $600 \text{ } ^\circ\text{C}$ , respectively). The data on which the choice of the heat transfer coefficients is based were taken from Böhm<sup>11</sup>; they have been corrected according to Schofield<sup>6</sup> and subsequently linearized. In  $h$  an extra constant term has been added to account for heat transfer by radiation.

Most of the computations were performed with a discretization of  $M = 8$ ,  $N = N' = 10$  which induces an estimated error in the gas and solid temperatures of a few tenths of a degree centigrade. (Note that the temperatures referred to here are average temperatures during a time-interval in the case of gas temperatures and mean temperatures over a layer in the case of solid temperatures (see Appendix).)

The following computations have been performed:

Optimization of staggered parallel system: In Fig. 4 results are given for four different combinations of hot blast flow rate and temperature demands. It is seen that as the load increases the efficiency decreases and the optimal duration of the cooling period shifts from 70 to 50 minutes. The sensitivity to variations in the duration of the cooling period is not very great but a reduction from two hours (a normal value in practice) to about one hour may yield an efficiency improvement of approximately 0.5 per cent. The dashed portions of the curves indicate solutions which are technologically inadmissible because near the end of the heating period the exit gas temperature



risks above  $300^{\circ}\text{C}$ .

A more refined computation was carried out for a desired hot blast temperature of  $1200^{\circ}\text{C}$  and a flow rate of  $84.1\text{ kg/s}$  by employing a discretization in time of  $N = N' = 20$  rather than 10. With a cooling period of 60 minutes (optimal for this case) an efficiency of 0.9207 was obtained. The corresponding optimal distribution of the gas flow rate in the cooling period is plotted in Fig. 3a.

Non-optimal operation of staggered parallel system: Non-optimal solutions of the staggered parallel system may be obtained by solving the equations (12) for non-minimal values of the gas flow rate  $V_0$  during the heating period. In Fig. 3b a non-optimal solution is shown for the same case as in Fig. 3a. The non-optimal solution of Fig. 3b constitutes an extreme situation because further reduction of the efficiency results in a negative gas flow rate at the beginning of the cooling period which corresponds to a physically unrealizable situation.

The thermal efficiency of this extreme non-optimal solution is 0.9199 as compared to 0.9207 for the optimal solution. This means that there is an extremely small range of admissible stationary periodic solutions. We conclude that optimization is not really a problem in this case: Any stationary periodic solution is very close to the optimal solution with the same period. Only by choosing the period correctly some gain may be achieved.

Four-stove serial operation: The staggered parallel system has been compared to two types of four-stove serial operation. In the 1-3 serial system only one stove is in the cooling period while the other three are being heated up. The hot blast from this single stove is mixed with cold blast to obtain blast of the desired temperature. In the 2-2 serial system two stoves simultaneously supply hot blast while the other two are in the heating phase. The stoves operate completely in parallel; there is no shift in time as in the staggered parallel system. The hot blast from the two stoves is mixed with cold blast to obtain the correct temperature.

For this type of operation a set of equations may be formulated which is analogous to (12). It can be made plausible that for optimal operation  $V'[N'] = V_0$ , i.e., just prior to the change-over to the

heating period the entire desired flow of hot blast is supplied by the stove or stoves in the cooling period. This extra equation reduces the optimization problem to the solution of  $N'$  nonlinear equations with  $N'$  unknowns.

The results of the computations for the 1-3 and 2-2 serial system are also represented in Fig. 4. We note that (i) the 2-2 system is more efficient than the 1-3 system except for very short cooling periods, (ii) extremely short durations of the cooling period are optimal, and (iii) the sensitivity of the efficiency to variations in the duration of the cooling period is much greater than for the staggered parallel system.

## 5. CONCLUSIONS

In this paper a numerical method to find the optimal operation of the four-stove staggered parallel system for supplying hot blast to a blast furnace has been developed. Also the efficiency of serial operation of the stoves has been calculated. The conclusions may be summarized as follows:

- (a) The efficiency of the staggered parallel system of operation is not very sensitive to variations in the duration of the cooling period. For the cases investigated the optimal duration of the cooling period varies from 50 to 70 minutes depending upon the load.
- (b) For any given duration of the cooling period the range of admissible solutions for stationary periodic operation of the staggered parallel system is extremely small. This together with (a) suggests that the operation of the staggered parallel system is not very critical.
- (c) Serial operation of the four stoves is considerably less efficient. The explanation for this is that the average exit temperature in the cooling period is much higher than for the staggered parallel system. The corresponding loss of efficiency is only very partially compensated by the lower average flow rate.

- (d) With serial operation the sensitivity to the duration of the cooling period is much greater. Optimal operation is achieved for impractically short durations of the cooling period (10 to 20 minutes for the cases investigated).

The conclusions of this investigation are quite favorable for the staggered parallel system of operation. Comparison with the customary three-stove serial system of operation would involve considerations concerning capital investment, depreciation, cost of fuel, etc., and is beyond the scope of this paper.

#### ACKNOWLEDGEMENT

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APPENDIX: Numerical Integration of Differential Equations and  
Computation of Initial Temperature Distribution

Consider M intervals of equal length  $\Delta y$  spaced along the y-direction and N intervals of equal length  $\Delta t$  spaced along the time-axis. Discretization of equations (1) and (2) according to the trapezoidal method yields

$$\frac{T_g[k,i] - T_g[k,i-1]}{\Delta y} = -\frac{h[k]a}{c V[k] \ell} \frac{1}{2} (T_g[k,i] + T_g[k,i-1] - T_s[k-1,i] - T_s[k,i]) \quad (A.1)$$

$$\frac{T_s[k,i] - T_s[k-1,i]}{\Delta t} = \frac{h[k]a}{mc_s} \frac{1}{2} (T_g[k,i] + T_g[k,i-1] - T_s[k-1,i] - T_s[k,i]) \quad (A.2)$$

with  $T_g[k,i]$  average gas temperature during the k-th time interval at  $y = i\Delta y$

$T_s[k,i]$  mean solid temperature of the i-th layer at  $t = k\Delta t$

$V[k]$  gas flow rate (constant) during k-th time interval

$h[k]$  heat transfer coefficient during k-th time interval (dependent upon  $V[k]$ ).

It can be shown that this representation is equivalent to that of Wilmott<sup>7</sup> provided that  $T_g[k,i]$  signifies the arithmetic mean of the gas temperature at  $t = (k-1)\Delta t$  and  $t = k\Delta t$  and likewise  $T_s[k,i]$  is the arithmetic mean of the solid temperatures at  $y = (i-1)\Delta y$  and  $y = i\Delta y$ . From the right-hand sides of the equations (A.1) and (A.2)  $T_g[k,i]$  and  $T_s[k,i]$  can be eliminated. Define

$$\beta[k] = \frac{\frac{h[k]a}{c V[k] \ell} \Delta y}{1 + \frac{1}{2} \frac{h[k]a}{mc_s} \Delta t + \frac{1}{2} \frac{h[k]a}{c V[k] \ell} \Delta y} \quad (A.3)$$

and

$$\gamma[k] = \frac{\frac{h[k]a}{mc_s} \Delta t}{1 + \frac{1}{2} \frac{h[k]a}{mc_s} \Delta t + \frac{1}{2} \frac{h[k]a}{c V[k] \ell} \Delta y} \quad (A.4)$$

With these equations (A.1) and (A.2) can be reduced to

$$T_g[k,i] = (1-\beta[k])T_g[k,i-1] + \beta[k]T_s[k-1,i] \quad (A.5)$$

$$T_s[k,i] = (1-\gamma[k])T_s[k-1,i] + \gamma[k]T_g[k,i-1] \quad (A.6)$$

Together with the appropriate initial and boundary conditions these equations allow step-by-step computation of successive gas and solid temperature distributions during heating and cooling period.

The stationary initial solid temperature distribution is computed as follows. Let  $T_{so}^{(0)}$  (a column vector\* with components  $T_s^{(0)}[0,i]$ ) and  $T_{so}^{(1)}$  (a column vector with components  $T_s^{(1)}[0,i]$ ) denote the initial temperature distributions for the heating period associated with two successive regenerator cycles. Since for fixed hot gas flow rate during the heating period and fixed blast flow rate during the cooling period the difference equations (A.5) and (A.6) are linear in  $T_g$  and  $T_s$  a relationship of the following type must hold

$$T_{so}^{(1)} = AT_{so}^{(0)} + b \quad (A.7)$$

where  $A$  is an  $M \times M$  matrix and  $b$  an  $M$ -dimensional column vector. The elements of the matrix  $A$  and the vector  $b$  can be obtained by repeated use of the difference equations (A.5) and (A.6). From (A.7) the stationary temperature distribution  $T_{so}$  can be found as  $T_{so} = (I-A)^{-1}b$  where  $I$  is the identity matrix. Closer investigation of the difference relations (A.5) and (A.6) reveals that the matrix  $A$  has a rather special structure. Its elements and those of the vector  $b$  can be found by solving equations (A.5) and (A.6) a number of times for different special sets of initial and boundary conditions.



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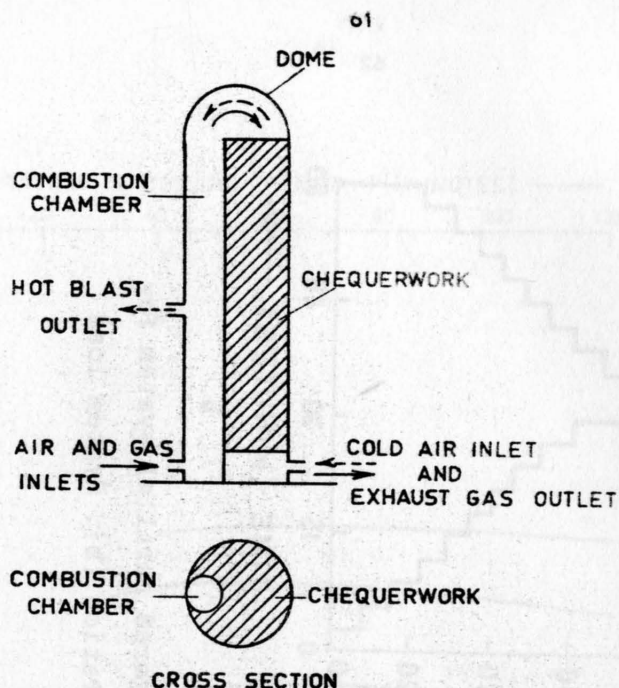


Fig. 1: Schematic diagram of Cowper Stove. During the heating period combustion is maintained in the combustion chamber and hot exhaust gases flow through the chequerwork. In the cooling period the direction of the flow is reversed.

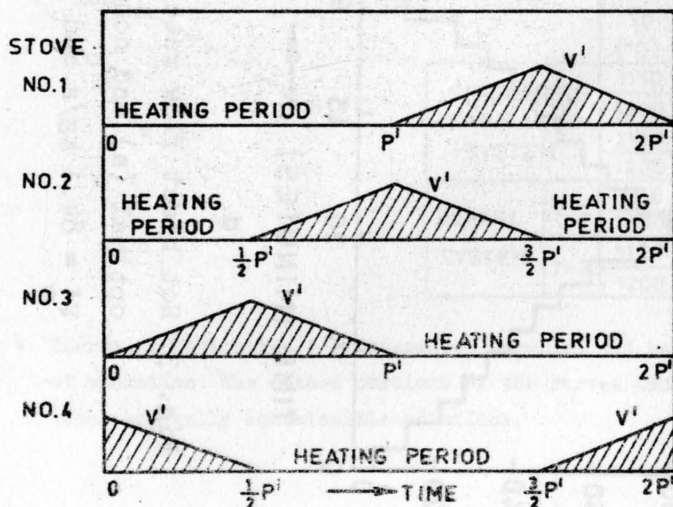
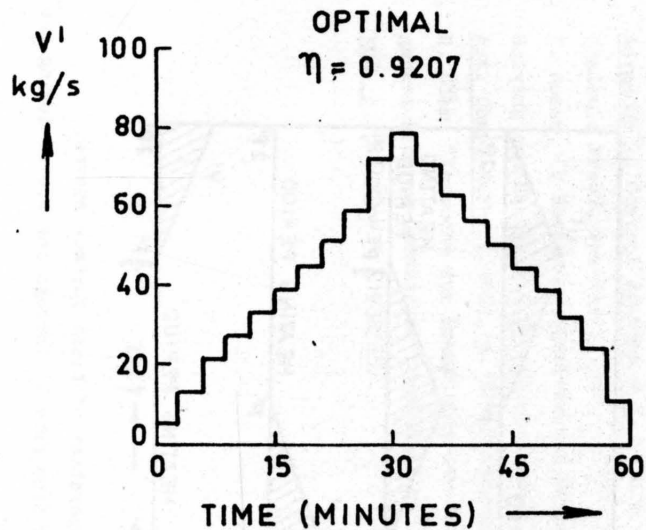
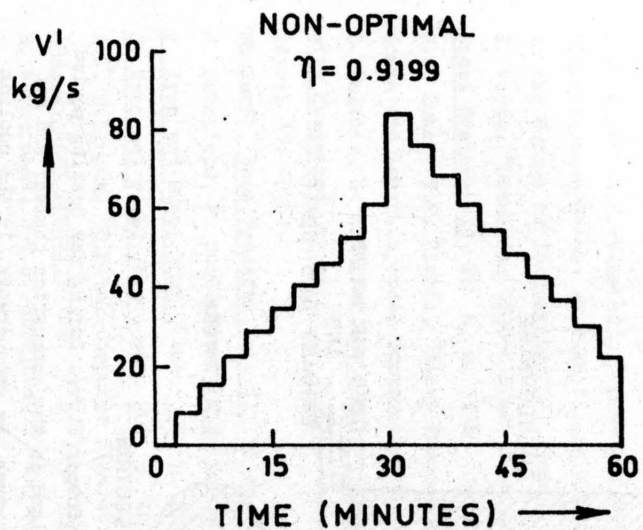


Fig. 2: Staggered parallel operation of blast furnace stoves. Indicated is the gas flow rate  $V'$  through the stoves during the cooling period.



a.



b.

Fig. 3: Hot blast flow rate  $V'$  for staggered parallel system for optimal (a) and non-optimal operation (b). System load  $V'_0 = 84.1$  kg/s and  $T_0 = 1200^\circ\text{C}$ .

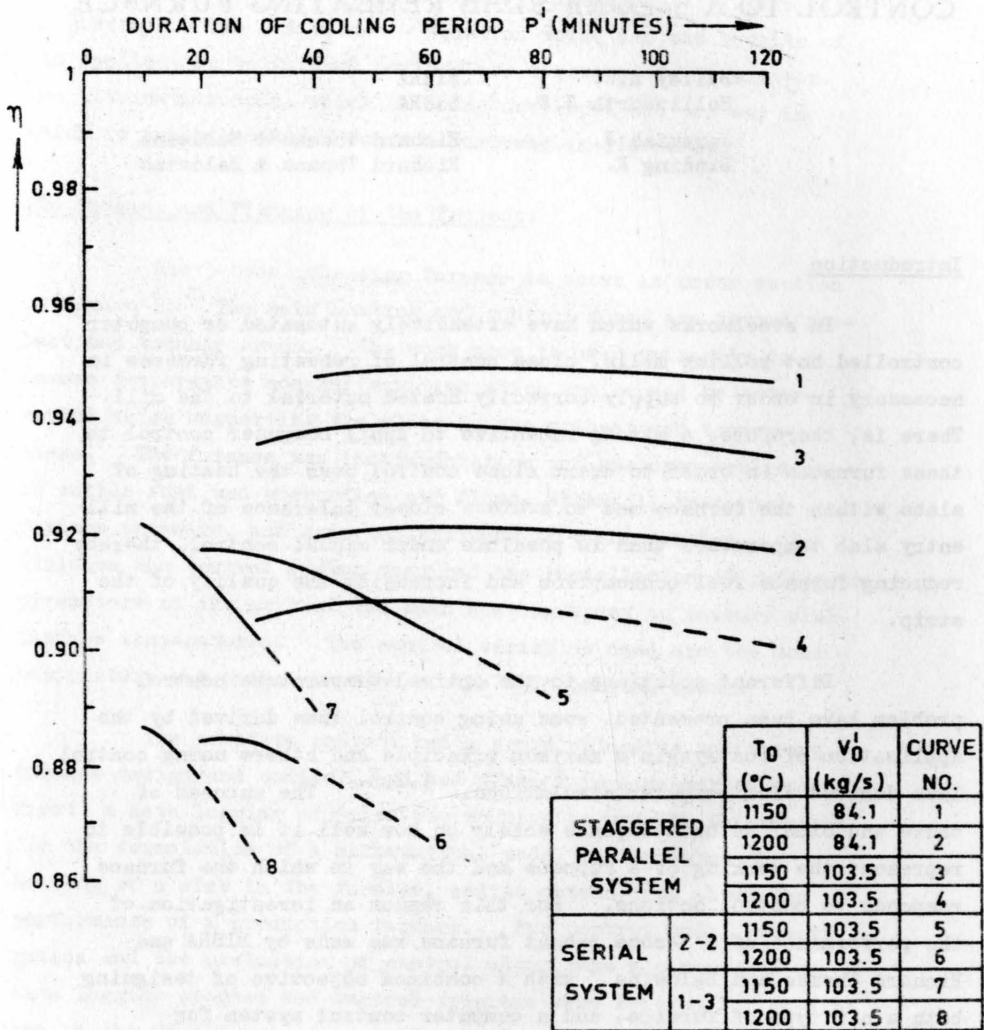


Fig. 4: Thermal efficiencies  $\eta$  of staggered parallel and serial system of operation. The dashed portions of the curves indicate technologically inadmissible solutions.

## DEVELOPMENT AND APPLICATION OF COMPUTER CONTROL TO A 5-ZONE SLAB REHEATING FURNACE

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### Introduction

In steelworks which have extensively automated or computer controlled hot rolling mills, close control of reheating furnaces is necessary in order to supply correctly heated material to the mill. There is, therefore, a strong incentive to apply computer control to these furnaces in order to exert close control over the heating of slabs within the furnace and to achieve closer tolerance of the mill entry slab temperature than is possible under manual control, thereby reducing furnace fuel consumption and increasing the quality of the strip.

Different solutions to the optimal temperature control problem have been presented, some using control laws derived by the application of Pontryagin's maximum principle and others using control laws derived from computer simulations.<sup>1,2,3.</sup> The success of these and other methods depends solely on how well it is possible to represent the working of a furnace and the way in which the furnace responds to control actions. For this reason an investigation of the performance of a 5-zone reheat furnace was made by BISRA and Richard Thomas and Baldwins<sup>4</sup> with a combined objective of designing both a new type of furnace, and a computer control system for operating existing furnaces.

Control studies were undertaken to determine the type of temperature disturbances arising in normal operation, their magnitude, the type of control system that was required, and the way in which control should be implemented. The result of these studies was the construction of a linear difference equation model of the furnace incorporating the main disturbances, and the successful application of this model on line. More recently work has started on the



construction of a model which is a closer representation of the physical furnace and which potentially offers greater flexibility of control. The difference - equation model and the results of its application to furnace control are described in this paper. The alternative model which is being developed and the way in which it can be used for control purposes is discussed.

### The Furnace and Planning of the Project.

The 5-zone reheating furnace is shown in cross section in Figure 1. The main heating and control zones are termed pre-heat and tonnage zones. The soak zone is used primarily to remove temperature non-uniformities which are caused by water-cooled skids supporting the slabs in the preheat and tonnage zones. The furnace has instrumentation and analogue equipment to enable fuel and combustion air flows, steam/oil ratioing, furnace pressure, and zone temperatures to be controlled. In addition the control system required the installation of radiation pyrometers at the exit of the main heating zones to measure slab surface temperatures. The control variables used are the zone temperature set points in each of the main heating zones.

The complete project had a broad objective to improve furnace design and control, and was planned in a number of stages. First, a data logging exercise was mounted to provide information for the formulation of a mathematical model to represent the heating of a slab in the furnace, and to obtain data about the performance of a production furnace. For control model investigation and the evaluation of control algorithms a comprehensive data logging program and control programs were written for on-line use in the GE 412 hot strip mill computer. The final stage which has not yet been reached is to develop the control system to the point where all the reheat furnaces in the mill can be automatically controlled under all conditions of operation.

### The Initial Stages

The first stage of the project was data logging of the furnace performance during normal production. The data logging involved the recording of approximately 140 sensors each minute to

give information about roof and hearth temperature profiles, fuel and combustion air flows, waste gas temperature, skid cooling losses and slab temperature at the exit of the preheat, tonnage and soak zones. In addition thermocouples were placed in certain slabs to measure slab internal temperature during their passage through the furnace.

The collection of these data enabled the relation between zone temperature and zone roof and hearth profiles to be established so that a simulation model <sup>5</sup> could be built. This simulation model allowed computations of slab temperature to be made at different zone temperatures and throughputs. In this way it was possible to examine the magnitude of slab temperature variations which resulted from changes from one steady state level of throughput to another, and from one steady state zone temperature to another.

#### Preliminary Data Analysis and Construction of the First Control Model.

To be able to control a distributed parameter process in which long time constants occur, it is necessary to use a predictive form of control. This requires the construction of a model of the process so that the temperature variations caused by frequent changes in slab sizes and throughputs can be predicted in advance and the requisite control actions taken in time to correct them. To construct such a model the type of response and dynamic effects known to exist in the furnace were expressed in a suitable mathematical form. These effects were then combined to yield a statistical model, relating output slab temperature to the inputs of zone temperature set point and furnace throughput.

If a change in zone temperature set point  $\bar{U}$  is made at time  $t$  the response of the zone temperature  $U$  to the new level approximates to a lag followed by an exponential rise or fall to the new level. If the lag is represented by a finite number of sample intervals  $p$  and an amount less than a sample interval then it is possible to represent this form of response by a simple difference equation of the form: -

$$(1 + a_1 \nabla) U_{t+p} = g_1 (1 + b_1 \nabla) \bar{U}_t \quad (1)$$

If the response of slab temperature  $X$  to a change in zone temperature  $U$  is represented in a similar way a difference equation of similar form results: -

$$(1 + c_1 \nabla) X_{t+q} = g_2 (1 + d_1 \nabla) U_t \quad (2)$$

To represent the response of slab temperature to a change in zone setpoint the two equations 1 and 2 are combined. This is done by putting  $t$  equal to  $t + p$  in equation 2 and substituting for  $U_{t+p}$  from equation 1. This gives an equation of the form: -

$$(1 + \xi_1 \nabla + \xi_2 \nabla^2) X_{t+r} = g_1 (1 + \eta_1 \nabla + \eta_2 \nabla^2) \bar{U}_t \quad (3)$$

A change of throughput in the furnace will also result in a variation in slab temperature. To account for this, the mass flow of steel through the hottest section of the zone is calculated, and its disturbance effect incorporated in the equation by an additional difference term. The final model, therefore, takes the form: -

$$(1 + \xi_1 \nabla + \xi_2 \nabla^2) X_{t+r} = k + G_1 (1 + \eta_1 \nabla + \eta_2 \nabla^2) \bar{U}_t + G_2 (1 + s_1 \nabla + s_2 \nabla^2) M_t \quad (4)$$

It is necessary to point out that this form of model is linear and at best an approximation to the real plant. In order to provide some form of adjustment to allow the model to cater for unmodelled effects an adjustment model was used of the form: -

$$Z_{t+r} = \epsilon_{t+r} + \gamma_1 \sum_{i=0}^{\infty} \epsilon_{t-i+r} \quad (5)$$

The relationship between the actual slab temperature  $T_{t+r}$  and the model prediction  $X_{t+r}$  and  $Z_{t+r}$  is

$$T_{t+r} = X_{t+r} + Z_{t+r} \quad (6)$$

In practice the aim of the control is to obtain a fixed output temperature  $T_D$  from the zone so that equation (6) becomes

$$X_{t+r} = T_D - Z_{t+r} \quad (7)$$

Since, as indicated earlier, a prediction of future output temperature is necessary to enable effective control action to be taken, it is necessary to use the latter equation (7) in predictive form. This may be achieved simply by equating the values of

$\epsilon_{t+1}, \epsilon_{t+2} \dots \epsilon_{t+r}$  not yet measured to zero<sup>10</sup>. If this is done a simple control algorithm results. The derivation of this is given in the appendix.

To determine whether a model of the type constructed is adequate it is necessary to try fitting it to data obtained from the furnace. This preliminary fitting was carried out by using a standard regression analysis program and examining the residuals of the model fit. The results of this preliminary fitting were satisfactory. The model and control equation were then implemented on the Number 2 furnace using the GE 412 process computer already installed on the Spencer Works hot strip mill.

#### Application of the Control Computer

The hot strip mill computer has an 8K, 20 bit word core store, a 56K drum and a cycle time of 20  $\mu$  secs. It is primarily concerned with material tracking through the mill and with control of hot strip width, thickness and temperature.

The information demanded for the control system on the velocity and on the distribution of thickness of slabs along the furnace length required the tracking of each slab through the furnace from the moment it was charged.

The rolling schedule is fed into the computer on paper tape and includes slab identity and sizes (which can vary between 12 to 25 cms in thickness and between 76 to 152 cms in width) and other data pertinent to each order. Just prior to a particular slab being charged into the furnace, the computer displays the slab identity in the charging pulpit, and the operator informs the computer, via his manual entry station, to which furnace the slab is being assigned. All the information relating to each slab is stored on drum in a furnace tracking table. In this way the sequence of slabs in the furnace and their time of entry into the furnace is recorded.

Slabs are sensed dropping out of the furnace by a mercury inertia switch. On receipt of a signal from this switch the displacement of steel through the furnace and mean thickness in zones is computed.

Many furnace variables are logged and may be output on a typer or punch every 4 minutes if the appropriate computer output routine is selected. For the logging program fuel and air flows, zone, slab, and air temperature are scanned every 2 minutes, and alarms are printed if values go out of limits. Current values of velocity, thickness in zones, identity and thickness of slabs under the pyrometers, and terms computed in the control equations are scanned every 4 minutes.

Control calculations in which new zone setpoints are computed, converted to analogue signals and transmitted to the appropriate controllers take place at 4 minute intervals.

In all, 2K words of storage are used for furnace logging and control calculations.

#### Preheat Zone Control.

The validity of the model parameters estimated was initially tested by comparing the setpoints computed with those conventionally applied by the operator. This test showed that the model coefficients were not correct. The preheat zone was then logged continuously and the model refitted. The coefficients were checked and the control again compared with manual practice. The setpoints calculated were now in fair agreement with those used by the operator. Some adjustment was made to the constant  $k$  in the control equation prior to implementing the control system on line. Improved slab temperature control was then achieved under closed loop control at furnace throughput rates up to approximately 80% of the rated design capacity. At higher throughputs (in excess of 190 tons per hour) full fuel flow limit was reached in the preheat zone and adequate temperature control could not be maintained.

Feedback control via the adjustment model has not proved suitable for on line use in the preheat zone under fuel limit conditions. This is because the feedback is intended only as a small correction to the set points calculated using the difference equation model and it is



essential that these setpoints are implemented. Also during mill delays a lower limit of  $1230^{\circ}\text{C}$  is placed on preheat zone temperature, the computed set points cannot be applied, and slabs heat up beyond their desired value. Therefore, feedback is inappropriate and predictive open loop control only has been used.

#### The Tonnage Zone Model.

The tonnage zone model was developed along lines similar to those used for preheat control. In this model, however, it was necessary to make provision for feedforward temperature deviations from the preheat slab exit temperature demanded. This type of correction is necessary even when the preheat zone is being controlled since the slow response of this zone to control actions means in practice that deviations from this temperature will inevitably occur. When due to the conditions described in the preceding paragraph, temperature control of the preheat zone cannot be maintained, the feedforward correction is essential.

The model used in this zone is, therefore, of the form: -

$$\begin{aligned} (1 + \xi_1 \nabla + \xi_2 \nabla^2) X_{t+s} = & k + g_1 (1 + \eta_1 \nabla + \eta_2 \nabla^2) \bar{U}_t \\ & + g_2 (1 + \xi_1 \nabla + \xi_2 \nabla^2) M_t \\ & + g_3 (1 + \delta_1 \nabla + \delta_2 \nabla^2) M (T - T_D)_t \quad (8) \end{aligned}$$

where  $T$  is the temperature of the slabs leaving the preheat zone.

The temperature  $T$  is measured by pyrometer or, in the absence of this measurement, predicted from past heating history, and tracked with the slabs as they proceed along the furnace. The tracked temperatures are then used to compute the correction value when the slabs reach the control point in this zone. The form of adjustment model used in the preheat zone was proposed so that the form of control equation was very similar to that used in the preheat zone.

The estimation of the parameters of the tonnage zone model proved to be more difficult than for the preheat model. Existing manual practice is to use full firing rates in the preheat zone during

normal running and to heat slabs to rolling temperature during delays. Under these conditions the tonnage zone acts rather like a soak zone and small correlation exists between output slab temperature and the control parameters. In order to identify the model a period of logging was selected in which continuous operation was maintained and in which slab entry temperature was controlled. This enabled an initial identification of the parameters of the model to be made. There were doubts about the accuracy of the parameters since the estimated gains  $g_1$  in the model were lower than simulation evidence suggested. The control program for the tonnage zone was written and the setpoints compared with those normally applied. This confirmed that the parameters of the model were not correct. The parameters of the model were then re-estimated and checked. Final adjustments were made to the control coefficients prior to implementation on line and improved slab temperature control then resulted.

During the course of the campaign further attempts were made to improve the estimation of the parameters of the tonnage zone model using a constrained hill climber <sup>7</sup>. The  $\xi_i$ ,  $\eta_i$  and  $g_1$  parameters varied little but some variation occurred in the other parameters. Investigation of these parameters showed that no benefit in control would result.

#### Estimation of the Parameters of the Models.

In order to estimate the parameters of the difference equation models it was necessary to make use of normal operating records, since it is undesirable to perform special experiments on a production furnace.

A number of different methods were investigated in order to obtain estimates of the parameters. Several methods are available for fitting this type of model. <sup>8,9,11</sup>. The application of such techniques in practice has not been very satisfactory. Our general experience has shown that care must be taken to ensure that good initial estimates of the parameters are available and that these are constrained so that stable and physically sensible models can be estimated. In addition the data has had to be checked in order to ensure that correlation between the input and output variables exist. If these precautions

are not taken then it is impossible to guarantee successful estimation of the parameters. Where poor initial parameters have been used it has been found that either the rate of convergence has been very slow or with some methods, for example with Aström & Bohlins method, it is possible for the method to diverge.

The procedure adopted to estimate the models was as follows: - First, a standard multiple correlation method was used in order to identify the delay required on the parameters. When this delay was determined the parameters were then estimated by a standard least squares program. To determine the adequacy of the model the error sequence was investigated by making a time plot of the residuals to detect if any trends were present<sup>12</sup>. This is usually more informative than the autocorrelation function, although this was also used in the initial stages to determine if the model was adequate<sup>6,10</sup>. Finally, a constrained hill climbing approach has been applied to improve the estimates of the parameters<sup>6</sup>.

The models calculated have been checked to ensure that they were physically sensible and that the gain factors  $g_1$  were comparable to those expected from simulation evidence. If these tests were satisfactory the models were tested on line and the final adjustment and tuning of the parameters made on the furnace.

#### The Results of the Control Trials.

Experience of using the statistical type of control model is currently limited, but it is possible to see a distinct improvement in the control of slab temperature which has resulted from the implementation of the control system. This is illustrated in the histogram in Figure 2 which shows the contrast between manual and automatic control in various stages of its implementation. The last histogram represents a 13 hour control period in which the preheat pyrometer was used to measure slab surface temperature for feedforward correction. The control system maintained a mean temperature of 1287°C with a standard deviation from the tonnage zone of 13°C.

As discussed above, the preheat zone is unable to respond to rapid control actions, so it is only possible to exert coarse control over slab temperature in this zone. Some control is essential to allow the tonnage zone control to function in a correct manner, viz. to

compensate for small deviations in slab entry temperature to this zone. At high throughput (in excess of 190 tons/hr.), a design constraint of the furnace is encountered in which full fuel limits are reached in the preheat zone. This means in practice that no control of temperature can be achieved from this zone at high throughput and the tonnage zone is required to correct for large deviations in slab temperature.

The tonnage zone control is far more responsive than the preheat zone so that rapid control actions are possible. The application of computer control has improved slab temperature control from this zone and at throughputs less than 190 tons/hr. good results have been achieved using the preheat zone pyrometer to measure slab temperature for feedforward correction. When the preheat zone is operating at its maximum output and slabs cannot be controlled there is heavy dependence on this measurement, and further practical difficulties have been encountered. Due to the reverse flow design of the furnace, when the tonnage zone uses high fuel input there is flame interference on the preheat pyrometer measurement of slab temperature. This problem has resulted in further work being initiated along two paths. One to develop a successful temperature predictor to replace this measurement at high throughput, and the second to improve the siting and reliability of the pyrometer.

The predictor in use at the present time has been derived from numerical solution of the Fourier Equation for heat conduction using finite differences. The results of off-line simulations using this method have been adapted for plant use by computing the mean slab temperature from curves which relate this quantity to thickness of slab, time spent in the preheat zone, and the average zone temperature. The results of this technique are proving satisfactory and it is now felt that the pyrometer readings will be used mainly for off-line analysis.

All tests of the control models have been made without the addition of feedback control. Nevertheless it is expected that feedback would make some improvement to the control of the tonnage zone. Since for reasons stated earlier the preheat zone cannot always be adequately controlled feedback is inappropriate and will not be used on this zone.



### Further Modelling Work.

In order to obtain a closer representation of the physical furnace under the wide range of operating conditions which occur in practice and to represent the operation of the preheat zone when set-points are not obeyed, further modelling work has been carried out. This has resulted in a model which uses recorded zone temperature, time spent in different portions of the heating zone, and slab thickness.

Each main heating zone is divided into 4 equal sections (Fig.3). For each section, information about zone temperature  $U$  and time spent in each section  $t_i$  for each slab is recorded. At the exit of each section, the temperature rise  $T_i$  of the slab is computed by an equation of the form: -

$$T_i = (\alpha_i U - T_{IN}) (1 - e^{-\beta_i t_i}) \quad (9)$$

where  $T_{IN}$  is the entry temperature to the section and  $\alpha_i$  and  $\beta_i$  are constants appropriate to the section.

This equation ensures that slab temperature can rise only to a fraction of the zone temperature appropriate to that particular section. It also ensures that a complete record of heating history is maintained for each slab in the furnace even when fuel limits are reached and set-points cannot be obeyed.

When a slab leaves the heating zone its surface temperature may be computed by adding its entry temperature to the temperature rises ( $T_1, T_2 \dots T_4$ ) it experiences in each section, so that

$$(T_{out})_{calc} = T_{ENTRY} + T_1 + T_2 + T_3 + T_4 \quad (10)$$

This temperature, in the case of the preheat zone, may be fed forward to the tonnage zone or a measured value used in its place.

Using a hill climbing technique<sup>13</sup> to minimise the sum of squares of the errors between the calculated temperature  $(T_{out})_{calc}$  and the measured temperature  $T_{out}$  the parameters of this model have been fitted. The application of this method using different sets of furnace



data has given valid consistent estimates of the parameters.

#### Use of the Alternative Model for Control.

This model may also be used for control purposes. The passage of slabs through the earlier sections 1 and 2 is recorded and their temperature rises  $T_1$  and  $T_2$  computed. The set point  $\bar{U}$  is calculated so that the slabs will reach the demanded exit temperature  $T_{DEM}$  after their passage through sections 3 and 4.

At the time of control the slabs in section 3 will have reached a temperature  $T$  given by

$$T = T_{ENTRY} + T_1 + T_2 \quad (11)$$

The temperature rises  $T_3$  and  $T_4$  through sections 3 and 4 will depend on the residence times  $t_3$  and  $t_4$  and the zone temperature  $U$ . It is necessary to predict the temperature rises using equation (9) so that

$$T_3 = (\alpha_3 \bar{U} - T) (1 - e^{-\beta_3 t_3}) \quad (12)$$

$$\text{and } T_4 = (\alpha_4 \bar{U} - T - T_3) (1 - e^{-\beta_4 t_4}) \quad (13)$$

The temperature rise required is  $T_{DEM} - T$  so that

$$T_{DEM} - T = T_3 + T_4 \quad (14)$$

If  $T_3$  and  $T_4$  from equations (12) and (13) are now placed in equation (14) the control setpoint may be obtained. This gives: -

$$\bar{U} = \frac{T_{DEM} - T e^{-\beta_4 t_4} - \beta_3 t_3}{\alpha_3 e^{-\beta_4 t_4} (1 - e^{-\beta_3 t_3}) + \alpha_4 (1 - e^{-\beta_4 t_4})} \quad (15)$$

To use this equation the residence times  $t_3$  and  $t_4$  need to be predicted. For greatest accuracy in prediction it is best to know what the future pushing interval will be, and then to use this with the knowledge of the dimensions of the slabs to be discharged from the furnace to estimate  $t_3$  and  $t_4$ . If the future pushing interval is not known, then the best that can be done is to predict this from the most recently observed push intervals.

### Conclusions and Future Work.

The work described in this paper has demonstrated that it is possible to control slab temperature. The application of the difference equation models has brought improved temperature control and has justified the use of this technique for modelling and control.

The result of applying control to the No.2 furnace at Spencer Works has clearly revealed some limitations in furnace design since the preheat zone fuel flows are inadequate for the control of slab temperature at high throughput. This is a problem which is likely to occur on similar 5-zone furnaces which are operated close to their design limits. Further experimental evidence is required to assess whether it is possible to lower the desired output temperature from the preheat zone. This has not yet been done at Spencer Works since the automatic control system has been designed to follow closely the present manual practice of furnace operation. It has certainly been demonstrated that temperature control can be achieved with the furnace operating within the constraints imposed by the preheat zone fuel limits.

The identification of the parameters of the difference - equation models have been found to present practical problems. This may be explained by the fact that the method is seeking a linear approximation to the plant and that, not unnaturally, non-linearities cause difficulty<sup>14</sup>. Certainly, great care has to be taken to get good initial estimates of the parameters, to submit the models to careful off-line analysis and to have a clear understanding of the dynamics of the process. Only in this way has it been possible to obtain satisfactory models.

The problems of estimation, the wide range of operating conditions met in practice and the problems posed by the fuel limits in the preheat zone have made the alternative model an attractive proposition. In concept it offers a more flexible type of control and early work in identifying the model parameters has been very encouraging. At the time of writing, this model is being evaluated to determine what extra benefits in control can be expected and how sensitive the model is likely to be to variation in its parameters. If these tests are satisfactory, it is likely that future examples of furnace control will be based upon this model.

The results of the experimental work carried out on this furnace have shown that improved controllability will require improvement in design, and design studies for new types of furnace are now being carried out in BISRA <sup>15</sup>. If our conclusions to date are generally valid existing furnaces could benefit by increased firing capacity in the preheat zone to allow control of slab temperature under the highest throughput conditions.

Other factors also affect temperature control. In particular it is necessary carefully to control slab scheduling so that a smooth transition in slab thicknesses occurs, since otherwise it is impossible to heat adjacent slabs of differing thickness correctly without restricting throughput. The control of mill pacing so that the furnace is pushed at regular intervals will reduce throughput disturbance and reduce the temperature variations this would otherwise occur.

Finally, future work by RTB will be directed towards the development of the existing finishing mill temperature model, the development of a roughing mill temperature model, and the linking of these to the furnace control model. By these means RTB expect to develop an optimal control system for complete temperature control of material from furnace entry to coiler.

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AJDB/CO.

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Appendix 1.Notation

$U_t$	Measured temperature of the zone (sidewall thermocouple).
$\bar{U}_t$	Set point of the heating zone.
$X_t$	Slab surface temperature predicted by the model at the exit of the heating zone.
$T_t$	Measured slab surface temperature at the exit of the heating zone.
$\epsilon_t$	error
$Z_t$	Error between the measured temperature and the model temperature.
$T_D$	Demanded slab temperature at the exit of the heating zone.
$M_t$	Mass flow of steel through the hottest section of the heating zone.
$\nabla$	Backward difference operator.
$G_i, \bar{G}_i$	Gain constants.
$\xi_i, \eta_i, \zeta_i, \delta_i, \gamma_i$	Constants
$k$	Level constant

Other Notation is described in the text.

Appendix 2.Derivation of the Control Algorithm

The difference equation model is

$$(1 + \xi_1 \nabla + \xi_2 \nabla^2) X_{t+r} = k + G_1 (1 + \eta_1 \nabla + \eta_2 \nabla^2) U_t + G_2 (1 + \zeta_1 \nabla + \zeta_2 \nabla^2) M_t \quad (16)$$

The adjustment model is

$$Z_{t+r} = \epsilon_{t+r} + \gamma_1 \sum_{i=0}^{\infty} \epsilon_{t+r-i} \quad (17)$$

The connection between the two models is found by measuring the output temperature  $T_{t+r}$  so that

$$T_{t+r} = X_{t+r} + Z_{t+r} \quad (18)$$

The purpose of the control is to obtain a constant temperature  $T_D$  from the zone.

$$T_D = X_{t+r} + Z_{t+r} \quad (19)$$

Since the control action taken at time  $t$  has to cancel out a predicted error it is necessary to predict the error in the adjustment model. This is done simply by equating the values of  $\epsilon_{t+1}$ ,  $\epsilon_{t+2}$  ....  $\epsilon_{t+r}$  not yet measured to zero. The estimated error is then given by

$$\hat{Z}_{t+r} = \gamma_1 \sum_{i=0}^{\infty} \epsilon_{t-i} \quad (20)$$

Therefore from (19) using (20)

$$X_{t+r} = T_D - \gamma_1 \sum_{i=0}^{\infty} \epsilon_{t-i} \quad (21)$$

Substituting (21) in (17)

$$(1 + \xi_1 \nabla + \xi_2 \nabla^2)(T_D - \gamma_1 \sum_{i=0}^{\infty} \epsilon_{t-i}) = k + G_1(1 + \eta_1 \nabla + \eta_2 \nabla^2)\bar{U}_t + G_2(1 + \xi_1 \nabla + \xi_2 \nabla^2)M_t \quad (22)$$

Solving equation (22) for  $\bar{U}_t$ , the required set point, the control algorithm resulting is

$$\begin{aligned} \bar{U}_t = & \frac{T_D - k}{G_1(1 + \eta_1 + \eta_2)} + \frac{\eta_1 + 2\eta_2}{1 + \eta_1 + \eta_2} \bar{U}_{t-1} - \frac{\eta_2}{1 + \eta_1 + \eta_2} \bar{U}_{t-2} \\ & - \frac{G_2(1 + \xi_1 + \xi_2)}{G_1(1 + \eta_1 + \eta_2)} M_t + \frac{G_2(\xi_1 + 2\xi_2)}{G_1(1 + \eta_1 + \eta_2)} M_{t-1} - \frac{G_2\xi_2}{G_1(1 + \eta_1 + \eta_2)} M_{t-2} \\ & - \frac{\xi_2\gamma_1}{G_1(1 + \eta_1 + \eta_2)} \nabla \epsilon_t - \frac{\xi_1\gamma_1}{G_1(1 + \eta_1 + \eta_2)} \epsilon_t - \frac{\gamma_1}{G_1(1 + \eta_1 + \eta_2)} \sum_{i=0}^{\infty} \epsilon_{t-i} \end{aligned} \quad (23)$$

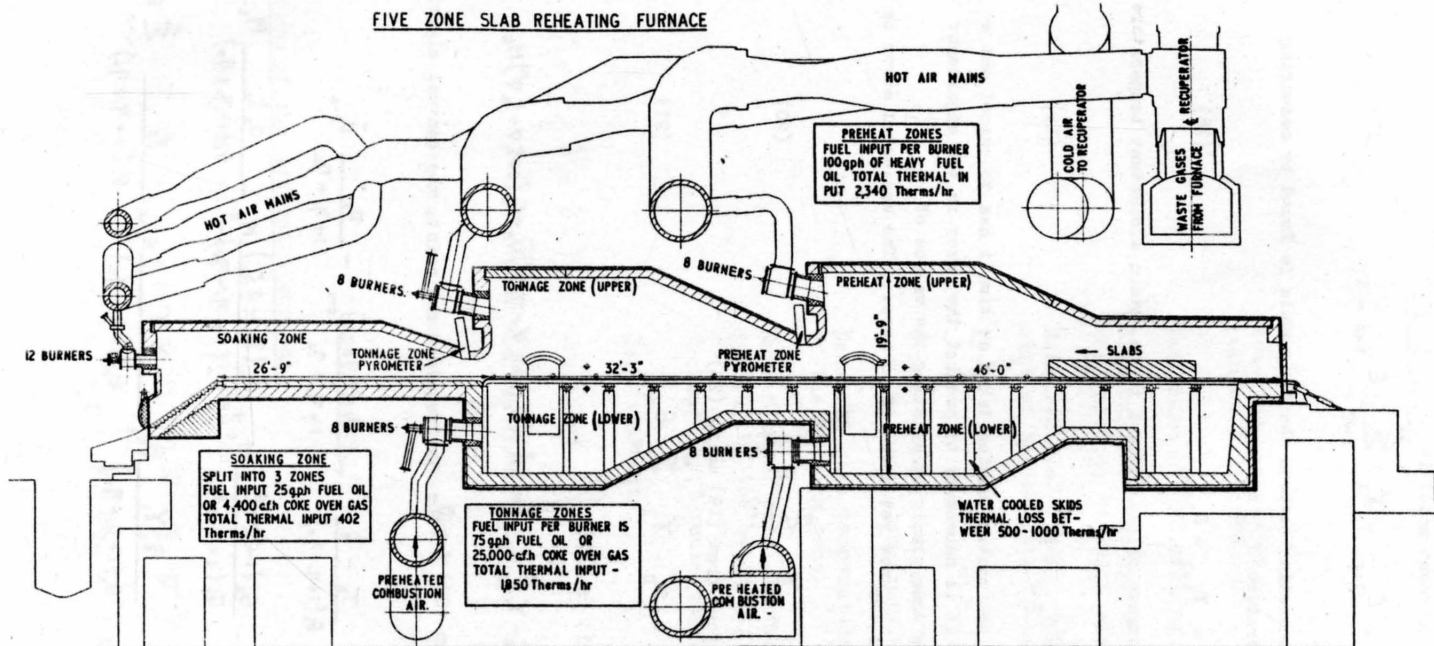


Fig. 1

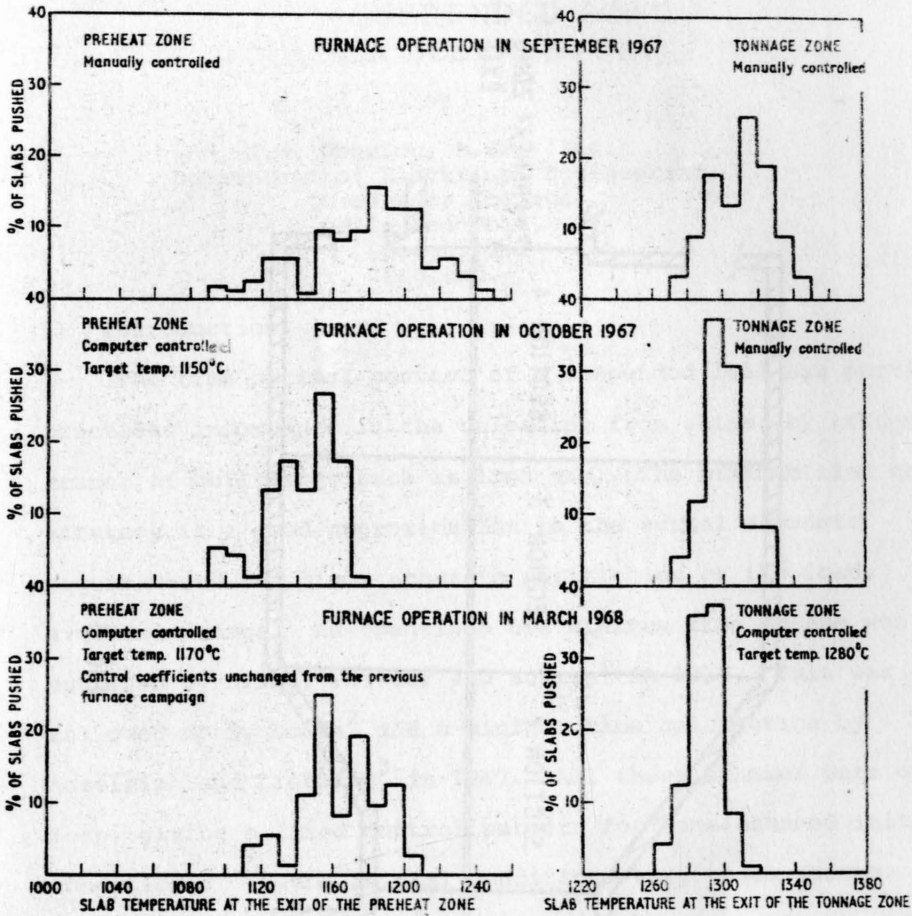


FIG2 THE STAGES OF IMPLEMENTATION OF AUTOMATIC CONTROL

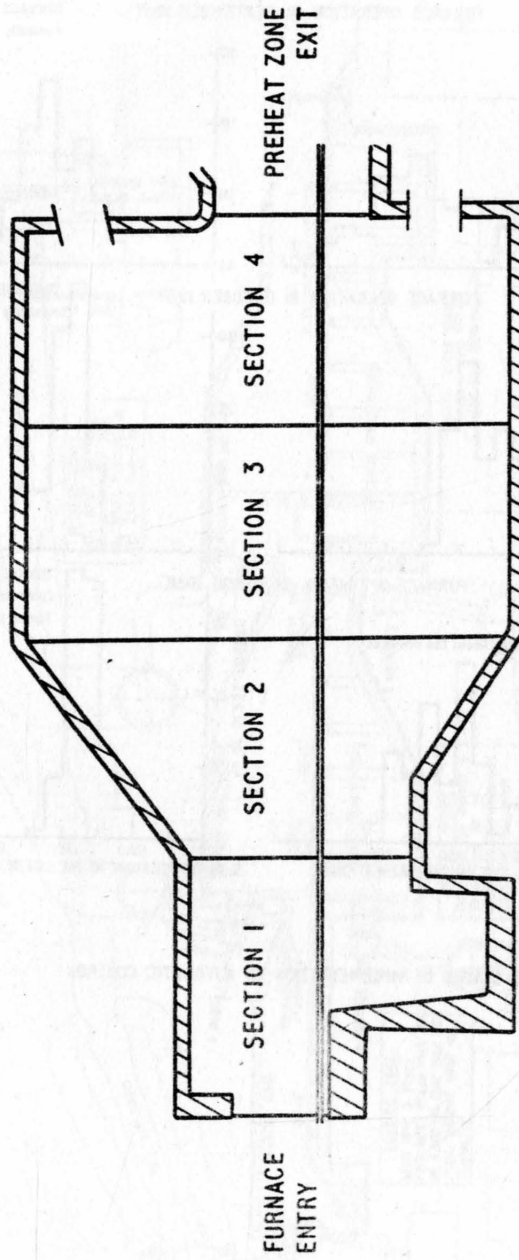


FIG. 3



# CLOSED-LOOP TIME OPTIMAL CONTROL OF A SUSPENDED LOAD A DESIGN STUDY

by

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## 1) Introduction.

The time optimal control of a suspended load has particular practical importance in the unloading from ships, by bridge crane, of bulk cargo such as iron ore. The minimum time control strategy is a good approximation to the actual economic requirements and gives accurate positioning of the load, avoiding damage. An open-loop non minimum time scheme was outlined by Alsop, Forster and Holmes<sup>1</sup> in 1965. This was followed up by Dodds<sup>2</sup> and a minimum time calculation by Anselmo and Liebling<sup>3</sup> in 1967. All these schemes were open loop, giving a fixed control pattern for some assumed initial conditions. If the initial conditions vary these schemes produce load swings and trolley position errors at the end of the motion which can be unacceptable. It is the purpose of this work to calculate the time optimal control, describe two means of implementing this as a demonstration of multiple nonlinear regression analysis and to compare them with conventional linear feedback. It should be stressed that although the techniques developed here are for a particular application, they could readily be developed for other control problems.

## 2) Equations.

A simplified diagram of a bridge crane is shown in figure 1.

$x$ , Horizontal position of centre of gravity.

$\dot{x}_2$  Horizontal velocity of centre of gravity.

$x_3$  Position of load relative to trolley.

$\dot{x}_4$  Velocity of load relative to trolley.

$M$  Effective mass of trolley, including referred inertias of motor and gearing.

$m$  Mass of load.

$f(x_2 - \frac{m}{m+M} \dot{x}_4)$  Feedback of trolley velocity, which may be nonlinear.

$u$  Force applied to trolley by motor drive.  $u_{\min} \leq u \leq u_{\max}$

$l$  Pendulum length.  $\ddot{l}$  Hoist acceleration.

The equations of motion are, assuming small angles of load swing:

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \left( u - f\left(x_2 - \frac{m}{m+M} \dot{x}_4\right) \right) / (M+m) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -\left(g + \ddot{l}\right) \left( \frac{1}{m} + \frac{1}{M} \right) x_3 / l - \left( u - f\left(x_2 - \frac{m}{m+M} \dot{x}_4\right) \right) / M\end{aligned}\quad (1)$$

The pendulum length is a function of time, depending on a pre-programmed hoist motion. The nonlinear velocity feedback could represent a velocity limit for the traverse motor. The control problem is to transfer the system represented by the above equations from any initial conditions in the range of interest to the origin in minimum time.

### 3) Maximum Principle.

Pontriagin's Maximum Principle<sup>4</sup> may be used to calculate the open loop control for specified initial conditions.

Following Pontriagin, we form the Hamiltonian:

$$\begin{aligned}
 H &= \sum_{i=1}^4 p_i \dot{x}_i - 1 \\
 &= p_1 x_2 + p_2 \left( u - f \left( x_2 - \frac{m}{M+m} x_4 \right) \right) / (M+m) \\
 &\quad + p_3 x_4 - p_4 \left( (g+l) \left( \frac{1}{m} + \frac{1}{M} \right) x_3 / l + \left( u - f \left( x_2 - \frac{m}{M+m} x_4 \right) \right) / M \right) - 1
 \end{aligned} \tag{2}$$

The response of the adjoint vector is given by:

$$\dot{p}_i = - \frac{\partial H}{\partial x_i} \quad \text{for } i = 1 \text{ to } 4 \tag{3}$$

The control input  $u$  is chosen to maximise  $H$ :

$$\begin{aligned}
 &\text{if } \left( \frac{p_2}{(M+m)} - \frac{p_4}{M} \right) > 0 \quad \text{then} \quad u = u_{\max} \\
 &\quad \quad \quad < 0 \quad \text{then} \quad u = u_{\min}
 \end{aligned} \tag{4}$$

Therefore the minimum time control is 'bang-bang'. The number of switches of the control input cannot initially be assumed for this nonlinear oscillatory system. We may however, bearing in mind the results for linear systems, consider it probable that three switches will be required. The vector of unknown initial conditions:

$$\underline{c} = \{ p_2(0), p_3(0), p_4(0), T \} \tag{5}$$

has now to be chosen so that integrating the equations from the known initial state  $\{x_1(0), x_2(0), x_3(0), x_4(0)\}$  and stopping at the time  $T$  satisfies the required final state  $\underline{x}(T) = 0$ .

## 4) NUMERICAL SOLUTION OF THE BOUNDARY VALUE PROBLEM

The Newton-Raphson method<sup>5</sup> may be used to correct an initial estimate of  $\underline{c}$ , giving changes in  $\underline{c}$  of:

$$\Delta \underline{c} = -k \left[ \frac{\partial \underline{x}(\tau)}{\partial \underline{c}} \right]^{-1} \underline{x}(\tau) \quad (6)$$

A digital computer program of this method gave good convergence for an initial guess of  $\underline{c}$  which produced three switches of  $u$ . In this problem and many others, however, the solution may already be known for a similar system. For example the problem may have been initially tackled for linearised dynamics, or a previous scheme may have been designed having different numerical parameters in the equations. A modified sensitivity technique<sup>6</sup> has been developed to solve the two point boundary value problem for systems of similar structure. Let the change from the initial system to the required system be represented by a scalar  $s$  such that  $s = s_0$  gives the initial system and  $s = s_1$  the required system. The technique calculated the changes in  $\underline{c}$  with  $s$ , the specified boundary conditions  $\underline{x}(0)$  and  $\underline{x}(\tau)$  remaining satisfied. It is straightforward to show that the required changes in  $\underline{c}$  are given by:

$$\Delta \underline{c} = - \left[ \frac{\partial \underline{x}(\tau)}{\partial \underline{c}} \right]^{-1} \frac{\partial \underline{x}(\tau)}{\partial s} \Delta s \quad (7)$$

Since these changes were based on first derivatives, the new value of  $\underline{c}$  was in error resulting in errors in  $\underline{x}(\tau)$  from

the required value of zero. These errors were reduced using a Newton-Raphson method, as equation 6, but with unity acceleration factor  $k$ . Thus the size of the step  $\Delta s$  in  $s$  was limited to keep the errors in  $\underline{x}(T)$  within the range from which the Newton-Raphson method with unity acceleration would converge. A numerical example of the application of the method is given in figure 2. This was for obtaining the solution for lowering of the load, starting from the given solution for a constant pendulum length. The graphs of load position and control input against time show that the response had no overshoot and that there were three switches of the control input. This was typical of the minimum time responses over the range of initial conditions. The same results could be obtained, without using the adjoint system of equation 3, by changing the three switching times and the final time directly. In this case the vector  $\underline{c}$  is given by:

$$\underline{c} = \{ t_1, t_2, t_3, T \} \quad (8)$$

The sensitivity method was also used for the introduction of a motor velocity limit, different masses of trolley and load and different limits of  $u$ .

#### 5) Regression Analysis

The nonlinear multivariable regression analysis program<sup>7,8</sup> described here is used in the next section to produce two possible control schemes based on data from the minimum time trajectories obtained using the Maximum Principle. Consider the problem of representing a dependent variable  $y$  as a linear



function of  $p$  independent variables  $z_1, z_2, \dots, z_p$ . A stepwise linear regression of the variables  $z_1, \dots, z_p$  on  $y$  proceeds as follows:

- 1) Form the equation  $y = a_0 + a_1 z_i$  where  $a_0$  and  $a_1$  are chosen in the least squares sense and  $z_i$  is the most significant single variable in explaining the variations of  $y$ .
- 2) Find another variable  $z_j$  which, of the remaining independent variables, most explains the residual variation of  $y$ .

Calculate a new set of coefficients:

$$y = a'_0 + a'_1 z_i + a'_2 z_j$$

Further steps continue in a similar manner, adding more independent variables to the expression for  $y$ . The stepwise procedure is terminated when either the number of independent variables in the expression has reached a specified limit, or when there are no further variables of sufficient significance remaining. It is possible that a variable previously entered into the expression is made less significant by the addition of further variables. A test is therefore included to remove a variable which falls below a given significance level. A flow diagram of this stepwise regression procedure is given in figure 3.

Consider now the problem of determining a nonlinear relationship between  $n$  independent variables  $x_1, \dots, x_n$  and  $y$ . Define another set of variables  $z_1, \dots, z_p$  such that  $z_i = f_i(x_1, \dots, x_n)$ . That is, we replace the original set of variables  $x_1, \dots, x_n$  by a set  $z_1, \dots, z_p$ , each of the  $z$  variables being a separate function of one or more of the  $x$  variables, e.g:

$$z_1 = x_1 x_2 \quad z_2 = x_1^2 x_3^{-5} \text{ etc.}$$

Each of the  $z$  variables is defined as a TERM, each term contains one or more FACTORS, the number of factors being the INTERACTION ORDER of that term. In the program each factor may be one of four FUNCTION TYPES:

- 1) positive powers  $x_j^k$
- 2) negative powers  $x_j^{-k}$
- 3) inverse powers  $x_j^{1/k}$
- 4) inverse negative powers  $x_j^{-1/k}$

Where  $k$  is a positive integer, the ORDER of the factor, and  $j$  is the VARIABLE NUMBER of the factor. Each factor is defined by the three numbers VARIABLE NUMBER, FUNCTION TYPE and ORDER. A term is defined by its INTERACTION ORDER together with the definition of each of its factors. A stepwise linear regression of  $z_1, \dots, z_p$  on  $y$  will produce a nonlinear relationship  $y = f(x_1, \dots, x_p)$ . The number of  $z$  variables increases very rapidly with the interaction order and order of the factors. A learning mechanism is therefore included in the program so that only a small number of the total  $z$  variables is taken for one trial of the stepwise regression and, based on the  $z$  variables chosen at that trial, the selection of them is modified for succeeding trials. The selection of terms is by a selection tree, figure 4. Initially there is an equal probability of going from a node of the tree down any of the adjacent branches, and thus the initial selection of, say, twenty  $z$  variables is at random. After the stepwise regression the probability of going down branches corresponding to variables in the equation is increased and for variables not in the equation it is decreased. In this way the program gains information on the sort of terms which best explain  $y$ , and convergence to a good fit may be

expected after sampling a relatively small number of the  $z$  variables (typically under 100 from about 10000). The basic program flow chart is shown in fig. 5

#### 6) Application of regression analysis

The range of initial conditions in this problem is too wide to use techniques based on updating the control using first derivatives of the errors.

Two possible means of designing a control scheme based on the minimum time trajectories produced in section 4 are:

- 1) A nonlinear feedback controller

$$u = f(x_1, x_2, x_3, x_4) \quad (9)$$

where the function  $f$  is determined by regression analysis on the data from the minimum time trajectories.

- 2) A controller defined by the three switching times of the control input  $t_1, t_2, t_3$  where the relationships

$$t_1 = f_1(x(\omega)) \quad t_2 = f_2(x(\omega)) \quad t_3 = f_3(x(\omega)) \quad (10)$$

are determined again by the regression analysis technique. Since in this case the control is determined by the initial state only, it is essentially an open loop scheme.

The relative advantages of the two schemes depend partly on the disturbances expected during the dynamic response and partly on the fact that the regression equation for the closed loop scheme would be more complex and require more data for its design. In either case it is unlikely that the regression equation could be simulated by analogue techniques, so that a small on-line digital computer would be essential.

The latter method, applied to the system of figure 2, with a maximum order of 5 and interaction order of 2, produced the following equations:

$$\begin{aligned}
 t_1 &= 2.12 - 0.0238 x_1(0) + 5.35 (0.001 x_1(0) x_3(0))^5 - 609 x_4(0) / (0.1 x_1(0))^5 \\
 &\quad + 0.132 x_4(0) / x_1^2(0) - 93.2 (0.1 x_3(0))^4 / x_1(0) - 12.9 x_3^2(0) / (0.1 x_1(0))^4 \\
 &\quad + 17.1 x_3^5(0) / x_1^3(0) . \\
 t_2 &= 3.69 - 0.0283 x_1(0) + 16.9 (0.001 x_1(0) x_3(0))^5 + 129 x_3^4(0) / x_1^3(0) \\
 &\quad - 0.126 x_4(0) / (0.1 x_4(0))^5 + 1.41 x_3^5(0) / (0.1 x_1(0))^4 + 163 x_3^3(0) / (0.1 x_1(0))^5 \\
 t_3 &= 4.28 - 0.0313 x_1(0) + 0.46 x_3^2(0) / x_1(0) - 1.94 x_3^3(0) / (0.1 x_1(0))^3 \\
 &\quad + 8.68 x_4(0) (0.001 x_1(0))^2
 \end{aligned}
 \tag{11}$$

Each equation was produced in less than five trials, in which about 40 of a possible total of 1800 terms were sampled. The typical residual standard error of  $y$  was 0.05. When used in a digital computer simulation this controller produced responses almost indistinguishable from the minimum time responses for initial conditions within the design range. The addition of a small linear feedback region near the origin of the state space would produce a usable scheme. A comparison of the minimum time control with conventional linear feedback is useful to determine the possible performance advantages. The comparison is complicated, however, by the choice of the region near the origin of the state space within which the response is considered

to have terminated. The size of this region would depend on accuracy considerations. Figure 6 shows comparison of minimum time and linear feedback analogue simulation performance averaged over the range of initial conditions for different values of this final error region.

## 7) Conclusions

This paper has outlined the design of a control scheme using multiple nonlinear regression analysis with a learning mechanism. For the minimum time control of a suspended load the technique of obtaining the switching times as a function of the initial state gave good performance. In other problems, however, particularly where the optimal control is not 'bang-bang', the first method of obtaining nonlinear feedback throughout the response could be used. The learning mechanism included in the regression program overcomes the problems previously met in applying multiple nonlinear regression analysis. With no prior information on the analytic form of the function convergence to a good fit is obtained whilst sampling only a very small number of the total available nonlinear functions.



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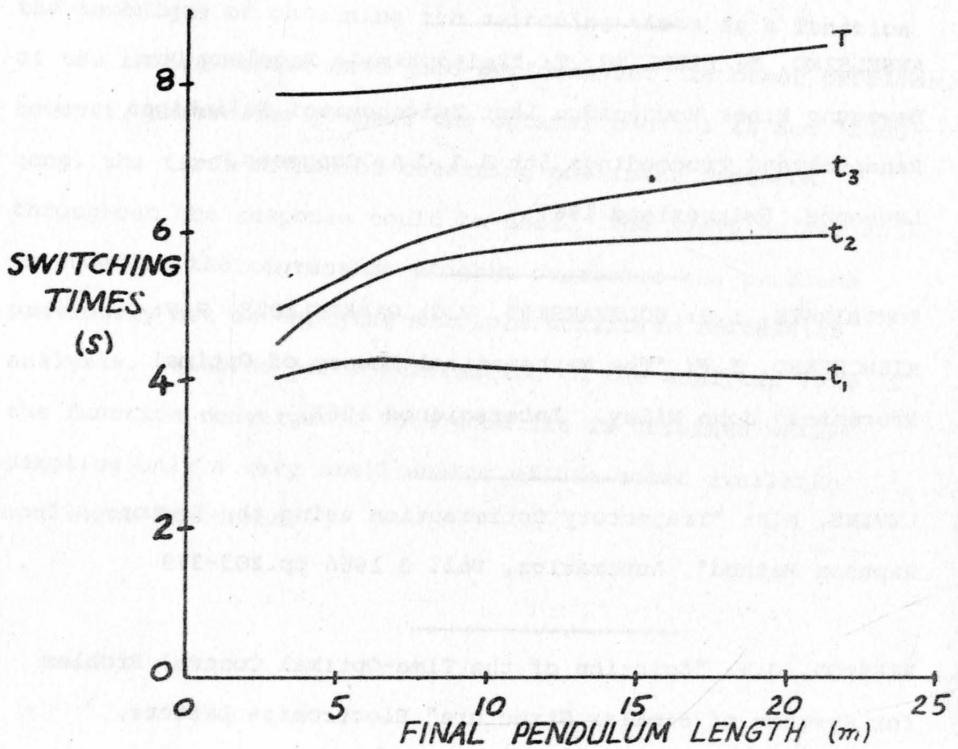
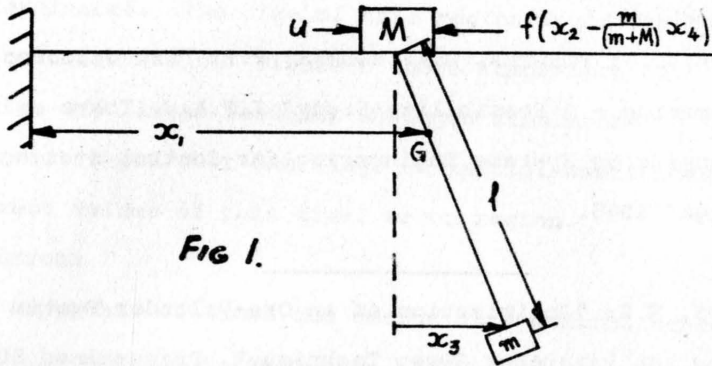
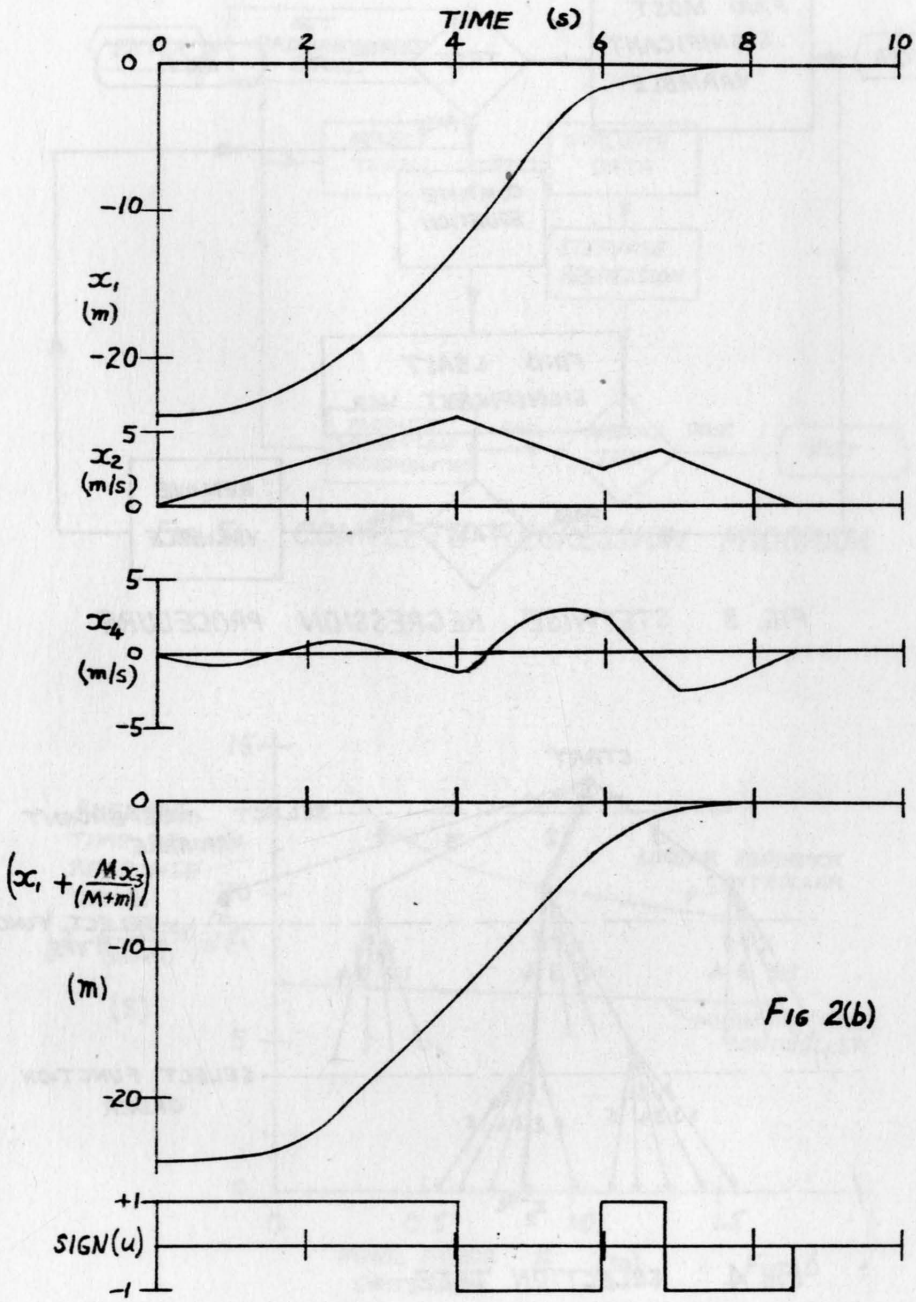


Fig 2(a)

$M = 67.1$  tonne.  $m = 14.22$  tonne.

$f(x_2 - \frac{m}{(m+M)}x_4) \equiv 0$ .  $u_{\min} = -15.5 \times 10^4$ ,  $u_{\max} = 11.57 \times 10^4$  kg.m/s<sup>2</sup>

INITIAL PENDULUM LENGTH (CONSTANT) = 3.04 m.



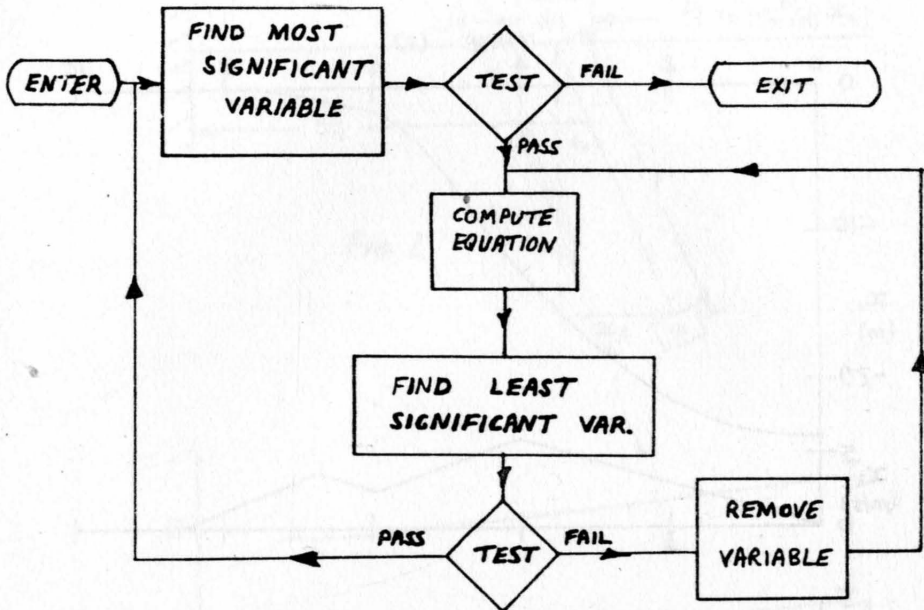


FIG. 3 STEPWISE REGRESSION PROCEDURE

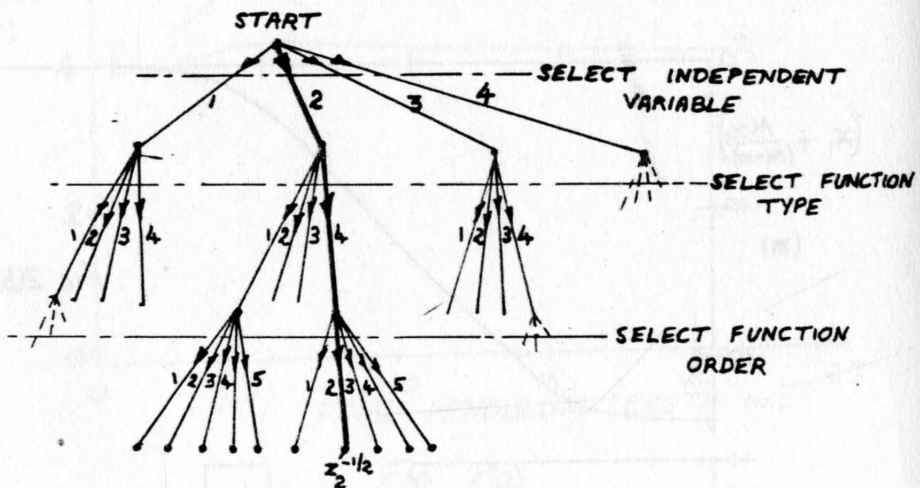


Fig 4 SELECTION TREE

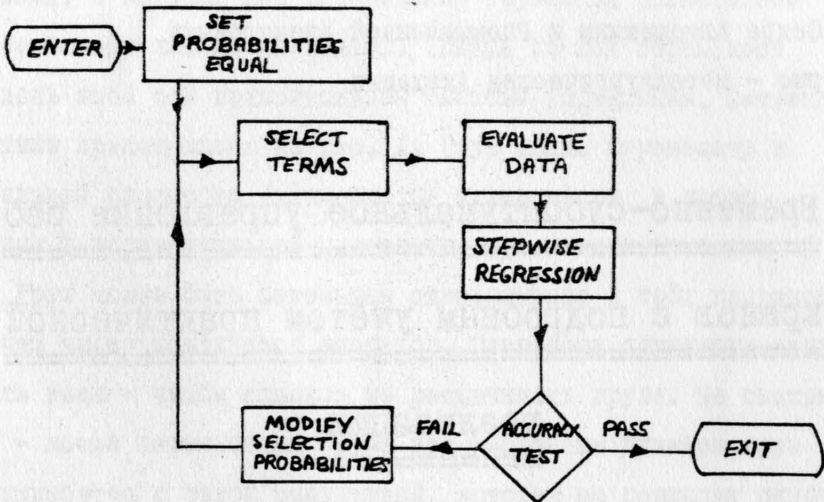


FIG. 5. COMPLETE REGRESSION PROGRAM

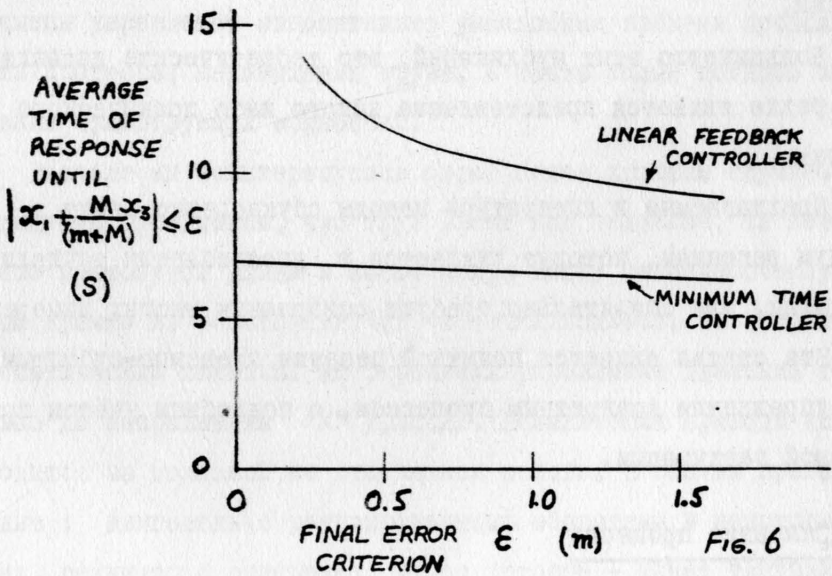


FIG. 6



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## Временно-субоптимальное управление работой

## кранов с подробным учётом практической

## реализации

### 1. Вступление

В настоящее время находится уже богатая литература относительно оптимального управления и постоянно появляются новые отрасли.

Большинство этих публикаций, это теоретические издания, и из редка является представление какого либо пракического решения.

Предлагаемые в литературе методы обукновенно ведут к сложным решениям, которые нуждаются в употреблении вучислительных машин, или обязательно требуют совершения многих измерений.

Эта статья является попыткой решения временно-субоптимального управления конкретным процессом, с подробным учётом практической реализации.

### 2. Описание процесса

звёмся за проектированием системы к оптимальному - в значении "минимум времени" - управлению работой крана. Мы ограничимся до отыскивания оптимального управления мостовых кранов.

т.е. таких, в которых для перемещения груза, мы располагаем тремя самостоятельными приводами. Каждый из них перемещает груз вдоль иной оси прямоугольной системы управления. Ситуация этого типа представлена на рис. 1. Груз можно перемещать в вертикальной плоскости /движение С/ и переместить в любое положение в горизонтальной плоскости, при помощи приводов А и В. Груз может быть перемещен одновременно в трёх направлениях, чем часто пользуется оператор. Операторы принимают малую скорость езды - чтобы слишком не раскачивать груза. Не смотря на это - после перемещения груза над выбранным пунктом груз будет колебаться с такой амплитудой, которая не позволит опустить его без усгояения колебания.

В случае постоянного использования работы крана, например в учреждениях, которые занимаются перегрузкой товаров, как порты - следовало бы подумать над возможностью модернизации управления, перемены управления относительно уменьшения времени продолжительности процесса, перемещения груза, а также более полного использования существующих мощностей.

Вначале мы заинтересуемся более более простым случаем, чем предыдущий. предположим, что груз висит под тележкой, на невесовом канате постоянной длины и имеет целую массу сосредоточенную в одной точке на расстоянии "l" от точки подвеса, т.е. составляет математический маятник. Мы ограничим управление движения тележки только до направления "х" /рис.2/. Локализация привода тележки находится за тележкой на стабильной основе. В состав привода входит : двигатель с регулирующими оборотами в некоторых пределах, редуктор и сцепление, через которые - канат тянущий тележку - сцепляется с действующим приводом или тормозом. Кроме этого предположим, что масса тележки ничтожно мала по сравнению с инерцией

масс находящихся в движении. Прямое расстояние от точки зацепления от не-  
которого первоначального положения, обозначим через "у", а пе-  
ремещение груза вдоль оси "х" через "х" /Рис.3/.

На груз в движении, действуют следующие силы : сила тяжести  
" $G$ ", сила инерции " $B$ ", сопротивление динамического трения  
" $T$ " и сила реакции каната " $R$ ". Напишем условия равновесия  
проекции сил на нормальное направление к пути движения т.е. вдоль  
каната.

$$G_n - R = 0 \quad 2.1.$$

где :  $G_n$  - проекция силы тяжести " $G$ " на нормальное направление  
к пути движения ;

$R$  - реакция в канате ;

напишем следующее условие равновесия сил на направление "х"

$$-B_x - T_x + G_s \cdot \cos \alpha - G_n \sin \alpha + R \sin \alpha = 0 \quad 2.2.$$

$B_x$  - составляющая сила инерции к направлению "х". Она  
пропорциональна массе груза " $G$ " и ускорению в  
направлении этой оси.

$$B_x = m x'' \quad 2.3.$$

$T_x$  - составляющая сопротивлений движения пропорциональная  
к скорости груза в направлении оси "х"

$$T_x = r x' \quad 2.4.$$

где "r" - коэффициент сопротивления движения

$G_s$  - составляющая силы тяжести " $G$ " касательная к пути  
движения

$$G_s = G \sin \alpha \quad 2.5.$$

учитывая уравнение 2.2. и зависимость 2.1. - 2.5. можем написать :

$$- mx'' - gx' + G \sin \alpha \cdot \cos \alpha = 0 \quad 2.6.$$

Для малых углов " " можно принять :

$$\cos \alpha = 1 \quad 2.7.$$

с рис.3. видно :

$$\sin = \frac{y - x}{l} \quad 2.8.$$

следовательно для малых отклонений 2.6. примет вид :

$$- mx'' - gx' + G \cdot \frac{y - x}{l} = 0 \quad 2.9.$$

в последствии получим :

$$x'' + \frac{g}{m} x' + \frac{G}{m \cdot l} /x - y/ = 0. \quad 2.10.$$

Так как :

$$G = m \cdot g \quad 2.11.$$

затем :

$$x'' + \frac{g}{m} x' + \frac{g}{l} x = \frac{g}{l} y \quad 2.12.$$

обозначая :

$$\frac{g}{m} = a \quad ; \quad \frac{g}{l} = b \quad 2.13.$$

получим :

$$x'' + ax' + bx = by \quad 2.14.$$

Так как мы управляем скоростью тележки "x" , затым её перемещение равняется :

$$y = \int v \cdot dt \quad 2.15.$$

Скорость езды тележки может принимать величину в пределах от

$$0 - v_{\max} = u \quad 0, \text{ учитывая в } 2.14.$$

$$x'' + ax' + bx = \int_0^t bv \, dt \quad 2.16.$$

Дифференцируя обе стороны 2.16. относительно "t" получим:

$$x'''' + ax'' + bx' = bv \quad 2.17.$$

$$0 \leq v \leq v_{\max} = u$$

вышеуказанных объект надо перенести с первоначального состояния

$$x'' / 0 / = x' / 0 / = x / 0 / = 0 \quad 2.18.$$

в моменте t - 0 к окончательному состоянию

$$x''/t_k' = x'/t_k' = 0 ; \quad x/t_k' = x_k \quad 2.19.$$

в моменте " $t_k$ ", в возможно короткое время.

### 3. Применение "принципа максимум"

При линейном уравнении 2.17. - управляемая функция /чтоб получить процесс временно-оптимальный/ должна быть - согласна с "принципом максимум" - постоянна интервалами, и должна принимать на смену крайние значения. "Принцип максимум" составляет, в этом случае, необходимое условие оптимальности в смысле минимум времени.

Уравнение 2.17. третьего порядка превращаем в систему первого порядка. Мы это получим подставляя

$$x_1 = x ; \quad x_2 = x' ; \quad x_3 = x'' \quad 3.1.$$

и получим :

$$\begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ x_3' = -b \cdot x_2 - ax_3 + bv \end{cases} \quad 3.2.$$

Функция Гамильтона :

$$H = \sum_{k=1}^3 \psi_k \frac{dx_k}{dt} \quad 3.3.$$

если возьмём во внимание 3.2. мы получим :

$$H = \psi_1 \cdot x_2 + \psi_2 \cdot x_3 + \psi_3 / -bx_2 - ax_3 + bv / \quad 3.4.$$

Эта функция достигнет максимум при управлении, в котором :

$$v = \text{sign} / b \psi_3 / \quad 3.5.$$

Это в нашем случае примет вид :

$$v = \begin{cases} u & \text{когда } \psi_3 > 0 \\ 0 & \text{когда } \psi_3 \leq 0 \end{cases} \quad 3.6.$$

Для обозначения  $\psi_3$  в функции времени мы воспользуемся зависимостью

$$\frac{d\psi_i}{dt} = - \frac{\partial H}{\partial x_i} \quad / \psi_i = 1, 2, 3 / \quad 3.7.$$

Мы должны решить систему уравнений



$$\begin{cases} \psi_1' = 0 \\ \psi_2' = -\psi_1 + b\psi_3 \\ \psi_3' = -\psi_2 + a\psi_3 \end{cases} \quad 3.8.$$

следовательно :

$$\begin{cases} \psi_1'' = c_1 \\ \psi_2'' = -\psi_2' + a\psi_3' \\ \psi_3'' = \psi_1' - b\psi_3' + a\psi_3' \end{cases} \quad 3.9.$$

Окончательно мы должны решить уравнение :

$$\psi_3'' - a\psi_3' + b\psi_3' = c_1 \quad 3.10.$$

Функция  $\psi_3$  после решения уравнения 3.10. выражается формулой :

$$\begin{aligned} \psi_3 / t / = & \frac{s_1^2 c_3 + s_1 c_2 - s_1 a c_3 + c_1}{3s_1^2 - 2as_1 + b} e^{s_1 t} + \\ & + \frac{s_2^2 c_3 + s_2 c_2 - s_2 a c_3 + c_1}{3s_2^2 - 2as_2 + b} \cdot e^{s_2 t} + \frac{c_1}{b} \end{aligned} \quad 3.11.$$

где

$$\begin{aligned} c_2 &= \psi_3' / 0 / & s_1 &= \frac{a + \sqrt{a^2 - 4b}}{2} \\ c_3 &= \psi_3 / 0 / & s_2 &= \frac{a - \sqrt{a^2 - 4b}}{2} \end{aligned} \quad 3.12.$$

Предложим  $a = 0$ , тогда функция  $\psi_3$  примет более простую форму. В нашем случае, мы можем это сделать смело - потому что амортизация движения через сопротивление воздуха - минимальная.

$$\psi_3 / t / = \frac{c_1}{b} / 1 - \cos \sqrt{b} t / + c_3 \cos \sqrt{b} t + c_2 \frac{1}{\sqrt{b}} \sin \sqrt{b} t \quad 3.13.$$

обозначая

$$\sqrt{b} = \omega \quad 3.14.$$

$$\psi_3 / t / = \frac{c_1}{\omega^2} / 1 - \cos \omega t / + c_3 \cos \omega t + c_2 \frac{1}{\omega} \sin \omega t. \quad 3.15.$$

На основании уравнений 3.8. и 3.15. мы можем определить остальные вспомогательные функции

$$\begin{cases} \psi_1 = c_1 \\ \psi_2 = / c_3 - \frac{c_1}{\omega} / \sin \omega t + c_2 \cos \omega t \\ \psi_3 = \frac{c_1}{\omega^2} + / c_3 - \frac{c_1}{\omega^2} / \cos \omega t - c_2 \frac{1}{\omega} \sin \omega t \end{cases} \quad 3.16.$$

Систему уравнений 3.16. мы можем представить :

$$\left. \begin{aligned} y_1 &= c_1 \\ y_2 &= \omega R \sin / \omega t + \alpha / \\ y_3 &= \frac{c_1}{2} + R \cos / \omega t + \alpha / \end{aligned} \right\} \quad 3.17.$$

где  $R = \frac{1}{\omega} \sqrt{c_3 \omega - \frac{c_1}{\omega} / 2 + c_2^2}$

$$\alpha = \arctg \frac{c_2}{c_3 \omega - \frac{1}{\omega}}$$

Параметрическое уравнение 3.17 представляет в прямоугольной системе оси  $y_1, y_2, y_3$  эллипс находящийся в перпендикулярной плоскости к оси  $/1$ ; Эта плоскость пересекает ось в точке  $y_1 = c_1$ . В системе оси  $\frac{y_2}{\omega}, y_3$  возникает круг зачёркнутый радиусом "R" центр которого находится на оси  $/3$  в точке  $y_3 = \frac{1}{\omega^2}$ , Рис.4.

Вышеуказанные рассуждения указывают какой характер имеет функция  $y_3$ . Благодаря этому мы знаем характер переключаемой функции "v". К сожалению метод понтригина не представляет нам достаточных информации о величине первоначальных условий  $c_1, c_2, c_3$ ; и так мы знаем что функция "v" будет повторяться с периодом "T", но не знаем длины интервала включений, а также времени, после которого наступит первое и последнее переключение.

По характеру управляемой функции видно, что можно её принять как сумму перемещенных во время постоянных вынуждений стоимости "u".

Если первоначальные величины равняются нулю - то управляемая величина  $x/t$  равняется сумме ответа на каждый входящий сигнал.

На основании формулы 2.17. можем вычислить ответ системы на ступенчатое принуждение

$$v = \begin{cases} 0 & \text{как } t < 0 \\ u & \text{как } t \geq 0 \end{cases} \quad 3.18.$$

равняется

$$\left\{ \begin{aligned} h / t / &= u / t - \frac{1}{\omega} \sin \omega t / \\ h' / t / &= u / 1 - \cos \omega t / \\ h'' / t / &= \omega \cdot u \sin \omega t \end{aligned} \right. \quad 3.19.$$

Ответ системы на управляющий сигнал, сложный с интервалов постоянно функции, которая равняется, попеременно "u" и "0" - будет представляться :

$$x(t) = u \sum_{k=0}^n (-1)^k \cdot h(t - t_k) \quad 3.20.$$

#### 4. Оптимальное управление

Для определения оптимального управления нужна добавочная информация. В случае характеристического уравнения второго порядка, удобно рассмотреть фазовые траектории на плоскости  $x, x'$ . В случае уравнения третьего порядка, следовало бы рассматривать эти траектории в пространстве  $x, x', x''$ , но этого мы не сможем достаточно ясно представить на плоскости.

Уравнение описывающее рассматриваемую систему не полное, ибо значение коэффициента при  $x$  равно нулю, благодаря чему, наблюдая поведение системы на плоскости  $x, x'$  мы имеем полную информацию о всех производных и изменении значения функции.

На плоскости  $x', \frac{x''}{\omega}$ , траектории рассматриваемой системы являются кругами с центром в точке  $x = v, \frac{x''}{\omega} = 0$ .

В оптимальном управлении принимают участие только траектории с центром  $v = 0$  и  $v = u$ , а с начальных и окончательных условий возникает, что процесс начинается и кончается траекторией, которая переходит через начало системы координат  $x', \frac{x''}{\omega}$ , а  $x$  переходит от  $x_0$  к  $x_k$ , чего уже не видно на рис.5.

Если мы подвергнем систему действию входной величины  $=u$  то увидим, что через время  $t = T$ , точка состояния на плоскости  $\frac{x''}{\omega}, x'$ , найдется вторично в центре системы координат. Груз будет находится вертикально под тележкой, так как в моменте старта. Передвижение тележки и груза будет равно одно другому

$$x_T = x_T = u \cdot T \quad 4.1.$$

Из этого видим, что для передвижения груза на расстояние

$$x = n \cdot x_T \quad n = 1, 2, 3, \dots \quad 4.2.$$

необходимое управление будет ограничено до одного периода

включения. Этот итог согласен с результатом полученным с помощью метода Понтрягина. Это особенный случай, где период включения равняется периоду функции, а времена постоя тележки сократились к нулю.

Количество переключений во время оптимального управления равняется:

$$i = 2 \left[ E^* / \frac{x_k}{x_T} + 2 \right] \quad 4.3.$$

На рис.5. представлено в качестве примера фазовую траекторию для  $x_T > x_k > 0$ .

Благодаря существованию зависимости :

$$\alpha = \omega t \quad 4.4.$$

и наблюдению, что согласно с первоначальными и окончательными условиями путь передвижения груза равна пути переезда тележки, а время езды соответствует времени перемещения точки состояния по круговой траектории с центром  $x' = u$ , истинно :

$$x_k = u / T_1 + T_3 - T_2 / \quad 4.5.$$

и

$$\alpha = \pi - \gamma \quad 4.6.$$

а принимая во внимание 4.4.

$$\begin{cases} T_2 - T_1 = \frac{\pi}{\omega} - T_1 \\ T_2 = \frac{\pi}{\omega} \end{cases} \quad 4.7.$$

подобным образом

$$T_k - T_1 = \frac{\pi}{\omega} \quad 4.8.$$

Окончательно получаем

$$\begin{cases} T_1 = \frac{x}{2u} \\ T_2 = \frac{\pi}{\omega} \\ T_k = \frac{x}{2u} + \frac{\pi}{\omega} \end{cases} \quad 4.9.$$

Определение оптимального процесса для  $2x_T > x_E > x_T$  можно перевести на основании фазовой характеристики полагаясь

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\*/  $E/z/$  - обозначает целую часть числа "z".



приобретённой раньше информации. Пример представлено на рис.6.

Основное указание при определении оптимальной траектории, это информация что исключая первый и последний промежуток времени сумма двух очередных переключений согласно с методом Понтрягина равняется периоду  $T$ .

На основании рис.6. можно вывести зависимость угла от  $\delta$  :

$$\alpha = \frac{1}{2} \arccos \frac{2 - \cos \delta}{\sin \delta} \quad 4.10.$$

а для  $x$  находящегося в интервале  $(n-1) \cdot x_T \leq x \leq n \cdot x_T$

$$\alpha_n = \frac{1}{2} \arccos \frac{n - \cos \delta}{\sin \delta} \quad 4.11.$$

Зная значение угла  $\alpha$  соответствующее отрезком времени постоя и  $\delta$  соответствующее первому и последнему отрезку времени езды, не трудно вычислить времена очередных переключений. Полное время исполнения операции равняется :

$$T_{kn} = \frac{T}{360^\circ} [2\delta + \alpha + (n-1) \cdot 360^\circ] \quad 4.12.$$

в это-же время груз переместится на заданное расстояние

$$x_k = \frac{x_T}{360^\circ} [2\delta + (n-1) \cdot 360^\circ - \alpha] \quad 4.13.$$

Если заданное расстояние увеличивается /возрастает число "n"/ то  $\alpha$  стремится к нулю, а время исполнения операции стремится к суммарному времени езды  $T_j$  которое выносит

$$T_j = \frac{x_k}{u} \quad 4.14.$$

Так как это самое краткое время в котором тележка сможет переехать заданное расстояние независимо от колебания груза, возможное симметричное ограничение управляющей функции  $-u \leq \dot{x} \leq u$  при большом расстоянии  $x_k$  не сокращало бы времени исполнения процесса.

Увеличивая  $x_k$  - время постоя сокращается, а это приводит к невозможности реализации временно-оптимального управления.

Кроме того встречаются значительные трудности, если надо ввести поправки, программу, которые учитывают неточное исполнение предложений.

## 5. Временно-субоптимальное управление

Из практической точки зрения в некоторых случаях стоит



отступить от строки оптимального управления и удовлетворится управлением уступающим немного, в смысле быстрогодействия, оптимальному, но зато более способному по практической реализации. Кроме того следует обратить внимание на возможность реализации управления, которое ограничивалось бы балансирование груза во время его езды, а особенно при преодолении большого расстояния. Управление которое обеспечивает при преодолении большого расстояния езду без колебаний, легко обозначить из фазовой плоскости  $\frac{x'}{\omega}$ ,  $x'$  рис.7.

Поэтому надо скорость груза сравнить со скоростью тележки, которая равняется "u". Точку на фазовой траектории надо провести сначала составления координат в точку "u" на оси "x". Это состоится за наименьшее время если мы применим круговую траекторию которая переходит через начало составления координат, и середина которой находится в точке "u", а потом из показанной на рисунке траектории зачёркнутой вокруг начала составления координат и совпадающей в точку "u".

Из зависимости очевидных с рисунка, и так с равенства сторон треугольника возникает, что угол

$$\alpha_1 = \alpha_2 = \frac{1}{6} 2\pi \quad 5.1.$$

а это отвечает

$$T_1 = T_2 - T_1 = \frac{1}{6} T \quad 5.2.$$

Таким образом управление в котором сравнивается скорость перемещения груза со скоростью езды тележки, будет полагаться на включении привода в периоде  $T_1 = \frac{1}{6}T$ , потом задержании его в периоде  $T_2 - T_1 = T_1 = \frac{1}{6}T$ , после чего наступает спокойный проезд тележки в любое продолжение периода времени  $T_3 - T_2$ . С целью задержки груза нам надо перевести ту-же самую операцию т.е. задержать тележку в периоде  $T_4 - T_3 = \frac{1}{6}T$ , а потом ещё один раз переместить её с максимальной скоростью "u" во время  $T_k - T_4 = \frac{1}{6}T$ . Груз будет спокойно задержан, а путь которую он проехал будет равняться :

$$x_k = / T_k - T_4 / + / T_3 - T_2 / + T_1 u \quad 5.3.$$

или

$$x_k = / T_k - \frac{1}{3} T / u \quad 5.4.$$

Это уравнение мы можем применять только для:

$$x_k = \frac{1}{3} T \cdot u \quad 5.5.$$

Можно оценить что расница между продолжением времени процесса в оптимальном  $T_0$  в наиболее плохом случае равна времени задержки, согласно с уравнением

$$T_k - T_0 = \frac{1}{3} T \quad 5.6.$$

В случае большого расстояния проезда и небольшого периода собственных колебаний это составляет ничтожный процент потери времени. На рис.8 представлено в относительных величинах время существования процесса при субоптимальном управлении, и оптимальном управлении с симметрическим и несимметрическим ограничением функции управления. Описанное управление в котором четыре из пяти отрезков времени равны между собой и являются только функцией одной величины "Т", а пятый отрезок времени выражается простым уравнением

$$T_3 - T_2 = \frac{x_k}{u} + \frac{1}{3} T \quad 5.7.$$

будет очень простое в технической реализации.

Согласно с рис.2. сделано в лаборатории Кафедры Автоматики и Промышленной Электроники, Горно-Металлургической Академии в Кракове - устройство управления действующего согласно с выше описанной программой. Груз 4,5 кг подвешенный на цепке длина которого 1,75 м - перемещается на расстоянии до 3,5 м с максимальной скоростью тележки 0,25 м/сек. По достижении точки цели амплитуда колебаний не превышает 0,5 см. Управляя вручную процессом очень трудно усмирить колебания с одновременным окончанием процесса в определённой точке. Потеря времени в сравнении с оптимальным по быстродействию управлением, зависит от опыта оператора, но обыкновенно превышает значительно двукратную потерю времени в случае программного управления.

Интересно отметить что можно не изменять программы в случае необходимости обхождения вертикальной преграды если вертикальные движения /вверх и вниз до первоначального состояния/ будут реализованы во время езды когда отсутствуют колебания т.е. в интервалах времени от  $T_2$  до  $T_3$ . Если точка цели находится на другом уровне, тогда достаточно два последние отрезки времени принять равным  $\frac{1}{3}$  нового периода собственных колебаний. Если для передвижения груза необходимо использовать

Если для передвижения груза необходимо использовать два привода, А и Б /рис.1./ то программа сохранит простую форму в противоположности к временно-оптимальной программе, в которой управление одного из приводов зависит от характера других движений.

### Л и т е р а т у р а

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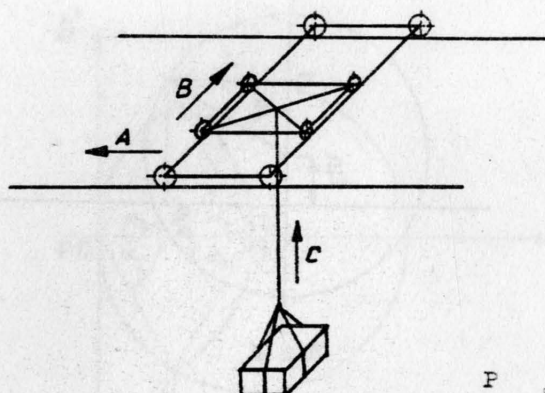
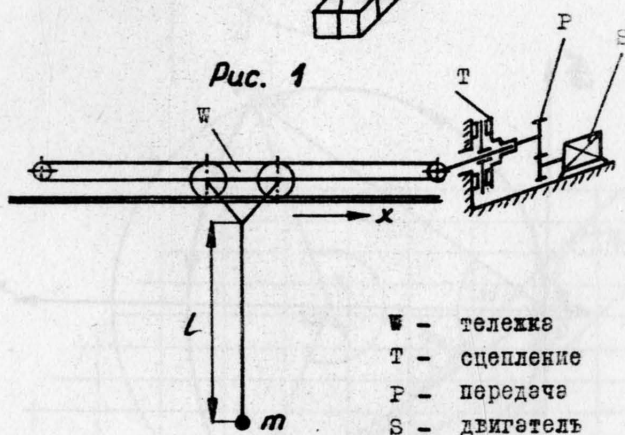


Рис. 1



W - тележка  
T - сцепление  
P - передача  
S - двигатель

Рис. 2

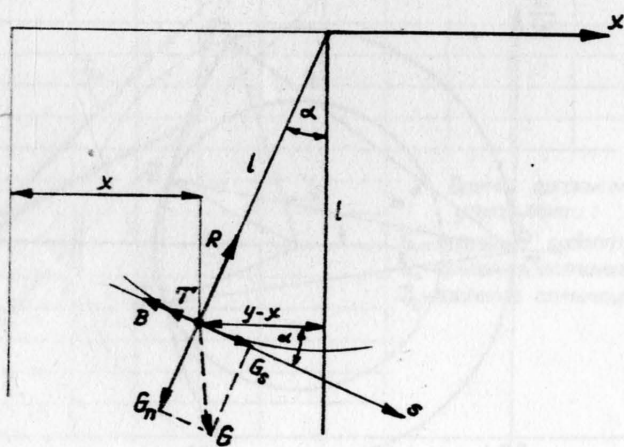


Рис. 3

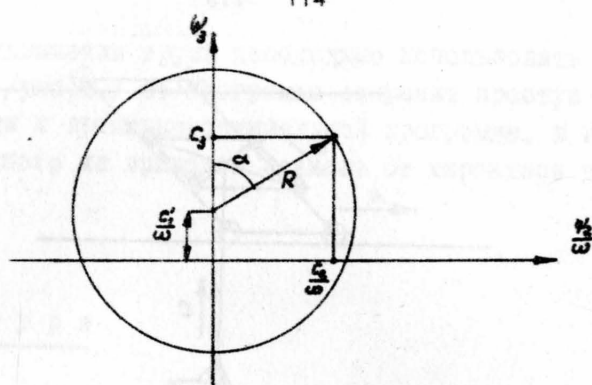


Рис. 4

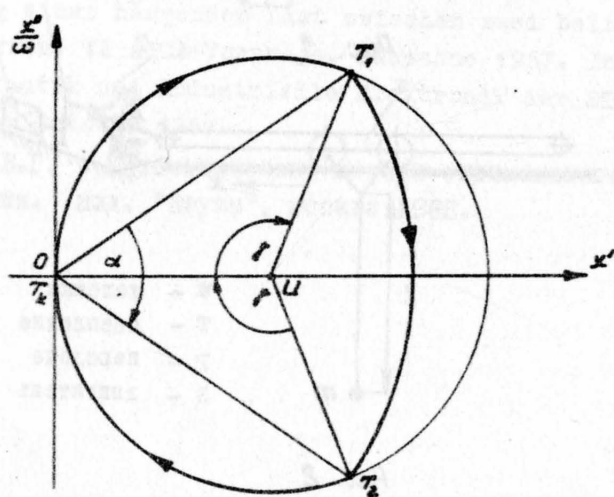


Рис. 5

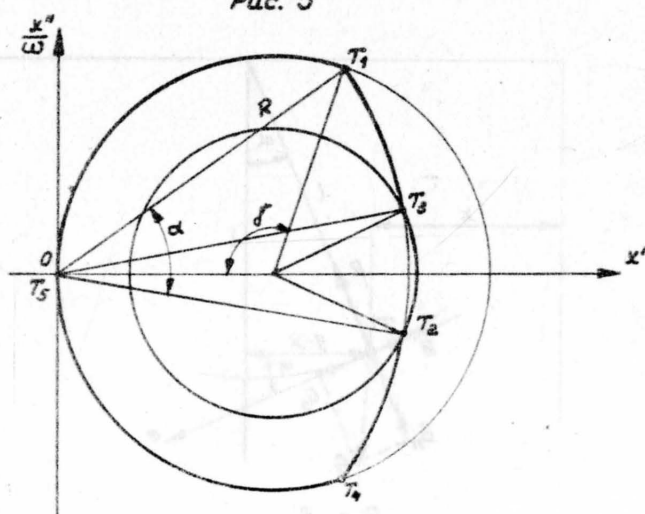


Рис. 6



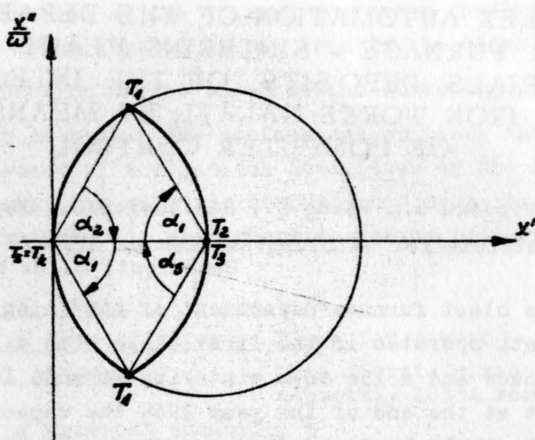


Рис. 7

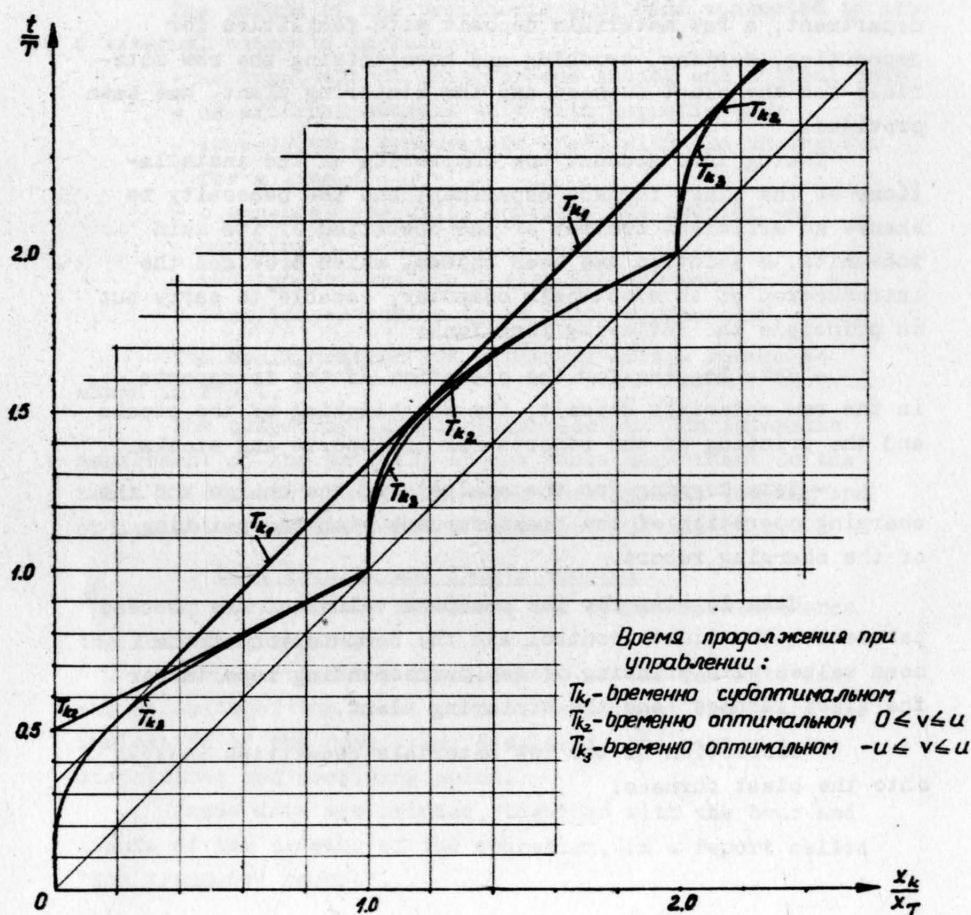


Рис. 8

# COMPLEX AUTOMATION OF THE DEPARTMENT BLAST FURNACE - SINTERING PLANT - RAW MATERIALS DEPOSITS, OF THE INTEGRATED IRON WORKS GALATI, BY MEANS OF COMPUTER CONTROL

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The blast furnace department of the Integrated Iron Works Galati, operates in the first stage with a 1700 cub.m blast furnace and a 150 sq.m sintering strand. It is understood that at the end of the year 1969 the capacity of the department should be doubled.

For the supply with raw materials of the blast furnace department, a raw materials deposit with facilities for depositing, sorting, crushing and homogenizing the raw materials for the blast furnace and the sintering plant, has been provided.

Taking into account the complexity of the installations of the blast furnace department and the necessity to ensure an efficient control of the operation of its main sub-units, a solution has been chosen, which provides the introduction of an electronic computer, capable to carry out in principle the following functions:

- data logging for the operation of the transports in the raw materials deposit, the establishing of the stocks and the printing of the reports for transports and stocks.

- data logging for the analysis of the charge and the charging operation of the blast furnace with the printing of the charging report.

- data logging for the measured values of the process parameters, the limits control and the computation of the mean values with printing of the corresponding reports for the blast furnace and the sintering plant.

- correction of the raw materials quantities charged into the blast furnace.

- optimization of the thermal operating conditions of the process of production of pig iron in the blast furnace
- optimization of the speed of the sintering strand.

In order to obtain the achievement of these targets and the improvement of the general activities of the blast furnace department, an electric computer has been installed during the year 1968, its main characteristics being as follows:

<u>Length of a word:</u>	24 bits
<u>Length in time of a cycle:</u>	33 $\mu$ s for a word
<u>Internal storage:</u>	with ferrite cores capacity 16384 words
<u>Number of external channels:</u>	6
<u>Number of instructions:</u>	31

The volume of the peripheric equipment connected to the 6 external channels includes:

- an input-output unit for the analog and digital data
- an external storage unit with magnetic drum
- two electric typewriters (30") with 330 characters for a line.
- a punched tape unit, equipped for reading and punching.
- two typewriters (10") with 104 characters for a line (an extension up to four is possible).

The configuration and situation of the system is shown in Fig.1.

The computing equipment carries out the automatic management of the activity of the whole department on the basis of distinct functions which include data logging and optimization computations as described further below.

#### 1. Data logging and limits control

As part of this function, the computer establishes the following reports:

a) Report on the displacement of the raw materials of the deposits of the blast furnace department including the indication of the quantities displaced, the sort and the dispatching and receiving point.

These data are printed, together with the hour and minute of the closing of the operation, in a report called "the transport report".

The quantities of displaced raw materials are printed on the basis of the information delivered by a number of 14 electronic belt weighers which transmit the information to the computer under the form of impulses (1 impulse = 100 kg)

The other data are introduced manually by means of a decimal keyboard, mounted on the desk of the raw materials dispatcher. These data include: number of the belt weigher, code number of the dispatching point, code number of the receiving point, code number of the sort and the quantity of raw materials which is to be displaced.

The manually introduced data are displayed at the desk of the raw materials dispatcher and at the desk of an operator, who is charged to build up the sequence of belt conveyors, which is necessary for carrying out the desired transport.

The constituted belt conveyor sequence is also displayed on the light panel of the raw materials dispatcher, who is able to verify in this way the correspondance between the formed belt conveyor sequence and the desired sequence, as previously formulated.

If the formed sequence is equal to the desired sequence, the dispatcher pushes the button for data input, and the consequence is the taking over by the computer of the introduced data and the lightening up of a lamp on the operator's desk who receives subsequently the information that he may start the transport of raw materials on the belt conveyors sequence, previously formed. From this moment, the impulses transmitted by the belt weighers are interpreted by the computer together with the manually introduced data.

All the data concerning the transport request are printed in red in the transport report, while the data concerning the actual transports are printed in black, at the moment when the transport is completed (this moment is marked by the stopping of the conveyor on which the programmed belt weigher is mounted).

b) On the basis of the data accumulated during a period of eight hours, the computer draws up the report for stocks of raw materials according to sorts and quantities for each depositing point. The corresponding data are printed in a report, called "the balance report" draft in a

form of chess board, which permits the identification of all quantities of raw materials, received, dispatched or in stock, for each sort and for each depositing point.

This balance report is automatically printed at the end of each period of eight hours. It is also automatically printed in a cumulative form at the end of each day (3 shifts of 8 hours) and at the end of each month (totalizing on 28, 29, 30 or 31 days according to the length of the months).

c) Data logging for the charging of the blast furnace and printing of the charging report.

For the weighing of the raw materials charged into the blast furnace, 7 bunker weighers have been provided, with the following destination:

- 2 bunker weighers for iron ore (left and right)
- 2 bunker weighers for coke (left and right)
- 2 bunker weighers for sinter (left and right)
- 1 bunker weigher for charge additives, which is able to pour out either in the left or in the right skip car, by means of butterfly valve with remote control.

Each bunker weigher is provided with one or more index setters, intended to prescribe the value of the weighed quantity, the number of index setter being equal to the number of the weighed sorts.

The bunker weighers for ore have three index setters, the one for additives has two and those for coke and sinter have only one index setter.

The bunker weighers can operate in two conditions, which are established by means of a two positions selector:

- manual operation (without correction by computer)
- automatic operation (with correction by computer).

In the case of the manual operation, the index setter is firstly settled at the quantity which is to be weighed. This value is transmitted to the electronic weigher in tetradic code, together with the actual value of the quantity of raw materials weighed in the bunker weigher. Each bunker weigher consists of three load cells which transmit a signal proportional to the weight to an electronic equipment.

When the actual value of the weight is equal to the prescribed value, a signal is emitted which is used for stopping the extraction of the raw material from the bunker.

After a delay of approximately 30 seconds during which



time the raw material existing on the conveyor is emptied into the weighing bunker, a second signal is emitted, which is intended to make the computer to accept the actual value of the weighed quantity.

The computer utilizes these values for the printing of the charging report of the blast furnace, in which are printed the weight of the raw materials charged by sorts in each skip car, together with the time (hour and minute), the number of the skip car, the position of the skip car (left or right), the type of the charge and the total weight of the materials charged into a skip car.

A cumulative report is also printed, at the end of each shift and at the end of each day.

d) Data logging of the analog values for the blast furnace and the sintering plant and printing of the mean values report.

The computer receives from the blast furnace the following analog signals:

- temperature of the hot blast in the main aduction pipe (1 measuring point)
- temperature of the hot blast in the ring pipe
- temperature in the blast furnace at the blow in points (4 measuring points)
- temperature of the top gas before cleaning (4 measuring points)
- pressure in the blast furnace (4 measuring points)
- humidity of the blast (1 measuring point)
- analysis of the top gas  $\text{CO}_2$ ,  $\text{CO}$ ,  $\text{H}_2$  (3 measuring points)
- cold blast flow rate (1 measuring point)
- methane gas flow rate (1 measuring point)
- top gas flow rate (1 measuring point).

The data logging of the following analog signals is provided for the sintering plant:

- temperature in the wind boxes (4 measuring points)
- depression by the exhaustor (1 measuring point)
- water flow rate at the primary and secondary mixing drum ( 2 measuring points)
- speed of the sintering strand (1 measuring point).

For all the above mentioned measuring points, the computer calculates the mean values, which are subsequently printed

in the mean values report.

Some measuring points which are of particular interest for the normal operation of the technological equipment are also controlled from the point of view of the admissible limits.

The values which are over the superior limit or under the inferior limit are printed in an alarm report, together with the emitting of an acoustical and optical signal which may be interrupted from the control desk.

The coming back of the controlled parameters within the range of the admissible values is also printed in the alarm report.

For the totalizing of the consumptions of air, water, methane gas, top gas etc. a range of impulse inputs are provided which totalize the impulses emitted by the flow rate counters of the installations. The totalized values over a period of 8 hours are printed in a report. Cumulative reports are also printed for periods of a day and a month.

It is also provided the possibility for introduction of numerical data by means of a keyboard. In this way are introduced for example some data concerning the analysis of the pig-iron, the analysis of the slag, the weight of the pig iron and some data necessary for the mathematical model.

For the individual control of the different analog measuring points, an optical display device has been provided. After choosing the desired measuring point by means of a decimal keyboard, the numerical value of the last measurement is displayed on a panel with light digits.

## 2. Correction of the raw materials quantities to be charged into the blast furnace:

For the weighing of the raw materials charged into the blast furnace, a number of 7 electronic bunker weighers has been provided, their operation being previously described. It is easy to remark that the quantities of raw materials charged into the blast furnace, allways differ from the quantities settled by the index setters due to the different position of the bunkers as related to the weighers and to the fact that the conveyors feeding the weighers are to be emptied at each weighing process.

For the compensation of this error, the difference is fed into the computer, which adds it, with the corresponding sign, to the value fixed by the manual index setter, at the next

weighing process, programmed for the same sort and on the same bunker weigher.

The calculation of the corrected prescribed value is made by means of a list in which are memorized the differences between the prescribed and measured values of the last weighings for the different sorts and bunker weighers.

The established difference for a determined sort and bunker weigher is added to the manually fixed value, being emitted in form of digital signals 12 V, 100 mA, which operate on the relay sets existing in the panels with electronic equipment of the bunker weighers. When the equipment of the weigher establishes the equality between the new prescribed value emitted by the computer and the measured value, a signal is emitted which is used for the closing of the bunker valve, which stops the delivery of the material. The conveyor continues to discharge into the bunker weigher, until the existing material is evacuated. After 30 seconds from the stopping of the delivery, a signal is given for the blocking of the measuring disc of the weigher and 3 seconds after this signal, the measured value is taken by computer which uses it for the establishing of the new difference according to the list and for the printing of the charging report of the blast furnace.

### 3. Optimization of the thermal condition of the elaboration process in the blast furnace.

The utilization of the electronic computer for data logging, may be considered as the first step in the direction of the automatic control of the process in the blast furnace. The second step is accomplished by connecting the computer to the process, for the purpose of improving the operation of the blast furnace, on the basis of the data continuously coming from the process and of a mathematical model of the technological process which periodically works out the data received from the process and produces index values for the automatic control of some process variables.

The mathematical model utilized for the blast furnace nr.1 from the Integrated Iron Works Galați is delivered by the contractor of the electronic computing equipment and consists in principle <sup>of</sup> a set of thermal and chemical balances by means of which the thermal regime and the characteristics of the elaborated charge may be predetermined.

The blast furnace is considered for this purpose as

being divided into a superior zone, where the indirect reduction process is predominant, and an inferior zone, where the direct reduction process takes place.

In the case of the mathematical model offered by the contractor, the following elements are utilized for the control of the thermal regime of the blast furnace: the enthalpy of the hot blast, the humidity of the hot blast and the blown in methane gas flow rate.

The indirect reduction process is especially influenced by the quantity of hydrogen contained in the humidity added to the hot blast and in the blown in methane gas, which leads to a reduction of the quantity of coke utilized in the reduction process. This aspect has a considerable economic significance, especially for the countries showing a deficit of coke and it is expected that the investments made should be amortized in a few years.

On the basis of the data delivered by the gas analyzers, the following balances are drawn up:

- the oxygen balance
- the hydrogen balance
- the carbon balance
- the nitrogen balance.

The accuracy degree of these balances depends in a decisive degree from the accuracy of the results obtained from the top gas analysis. For this reason the gas analyzers are very cautiously inspected and verified before being put into operation and the measuring elements of the analyzers are placed in a thermostate. The gas analyzers are also periodically calibrated with standard gases, utilizing the control signals emitted by the computer. The results of the analysis are directly transmitted to the computer as analog signals of unified current.

under It is necessary that the measuring errors should remain 1 % in order to avoid that the balances drawn up by the computer should lead to incorrect control values for the process.

The thermal balance of the inferior part of the blast furnace is based on the supposition that the direct reduction of the iron oxides takes place exclusively in this zone and that in the upper part of the considered zone, the temperature is, aproxim.  $1000^{\circ}\text{C}$ , practically the same for the gases and for the solid materials.

This thermal balance considers on the one side the heat



contributed by the enthalpy of the hot blast and by the integral utilization by burning of the  $O_2$  quantity contained in the hot blast, and on the other side the quantities of heat necessitated by the evolution of the thermal process in the blast furnace, including the enthalpy of the final products (pig iron and slag) and the heat losses.

For the control of the quantity of heat available at the lower part of the blast furnace, the computer puts at the disposal of the automatic control equipment of the blast furnace, three prescribed values (temperature of the hot blast, humidity of the hot blast and the quantity of the blown in methane gas) on the basis of the data calculated from the thermal balance and of the results obtained in the last three charges, by introducing in the equations of the deviations of the silicon contents in the slag as compared to the prescribed values.

#### 4. Automatic control of the speed of the sintering strand

It is well known that the sintering process evolves in a progressive way in vertical direction, due to the air which penetrates through the material layer in all the length of the sintering strand, from the burner up to evacuating end.

The experience has shown that of great importance in the evolution of the process is the conduct of the burning process along the sintering strand so that it should develop regularly and be finished at the evacuating end of the strand, because in that case the best utilization index of the installation is obtained.

Because of the fact that it is not possible to ensure permanently constant characteristics of the material on the strand (humidity, granulation, chemical analysis etc.), a permanent variation results of the place where the sintering process may be finished.

In order to avoid this inconvenience, the idea

of the variation of the speed of the s i n t e r i n g strand has been adopted, which should automatically follow the variations of the physical and chemical conditions of the material on the strand, so that the place of the ending of the sintering process should be kept constant at the very evacuating end of the strand.

This thing is perfectly achievable if as leading element is taken the temperature of the material on the strand, which is proportional to the temperature of the flue gases which can be easily measured.



It is experimentally proved that when the sintering process is optimal (when it is ready at the evacuating end of the strand), the curve of the temperature of the flue gases has a maximum value opposite the last but one aspiration chamber.

It has been possible to draw up a curve of the temperatures in the aspiration chambers and to establish the relations for the positioning of the maximum of this curve.

In fact, at the computer arrive continuously as standard signals, the values of the temperatures in the last aspiration chambers, and on this basis the computer determines the position of the maximum of the curve of the temperatures and the difference between this point and the optimal position deduced in an experimental way. This difference constitutes the perturbing error of the controller for the speed of the sintering strand so that the controller acts with the aim of making this error equal to zero.

In this way the computer creates the possibility of obtaining an optimal speed of the sintering strand and in the same time a growth of the general productivity of the sintering plant.

It has been mentioned above the way in which has been designed the installation for complex automation of the blast furnaces department of the Integrated Iron Works Galați for the stage of 2.5 million tons steel/year.

Because of the fact that at the time of the drawing up of this report, the computer was on the way of being installed, it follows that during the period IV quarter 1968 - 1st quarter 1969, it will be possible to verify in practice all the considerations which have stood at the basis of our project.

