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MUSIC AND MATHEMATICS IN PRIMARY EDUCATION. DEVELOPMENT OF KNOWLEDGE AND SKILLS CONCERNING MAGNITUDE FEATURES

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Keywords: primary education, early childhood education, music, mathematics.

The article aims to present the possibilities of developing knowledge and skills related to magnitude characteristics based on integrating music and mathematics in primary education. Early childhood education specialists agree that creative combinations of various issues are essential to support and stimulate harmonious development. The first part presents information on the relationship between music and mathematics, which was noticed already in antiquity. The following focuses on examples and possibilities of shaping mathematical concepts in quantitative features through various musical activities at individual stages.

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MUZYKA I MATEMATYKA W EDUKACJI WCZESNOSZKOLNEJ. ROZWÓJ WIEDZY I UMIEJĘTNOŚCI DOTYCZĄCYCH CECH WIELKOŚCIOWYCH

Słowa kluczowe: edukacja wczesnoszkolna, muzyka, matematyka, cechy wielkościowe.

Celem artykułu jest zaprezentowanie możliwości rozwijania wiedzy i umiejętności dotyczących cech wielkościowych opartych na integracji muzyki i matematyki w edukacji wczesnoszkolnej. Specjaliści z zakresu kształcenia dzieci etapie wczesnoszkolnym są zgodni, że twórcze połączenia różnorodnych zagadnień są niezbędne do wspierania i stymulowania harmonijnego rozwoju. W pierwszej części zaprezentowano informacje na temat związków muzyki i matematyki. W kolejnych skoncentrowano się na przykładach i możliwościach kształtowania pojęć matematycznych z zakresu cech wielkościowych przez różnorodne działania muzyczne.

Introduction

The article aims to present the possibilities of developing knowledge and skills related to size characteristics based on integrating music and mathematics in primary education (first grade). The research results show that teachers have considerable problems combining educational levels (Łuczak 2020a, pp. 69-107). The possibilities of using the integration of music and mathematics have been illustrated on the example of the development of knowledge and skills in the field of spatial orientation (Gruszczyk-Kolczyńska 2014, pp. 47-51), cause and effect reasoning (ibidem, pp. 133-139) and size feature Gruszczyk-Kolczyńska 2021, (ibidem, pp. 139-150) using elements of music teaching.

In the discourse on early childhood education, the integration of various contents is treated as a non-negotiable approach. Early childhood education specialists agree that creative combinations of various issues are essential to support and stimulate harmonious development. However, the integrated approach is often characterised by a tendency to associate the material stereotypically. Also, the way of thinking given in school textbooks is not conducive to creative solutions to problems. Moreover, most teachers underestimate music classes (in the arts) that integrate mathematical concepts (in science subjects). At the stage of the initial classes, children create intuitions, outlines of concepts and mathematical skills, which, when improved and expanded, become concepts (ibidem 2021, pp. 127-150, 219-235).

Understanding music as a mathematical discipline has a very long tradition. Such connections were noticed already in antiquity (Sudak 1992, Papadopoulos 2002). It can be said, with some simplification, that there is a great deal of math in music and vice versa; math can be very musical. If so, why do teachers seem to underestimate this type of integration?

The first part presents information on the relationship between music and mathematics, which was noticed already in antiquity. Then, the author focuses on the advantages and possibilities of shaping mathematical concepts in quantitative features through musical activities.

Music and math

Already in antiquity, the connections between music and mathematics were noticed (Papadopoulos 2002, Sudak 1992, pp. 15-50). According to the contemporaries of the time, the traits that united both fields were harmony and beauty. Regularities and proportions were noticed in both. Both domains were characterised by a specific language and structures (Periton 2013). It was also emphasised that both areas require logical-formal and abstract thinking (Łuczak 2018, p. 246).

Today, mainly four dimensions of music and mathematics are mentioned: space, rhythm, structure and symbolisation (Watson 2005, pp. 16-18). Music facilitates exploring and understanding two-dimensional space (e.g. through drawing) and three-dimensional space (through movement/dance – the so-called body art), helping to understand and assimilate figures/shapes (Łuczak 2020b).

On the other hand, regularity and variability can be known through the rhythm (Łuczak 2016, 2017, 2018, 2019, 2020a, 2020b). On this basis, access to appropriate musical material is valuable in learning and organising 'non-musical' material. It is crucial in the education process. Properly selected musical material, e.g. songs with different rhythms, can facilitate the learning and analysis of diversity (Keeler 2009).

The creation of musical works can be compared to mathematical constructions (Pesic 2013). What can significantly develop the combination of music and mathematics is the abstract thinking and the ability to use it in practical activities. Both areas are the pillars of thinking in understanding and learning about the world.

In education, the use of mathematics to analyse the phenomena of the surrounding world is highly appreciated. The ability to count, which is developed in all areas of a child's math education, derives from rhythms and (similarly) from the regularity of the decimal numeral system. In order to improve numeracy, the intuitions of cyclicality are used

(Gruszczyk-Kolczyńska 2021, pp. 49-85), which are visible in the structure of the calendar (seasons, months, weeks, days), in the rhythm of organising life activities during the day (morning, noon, afternoon, evening, night). The above math concepts can become more accessible for children to remember and understand thanks to a variety of musical reinforcements.

Furthermore, just as mathematics facilitates the exploration of the world, musical material through abstract thinking supports the perception of various phenomena and strengthens the ability to analyse and compare them. Among the mechanisms of perception and auditory memory, the aforementioned author lists the following elements: physiological and phonemic hearing, auditory analysis and memory, ability to associate the words heard and to see the relationship between them (Gruszczyk-Kolczyńska, Zielińska 2009, p. 164). Gruszczyk-Kolczyńska mentions that it is essential to develop attentive listening in children. It can be pointed out that math and music classes also facilitate the development of attentive listening; additionally, it happens during play and task situations in which children like to participate.

All this contributes to a more profound perception of the world, facilitating the potential application of knowledge and skills. It is worth mentioning one more dimension of combining music and mathematics, namely reducing potential stress during education. Preparation of creative activities which engage students, releasing their joy (accompanying play), means that negative associations with mathematics (it is difficult, tedious and boring) will not be able to consolidate and children will be more openly developing their mathematical competencies (Arehbay et al. 2013).

The importance of education which integrates music and mathematics

Many authors emphasise the importance of education supporting various spheres of children's development (Cackowska 2000; Hannaford 2003; Kisiel 2007; Plewka, Taraszkiewicz 2010, pp. 105-107). Particular attention is paid to the development of key competencies (Łuczak 2020a, pp. 34-49). In the original concept of 'children's mathematics', developed by Gruszczyk-Kolczyńska, the importance of supporting a child at the preschool stage in the field of mathematical education combined with supporting the mental development of children is emphasised (Gruszczyk-Kolczyńska 2015, pp. 9-58, pp. 129-134, pp. 173-194). Looking at the above issues from a different perspective, the development of key competencies also deserves attention (Łuczak 2020a, pp. 34-49).

The preparation and implementation of activities that will simultaneously shape many spheres of a child's development (emotional, mental, physical, moral and mental) is a complicated task. The main goal is to develop mathematical concepts as well as social competencies and various practical skills. Therefore, music education, saturated with various expressive and active forms, gives a chance for the complex process of acquiring and remembering knowledge to be effective, understandable, and joyful.

A constructive way to achieve such complex goals is through multi-sensory educational interaction. Thanks to music, combining aesthetic and emotional experiences with the expressive possibility of self-expression is more robust. It is also worth emphasising the inspiring role of music pedagogy in enriching educational activities with games with the use of rhythmics, improvisation and creativity (Łuczak 2020b, pp. 182-183). Other sources of inspiration for the integrated education trend (Duraj-Nowakowa 1998, Kisiel 2005) can be found in educational kinesiology (Dennison 1998), the good start method (Bogdanowicz 2014), or the use of movement in the education process (Bogdanowicz, Okrzesik 2005).

Cognitive processes run most effectively when accompanied by children's activity, as shown by the results of research on the use of music in teaching other subjects (Keeler 2009, p. 26). Creative math classes with music increase the effectiveness of the acquired material (Cooper 2011). Thanks to this, the whole organism is involved, and not only the intellect, as in passive teaching of mathematics.

Peter Sulivan (2011) highlights five fundamental principles that are important to learning mathematics effectively. They include: a) clearly defined goals of the classes and ways of achieving them; b) creating links between mathematical material and real life known to students; c) strengthening students' involvement in activities and activities that are of interest to them; (d) different challenges in terms of joint action as well as opportunities for questioning and discussing; e) preparation of the structure of classes; f) strengthening the level of advancement through regular exercise and repetition.

It is also essential to create conditions for transferring knowledge and skills (Sulivan 2011, pp. 24-30). Activities rich in various forms significantly increase the pleasure of learning about mathematics and the effectiveness of science (Arehbay et al. 2013). Therefore, it is worth taking care of enriching math classes with physical activity and sensory-stimulating activities. The implementation of ambitious integrated classes focused on shaping mathematical concepts will become much easier with musical activities

(Brzozowska-Kuczkiewicz 1991; Klopper, Vliex 1995; Watson 2005).

Preparing mathematics and music classes that affect many senses is a difficult task. It requires the teacher to know the models of shaping knowledge and mathematical skills and the ability to diagnose cognitive preferences of children.

Preparing mathematics and music classes that stimulate many senses is a difficult task. It requires a great deal of knowledge, experience and the ability to diagnose children's cognitive preferences (including a whole range of talents) on the part of the teacher. In addition, there remains the issue of a skilful, individualised approach to the transfer of knowledge and skills with the enormous developmental differences of students, remembering that students are characterised by different levels of abilities and a different learning strategy (Plewka, Taraszkiewicz 2010, p. 86).

The dynamically developing cognitive neuroscience offers numerous hints about education, as it allows for a relatively precise understanding of which parts of the brain are activated during specific activities (Spitzer 2011; Petlák, Zajacová 2010). Both hemispheres can be activated at different moments in cognitive processes, which significantly strengthens the memorisation of the material (Plewka, Taraszkiewicz 2010, p. 110). Any educational activities that use the natural need for movement in children reduce stress related to school stay. It can also support cognitive processes.

A Danish educational experiment tested the effect of movement on the effectiveness of the process of learning maths (Beck et al. 2016). Although the children's results did not indicate a general increase in the effectiveness of teaching through physical activity in the classroom, they did show that this way of learning makes it easier to understand and remember mathematical material for most gifted students. (Beck et al. 2016, p. 3). Less able students found it more difficult to concentrate during this lesson; therefore, their physical activity should be adjusted to their abilities (Bogdanowicz 2014; Beck et al. 2016, p. 13).

The importance of music in the development and training of a pupil's math skills

The versatility of music lies in its multidimensionality. Its crucial function in education is in the field of communication and integration. The very nature of music, its potential emotionality, and various forms mean that musical content can be combined with other content from various subjects (Jones, Pearson 2013). "Art is an indispensable means of communication between an individual and a group, a means of his unlimited socialisation" (Fischer 1961, p. 5).

Howard Gardner (2009) draws attention to the multifaceted interaction through music. Students' creative potential revealed during various musical activities enhances their joy and faith in the fixed capital of opportunities (Cooper 2011). The importance of joint expressive music therapy interactions is emphasized by Cylulko (1998), Lewandowska (1996), Metera (2002), Kataryńczuk-Mania (2005, pp. 55-65), and Łuczak (2009, 2016, 2020a). The influence of art on the educational and didactic process opens the way for development for "a comprehensively developed, rational, creative and sensitive man" (Wojnar 1965, p. 148).

The results of research on motor expression and its impact on students' cognitive development confirm its effectiveness in acquiring knowledge in mathematics (Beck et al., 2016). Weaving dance elements into mathematical education helps to understand many mathematical issues. The use of various expressions awakens the students' creativity and creative approach to problem-solving (Keeler 2009) while giving children many joyful experiences (Nordoff, Robbins 2008).

The use of arts education, especially music as a tool for the creative transfer of knowledge and skills in integrated teaching, is prevented by the outdated structure of the traditional education model (Uszyńska-Jarmoc 2011, pp. 14-15). Without a dialogue saturated with joy and adventure in discovering knowledge, we will not change the students' approach to school (Mazur 2014, p. 11).

Taking into account the development of mathematical thinking of students in grades 1-3, "practising the same calculation skills, which unfortunately too often occurs in primary education [...], stiffens mathematical thinking, blocking the ability to develop it in the future. Teachers and authors of many textbooks forget that mathematics cannot be reduced to the list of learned technical skills" (Klus-Stańska, Kalinowska 2004, p. 21).

All areas of music and mathematics education (spatial orientation, size features, geometric figures, sets and their classification, arithmetic operations, determining the weight of objects and vessel capacity, noticing regularities over time) have been developed and presented in stages by the author of this article, among others in the book "Music and mathematics in child education" (Łuczak 2020a) and other publications (Łuczak 2016, 2017, 2019, 2020b).

Musical activities supporting the formation of mathematical concepts in the field of magnitude features

The formation of mathematical concepts in the field of magnitude features requires a stepwise approach. First, the children get acquainted with the outline of concepts so that they can understand and remember them in later stages through specific musical activities. Table 1 provides an abridged description of such training.

Table 1

Content of mathematical education that can be expanded and deepened in the musical education of first-grade students

Stage I	**Stage II**	**Stage III**	**Stage IV**
Mathematical knowledge and skills			
Noticing the differences during games/tasks and using the following terms: large-small, thick-thin, long-short.	Differentiate items for size characteristics that show slight differences by naming: big-bigger, small-smaller, thick-thicker, thin-thinner, long-longer, short-shorter, narrow-narrower, wide-wider, high-taller, low-lower, the same as. Try to measure any length in steps or feet.	The use of terms regarding the size of objects in tasks/games by differentiating, comparing and ordering according to the increasing or decreasing selected feature, e.g. small-smaller-the smallest. Ability to use simple methods of measurement.	Proper use of terms in the scope of size, length, width, height, thickness, such as: higher than, lower than, narrower than, wider than, the same. Improving the ability to use a specific measure.
Musical educational activities			
Shaping the right intonation and sonic imagination.	Shaping the sense of rhythm.	Shaping musical sensitivity (in terms of rhythm and melody).	Shaping musical sensitivity and kinesthetic at the level of conceptual organisation.
Types of expressive activities			
Verbal-rhythmic and vocal-rhythmic expression of concepts and rhymes	Vocal-rhythmic expression of rhymes with the use of concepts.	Vocal-rhythmic expression of stories, and stories with the use of concepts.	Vocal-rhythmic expression of fairy tales with the use of concepts.
Getting to know the timbre of the sounds of various instruments during fun and games.	Learning about different ways to play instruments during group tasks/exercises.	Vocal-instrumental expression realised in small groups and individually.	Vocal-instrumental expression carried out according to a set rule.

Shaping the sense of rhythm through motor expression.	Inhibition-incitation exercises.	Motor-instrumental expression as a complement, or accompaniment to songs.	Movement improvisations to the music heard (the so-called animated visuals).
Vocal performances with the use of instruments. Shaping the sound imagination.	Vocal and rhythmic performances with the use of instruments.	Vocal-rhythmic-movement improvisations with the use of instruments on any topic.	Vocal-rhythmic-movement improvisations with the use of instruments on any topic with the use of concepts.
Shaping musical taste and perceptual awareness of the relationship between sounds.	Shaping perceptual awareness of sound-rhythmic structures and sound imagination.	Building musical associations.	Shaping musical reflection and emotional experiences related to music.

Source: author's elaboration.

The content of music and mathematics education for children in grade 1 should be implemented through forms of music education, and examples are presented in Table 2 below.

Table 2

Teaching material content with the support of musical education content and forms

Forms of musical activities	**Teaching material content which develops knowledge and skills related to magnitude features**
Singing and speech exercises	• Recitation of musical and mathematical concepts in nursery rhymes and count-ups in a specific rhythm using tataisation (with various dynamics and agogics). • Singing and comparing different melody patterns with pitching. • Singing the C major scale (solmisation and lettering) with phonogestics from the lowest to the highest sound). • Ordering the values of rhythmic notes and rests from the shortest to the longest and their implementation by tataisation. • Singing a song ("Kolorowe kredki" [Coloured pencils]).

Movement with music	• Fun with the song (e.g. "crayons for a box"). Children are assigned specific colours of crayons. Each group has a designated area on the floor (it is to be a "crayon box"). During the accompaniment of jumping or running, children move around the room. Upon hearing the song's melody, the children lay down next to each other on the floor "like crayons in a box". Then, on the marked signal – only the girls (the smallest or the largest) are put into the box; in the next version – girls with long or short hair, or boys in black or blue pants; etc. • Colour counts-up A group of students (around 8-12) gets coloured pencils, one for each child. Children with crayons stand in a row next to each other, holding them in front of them. One of the children 'lists' standing colleagues, saying the following text: "take only one crayon and paint what you want" in a syncopated rhythm. The child who receives the last word of the counting sheet shows his or her crayon and quickly says what he or she can draw with the colour, for example: – blue crayon – sky, forget-me-not, cornflower, – red crayon – poppy seed, red toadstool, tomato. While still playing, the child leaves the row and 'lists' the other friends with the crayons. After completing the counting, he/she goes to the side. There is one child less in the row each time. The fun lasts as long as there are two children in the group.
Playing musical instruments	• Five girls play percussion accompaniment to the song with tambourines and three boys with rattles. • Children play high and low sounds (legato or staccato) on the bells – only girls in skirts and with pigtails. There are cardboard boxes with notes and instruments on each desk. Children identify, compare and organise them in terms of two and three characteristics (e.g. colour, size, duration, height). • Children play accompaniment to the song on the bells. Children with red sticks perform the first verse, the chorus – only girls play the next verse – boys with green sticks.
	• Playing the bells two tones higher and longer than those given by the teacher. • Children receive cards with pictures of instruments. For a given sign, they lift cards with an illustration of the largest or smallest string instrument.
Making music	• Singing sounds of varying pitches from higher to lower. • Singing notes higher than 'mi' and longer than a quarter note, then lower than 're' and shorter than a quarter note. • Playing a fragment of a song on the bells: a) fast and slow; b) quiet and loud; c) legato and staccato.

Perception of music	• The teacher presents the students with three different cards. Each one has a fragment of a well-known melody. The children's task is to compare them and identify the differences between them: time signatures, rhythmic values of notes and rests, and others. • Children listen to two songs. They are to recognise which of them is dominated by low and high voices. • While listening to the piece, the children organise the illustrations of the instruments according to the division: strings, keyboards, etc. • The teacher presents various melodies. Students define the timbre of the sound by assigning it to the appropriate instrument.
Scope of terms: **MUSICAL NOTIONS:** The names of the sounds (lettering and solmisation) in the C major scale; Harmonic triad: tonic, subdominant, dominant; rhythmic values of notes, and rests; types of female and male voices; musical instruments. **MATHEMATICAL TERMS:** size, thickness, length, width, height.	1. Shaping the ability to identify objects in terms of the distinguished size feature, e.g. • long – short sound / piece or its fragment; • large – small instrument; • heavy – light (piano – flute); • even – unequal (recorder – transverse flute); • thick – thin (drum – barrel); • high – low (register of sounds on various instruments, bass – soprano); • fast – slow (tempo: allegro – adagio). 2. The teacher presents the students with two different cuckoo clocks. Which cuckoo clock will make a sound longer? Colour the clock which makes the sound longer brown. 3. Developing the ability to identify objects in terms of the two outstanding size features. Highlight the underlining in the notation: a) long notes in red, b) short notes in green and count how many there are.

Source: author's own elaboration.

Conclusion

As shown in the text, music and mathematics taken together can and should be an essential element of primary education. This combination is especially beneficial when it comes to reducing the educational stress in children learning mathematics.

Music increases the potential of learning through play. On the example of developing intuition and outline concepts in spatial orientation, supporting children in cause-and-effect reasoning and shaping the ability to measure continuous quantities (length, time), this article shows how music can help shape them. When children are allowed to learn mathematical content, combined with movement and music expression, school mathematics activities will become a joyful adventure for them.

Moreover, it will contribute to supporting children in the development of mathematical and musical talents.

Literatura | References

AREHBAY B., NAI-QING SONG, IBRAIMOV S. (2013), Some aspects of effective mathematics lessons in primary school, "International Journal of Academic Research", Part A, 5(6), pp. 216-219.

BECK M. M., LIND R R., GEERTSEN S. S., RITZ C., LUDBYE-JENSEN J., WIENECKIE J. (2016), Motor-Enriched Learning Activities Can Improve Mathematical Performance in Preadolescent Children, "Frontiers in Human Neuroscience", 10, pp. 1-14.

BOGDANOWICZ M. (2014), Metoda dobrego startu we wspomaganiu rozwoju, edukacji i terapii pedagogicznej, Wydawnictwo Harmonia, Gdańsk.

BOGDANOWICZ M., OKRZESIK D. (2005), Opis i planowanie zajęć według Metody Ruchu Rozwijającego Weroniki Sherborne, Wydawnictwo Harmonia, Gdańsk.

BRZOZOWSKA-KUCZKIEWICZ M. (1991), Emil Jaques-Dalcroze i jego rytmika, Wydawnictwa Szkolne i Pedagogiczne, Warszawa.

COOPER PH. (2011), Creative Mathematics In the Primary Classroom, "Mathematics Teaching", 223, July.

DENNISON P. (1998) Kinezjologia Edukacyjna. Polskie Stowarzyszenie Kinezjologów.

DURAJ-NOWAKOWA K. (1998), Integrowanie edukacji wczesnoszkolnej, Kraków.

GRUSZCZYK-KOLCZYŃSKA E. (ED.) (2014), Edukacja matematyczna w klasie I. Książka dla nauczycieli i rodziców. Cele i treści kształcenia, podstawy psychologiczne i pedagogiczne oraz opisy zajęć z dziećmi, Wydawnictwo CEBP 24.12, Kraków.

GRUSZCZYK-KOLCZYŃSKA E. (2015), Dziecięca matematyka – dwadzieścia lat później. Książka dla rodziców i nauczycieli starszych przedszkolaków, Wydawnictwo CEBP 24.12, Kraków.

SEMADENI Z., GRUSZCZYK-KOLCZYŃSKA E., TRELIŃSKI G., BUGAJSKA-JASZCZOŁT B., CZAJKOWSKA M. (2015), Matematyczna edukacja wczesnoszkolna. Teoria i praktyka, Wydawnictwo Pedagogiczne ZNP, Kielce.

Gruszczyk-Kolczyńska E., Zielińska E. (2016), Nauczycielska diagnoza gotowości do podjęcia nauki szkolnej. Jak prowadzić obserwację dzieci, interpretować wyniki i formułować wnioski, Wydawnictwo CEBP 24.12, Kraków.

Gruszczyk-Kolczyńska E. (2021), Jak pomóc dziecku pokonać niepowodzenia w nauce matematyki? Podręcznik dla rodziców, terapeutów i nauczycieli z serii "Dziecięca matematyka", Wydawnictwo CEBP 24.12, Kraków.

Gruszczyk-Kolczyńska E., Zielińska E. (2009), Zajęcia dydaktyczno-wyrównawcze dla dzieci, które rozpoczną naukę w szkole, Wydawnictwo Edukacja Polska, Warszawa, pp. 164-178.

Jones S. M., Pearson Jr. D. (2013), Music: Highly Engaged Students Connect Music to Math, "General Music Today", 27(1), pp. 18-26.

Keeler S. (2009), Music for Music's Sake, "American Music Teacher", 58(5), pp. 26-27.

Kisiel M. (2005) Muzyka w zintegrowanej edukacji wczesnoszkolnej, Wydawnictwo Uniwersytetu Śląskiego, Katowice.

Klöppel R., Vliex R. (1995), Rytmika w wychowaniu i terapii, PNO, Warszawa.

Klus-Stańska D., Kalinowska A. (2004), Rozwijanie myślenia matematycznego młodszych uczniów, Wydawnictwo Akademickie "Żak", Warszawa.

Lewandowska K. (1996), Muzykoterapia dziecięca, Gdańsk.

Łuczak A. (2009), Terapeutyczny aspekt działań muzycznych w zintegrowanej edukacji wczesnoszkolnej, [in:] Sztuka wobec zakresów wolności człowieka liberalnego: pedagogiczne rozważania i doświadczenia, (ed.) M. Zalewska-Pawlak, Wydawnictwo Uniwersytetu Łódzkiego, Łódź, pp. 245-250.

Łuczak A. (2016), Od muzyki do matematyki. Kształtowanie pojęć matematycznych w edukacji muzycznej dzieci, Oficyna Wydawnicza UZ, Zielona Góra.

Łuczak A. (2017), Dlaczego warto łączyć nauczanie matematyki z ruchem i muzyką? Wybrane przykłady badań na temat łączenia różnych form w zintegrowanej edukacji wczesnoszkolnej, "Relacje. Studia z nauk społecznych", 4, pp. 191-201.

Łuczak A. (2018), Muzyka i matematyka w edukacji dzieci. Przełamywanie barier – punkty styczne, [in:] Socjologia muzyki w Polsce. Pęknięcia i kontynuacje, (eds.) M. Choczyński, A. Rozalska, K. Drzewek, Warszawskie Wydawnictwo Socjologiczne, Warszawa, pp. 241-255.

ŁUCZAK A. (2019), Łączenie treści muzycznych i matematycznych w edukacji wczesnoszkolnej - wielowymiarowość efektów [in:] Analiza wybranych problemów z zakresu nauk społecznych, (eds.) M. Śliwa, A. Surma, vol. 2, Wydawawnictwo Naukowe TYGIEL, Lublin.

ŁUCZAK A. (2020a), Muzyka i matematyka w edukacji dziecka. Teoretyczne i praktyczne aspekty nauczania muzyki i matematyki w edukacji wczesnoszkolnej, Oficyna Wydawnicza UZ, Zielona Góra.

ŁUCZAK A. (2020b), Wspomagająca rola działań muzyczno-plastycznych w kształtowaniu pojęć matematycznych z zakresu stosunków przestrzennych u dzieci, [in:] Konteksty kultury i edukacji muzycznej, (eds.) E. Parkita, A. Parkita, J. Sztejnbis-Zdyb, Wydawnictwo Uniwersytetu Jana Kochanowskiego, Kielce, pp. 179-197.

MAZUR A. (2014), Edukacja artystyczna – zadanie do wykonania czy radość tworzenia, [in:] Interpretacje i inspiracje do edukacji artystycznej, (eds.) A. Mazur, R. Tarasiuk, Lublin.

METERA A. (2002), Muzykoterapia. Muzyka w medycynie i edukacji, Drukarska Spółdzielnia Pracy, Leszno.

NORDEMANN K., ROTTMANN T. (2019), Embodied Learning in Mathematics Education – Fostering Children's Repeating Pattern Competencies by Using Whole-Body Movements, [in:] Opportunities in Learning and Teaching Elementary Mathematics, (eds.) J. Novotná, H. Moraová, Prague, pp. 261-270.

NORDOFF P., CLIVE R. (2008), Terapia muzyką w pracy z dziećmi niepełnosprawnymi, Impuls, Kraków.

PAPADOPOULOS A. (2002), Mathematics and Music Theory: from Pytagoras to Rameau, "The Mathematical Intelligencer", 24(1), pp. 65-73.

PERITON CH. (2013), Mathematics and Music, "Mathematics Teaching", 233, pp. 24-26.

PESIC P. (2013), Euler's Musical Mathematics, [in:] "Mathematical Intellingencer", 35(2), pp. 35-43.

PETLAK E., ZAJACOVÁ J. (2010), Rola mózgu w uczeniu się, PETRUS, Kraków.

PLEWKA CZ., TARASZKIEWICZ M. (2010), Uczymy się uczyć, Pedagogium – Wydawnictwo OR TWP w Szczecinie, Szczecin.

SPITZER M. (2011), Jak uczy się mózg, Wydawnictwo Naukowe PWN, Warszawa.

SUDAK B. (1992), Matematyczna koncepcja muzyki, Wyższa Szkoła Pedagogiczna im. T. Kotarbińskiego, Zielona Góra.

Sulivan P. (2011), Teaching Mathematics: Using research-informed strategies, "Australian Education Review", 59, pp. 24-30.

Uszyńska-Jarmoc J. (2011), Czego nie wiemy o twórczości w szkole? Obszary zdeformowane, ignorowane i/lub zaniedbane, [in:] Twórczość-ekspresja-aktywność artystyczna w rozwoju dzieci i młodzieży, (ed.) K. Krasoń, Katowice.

Watson A. (2005), Dance and Mathematics: Engaging Senses in Learning, "Australian Senior Mathematics Journal", 19(1), pp. 16-23.