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NUMERICAL ANALYSIS OF STEEL-REINFORCED CONCRETE COMPOSITE GIRDERS

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The article presents a numerical model of a typical composite girder which consists of a strip of a reinforced concrete slab resting on a steel beam. Compatibility of horizontal displacements is assumed in the slab-beam contact region. The structure is subjected to unidirectional bending. The cross-section is divided into layers, either steel or concrete, and then analysed by applying non-linear characteristics of materials. The problem is solved by employing the finite element method. For a nonlinear system of equations the modified Newton-Raphson method is used. Examples of verification and illustration are enclosed. The stages of the composite structures under increasing loads are analysed in details.

Keywords: composite structures, numerical modelling, finite element method, nonlinear analysis

1. INTRODUCTION

Composite structures with reinforced concrete slabs resting on steel girders are widely used in bridge and industrial construction. The solution enables performance of wide-span structures with relatively small heights of the sections of steel girders. The structures are subjected to unidirectional bending. Such formulated task allows the analysis of a single steel girder as well as an interacting strip of the reinforced concrete slab, independently from other girders. The system is reduced to a beam subjected to unidirectional bending.

The article considers a composite structure, connectors of which ensure a full union of the slab and the girder (the displacements of the bottom surface of the slab and the upper surface of the girder correspond with each other). A nu-

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merical model of the composite girder in which the loaded surface is the surface of symmetry of the cross section is described. Nonlinear relations between strains and stresses are assumed for the reinforced concrete part in the compressed zone whereas the theory of smeared cracks is applied in the tensioned zone. The phenomenon of tension stiffening effect is considered. It is also assumed that the relation σ–ε is of an elasto-plastic character, with isotropic hardening, for the steel part of the section and for the reinforcement of the reinforced concrete plate. The elaborated model is used for calculation of a typical composite girder, consisting of a steel beam and a reinforced concrete slab. The section of the girder is presented in fig.1.

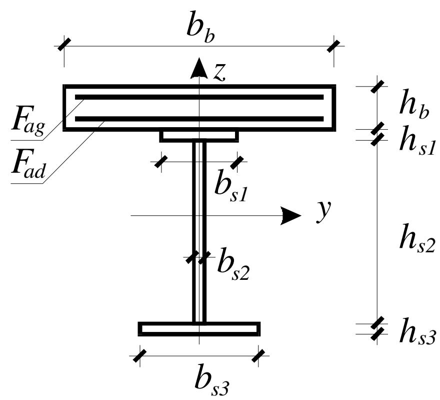


Fig.1. Cross section of a composite girder

The following assumptions are additionally made to the calculation model:
1. rheological and dynamical influences are neglected in the analysis,
2. loss of the system stability is not considered,
3. normals to the midsurface before deformation remains strait but not necessarily normal to the midsurface after deformation.

2. LAMINAR ELEMENT OF A COMPOSITE GIRDER

The paper considers a girder subjected to bending moment, a transverse force and an axial force. Timoshenko beam theory is used, which allows for transverse shear deformation effects.

Displacement of any point of a section is expressed by generalised displacements of the neutral axis of the beam $\overline{u}, \overline{w}, \overline{\varphi}$ (respectively: horizontal and vertical displacement, rotation of the normal), according to the following relations

$$
\begin{aligned}
u(x,z) &= \overline{u}(x) - z\overline{\varphi}(x)\,, \\
w(x,z) &= \overline{w}(x)\,, \\
\overline{\varphi}(x) &= \frac{d\overline{w}}{dx} - \beta\,,
\end{aligned}
\tag{2.1}
$$

where β is a rotation due to transverse shear deformation.

Trying to define relationships in a finite element of a girder, the generalised displacements of the beam are presented through nodal parameters (for node i : u_i, w_i, φ_i –fig.2A), with the use of equations $(2.1)_1$ and $(2.1)_2$ as well as interpolation functions N_i

$$
\begin{bmatrix} \overline{u} \\ \overline{w} \end{bmatrix} = \sum_{i=1}^{m} \begin{bmatrix} N_i & 0 & -zN_i \\ 0 & N_i & 0 \end{bmatrix} \begin{bmatrix} u_i \\ w_i \\ \varphi_i \end{bmatrix},
\tag{2.2}
$$

where m is a number of nodes of a given element.
The relationship between stresses and strains takes the form

$$
\begin{bmatrix} \sigma_x \\ \tau_{xz} \end{bmatrix} = \begin{bmatrix} E & 0 \\ 0 & G \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \gamma_{xz} \end{bmatrix}.
\tag{2.3}
$$

For small displacements the normal and shear strains are given as

$$
\begin{bmatrix} \varepsilon_x \\ \gamma_{xz} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial}{\partial x} & 0 \\ \dfrac{\partial}{\partial z} & \dfrac{\partial}{\partial x} \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix}.
\tag{2.4}
$$

Applying relations (2.2), (2.3) and (2.4) as well as the rule of virtual work, we obtain a relation determining the stiffness matrix. A block of the matrix referred to nodes i and j takes the form

$$
[K_{ij}] = \int_{l} \int_{-b/2}^{b/2} \int_{-h_d}^{h_g} \begin{bmatrix} N_{i,x}EN_{j,x} & 0 & -N_{i,x}zEN_{j,x} \\ 0 & N_{i,x}GN_{j,x} & -N_{i,x}GN_j \\ -N_{i,x}zEN_{j,x} & -N_iGN_{j,x} & N_{i,x}z^2EN_{j,x} + N_iGN_j \end{bmatrix} dzdydx.
\tag{2.5}
$$

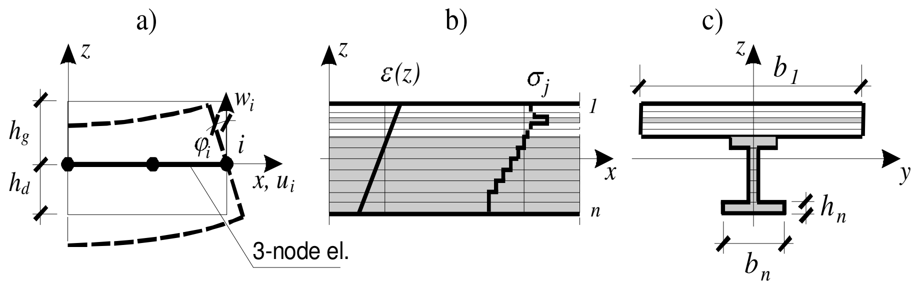


Fig.2. An element of a composite girder: a) degrees of freedom in node *i*, b)strains and stresses in layers, c) cross section of the element

The element is divided into layers along the height of the section. The purpose of the division is to select layers of reinforcing steel, layers of concrete and layers of the steel beam - fig. 2 b) and c) (Layers of steel are marked in grey whereas layers of concrete – in white). The number of layers n as well as the sequence of layers of steel and concrete may be arbitrary. Integration of equation (2.5) along the height of the section (variable z), is performed with the use of the rectangle method.

In further calculations a three-node finite element with parabolic-shape functions is used. In order to avoid shear locking, selective integration of the stiffness matrixes with respect to variable x is used (2 Gauss points for components connected with shear, 3 Gauss points for the others) [1].

3. PHYSICAL EQUATIONS OF STEEL AND CONCRETE

A hyperbolic relation between stress and strain is assumed [2], in the state of a uniaxial compression of concrete, given by equation

$$\sigma = \frac{E_0 \varepsilon}{1 + \left(\frac{E_0 \varepsilon_c}{f_c} - 2 \right) \frac{\varepsilon}{\varepsilon_c} + \left(\frac{\varepsilon}{\varepsilon_c} \right)^2}, \tag{3.1}$$

where: f_c –concrete compression strength, ε_c – strains corresponding to the concrete compression strength, E_0 – initial elasticity module of concrete
The diagram of the relation is presented in the bottom left part of fig.3A.

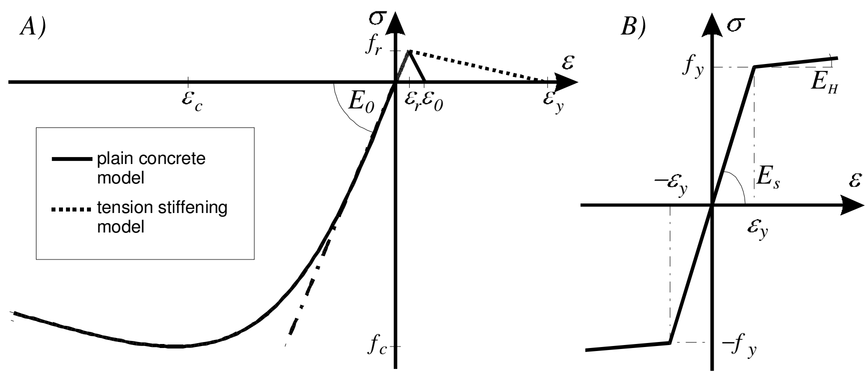


Fig. 3. Material models: A) concrete B) steel
ATTENTION scales on the axes in fig. A and B are different

The theory of smeared cracks is assumed in the tensioned zone, e.g.[2]. The state of the material is controlled in numerical integration points. It is assumed that cracks connected with a particular integration point do not disturb macroscopic continuity of the concrete, and decrease only the stiffness of the finite element in the area of occurrence, according to the relation presented by a continuous line in fig.3A, in the top right part.

The phenomenon of the reinforcement stiffening between the cracks, being the result of the interaction between steel and concrete, is considered *implicite*, through modification of constitutive compounds relations for tensioned concrete (so called tension stiffening model, presented by a broken line in fig.3A). After the value of strains exceeds the yield point for steel ε_y, the effects of stiffening vanish.

It is assumed that after cracking the structure, concrete may continue to carry the part of the shear stresses, because the edges of the cracks are rough. However, as the width of the crack increases this ability decreases. In order to allow for this phenomenon a linear reduction of the initial shear module is assumed to value 0.05 G, as in [3], according to equation.

$$G_{red} = \left(\frac{\varepsilon_y - \varepsilon}{\varepsilon_y - \varepsilon_r} \right) G \,. \tag{3.2}$$

A bilinear model of steel, that the part of the girder and the reinforcement are made of, is assumed - fig. 3B, where E_s and E_H are respectively the initial elasticity module and the strain hardening module.

4. SOLUTION METHOD

To solve the nonlinear task, where the stiffness matrix is a function of unknown displacements, a modified Newton-Raphson method is applied. Initial tangent stiffness matrix $\boldsymbol{K}_0$ is assumed. In each step of increment Δt a linearised set of equilibrium equations is solved in the form

$$\boldsymbol{K}_0\ \Delta\boldsymbol{r} = {}^{t+\Delta t}\boldsymbol{F}_{ext} - {}^{t}\boldsymbol{F}_{\text{int}}\ , \tag{4.1}$$

where: $\boldsymbol{F}_{ext}$ is a vector of nodal loads, $\boldsymbol{F}_{int}$ is a vector of internal forces, generated by stresses inside elements, $\Delta\boldsymbol{r}$ is a vector of generalised increments of the node displacements. Because of the form of constitutive equations the attitude known as *total formulation* [2] is applied. Total displacements and total strains are determined from the displacement increments for each step of increment. On this basis, applying suitable relations $\boldsymbol{\sigma} = \boldsymbol{D}(\boldsymbol{\varepsilon})\ \boldsymbol{\varepsilon}$ (e.g. equation (3.1) for compressed concrete layers), total stresses are determined and vector $\boldsymbol{F}_{int}$ is modified. A graphical explanation of this incremental – iterative procedure is presented in fig.4, where $[Z_{el}]$ is an incidence or location matrix and $[B]$ is a strain matrix.

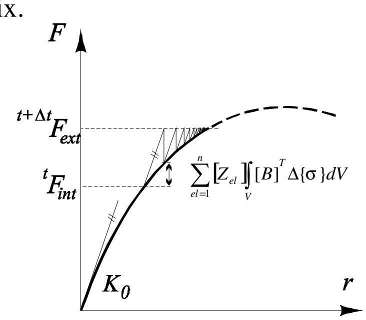


Fig.4. A modified Newton-Raphson method for one degree of freedom system

5. VERIFICATION AND ANALYSIS

In order to verify the elaborated composite structure model, numerical solutions are compared with the results of the experiment [4]. Figure 5A) presents a structural system, dimensions and loading of the tested girder. Figure 5C) presents a cross section and a slabs reinforcement, fig. 5B) presents a finite element discretization of the system. In the vertical direction each element is divided into 11 layers.

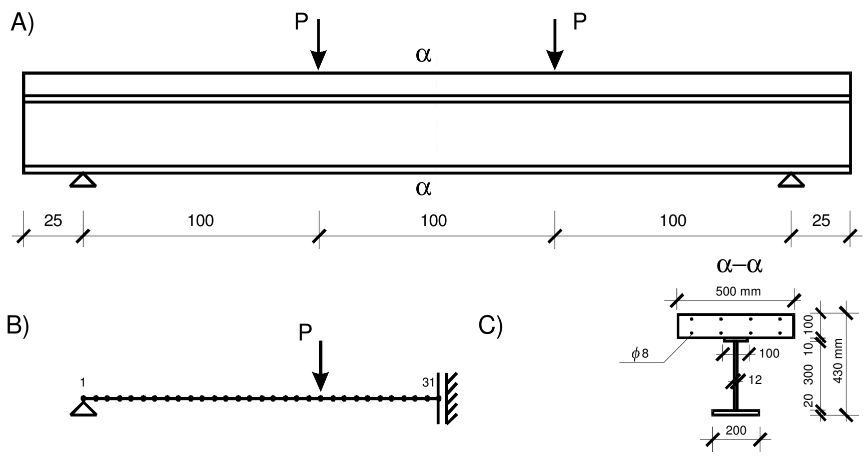


Fig. 5. A) Diagram of a composite girder, B) Division into finite elements, C)a cross section of the girder and the reinforcement of the slab

Values of deflections in section α–α obtained as a result of measurements are compared with linear and nonlinear FEM solutions (fig. 6). The nonlinear solution and the experimental results demonstrate a considerable conformance, what allows statement that the elaborated model of composite structure is correct.

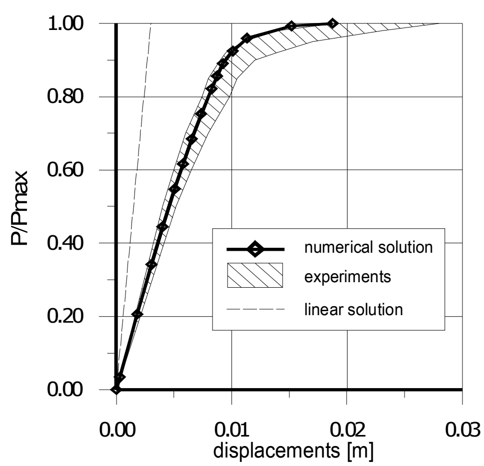


Fig.6. Deflection of the centre of the girder (section α–α, node 31)

To show the effect of internal forces redistribution a statically indeterminate two-span composite girder (fig.7A) is analysed in the next example. The spans are each 10m long. The section of the girder is presented in fig.1. A reinforced concrete slab is 0.2 m thick and a steel beam is 1.3 m high. The material parameters are assumed as the ones for concrete C30/37 and steel 18G2 according to the EC2. The girder is loaded by two vertical forces acting in the centre of each span. Taking the advantage of the symmetry of the task, the system is divided into 10 finite elements and the cross section into 11 layers.

Solutions obtained for the assumed linear-elastic features of the material, later called as linear (broken lines in the diagrams), are compared to solutions which consider all the described above nonlinear properties of steel and concrete, later referred to as nonlinear (continuous lines in diagrams).

Figure7B presents diagrams of bending moments obtained in the nonlinear analysis in a few subsequent load increments. For the last load increment an additional linear analysis is performed. The figure shows the redistribution of bending moments generated by concrete cracking above the central support and by steel yielding in zones of section 1-1 and section 2-2. There is also a zone where bending moment changes the direction while loading.

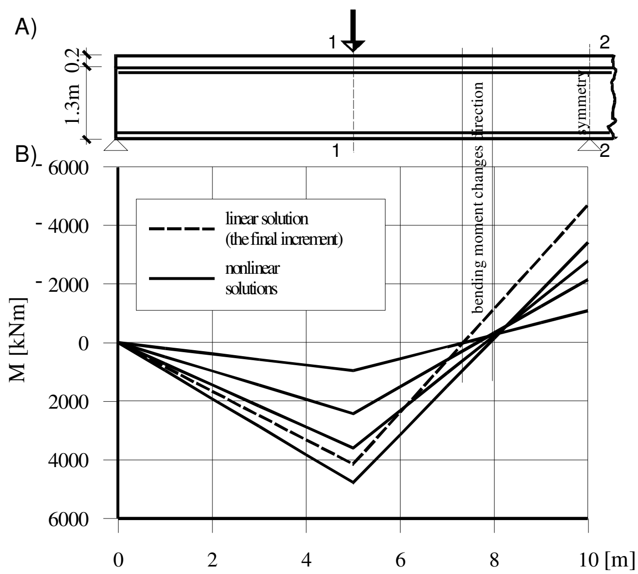
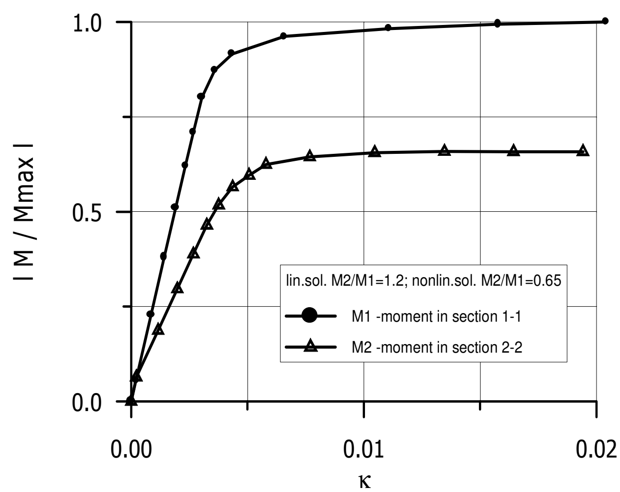


Fig.7.A) Structural system. B) Bending moments in the girder. Comparison of the linear and the nonlinear solutions in some subsequent increments of the load

It results from the linear solution that the proportion of moments M_2/M_1 equals 1.2. Because of a considerable stiffness reduction above the support and the redistribution of moments, the final value M_2/M_1 obtained in the nonlinear analysis equals 0.65.

Figure 8 presents the relation between the bending moments and the curvature of the beam in cross sections under the load and above the central support, obtained in subsequent steps of loading. The increment of moment M1, presented in the figure is nearly linear within a vast range of loading.



Rys. 8. Bending moment – curvature relations in sections 1-1 and 2-2

Figure 9 presents a more detailed analysis of the moment above the support (upper lines) and the moment in the centre of the span (bottom lines), as a load function. Values of loads, for which qualitative changes in the structure take place are marked with letters A, B, C, D and E. Point A refers to load $0.12P_{max}$, for which first cracks appear above the support. The cracked area is marked in grey. The first considerable stiffness reduction, connected with the propagation of the crack zone, takes place in point B. From this point on an increase in span moment is observed, in relation to the linear solution. The next characteristic value of load is marked as C (0.55 P_{max}). The second considerable stiffness reduction above the support takes place, this time connected also with yielding of reinforcing steel above the support and the loss of concrete load capacity within the area. The zone is marked in white. For load 0.83 P_{max} (point D) the upper flange of the steel girder above the support and the bottom flange under the force are beginning to yield. The changes are marked in black. Load equal 0.93 P_{max} causes widening of the cracked zone and propagation of the zone of the loss of concrete load capacity above the support. Considerable yield of the steel beam web above the support and under the force takes place. The solution of the iterative process does not appear to be convergent after a few subsequent load increments.

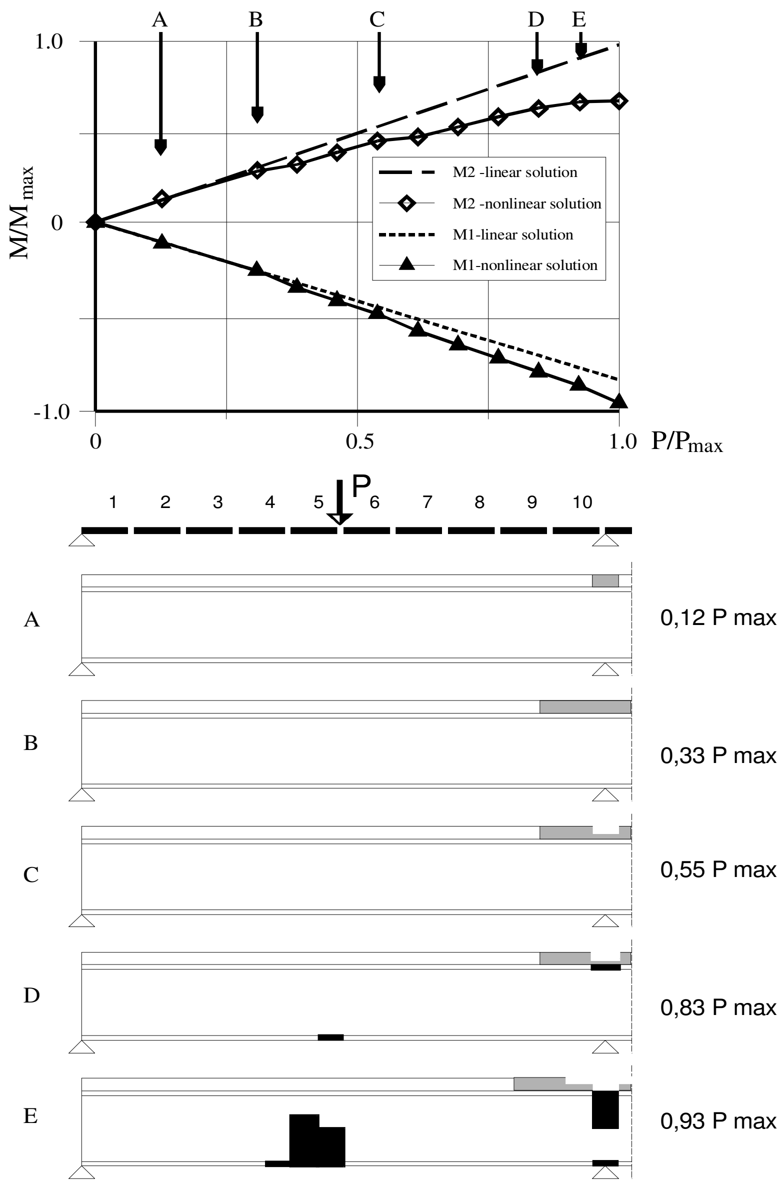


Fig.9. Bending moments redistribution and phases of girder degradation

6. SUMMARY AND CONCLUSIONS

The elaborated model allows the analysis of a steel beam-reinforced concrete slab composite structure analysis. The structure is discretised with beam finite elements. Relatively inconsiderable number of elements leads to results conforming the experiments. The analysed cross section is divided into layers b_j wide and h_j high. The number of layers may be arbitrary. For practical purposes the division into a few up to twenty layers is sufficient. Reinforcement of the reinforced concrete slab has to be replaced by an equivalent steel layer.

The model allows a qualitative and quantitative evaluation of the essential processes which take place in a composite girder subjected to increasing loads. It refers to the following processes: cracking of a reinforced concrete slab, stiffening of reinforcement between the cracks and yielding of zones of a steel beam. It also allows determination of distribution of normal stresses in an arbitrary cross section of the girder as well as determination of distributions of the section forces: bending moments, shear and axial forces, and displacements.

The presented results of the exemplary calculations of the composite beam demonstrate a considerable conformance with the experimental results. The cracking of concrete and yielding of steel layers that take place in subsequent stages of loading, cause redistribution of bending moments – which is a known phenomenon. Since the scope of the redistribution depends on individual geometric and material features of the analysed structures, it is difficult to formulate general conclusions. It seems more advisable to propose suitable software for engineer calculation.

The research should be followed by investigation of the influence of loss of local adhesion between the beam and the slab on internal forces and displacements.

Further research should be conducted to analyse the influence of flexibility of slab-beam interface on the behaviour of the composite girder.

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ANALIZA NUMERYCZNA STALOWO-ŻELBETOWYCH DŹWIGARÓW ZESPOLONYCH

S t r e s z c z e n i e

W artykule rozpatruje się konstrukcję zespoloną, składającą się z belki stalowej, na której wykonano płytę żelbetową. Zastosowane łączniki zapewniają pełne zespolenie płyty i dźwigara (zgodność przemieszczeń powierzchni dolnej płyty i powierzchni górnej dźwigara). Przedstawiono opis modelu numerycznego dźwiga zespolonego, w którym płaszczyzna obciążenia jest jednocześnie płaszczyzną symetrii przekroju. W stosunku do części żelbetowej przyjęto nieliniowe związki pomiędzy naprężeniami i odkształceniami w strefie ściskanej oraz teorię rys rozmytych w strefie rozciąganej z uwzględnieniem zjawiska usztywnienia zbrojenia pomiędzy rysami. Przyjmuje się jednocześnie sprężysto-plastyczny ze wzmocnieniem liniowym związek konstytutywny w odniesieniu do części stalowej przekroju i zbrojenia części żelbetowej. Fizycznie nieliniowe zadanie, rozwiązano wykorzystując metodę elementów skończonych w wersji przyrostowo-iteracyjnej. Opracowany model posłużył do wykonania obliczeń weryfikujących poprawność sformułowania, przez porównanie wyników obliczonych przemieszczeń z wynikami eksperymentu. Otrzymano dużą zgodność wyników. Ponadto wykonano obliczenia typowego dźwigara zespolonego. Schemat statyczny przyjęto w postaci belki dwuprzęsłowej. W wyniku przeprowadzonej analizy, zaobserwowano redystrybucję momentów zginających, wywołaną zmianami sztywności materiału dźwigara (zarysowanie betonu, uplastycznienie stali). Zamieszczono wyniki obliczeń w poszczególnych fazach zniszczenia układu.