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COMPOSITE ADAPTIVE SMC OF NONLINEAR BASE ISOLATED BUILDINGS WITH ACTUATOR DYNAMICS

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This paper addresses the problem of designing a robust active controller for nonlinear base isolated building structures in the presence of unknown seismic excitations, parametric uncertainties and actuator dynamics. A simple adaptation law is introduced to get available upper bounds for the unknown seismically excited nonlinearities. Adaptive composite output feedback sliding mode control schemes are proposed to drive the displacements of the base and structure to their zero equilibrium positions. A numerical simulation example is presented to illustrate the effectiveness of the proposed strategies to a ten-storey base isolated structure under the *El Centro* earthquake.

1. Introduction

One of the main objectives in the design of civil engineering structures is to keep the response of the structure within the limits defined by safety, service and human comfort conditions in the presence of seismic excitations. This objective can be achieved by applying traditional seismic design principles which assume that earthquakes act upon the structure across its fixed base, to assure partial dissipation of the induced energy. However, the plastic deformation of certain members can occur and, as a consequence, the structure is damaged to a certain degree. This disadvantage can be avoided by using passive control systems such as isolators (Kelly, 1986) to uncouple the structure from the seismic excitation by means of replaceable devices, placed between the structure and the foundation, capable of absorbing part of the energy induced by earthquakes and thus providing in certain circumstances a level of performance beyond the normal design requirements. The most important disadvantage of such systems is the dependence of their effectiveness on the frequency of the seismic excitation. Moreover, they cannot be applied in the case of tall or heavy structures, due to the size of the dynamic forces involved and to the risk of endangering the global stability of the structure.

In order to overcome the above problems, the idea of cooperatively combining both passive base isolators and active feedback controllers (applying forces to the base) has been increasingly considered in the last years (Dyke *et al.*, 1995; Kelly *et al.*, 1987). Since those active forces react to the absolute motion, they are able to

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supply an additional resistant scheme not attainable by purely passive means when the structure is under ground excitation. Within the above scheme, recent works have approached the problem of designing active controllers through the understanding of the interaction between the base isolation system and the structure by considering both as two coupled systems (Barbat *et al.*, 1995; Inaudi *et al.*, 1992; Luo *et al.*, 1995; 1996). In these works the objective was to ensure a form of stability of the overall system resulting in a significant reduction of the motion of both the structure and the base. Important issues of this approach are that the system parameters and the seismic excitation do not need to be known (only their bounds do) and the base isolation may behave nonlinearly, thus resulting in a robust control scheme. This paper goes in the same direction but it takes into account a new element: the actuator dynamics. The importance of the interaction of the actuator producing the active forces with the structure has been recognized in the literature and ways to account for it have been proposed (Ghaboussi and Joghataie, 1995; Nikzad *et al.*, 1996).

This paper addresses the problem of designing a robust active controller for a class of nonlinear base isolated buildings in the presence of unknown seismic excitation, parametric uncertainties and actuator dynamics. Composite output feedback SMC (sliding mode control) schemes are proposed by using some simple adaptation laws for the upper bounds of the unknown seismically excitated nonlinearities. The paper is organized as follows. In Section 2, the control problem is formulated and some preliminary results on the stability of the base isolated structure subsystem are given. In Section 3, design methods for composite adaptive output feedback SMC are proposed. Conditions for the generation of sliding motion and the asymptotic stability of the closed-loop system are given. In Section 4, a numerical simulation example is given for a 10-storey base isolated structure under the *El Centro* earthquake to show the effectiveness of the proposed control scheme. Finally, some conclusions end the paper.

2. Problem Formulation

Consider an active controlled nonlinear base isolated structure with a hydraulic actuator, as shown in Fig. 1, whose dynamic behavior is described by the following model composed of three coupled subsystems:

- Main Structure

$$\boldsymbol{M}\ddot{\boldsymbol{q}}_r(t) + \boldsymbol{C}\dot{\boldsymbol{q}}_r(t) + \boldsymbol{K}\boldsymbol{q}_r(t) = \boldsymbol{\rho}(q_c, \dot{q}_c) \tag{1a}$$

$$\boldsymbol{\rho}(q_c, \dot{q}_c) =: \boldsymbol{\rho}_1\dot{q}_c(t) + \boldsymbol{\rho}_2 q_c(t), \quad \boldsymbol{\rho}_1 = [c_1, 0, \ldots, 0]^T, \quad \boldsymbol{\rho}_2 = [k_1, 0, \ldots, 0]^T \tag{1b}$$

- Base Isolation

$$m_0\ddot{q}_c(t) + (c_0+c_1)\dot{q}_c(t) + (k_0+k_1)q_c(t) - c_1\dot{q}_{r1}(t) - k_1 q_{r1}(t) + f(q_c, \dot{q}_c, d, \dot{d}) = v(t) \tag{2a}$$

$$f(q_c, \dot{q}_c, d, \dot{d}) =: -c_0\dot{d}(t) - k_0 d(t) + f_N(q_c, \dot{q}_c, d, \dot{d}) \tag{2b}$$

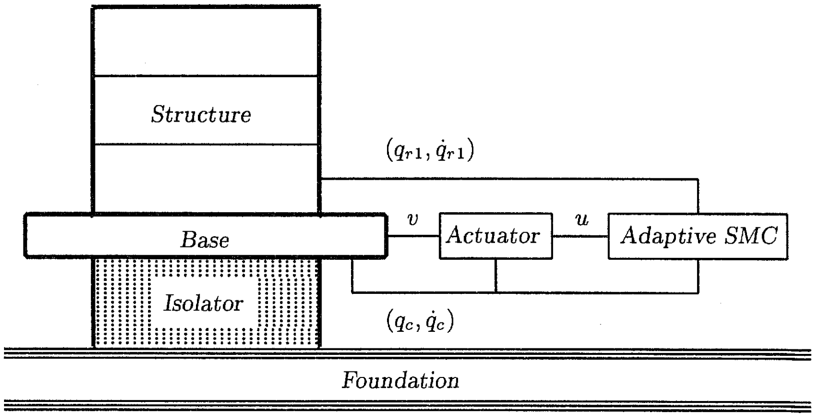

Fig. 1. Base isolated structure with active control.

- Hydraulic Actuator (Ghaboussi and Joghataie, 1995; Nikzad *et al.*, 1996)

$$P_v \dot{v}(t) + P_l v(t) + P_a \dot{q}_c(t) = u(t), \quad P_v =: \frac{C_v}{4\beta P_a} > 0, \quad P_l =: \frac{C_l}{P_a} > 0, \quad P_a > 0 \quad (3)$$

where $\boldsymbol{q}_r(t) = [\,q_{r1}(t)\,,\, q_{r2}(t)\,,\,\cdots\,,\, q_{rn}(t)\,]^T \in \mathbb{R}^n$ with $q_{ri}(t)$ $(i = 1, 2, \ldots, n)$ being the horizontal displacement of the i-th floor and $q_c(t) \in \mathbb{R}$ represents the horizontal displacement of the base with respect to an inertial frame, which can be measured by using some recently developed technique (Ida *et al.*, 1996). The coupling effect between the main structure subsystem (1) and the base isolation subsystem (2) is described by a vector function $\boldsymbol{\rho}(q_c, \dot{q}_c) \in \mathbb{R}^n$, as defined in eqn. (1b). Scalars m_0, c_0 and k_0 are the mass, damping and stiffness of the structural base, respectively. $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K} \in \mathbb{R}^{n \times n}$ are positive definite matrices corresponding to the mass, damping and stiffness of the main structure and are of the following form:

$$\boldsymbol{M} = \operatorname{diag}(m_i), \qquad i = 1, 2, \ldots, n \quad (4)$$

$$\boldsymbol{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 \\ -c_2 & c_2 + c_3 & -c_3 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & -c_n & c_n \end{bmatrix}, \qquad \boldsymbol{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & -k_n & k_n \end{bmatrix} \quad (5)$$

where m_i, c_i, k_i $(i = 0, \ldots, n)$ are unknown positive constant values with known upper bounds $\bar{m}_i$, $\bar{c}_i$, $\bar{k}_i$ $(i = 0, \ldots, n)$, respectively. Equation (3) represents the internal dynamics of a hydraulic actuator's chamber, with $v(t)$ being the average output actuator force, $u(t)$ the total fluid flow rate of the actuator's chamber, P_a the actuator effective piston's area, C_v the chamber's volume, β the bulk modulus of the hydraulic fluid and C_l the coefficient of leakage. For simplicity, in the subsequent sections $v(t)$ and $u(t)$ will be called the *actuator control force* and *actuator command control*, respectively. The seismic excitation is characterized by a displacement function $d(t)$ and its velocity $\dot{d}(t)$, which are assumed to be uniformly bounded for all $t \geq 0$. The scalar function $f_N(q_c, \dot{q}_c, d, \dot{d}) \in \mathbb{R}$ represents an additional horizontal force produced on the structural base by nonlinearities of the isolator.

Assumption 1. $f(q_c, \dot{q}_c, d, \dot{d}) \in \mathbb{R}$ is an unknown scalar function such that the following relationship holds:

$$\begin{cases} |f(q_c, \dot{q}_c, d, \dot{d})| \leq \eta_0 + \eta_1 \|z(t)\| \\ z(t) =: \left[q_c(t),\ \dot{q}_c(t)\right]^T \\ \|z(t)\| =: \left[q_c^2(t) + \dot{q}_c^2(t)\right]^{1/2} \end{cases} \tag{6}$$

with η_0 and η_1 being some unknown non-negative constants. The following results on the stability of the main structure subsystem are obtained.

Proposition 1. *The unforced main structure subsystem (1) (i.e., with zero coupling term $\rho(q_c, \dot{q}_c)$) is globally exponentially stable, provided that $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are positive definite matrices, as defined in eqns. (4) and (5).*

Proof. It follows from the assertion that no eigenvalues associated with a system mode can have positive real part. First, taking Laplace transforms with zero initial conditions in the unforced main structure subsystem (1), we obtain

$$(s^2\boldsymbol{M} + s\boldsymbol{C} + \boldsymbol{K})\boldsymbol{q}_r(s) = \boldsymbol{0} \tag{7}$$

where s is the Laplace operator. Consider any eigenmode $s_0 = \lambda + j\omega$. For any nonzero solution $\boldsymbol{q}_r(s)$ in (7), $\det(s^2\boldsymbol{M} + s\boldsymbol{C} + \boldsymbol{K}) = 0$ so that

$$\begin{aligned} &\operatorname{rank}\left[(\lambda^2 - \omega^2 + 2j\omega\lambda)\boldsymbol{M} + (\lambda + j\omega)\boldsymbol{C} + \boldsymbol{K}\right] \\ &\qquad = \operatorname{rank}\left[(\lambda^2 - \omega^2)\boldsymbol{M} + \lambda\boldsymbol{C} + \boldsymbol{K} + j\omega(2\lambda\boldsymbol{M} + \boldsymbol{C})\right] < n \end{aligned} \tag{8}$$

Thus the ranks associated with the real and imaginary parts in (8) should be less than n, i.e.,

$$\operatorname{rank}\left[(\lambda^2 - \omega^2)\boldsymbol{M} + \lambda\boldsymbol{C} + \boldsymbol{K}\right] < n \tag{9}$$

$$\operatorname{rank}\left[\omega(2\lambda\boldsymbol{M} + \boldsymbol{C})\right] < n \tag{10}$$

Consider two cases. Let $\omega \neq 0$ (i.e., $s_0 = \lambda + j\omega$). Then (10) implies $\lambda < 0$, since $\operatorname{rank}[\omega(2\lambda\boldsymbol{M} + \boldsymbol{C})] = n$ for all $\lambda \geq 0$, which contradicts (10) with $\boldsymbol{M}$ and $\boldsymbol{C}$ being positive definite matrices. For $\omega = 0$ (i.e., $s_0 = \lambda$) the same reasoning can be used with (9) to yield $\lambda < 0$ since $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are positive definite matrices. ■

For the forced main structure subsystem (1), the next stability result can be proved by using Proposition 1 about the exponential stability of the unforced main structure subsystem.

Proposition 2. *If the coupling term $\rho(q_c, \dot{q}_c)$ is uniformly bounded for all $t \geq 0$, then the main structure subsystem is stable and the state variables $\boldsymbol{q}_r(t)$ and $\dot{\boldsymbol{q}}_r(t)$ of the structure are uniformly bounded for all $t \geq 0$ and any bounded initial conditions.*

3. Main Results

The objective of robust control design is to drive the state variables $q_c(t)$, $\dot{q}_c(t)$ of the base and $\boldsymbol{q}_r(t)$, $\dot{\boldsymbol{q}}_r(t)$ of the structure to their zero equilibrium positions in the presence of unknown seismic excitations, parametric uncertainties and actuator dynamics. In the control design, the base isolated structure with actuator dynamics (eqns. (1)–(3)) is regarded as being formed by two cascade loops: the structure loop (i.e., the main structure subsystem (1) plus the base isolation subsystem (2)) and the actuator loop (i.e., the actuator subsystem (3)). Unlike the dynamic model without actuator dynamics, the virtual actuator control force $v(t)$ in eqn. (2) cannot be synthesized directly. Instead, $v(t)$ is the output of a first order actuator loop (eqn. (3)). In this paper, the control design is organized as a two-step procedure in accordance with the composite control strategy. First, $v(t)$ is regarded in the structure loop as a control variable for the subsystems (1)–(2) and a "desired" actuator control force $v_d(t)$ will be designed to make the base isolated structure without actuator dynamics to be asymptotically stable. Second, a robust command controller $u(t)$ will be designed in the actuator loop such that the "real" actuator control force $v(t)$ tracks asymptotically the "desired" actuator control force $v_d(t)$ and thus the global asymptotic stability is achieved in the base isolated structure with actuator dynamics. The overall closed-loop control system is shown in Fig. 2.

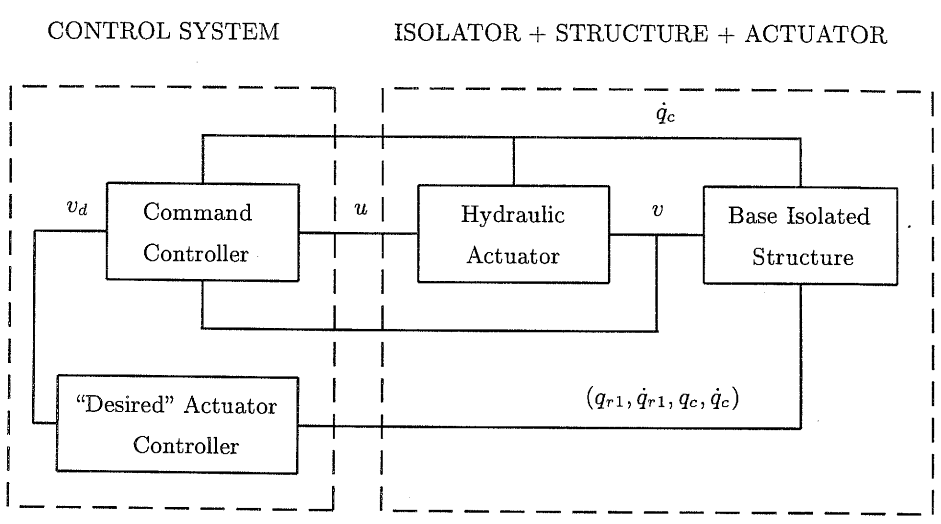

Fig. 2. Overall closed-loop control system.

3.1. Design of the "Desired" Actuator Control Force $v_d(t)$

Now, a "desired" actuator control force $v_d(t)$ is designed such that the control objective is achieved for the base isolated structure without actuator dynamics. A robust control scheme based on the sliding mode principle (Utkin, 1992) is chosen to drive the state variables $q_c(t)$ and $\dot{q}_c(t)$ of the base to zero so that the coupling term $\rho(q_c, \dot{q}_c)$ between the main structure subsystem (1) and the base isolation subsystem (2) vanishes asymptotically. Thus the state variables $\boldsymbol{q}_r(t)$ and $\dot{\boldsymbol{q}}_r(t)$ of the structure are uniformly bounded for all $t \geq 0$ and any bounded initial conditions according to Proposition 2.

The first step of the control design is to define a sliding function $\sigma_c(t) \in \mathbb{R}$ for the base isolation subsystem (2):

$$\sigma_c(t) = \dot{q}_c(t) + \delta q_c(t), \qquad t \geq 0 \tag{11}$$

where δ is a scalar to be chosen to guarantee the closed-loop stability of the base isolation subsystem (2). Suppose that a sliding motion is generated in the base isolation subsystem (2) at the time instant t_s. Then we have the following relationship:

$$\sigma_c(t) = \dot{q}_c(t) + \delta q_c(t) = 0, \qquad t \geq t_s \tag{12}$$

The equation of the motion of the base isolation subsystem in the sliding mode can be obtained by using the technique of equivalent control (Utkin, 1992). From eqns. (2a) and (11), an equivalent actuator control force $v_{eq}(t)$ is found as follows:

$$\begin{aligned} v_{eq}(t) &= (-\delta m_0 + c_0 + c_1)\dot{q}_c(t) + (k_0 + k_1)q_c(t) - c_1\dot{q}_{r1}(t) \\ &\quad - k_1 q_{r1}(t) - c_0\dot{d}(t) - k_0 d(t) + f(q_c, \dot{q}_c, d, \dot{d}), \qquad t \geq t_s \end{aligned} \tag{13}$$

The sustitution of $v(t) = v_{eq}(t)$ from (13) into (2a) leads to the following closed-loop state equation of the base isolation subsystem:

$$\ddot{q}_c(t) + \delta\dot{q}_c(t) = 0, \qquad t \geq t_s \tag{14}$$

From (12) and (14), one gets

$$q_c(t) = q_c(t_s)e^{-\delta(t-t_s)}, \quad \dot{q}_c(t) = -\delta q_c(t_s)e^{-\delta(t-t_s)}, \qquad t \geq t_s \tag{15}$$

Then it is known from (15) that by choosing $\delta > 0$ the base isolation subsystem (2) is exponentially stable when a sliding motion is generated. Moreover, the state variables $q_c(t)$ and $\dot{q}_c(t)$ are uniformly bounded for all $t \geq t_s \geq 0$ and converge exponentially to zero as $t \to \infty$.

Now, a "desired" actuator controller is designed for the generation of a sliding motion in the base isolation subsystem (2). Note that if the upper bounds η_0 and η_1 related to the unknown seismically excited nonlinearity $f(q_c, \dot{q}_c, d, \dot{d})$ were known *a priori*, the following output feedback SMC law could be chosen for the "desired" actuator control force $v_d(t)$ by using $q_c(t)$, $\dot{q}_c(t)$, $q_{r_1}(t)$ and $\dot{q}_{r_1}(t)$:

$$v_d(t) = -\Big[\eta_0 + \eta_1\big\|z(t)\big\| + \eta_2\big\|\boldsymbol{x}(t)\big\|\Big]\,\mathrm{sgn}[\sigma_c(t)] \tag{16}$$

where

$$\begin{cases} x(t) = \left[q_c(t),\ \dot{q}_c(t),\ q_{r1}(t),\ \dot{q}_{r1}(t)\right]^T \\ \|x(t)\| =: \left[q_c^2(t) + \dot{q}_c^2(t) + q_{r1}^2(t) + \dot{q}_{r1}^2(t)\right]^{1/2} \end{cases} \tag{17}$$

$$\begin{aligned} \eta_2 \geq \max \Big[& k_0 + 2k_1 + c_1 + |c_0 + c_1 - \delta m_0| : \\ & k_i \in [0, \bar{k}_i],\ c_i \in [0, \bar{c}_i],\ m_0 \in [0, \bar{m}_0],\ i = 0, 1 \Big] \end{aligned} \tag{18}$$

and

$$\operatorname{sgn}(\cdot) = \begin{cases} 1 & \text{if } (\cdot) > 0 \\ 0 & \text{if } (\cdot) = 0 \\ -1 & \text{if } (\cdot) < 0 \end{cases} \tag{19}$$

with η_0, η_1, $z(t)$ and $\|z(t)\|$ defined by eqn. (6). Note that the implementation of the SMC law (16) requires the knowledge of the upper bounds η_0 and η_1 related to the unknown seismically excited nonlinearity $f(q_c, \dot{q}_c, d, \dot{d})$, which may not be easily obtained in practice. In order to avoid such a requirement, some simple adaptation laws are proposed below to obtain upper bounds for the unknown scalars η_0 and η_1 and then these adaptive upper bounds are used in the following SMC law:

$$v_d(t) = -\lambda\sigma_c(t) - \left[\bar{\eta}_0(t) + \bar{\eta}_1(t)\,\|z(t)\| + \eta_2\,\|x(t)\|\right] \operatorname{sgn}\left[\sigma_c(t)\right] \tag{20}$$

where λ is a positive scalar; $\bar{\eta}_0(t)$ and $\bar{\eta}_1(t)$ are adaptive upper bounds for the unknown scalars η_0 and η_1. Note that the "desired" actuator control force $v_d(t)$ is uniformly bounded for all $t \geq 0$ since the uniform boundedness of the equivalent control ensures that of the corresponding SMC. Define

$$\tilde{\eta}_0(t) =: \bar{\eta}_0(t) - \eta_0, \qquad \tilde{\eta}_1(t) =: \bar{\eta}_1(t) - \eta_1 \tag{21}$$

as the parameter adaptation errors and choose the following simple adaptation laws to get upper bounds for the unknown scalars η_0 and η_1 such that

$$\dot{\tilde{\eta}}_0(t) = h_0|\sigma_c(t)|, \qquad \dot{\tilde{\eta}}_1(t) = h_1|\sigma_c(t)|\,\|z(t)\| \tag{22}$$

where $h_0 > 0$ and $h_1 > 0$ are adaptation gains. Since η_0 and η_1 are positive constants, one has $\dot{\tilde{\eta}}_0(t) = \dot{\bar{\eta}}_0(t)$ and $\dot{\tilde{\eta}}_1(t) = \dot{\bar{\eta}}_1(t)$. Thus the above parameter adaptation laws can be written as

$$\dot{\bar{\eta}}_0(t) = h_0|\sigma_c(t)|, \qquad \dot{\bar{\eta}}_1(t) = h_1|\sigma_c(t)|\,\|z(t)\| \tag{23}$$

Then the adaptive parameters $\bar{\eta}_0(t)$ and $\bar{\eta}_1(t)$ can be obtained by integrating (23). The rate of parameter adaptation can be adjusted by choosing properly the initial values of $\bar{\eta}_i(0)$ and h_i $(i = 0, 1)$.

Theorem 1. *Under Assumption 1 and by using adaptive SMC laws (20) and (23) with $\lambda > 1/\delta$, a sliding motion is asymptotically generated in the base isolation subsystem (2) and the state variables $q_c(t)$ and $\dot{q}_c(t)$ of the base are uniformly bounded for all $t \geq 0$ and any bounded initial conditions. They converge asymptotically to zero as $t \to \infty$.*

Proof. Consider the following Lyapunov function candidate:

$$V_1(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1) = \frac{1}{2} m_0 \sigma_c^2(t) + 2q_c^2(t) + \frac{1}{2} h_0^{-1} \tilde{\eta}_0^2(t) + \frac{1}{2} h_1^{-1} \tilde{\eta}_1^2(t) \quad (24)$$

Differentiating $V_1(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1)$ with respect to time t yields

$$\begin{aligned} \dot{V}_1(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1) &= m_0 \sigma_c(t) \dot{\sigma}_c(t) + 4q_c(t) \dot{q}_c(t) \\ &\quad + h_0^{-1} \tilde{\eta}_0(t) \dot{\tilde{\eta}}_0(t) + h_1^{-1} \tilde{\eta}_1(t) \dot{\tilde{\eta}}_1(t) \end{aligned} \quad (25)$$

Note from (11) that

$$\dot{q}_c(t) = \sigma_c(t) - \delta q_c(t) \quad (26)$$

$$\dot{\sigma}_c(t) = \ddot{q}_c(t) + \delta \dot{q}_c(t) \quad (27)$$

According to (2), (6), (18), (20)–(21) and (27), one gets

$$\begin{aligned} m_0 \sigma_c(t) \dot{\sigma}_c(t) &= -\lambda \sigma_c^2(t) - \sigma_c(t) \Big[\hat{\eta}_0(t) + \hat{\eta}_1(t) \|z(t)\| + \eta_2 \|x(t)\| \Big] \operatorname{sgn}[\sigma_c(t)] \\ &\quad + \sigma_c(t) \Big[(-\delta m_0 + c_0 + c_1) \dot{q}_c(t) + (k_0 + k_1)) q_c(t) \\ &\quad - c_1 \dot{q}_{r1}(t) - k_1 q_{r1}(t) - c_0 \dot{d}(t) - k_0 d(t) + f(q_c, \dot{q}_c, d, \dot{d}) \Big] \\ &\leq -\lambda \sigma_c^2(t) - \Big[\hat{\eta}_0(t) + \hat{\eta}_1(t) \|z(t)\| + \eta_2 \|x(t)\| \Big] |\sigma_c(t)| \\ &\quad + \Big[\eta_0 + \eta_1 \|z(t)\| + \eta_2 \|x(t)\| \Big] |\sigma_c(t)| \\ &= -\lambda \sigma_c^2(t) - \Big[\tilde{\eta}_0(t) + \tilde{\eta}_1(t) \|z(t)\| \Big] |\sigma_c(t)| \end{aligned} \quad (28)$$

Substitution of (26) and (28) into (25) leads to

$$\begin{aligned} \dot{V}_1(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1) &\leq -\lambda \sigma_c^2(t) + 4\sigma_c(t) q_c(t) - 4\delta q_c^2(t) \\ &\quad + \tilde{\eta}_0(t) h_0^{-1} \Big[\dot{\tilde{\eta}}_0(t) - h_0 |\sigma_c(t)| \Big] \\ &\quad + \tilde{\eta}_1(t) h_1^{-1} \Big[\dot{\tilde{\eta}}_1(t) - h_1 |\sigma_c(t)| \, \|z(t)\| \Big] \end{aligned} \quad (29)$$

Taking (23) (i.e., (22)) as the adaptation laws and noticing that $\lambda > 1/\delta$, we get

$$\dot{V}_1(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1) \leq -\lambda \sigma_c^2(t) + 4\sigma_c(t) q_c(t) - 4\delta q_c^2(t) < 0 \quad (30)$$

for all $[\sigma_c(t), q_c(t)]^T \neq \mathbf{0}$ and any $\tilde{\eta}_0(t)$ and $\tilde{\eta}_1(t)$. Thus eqn. (30) is negative semidefinite in the $(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1)$ space and the global stability of the origin is implied. Therefore, a sliding motion is asymptotically generated in the base isolation subsystem (2). Note that $V_1(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1)$ is uniformly bounded, so the state variables $q_c(t)$ and $\dot{q}_c(t)$ are uniformly bounded for all $t \geq 0$ and any bounded initial conditions. Since $V_1(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1) \to 0$ as $t \to \infty$, we have $\sigma_c(t) \to 0$ and $q_c(t) \to 0$ as $t \to \infty$, which implies that $\dot{q}_c(t) \to 0$ as $t \to \infty$ from (26). ■

3.2. Design of the Command Control $u(t)$

Now, a command control law $u(t)$ is designed for the base isolated structure with actuator dynamics such that the "real" actuator control force $v(t)$ tracks asymptotically the "desired" actuator control force $v_d(t)$ obtained in the previous section. Thus the global stability is achieved in the overall base isolated structure with actuator dynamics. The following two cases will be studied:

- The parameters of the actuator subsystem are known constants, and
- The parameters of the actuator subsystem are unknown constants but with known upper bounds.

3.2.1. Actuator with Known Parameters

Let

$$\tilde{v}(t) =: v(t) - v_d(t) \tag{31}$$

Suppose that the actuator parameters P_v, P_l and P_a are known positive constants and the absolute accelerations $\ddot{q}_c(t)$ of the base and $\ddot{q}_{r_1}(t)$ of the first floor are measurable. Then the following command control law is proposed:

$$u(t) = P_v \dot{v}_d(t) + P_l v_d(t) + P_a \dot{q}_c(t) \tag{32}$$

with $v_d(t)$ defined by eqns. (20) and (23).

Remark 1. Note that the control law (32) contains impulses at the time instants when $\sigma_c(t)$ changes its sign from (20). However, since $\sigma_c(t)$ is continuous from (11), its sign only changes after finite time intervals. As a result, there is no accumulative point (time instant) of impulses. Note that, in general, isolated control impulses are admissible by two reasons, namely, (a) from a conceptual and theoretical point of view, they only contribute to the system output with finite isolated discontinuities from the properties of Dirac delta, which describes impulses and is used for computing derivatives of discontinuous function (Simeonov and Bainov, 1985), and (b) from a practical point of view, impulses are implemented via control actions with bounded magnitude and very short duration. In active control of structures this kind of bounded impulse control has been considered and tested in experimental settings (Miller *et al.*, 1988). Furthermore, note that the real control law (42) generated (see Section 3.3) is approximately impulsive and does not exhibit any chattering since sgn$[\sigma_c(t)]$ is replaced with $\sigma_c(t)/(\sigma_c(t)+\epsilon)$, which is a very common approach used in variable structure control systems. Also, it is common in practice to generate real impulses as derivatives of step functions even on analogue computers.

A direct extension of Theorem 1 for the command control law (32) is given below.

Theorem 2. *By applying the command control law (32) to the base isolated structure with actuator dynamics (eqns. (1)–(3)) and choosing $\lambda > 1/\delta$ in (20), a sliding motion is asymptotically generated in the base isolation subsystem (2) and the state variables $q_c(t)$ and $\dot{q}_c(t)$ of the base are uniformly bounded for all $t \geq 0$ and any bounded initial conditions. They converge asymptotically to zero as $t \to \infty$.*

Proof. Consider the Lyapunov function candidate

$$V(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1, \tilde{v}) =: V_1(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1) + V_2(\tilde{v}), \qquad V_2(\tilde{v}) =: \frac{1}{2}P_v\tilde{v}^2(t) \tag{33}$$

where $V_1(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1)$ is defined by eqn. (24). By differentiating $V_2(\tilde{v})$ with respect to time and using (3) and (31)–(32), one has

$$\begin{aligned}
\dot{V}_2(\tilde{v}) &= \Big[P_v\dot{v}(t) - P_v\dot{v}_d(t)\Big]\tilde{v}(t) \\
&= \Big[-P_lv(t) - P_a\dot{q}_c(t) + u(t) - P_v\dot{v}_d(t)\Big]\tilde{v}(t) \\
&= \Big[-P_lv(t) - P_a\dot{q}_c(t) + P_v\dot{v}_d(t) + P_lv_d(t) + P_a\dot{q}_c(t) - P_v\dot{v}_d(t)\Big]\tilde{v}(t) \\
&= -P_l\tilde{v}^2(t)
\end{aligned} \tag{34}$$

Thus, using (30) and (34), we get

$$\begin{aligned}
\dot{V}(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1, \tilde{v}) &\leq -\lambda\sigma_c^2(t) + 4\sigma_c(t)q_c(t) - 4\delta q_c^2(t) - P_l\tilde{v}^2(t) \\
&\leq -P_l\tilde{v}^2(t) < 0
\end{aligned} \tag{35}$$

for all $[\sigma_c(t), q_c(t), \tilde{v}(t)]^T \neq \mathbf{0}$ and any $\tilde{\eta}_0(t)$ and $\tilde{\eta}_1(t)$ since $\lambda\delta > 1$. Thus eqn. (35) is negative semidefinite in the $(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1, \tilde{v})$ space and the global stability of the origin is implied. Here $\tilde{v}(t)$ converges asymptotically to zero as $t \to \infty$ and thus the "real" actuator control force $v(t)$ tracks asymptotically the "desired" actuator control force $v_d(t)$ such that the sliding motion is asymptotically generated in the base isolation subsystem (2). Note that $V(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1, \tilde{v})$ is uniformly bounded, so the state variables $q_c(t)$ and $\dot{q}_c(t)$ are uniformly bounded for all $t \geq 0$ and any bounded initial conditions. Since $V(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1, \tilde{v}) \to 0$ as $t \to \infty$, we have $\sigma_c(t) \to 0$ and $q_c(t) \to 0$ as $t \to \infty$, which implies that $\dot{q}_c(t) \to 0$ as $t \to \infty$ from (26). ■

The following result is a direct consequence of Theorem 2 and Proposition 2.

Corollary 1. *The state variables $\mathbf{q}_r(t)$ and $\dot{\mathbf{q}}_r(t)$ of the structure are uniformly bounded for all $t \geq 0$ and any bounded initial conditions.*

3.2.2. Actuator with Unknown Parameters

Now, suppose that the absolute accelerations $\ddot{q}_c(t)$ of the base and $\ddot{q}_{r_1}(t)$ of the first floor are measurable and the actuator parameters P_v, P_l and P_a are unknown positive constants which satisfy the following relationships:

$$|P_v - \overline{P}_v| \le \sigma_v, \quad |P_l - \overline{P}_l| \le \sigma_l, \quad |P_a - \overline{P}_a| \le \sigma_a \tag{36}$$

where $\overline{P}_v$, $\overline{P}_l$, $\overline{P}_a$, σ_v, σ_l and σ_a are some known positive constants. In this case, it is assumed that the "real" output actuator control force $v(t)$ is measurable. Then the following command control law is proposed:

$$\begin{aligned} u(t) &= \overline{P}_v \dot{v}_d(t) + \overline{P}_l v_d(t) + \overline{P}_a \dot{q}_c(t) \\ &\quad - \Big[\sigma_v|\dot{v}_d(t)| + \sigma_l|v_d(t)| + \sigma_a|\dot{q}_c(t)|\Big] \operatorname{sgn}\big[\tilde{v}(t)\big] \end{aligned} \tag{37}$$

with $v_d(t)$ and $\tilde{v}(t)$ defined by (20) and (31), respectively.

Theorem 3. *By applying the command control law (37) to the base isolated structure with actuator dynamics (eqns. (1)–(3)) and choosing $\lambda > 1/\delta$ in (20), the sliding motion is asymptotically generated in the base isolation subsystem (2) and the state variables $q_c(t)$ and $\dot{q}_c(t)$ of the base are uniformly bounded for all $t \ge 0$ and any bounded initial conditions. They converge asymptotically to zero as $t \to \infty$.*

Proof. Consider the Lyapunov function candidate (33). Differentiating $V_2(\tilde{v})$ with respect to time and using (3), (31), (36) and (37), we have

$$\begin{aligned} \dot{V}_2(\tilde{v}) &= -P_l\tilde{v}^2(t) + \Big[(\overline{P}_v - P_v)\dot{v}_d(t) + (\overline{P}_l - P_l)v_d(t) + (\overline{P}_a - P_a)\dot{q}_c(t)\Big]\tilde{v}(t) \\ &\quad - \Big[\sigma_v|\dot{v}_d(t)| + \sigma_l|v_d(t)| + \sigma_a|\dot{q}_c(t)|\Big]|\tilde{v}| \le -P_l\tilde{v}^2(t) \end{aligned} \tag{38}$$

Thus, using (30) and (38), we get

$$\begin{aligned} \dot{V}(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1, \tilde{v}) &\le -\Lambda\sigma_c^2(t) + 4\sigma_c(t)q_c(t) - 4\delta q_c^2(t) - P_l\tilde{v}^2(t) \\ &\le -P_l\tilde{v}^2(t) < 0 \end{aligned} \tag{39}$$

for all $[\sigma_c(t), q_c(t), \tilde{v}(t)]^T \neq \mathbf{0}$ and any $\tilde{\eta}_0(t)$ and $\tilde{\eta}_1(t)$ since $\lambda\delta > 1$. Thus eqn. (39) is negative semidefinite in the $(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1, \tilde{v})$ space and the global stability of the origin is implied. Here $\tilde{v}(t)$ converges asymptotically to zero as $t \to \infty$ and thus the "real" actuator control force $v(t)$ tracks asymptotically the "desired" actuator control force $v_d(t)$ such that the sliding motion is asymptotically generated in the base isolation subsystem (2). Note that $V(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1, \tilde{v})$ is uniformly bounded, so the state variables $q_c(t)$ and $\dot{q}_c(t)$ are uniformly bounded for all $t \ge 0$ and any bounded initial conditions. Since $V(\sigma_c, q_c, \tilde{\eta}_0, \tilde{\eta}_1, \tilde{v}) \to 0$ as $t \to \infty$, we have $\sigma_c(t) \to 0$ and $q_c(t) \to 0$ as $t \to \infty$, which implies that $\dot{q}_c(t) \to 0$ as $t \to \infty$ from (26). ■

The following result is a direct consequence of Theorem 3 and Proposition 2.

Corollary 2. *The state variables $\boldsymbol{q}_r(t)$ and $\dot{\boldsymbol{q}}_r(t)$ of the structure are uniformly bounded for all $t \ge 0$ and any bounded initial conditions.*

3.3. Continuous SMC Schemes

In the implementation of discontinuous SMC laws (20) or (37), non-ideal effects may cause chattering and high control activity which can sometimes excite high-frequency unmodelled dynamics in the system. In order to alleviate the chattering phenomenon, continuous SMC laws can be obtained by making the following substitution in the corresponding discontinuous ones:

$$\text{sgn}\left[\sigma_c(\cdot)\right] \longleftrightarrow \frac{\sigma_c(\cdot)}{|\sigma_c(\cdot)| + \epsilon_1} \tag{40}$$

$$\text{sgn}\left[\tilde{v}(\cdot)\right] \longleftrightarrow \frac{\tilde{v}(\cdot)}{|\tilde{v}(\cdot)| + \epsilon_2} \tag{41}$$

with ϵ_1 and ϵ_2 being a small positive constants. Thus the control laws (20) and (37) become

$$v_d(t) = -\lambda\sigma_c(t) - \left[\bar{\eta}_0(t) + \bar{\eta}_1(t)\|z(t)\| + \eta_2\|x(t)\|\right]\frac{\sigma_c(t)}{|\sigma_c(t)| + \epsilon_1} \tag{42}$$

$$\begin{aligned} u(t) &= \overline{P}_v\dot{v}_d(t) + \overline{P}_l v_d(t) + \overline{P}_a\dot{q}_c(t) \\ &\quad - \left[\sigma_v|\dot{v}_d(t)| + \sigma_l|v_d(t)| + \sigma_a|\dot{q}_c(t)|\right]\frac{\tilde{v}(t)}{|\tilde{v}(t)| + \epsilon_2} \end{aligned} \tag{43}$$

4. Numerical Example

Consider a ten-storey base isolated building structure described by eqns. (1)–(3). The mass of each floor, including that of the base, is 6.0×10^5 kg. The stiffness of the base is 1.184×10^7 N/m and its damping ratio is 0.1. The stiffness of the structure varies in 5.0×10^7 N/m between floors, from 9.0×10^8 N/m on the first one to 4.5×10^8 N/m on the top one with the damping ratio 0.05. The actuator dynamics is described by eqn. (3) with $C_v = 1.518 \times 10^{-3}\,\text{m}^3$, $\beta = 2.1 \times 10^3\,\text{N/m}^2$, $C_l = 1.0 \times 10^{-6}\,\text{m}^5/\text{Ns}$ and $P_a = 5.06 \times 10^{-2}\,\text{m}^2$. A frictional device is used for the base isolation, so that the nonlinear force $f_N(q_c, \dot{q}_c, d, \dot{d})$ is described by the following equation:

$$f_N(q_c, \dot{q}_c, d, v) = -\,\text{sgn}\left[q_c(t) - d(t)\right]\left\{\mu_{\max} - \Delta\mu e^{-\nu|\dot{q}_c(t) - \dot{d}(t)|}\right\}Q \tag{44}$$

where Q is the force normal to the friction surface, $\nu = 2.0$, μ stands for the friction coefficient, $\mu_{\max} = 0.185$ is the coefficient for a high sliding velocity, $\Delta\mu = 0.09$ denotes the difference between $\mu_{\max}$ and the friction coefficient for a low sliding velocity. In the simulation, the seismic excitation has been that of the *El Centro* (1940) earthquake (Kelly *et al.*, 1987) as shown in Fig. 7. Then we have the following relationship:

$$|f(q_c, \dot{q}_c, d, \dot{d})| = |-c_0\dot{d}(t) - k_0 d(t) + f_N(q_c, \dot{q}_c, d, \dot{d})| \le \eta_0 \tag{45}$$

where η_0 is an unknown constant. The composite adaptive SMC laws (42), (23) and (32) are used with $\delta = 387.3$, $\Lambda = 1.0$, $\eta_0(0) = 1.0 \times 10^6$, $h_0 = 10.0$, $\eta_1(0) = h_1 = 0$, $\eta_2 = 1.0 \times 10^9$, $\epsilon_1 = 0.1$, $P_v = 3.57 \times 10^{-3}$, $P_l = 1.99 \times 10^{-5}$ and $P_a = 5.06 \times 10^{-2}$. Both the passive case (pure base isolation) and the hybrid case (base isolation plus composite adaptive SMC) are studied. In Figs. 3 to 6, the time histories of the absolute displacement of the base and the relative displacement of the top floor are shown. The actuator control force $v(t)$ is plotted in Fig. 8. The absolute acceleration of the base and top floor are shown in Figs. 9 and 10, respectively. It is seen from the simulation results that, by using the proposed composite adaptive SMC schemes, the absolute displacement of the base, the relative displacements and absolute accelerations of the structure have been significantly reduced and the steady dynamics of the absolute base acceleration has been improved as compared with the purely passive case. The supplied control force is of a reasonable magnitude when compared with the mass of base where it is applied.

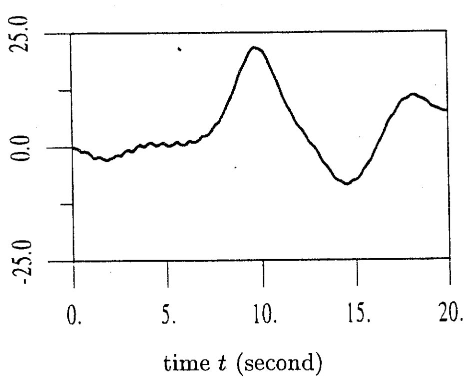

Fig. 3. Absolute base displacement [cm] (passive case).

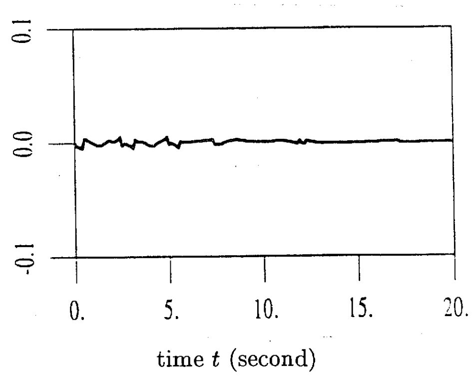

Fig. 4. Absolute base displacement [cm] (hybrid case).

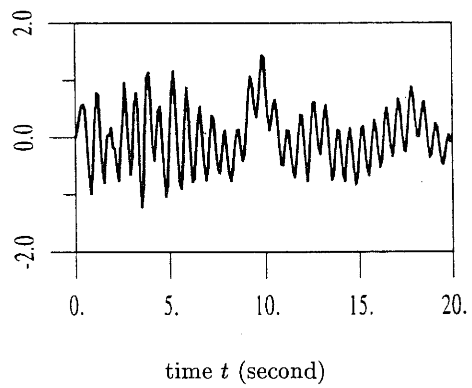

Fig. 5. Relative displacement of the top floor [cm] (passive case).

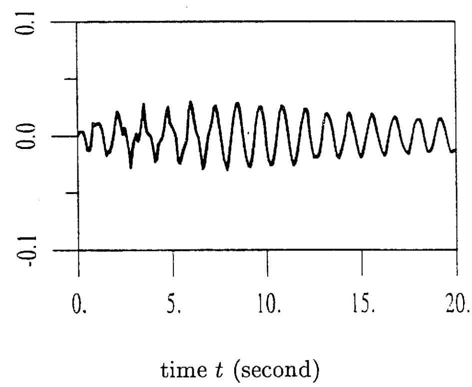

Fig. 6. Relative displacement of the top floor [cm] (hybrid case).

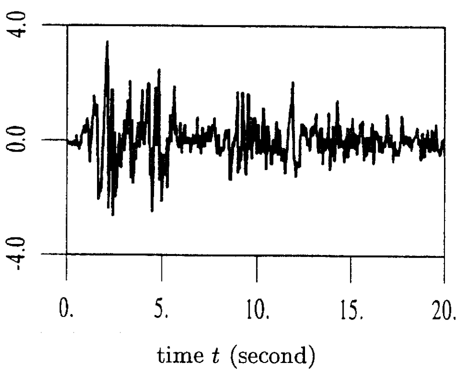

Fig. 7. *El Centro* earthquake ground acceleration [m/s^2].

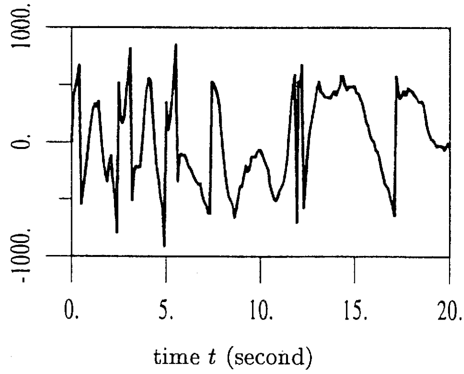

Fig. 8. Actuator control force [kN] (hybrid case).

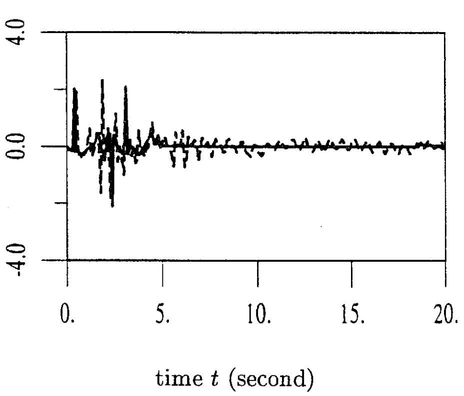

Fig. 9. Absolute acceleration of the base [m/s^2]: passive case (discontinuous line) and hybrid case (continuous line).

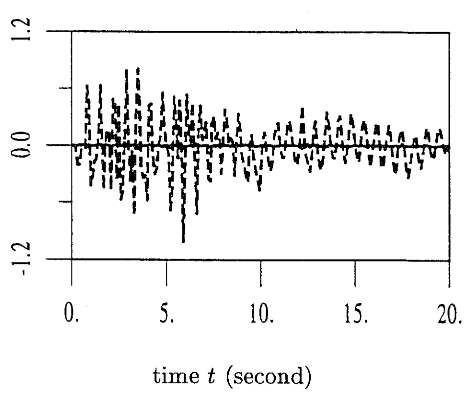

Fig. 10. Absolute acceleration of the top floor [m/s^2]: passive case (discontinuous line) and hybrid case (continuous line).

5. Conclusions

In this paper, composite adaptive output feedback sliding mode control schemes have been proposed for a class of nonlinear base isolated structures in the presence of unknown seismic excitations, parametric uncertainties and actuator dynamics. Only the information on the state variables of the base and the first floor has been used in the control design. It is shown by a numerical simulation for a ten-storey base isolated structure under the *El Centro* earthquake that the absolute displacement of the base, relative displacements and absolute accelerations of the structure under seismic excitations have been significantly reduced by using the proposed composite adaptive SMC schemes, as compared with the purely passive case.

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