INPUT RECONSTRUCTION BY MEANS OF SYSTEM INVERSION: A GEOMETRIC APPROACH TO FAULT DETECTION AND ISOLATION IN NONLINEAR SYSTEMS

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In this paper the classical detection filter design problem is considered as an input reconstruction problem. Input reconstruction is viewed as a dynamic inversion problem. This approach is based on the existence of the left inverse and arrives at detector architectures whose outputs are the fault signals while the inputs are the measured system inputs and outputs and possibly their time derivatives. The paper gives a brief summary of the properties and existence of the inverse for linear and nonlinear multivariable systems. A view of the inversion-based input reconstruction with special emphasis on the aspects of fault detection and isolation by using invariant subspaces and the results of classical geometrical systems theory is provided. The applicability of the idea to fault reconstruction is demonstrated through examples.

Keywords: fault detection and isolation, input reconstruction, linear systems, nonlinear systems, system inversion

1. Introduction and Problem Formulation

In the solution of the problem of fault detection and isolation (FDI), the principle of analytical redundancy is usually used when direct measurements from the system are not available. One method to infer the component fault status and analytically detect the existence of a fault is to look for anomalies in the plant's output relative to a model-based estimate of that output. Plant models, however, are generally incomplete and inaccurate. Moreover, the fault detection and isolation algorithms often assume the presence of a particular failure mode. These plant dynamics and failure mode modelling errors can either cause a high false alarm rate, or make it difficult to detect or isolate the faults. Any robust detection and isolation method that is designed to overcome the problems associated with these modelling errors must be able to distinguish among model uncertainties, disturbances and fault signals in order to avoid excessive false alarms or missed detections.

One possible approach to robustness relies on the use of models that describe the behaviour of the plant more precisely. This often leads to varying structure, time dependent or nonlinear models whose successful treatment depends on the development of new, more complex theories. Starting with nonlinear system models, however, may lead to difficulties not only from the point of view of theoretical complexity but also realizability. Besides these difficulties, one of the underlying problems with the application of nonlinear approaches is that most of the standard results established in linear systems theory must be relinquished, even if they comprise the basics for our understanding of dynamical systems. Nevertheless, it has already been widely recognized that the formulation and solution of many control and filtering problems in the framework of nonlinear theory of systems is much more a matter of necessity than of pure mathematical virtuosity.

In model-based FDI, the fault detection and isolation problem can be characterized as a two-step procedure: The first and basic problem is the detection and isolation of faults on the basis of the residual signal generated by a filter or detector. In some cases providing information about the real magnitude of the fault signal is required. This is usually referred to as fault estimation. In the second stage of the procedure the validation of the fault effects, i.e., the evaluation of the actual failure situation is accomplished by using a special logic or hypothesis testing.

The main objective addressed in this paper is the design and analysis of a residual generator for classes of nonlinear input affine systems subject to multiple, possibly simultaneous faults given in the most general form in the state space as

$$\begin{aligned} \dot{x}(t) &= f(x,u) + \sum_{i=1}^{m} g_i(x,u)\nu_i \\ y(t) &= h(x,u) + \sum_{i=1}^{m} \ell_i(x,u)\nu_i, \end{aligned} \tag{1}$$

where f, g, h, ℓ are analytic functions and $x(t) \in \mathcal{X} \subset \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ being the vector valued state, input and output variables of the system, respectively, $\nu(t)$ is the fault signal $(\nu_1, \ldots, \nu_m)^T$ whose elements $\nu_i : [0, +\infty) \to \mathbb{R}$ are arbitrary functions of time. The fault signals ν_i can represent both actuator and sensor failures, in general. The goal is to detect the occurrence of the components ν_i of the fault signal independently of each other and identify which fault component specifically occurred.

Along with the discussion of this paper, linear and nonlinear problems will be treated in parallel to each other. Results for linear time invariant (LTI) systems will always be viewed as special cases of the results obtained for the nonlinear problems specified by the general system model (1).

In our approach a detector, i.e., another dynamic system is constructed with outputs ν and with inputs u, y and possibly their time derivatives or integrals which, in the most general form, can be thought of as

$$\begin{aligned} \dot{\zeta}(t) &= \varphi(\zeta, y, \dot{y}, \ldots, u, \dot{u}, \ldots), \\ \nu(t) &= \omega(\zeta, y, \dot{y}, \ldots, u, \dot{u}, \ldots), \end{aligned} \tag{2}$$

with the state variable $\zeta(t)$ assuming φ, ω are arbitrary analytic time functions. The filter reproduces the fault signal at its output that is zero in the normal system operation, while it differs from zero if a particular fault occurs.

This detector should satisfy a number of requirements. It should distinguish among different failure modes ν_i, e.g., between two independent faults in two particular actuators. Moreover, it is aimed to completely decouple the faults from the effect of disturbances and also from the input signals. Note that for LTI systems the filter (2) traditionally serves as a residual generator which assigns the fault effects and the disturbances to disjoint subspaces in the detector output space.

Various solutions are known for generating residuals. The traditional methods of residual generation are based on the error dynamics of a state observer, see, e.g., the geometric approach of detection filters as initiated by (Massoumnia, 1986) for LTI systems. The parity space approaches were discussed in (Gertler, 1998), the unknown input observer in (Chen and Patton, 1998), the multiple model and the generalized likelihood ratio approaches in (Basseville and Nikiforov, 1993), just to mention a few. These approaches are used in a number of situations differing in the assumptions about noise, disturbances, robustness properties and in the specific design methods. For comparison, see some representations in the literature such as (Mangoubi, 1998; Mangoubi and Edelmayer, 2000).

It will be shown in this paper that residual generation for both linear and nonlinear systems can be viewed as an input reconstruction process and can be solved by using the idea of system inversion. The close relation of input reconstruction with the inverse problem was recognized by many authors earlier, see, e.g., (Hou and Patton, 1998), but the application of the idea to FDI was first considered by Szigeti *et al.* (2000). In the past few years, the solution of various types of inverse problems became particularly important in control and filtering. Inversion, which is a key to our approach, was studied, e.g., in the early works (Silverman, 1969; Hirschorn, 1979) for LTI, and was also considered in (Fliess, 1986; Isidori, 1995) for nonlinear systems. On-line dynamic inversion methods were successfully applied to many interesting problems in aerospace and aviation, such as e.g. (Krupadanam *et al.*, 2002). A summarizing study on related ideas was published in (Goodwin, 2002).

Input reconstruction addresses the problem of designing a filter or detector which, on the basis of the input and output measurements, returns the unknown inputs of the original system by utilizing its inverse representation, see Fig. 1. Though the solution of the inverse problem received considerable attention in the past years, there remained a number of open problems in this area, especially from the point of view of fault detection problems. Earlier approaches to system inversion (Silverman, 1969) considered the properties and calculation of the inverse as guaranteeing neither minimality (or observability, de-

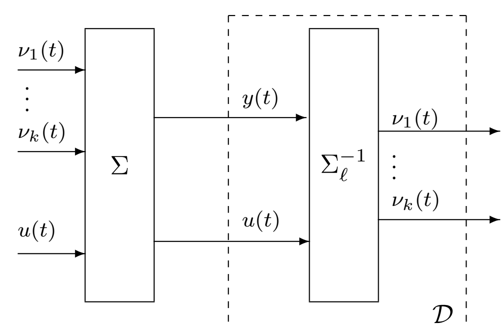

Fig. 1. Input (fault) reconstruction and the idea of system inversion: Σ is the plant, $\mathcal{D}$ is the detector which, most conveniently, can be obtained as the (left) inverse Σ_ℓ^{-1} of the original system.

tectability) nor stability properties of the resulting inverse system.

The applicability of system inversion to FDI in LTI systems was first demonstrated in (Szigeti *et al.*, 2001). Additional issues of inverse computation for the FDI problem can be found, e.g., in (Szigeti *et al.*, 2002), as well as in (Varga, 2002). One of the advantages of the inversion approach is that the extension of the idea to nonlinear systems is possible. It will be shown in this paper that, by using this concept, linear and nonlinear problems can be treated in the same theoretical framework. In most fault detection and residual generation methods developed for LTI systems this generalization cannot be made.

The content and organization of the paper is as follows. If we want to reconstruct the unmeasured fault signal at the output of the detector, the property of input observability is an important quality of the system. Therefore, we begin with the background of input observability of LTI systems. Some issues of input observability for linear systems were discussed, e.g., in (Hou and Patton, 1998). These preliminary results are briefly reviewed and the idea is related to system invertibility in Section 2.

In the past years geometric approaches proved to be particularly useful and successful means for the design and analysis of FDI methods. They provided fundamental tools for the design of residual generators aimed at providing structured and directional residuals, i.e., detection filters. Most of the results obtained for the classical detection filter theory were made available on the geometric platform, see, e.g., the results of (Massoumnia, 1986; White and Speyer, 1987; Massoumnia *et al.*, 1989) for LTI, (Edelmayer *et al.*, 1997) for linear time varying (LTV), and (Hammouri *et al.*, 1999) for bilinear systems based on geometric theory originated in (Wonham, 1992).

Efforts to extend geometric concepts to nonlinear problems were made, e.g., in (De Persis and Isidori, 2001). The generalization of geometric ideas to nonlinear systems, such as invariant subspaces used for LTI systems in a standard way, may prove to be cumbersome from several points of view in practice. Our approach attempts to avoid difficulties stemming from nonlinear invariant subspace theory and invariant distributions. It will be shown that the inverse problem for nonlinear systems can be dealt with with relative ease on the basis of standard geometric concepts introduced in (Wonham, 1992) and partly in (Isidori, 1995).

Therefore, it makes sense to relate the inversion problem to the classical results of geometric detection filter theory in Section 3. Section 3.1 gives geometric interpretation of the inverse problem in LTI systems. Then, we continue with input observability properties in the nonlinear framework. The generalization of the concepts obtained in the previous sections to nonlinear problems is discussed and geometric interpretation of inversion-based fault reconstruction in nonlinear systems is given in Section 4. This geometric approach not only proved to be useful from the point of view of better understanding the idea, but it also creates a theoretical basis for constructing efficient inversion algorithms. The technique is applied to simple demonstrative examples for both LTI and nonlinear systems.

2. Input (Fault) Observability of LTI Systems

Consider the minimal state space representation of the LTI system

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx + Du. \end{aligned} \tag{3}$$

Input observability of the linear dynamical systems (3) is closely related to their invertibility. In order to show this property, let us summarize some important results from the literature by considering the following proposition.

Definition 1. (Hou and Patton, 1998) The input $u(t)$ is said to be *observable* if $y(t) = 0$ for $t \geq 0$ implies $u(t) = 0$ for $t > 0$ provided that $x(0) = 0$.

Definition 2. (Basile and Marro, 1973) A linear system is called *left invertible* if the input $u(t)$ can be recovered from the knowledge of the output function $y(t)$ and the initial state $x(0)$.

Remark 1. For any known initial condition $x(0)$, input observability implies left invertibility.

Let us denote by Ω the set of all possible inputs of (3) and assume that they are at least n times differentiable.

Proposition 1. By taking the restriction of the input set

$$\begin{aligned} \Omega_o = \{u \in \Omega : \quad & u(0) = 0, \\ & \dot{u}(0) = 0, \dots, u^{(n-1)}(0) = 0\} \end{aligned}$$

and considering the system (3) over Ω_o, left invertibility and input observability are equivalent.

Proof. By successively taking the derivatives of the output, one has the equations

$$\begin{aligned} y &= Cx + Du, \\ \dot{y} &= CAx + CBu + D\dot{u}, \\ &\vdots \\ y^{n-1} &= CA^{n-1}x + CA^{n-2}Bu + \cdots \\ &\quad + CBu^{(n-2)} + Du^{(n-1)}, \end{aligned}$$

and for $t = 0$ on Ω_o we get

$$
\begin{aligned}
y &= Cx(0), \\
\dot{y} &= CAx(0), \\
&\vdots \\
y^{n-1} &= CA^{n-1}x(0).
\end{aligned}
$$

It follows that the output function $y(t)$ determines uniquely the initial state $x(0)$ which, according to Remark 1, means that left invertibility and input observability are equivalent on Ω_o. ■

Remark 2. (Fault observability and invertibility.) In case we work with fault detection problems, i.e., we consider systems of the type

$$
\begin{aligned}
\dot{x} &= Ax + Bu + L_1\nu, \\
y &= Cx + Du + L_2\nu, \qquad (4)
\end{aligned}
$$

where the fault signals $\nu \in \mathbb{R}^q$ may represent both actuator and sensor faults as reflected in the structure of the matrices L_1, L_2, all derivatives of the fault signals in the diagnostic system models will be zero for $t = 0$, since it is always supposed that $\nu(t) = 0$ if $t \leq t_o > 0$. It follows that the residual system is invertible iff it is input observable. Clearly, if L_2 is a full rank matrix, the inverse can be obtained by simple algebraic calculations. For treating more general cases, however, we need to consider the properties of invertibility in greater detail in the next sections.

3. Invertibility and the Relative Degree of Linear Systems

Consider the LTI system $\mathcal{S}$ given in (3) and the construction of its inverse representation. The system $\mathcal{S}$ is said to be left invertible (i.e., it has a left inverse denoted by $\mathcal{S}^{-1}$) if there exists a corresponding system representation such that the composition, shown in Fig. 2, will result in the identity for each pair (u, y) (cf. Definition 1).

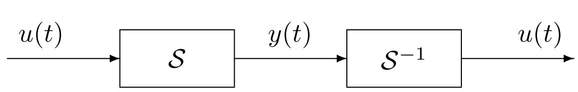

Fig. 2. The composition of systems $\mathcal{S}$ and $\mathcal{S}^{-1}$ resulting in the identity.

Let us consider the left invertible LTI system

$$
\begin{aligned}
\dot{x} &= Ax + Bu, \\
y &= Cx. \qquad (5)
\end{aligned}
$$

The derivatives of the measurement vector can be written as

$$
\begin{aligned}
\dot{x} &= Ax + Bu, \\
y &= Cx, \\
\dot{y} &= C\dot{x} = CAx + CBu, \\
\ddot{y} &= CA^2x + CABu + CB\dot{u}, \\
&\vdots \\
y^{(k)} &= CA^kx + CA^{k-1}Bu + \ldots + CBu^{(k-1)},
\end{aligned}
$$

where $k \geq 0$.

Let us denote by c_i the rows of the matrix C. If there exist integers $r_i > 0$, such that

$$
c_iA^kB = 0, \quad c_iA^{r_i-1}B \neq 0, \forall\, k < r_i - 1, \qquad (6)
$$

and

$$
\operatorname{rank} \begin{bmatrix} c_1A^{r_1-1}B \\ \vdots \\ c_pA^{r_p-1}B \end{bmatrix} = m, \qquad (7)
$$

then r_i is called a relative degree of the system. Based on the individual components r_i, the vector relative degree r is defined as $r = [\, r_1, \ldots, r_p \,]$. Then one can construct the equations

$$
\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_p^{(r_p)} \end{bmatrix} = \begin{bmatrix} c_1A^{r_1} \\ \vdots \\ c_pA^{r_p} \end{bmatrix} x + \begin{bmatrix} c_1A^{r_1-1}B \\ \vdots \\ c_pA^{r_p-1}B \end{bmatrix} u. \qquad (8)
$$

Obviously, from the representation (8) the input variable u can be obtained by inversion. The inverse system can be given in the possible nonminimal form:

$$
\begin{aligned}
\dot{\eta} &= A_{inv}\eta + B_{inv}v_{inv} \\
u &= C_{inv}\eta + D_{inv}v_{inv},
\end{aligned}
$$

where

$$
v_{inv} = \begin{bmatrix} y_1 & \cdots & y_1^{(r_1)} & \cdots & y_p & \cdots & y_p^{(r_p)} \end{bmatrix}^T. \qquad (9)
$$

If the realization of the inverse system is minimal, then A_{inv} gives the so-called zero dynamics of (A, B, C). Throughout this paper, it will be assumed that the zero dynamics of the system is asymptotically stable, i.e., the residual system is minimum phase. If this condition does not hold, the method presented here is not applicable.

3.1. Geometrical Properties of the Inverse in LTI Systems

In order to show the existence of a left inverse system in LTI systems consider the following results.

Proposition 2. *The system $\mathcal{S}$ given in the state space form (A, B, C) is left invertible iff B is monic and*

$$\mathcal{V}^*(B) \cap \operatorname{Im} B = 0, \tag{10}$$

where $\mathcal{V}^*$ is the supremal (A, B)-invariant subspace in $\ker C$ and F is the feedback, such that $(A + BF)\mathcal{V}^* \subseteq \mathcal{V}^*$, i.e., $(A+BF)$ is maximally unobservable, see (Wonham, 1992).

An equivalent description of the invertibility can also be given by the following proposition:

Proposition 3. *The system $\mathcal{S}$ is invertible iff for the maximal controllability subspace $\mathcal{R}^*$ contained in $\ker C$, the condition $\mathcal{R}^* = 0$ holds, see (Morse and Wonham, 1971).*

Remark 3. The subspace $\mathcal{V}^*$ can be calculated by using the (A, B)-invariant subspace algorithm without explicitly constructing F.

Proposition 4. *Consider the left invertible system $\mathcal{S}$: (A, B, C). The dynamics of the (left) inverse can be given as the restriction of $(A + BF)$ on $\mathcal{V}^*$, i.e.,*

$$A_{inv} = (A + BF) \,|\, \mathcal{V}^*. \tag{11}$$

Corollary 1. *The dimension of the state-space for the inverse system is $n_{inv} = \dim \mathcal{V}^* = n - \rho(r)$, where n is the state dimension of $\mathcal{S}$, r is its (vector) relative degree and $\rho(r) = \sum_{i=1}^{p} r_i$.*

Proof. Let us denote by V^* the insertion map of $\mathcal{V}^*$, and consider the state transform defined by $z = Tx = [\,\xi \quad \eta\,]^T, \quad \xi \in \mathcal{V}^{*\perp}, \quad \eta \in \mathcal{V}^*$, where

$$T^{-1} = \begin{bmatrix} B & \Lambda & V^* \end{bmatrix}, \quad \text{and} \quad \operatorname{Im}\Lambda \subset \mathcal{V}^{*\perp}. \tag{12}$$

From the invertibility condition $\mathcal{V}^* \cap \operatorname{Im} B = 0$, it follows that $\mathcal{V}^* \subset (\operatorname{Im} B)^{\perp}$, i.e., the transformation T is well defined.

In the new coordinate system the state matrices will take the form of

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \underbrace{\bar{A}_{21}}_{\rho} & \underbrace{\bar{A}_{22}}_{n-\rho} \end{bmatrix} \begin{matrix} \}\,\rho \\ \}\,n-\rho \end{matrix}, \quad \bar{B} = \begin{bmatrix} \bar{B}_1 \\ 0 \end{bmatrix} \begin{matrix} \}\,\rho \\ \}\,n-\rho \end{matrix},$$

$$\bar{C} = \begin{bmatrix} \bar{C}_1 & \bar{C}_2 \end{bmatrix},$$

where $\rho = \dim(\mathcal{V}^{*\perp})$.

Since $\mathcal{V}^* \subset \ker C$, the matrix $\bar{C}_2$ should be zero, i.e.

$$\bar{C} = \begin{bmatrix} \underbrace{\bar{C}_1}_{\rho} & \underbrace{0}_{n-\rho} \end{bmatrix}.$$

Also, since $\bar{A}\mathcal{V}^* \subseteq \mathcal{V}^* + \operatorname{Im}\bar{B}$, it follows that

$$\begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} 0 \\ \mathcal{V}^* \end{bmatrix} = \begin{bmatrix} \bar{A}_{12}\bar{\mathcal{V}}^* \\ \bar{A}_{22}\bar{\mathcal{V}}^* \end{bmatrix} \subseteq \begin{bmatrix} \operatorname{Im} B \\ \mathcal{V}^* \end{bmatrix},$$

i.e., there exists a unique matrix F_2 (since $\bar{B}_1$ is monic) such that

$$\bar{B}_1 F_2 = -\bar{A}_{12}.$$

By choosing

$$F = \begin{bmatrix} 0 & F_2 \end{bmatrix},$$

we get

$$\bar{A} + \bar{B}F = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} + \begin{bmatrix} \bar{B}_1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & F_2 \end{bmatrix}$$
$$= \begin{bmatrix} \bar{A}_{11} & 0 \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}.$$

To simplify the notation, the matrices $\bar{B}_1$ and $\bar{C}_1$ will be referred to as $\bar{B}$ and $\bar{C}$, respectively. By the selection of T, one has $\bar{B} = [\, I_m \quad 0\,]^T$.

Applying the feedback $u = F_2\eta + v$ to the transformed system, one gets the following equations:

$$\dot{\xi} = \bar{A}_{11}\xi + \bar{B}v, \tag{13}$$
$$y = \bar{C}\xi.$$

One can prove by induction that from $c_i A^k B = 0$, it follows that $\bar{c}_i \bar{A}_{11}^k \bar{B} = 0$ and $\bar{c}_i \bar{A}_{11}^{r_i - 1} \bar{B} \neq 0$ for $k < r_i - 1$.

Since $\dim(\mathcal{V}^{*\perp}) = \sum_{\mathrm{i}=1}^{\mathrm{p}} \mathrm{r_i}$, see (Wonham, 1992), one can define a state transform S for (13) such that

$$w = \begin{bmatrix} y_1 \\ \vdots \\ y_1^{(r_1-1)} \\ \vdots \\ y_p \\ \vdots \\ y_p^{(r_p-1)} \end{bmatrix} = S\xi,$$

where

$$S = \begin{bmatrix} \bar{c}_1 \\ \vdots \\ \bar{c}_1 \bar{A}_{11}^{r_1 - 1} \\ \vdots \\ \bar{c}_p \\ \vdots \\ \bar{c}_p \bar{A}_{11}^{r_p - 1} \end{bmatrix}. \quad (14)$$

It follows that

$$v = \bar{B}^{(-1)} S^{-1} (\dot{w} - S \bar{A}_{11} S^{-1} w), \quad (15)$$

where $S\bar{A}_{11}S^{-1}$ is exactly the observer canonical form of $\bar{A}_{11}$. From

$$\begin{aligned} \dot{\eta} &= \bar{A}_{22}\eta + \bar{A}_{21} S^{-1} w, \\ u &= F_2 \eta + v, \end{aligned} \quad (16)$$

one may get the matrix

$$\bar{A}_{22} = (A + BF)|\mathcal{V}^* = A_{inv} \quad (17)$$

in the basis represented by T, which proves Proposition 4. ■

Corollary 2. *The inverse dynamics of (A, B, C) can simply be obtained by calculating $\mathcal{V}^*$ using the (A, B)-invariant subspace algorithm. Choose a basis for $\mathcal{V}^*$ and compute the state transformation matrix T as defined by (12).*

Let us introduce the vector $v_{inv} = \begin{bmatrix} w^T & y_1^{(r_1)} & \cdots & y_p^{(r_p)} \end{bmatrix}^T$ as the input of the inverse system where w is defined by (14). Then, one can define

$$A_{inv} = \bar{A}_{22}, \quad B_{inv} = \begin{bmatrix} \bar{A}_{21} S^{-1} \\ 0 \end{bmatrix}, \quad (18)$$

where (A_{inv}, B_{inv}) describes the zero dynamics as

$$\dot{\eta} = A_{inv}\eta + B_{inv} v_{inv}. \quad (19)$$

The input u can be obtained from the equations

$$u = C_{inv}\eta + D_{inv} v_{inv}, \quad (20)$$

where $C_{inv} = F_2$. Moreover,

$$D_{inv} = Z - \begin{bmatrix} S\bar{A}_{11}S^{-1} & 0 \\ 0 & 0 \end{bmatrix}. \quad (21)$$

The matrix Z is given as

$$Z = \begin{bmatrix} Z_1 & 0 & \cdots & 0 & E_1 \\ 0 & Z_1 & \cdots & 0 & E_p \\ \vdots & & & & \\ 0 & 0 & \cdots & Z_p & E_p \end{bmatrix}, \quad (22)$$

where

$$Z_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \quad E_i = \begin{bmatrix} 0 \\ e_i^T \end{bmatrix}, \quad (23)$$

with e_i being the i-th unit vector in $\mathbb{R}^p$.

To conclude this section let us recall the characterization of the transmission zeros of linear systems:

Proposition 5. *(Transmission zeros.) The transmission zeros of (A, B, C) are the poles of the inverse dynamics, i.e., the eigenvalues of $(\bar{A} + \bar{B}F)|\mathcal{V}^* = \bar{A}|\mathcal{V}^*$.*

Example 1. In order to demonstrate the inverse calculation in LTI systems based on geometric characterization of the procedure presented in the previous section, consider the system representation (4) given by the matrices

$$A = \begin{bmatrix} -1 & 0 & -1 & 1 \\ 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

assuming that D and M are zero. It is simple to arrive at $\mathcal{V}^* = \ker C$, i.e., $\dim \mathcal{V}^* = 2$. Since $\dim \mathcal{V}^* = n - \rho$, it follows that the relative degree of the system is $\rho = 4 - 2 = 2$. Indeed, a simple calculation reveals that the relative degree $r = [\,1 \quad 1\,]$, i.e., $r_1 = 1$, $r_2 = 1$ and, therefore, $\rho = 1 + 1 = 2$. Since $\mathcal{V}^* \cap \mathrm{Im} L = 0$, (A, L, C) is left invertible.

The calculation of $\mathcal{V}^{*\perp}$ can be carried out from the span of the rows of C, i.e.,

$$\mathcal{V}^{*\perp} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = C$$

and one can choose

$$L^{\perp} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The state transform can be written as a simple change of coordinates x_i:

$$T = \begin{bmatrix} \mathcal{V}^{*\perp} \\ L^{\perp} \end{bmatrix} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right].$$

Then the coordinate transforms $z = Tx$, $\bar{B} = TB$ and $\bar{L} = TL$ are written as

$$z = \begin{bmatrix} x_3 \\ x_4 \\ x_1 \\ x_2 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \bar{L} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Moreover, with

$$\bar{C} = CT^{-1} = CT^{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

and $\bar{A} = TAT^{-1} = TAT^{T}$

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} = \left[\begin{array}{cc|cc} 0 & -1 & 0 & 1 \\ 2 & 0 & 0 & 0 \\ \hline -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & -2 \end{array}\right],$$

one arrives at $A_{inv} = A|\mathcal{V}^* = A_{22}$. Then the transformed state space system can be written in the form

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 2 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} u - \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \nu. \quad (24)$$

As the zero dynamics is $\eta = [z_3, z_4]^T$, the inverse system can be represented as

$$\dot{\eta} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \eta + \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} u.$$

Since $r_1 = 1$, $r_2 = 1$ and S is the identity matrix, by (15) the unknown inputs ν_1 and ν_2 can be derived from the first two equations of (24) as

$$\begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \left(\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \eta - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \right)$$

which, by using the identities $y_1 = z_1$, $y_2 = z_2$, $z_3 = x_1$ and $z_4 = x_2$, can be expressed in the form

$$\nu_1 = -\frac{1}{2}(-\dot{y}_1 - \dot{y}_2 + y_2 - z_4 - u),$$
$$\nu_2 = -\frac{1}{2}(-\dot{y}_1 + \dot{y}_2 + 2y_1).$$

♦

4. Geometric Characterization of Inversion-Based Input Reconstruction for Nonlinear Systems

To create a basis for further discussions, let us recall some elementary facts and definitions from nonlinear system theory as found, e.g., in (Isidori, 1995; Nijmeijer and van der Schaft, 1991).

Consider the nonlinear input affine system written in the form

$$\dot{x} = f(x) + g(x)u, \quad g(x) = \sum_{i=1}^{m} g_i(x)u_i,$$
$$u \in \mathbb{R}^m, \quad y \in \mathbb{R}^p,$$
$$y_j = h_j(x), \quad j = 1, \dots, p. \quad (25)$$

A smooth connected submanifold M which contains the point x_o is said to be *locally controlled invariant* at x_o if there exists a smooth feedback $\alpha(x)$ and a neighborhood U_o of x_o such that the vector field $\tilde{f}(x) = f(x) + g(x)\alpha(x)$ is tangent to M for all $x \in M \cap U_o$, i.e., M is locally invariant under $\tilde{f}$.

A smooth connected submanifold M that is locally controlled invariant at x_o and with the property that $M \subset h^{-1}(0)$ is called an output-zeroing submanifold of Σ. This means that for some choice of the feedback control $\alpha(x)$ the trajectories of Σ which start in M stay in M for all t in a neighborhood of $t_o = 0$ while the corresponding output is identically zero.

A submanifold M is said to be an integral submanifold of a distribution Δ if for every $x \in M$ and the tangent space T_xM to M at x one has $T_xM = \Delta(x)$. The maximal locally controlled invariant output-zeroing submanifold for a system Σ can be determined by the following *zero dynamics algorithm*:

Let U_o be a neighborhood of x_o and

1. $M_o = h^{-1}(0) \cap U_o$,
2. assume that M_k is a submanifold through x_o and define M_{k+1} as

$$M_{k+1} = \{x \in M_k \,|\, f(x) \in \operatorname{span}\{g_i(x)\} + T_x M_k\}.$$

If there is a U_o such that M_k is a smooth submanifold through x_o for each $k \geq 0$, then x_o is called a regular point of the algorithm and there is a k^* such that $M_{k^*+l} = M_{k^*}$ for all $l \geq 0$. If, in addition,

$$\dim \operatorname{span}\{\, g_i(x_o) \,|\, i = 1, m\} = m, \tag{26}$$

and

$$\dim \operatorname{span}\{\, g_i(x) \,|\, i = 1, m\} \cap T_x M^c_{k^*}$$

is constant for all $x \in M^c_{k^*}$, then the maximal connected component of M_{k^*}, denoted by Z^*, is the locally maximal output-zeroing submanifold of Σ. Moreover, if

$$\dim \operatorname{span}\{\, g_i(x) \,|\, i = 1, m\} \cap T_x M^c_{k^*} = 0, \tag{27}$$

then there is a unique smooth feedback control α^* such that $f^*(x) := f(x) + g(x)\alpha^*(x)$ is tangent to Z^*.

An algorithm for computing Z^* in general cases can be found in (Isidori, 1995) and (Nijmeijer and van der Schaft, 1991). However, in some cases Z^* can be determined easily by relating it to the maximal controlled invariant distribution Δ^* contained in $\ker(\mathrm{d}h)$, given by the following controlled invariant codistribution algorithm (CIcDA):

$$\Omega_1 = \operatorname{span}\{\mathrm{d}h_i \,|\, i = 1, p\}$$

$$\Omega_{k+1} = \Omega_k + L_f(\Omega_k \cap g^\perp) + \sum_{i=1}^{m} L_{g_i}(\Omega_k \cap g^\perp). \tag{28}$$

Moreover, $\Delta^* = \Omega_*^\perp$.

Theorem 1. *(Isidori, 1995) Suppose that x_o is a regular point regarding the controlled invariant codistribution algorithm and $\dim g(x_o) = m$. Also suppose that*

$$L_{g_i}(\Omega_k \cap g^\perp) \subset \Omega_k$$

for all $k \geq 0$. Then, for all x in a neighborhood of x_o, one has

$$\Delta^*(x) = T_x Z^*.$$

Remark 4. Conditions of Theorem 1 are trivially satisfied for linear systems, therefore, $\mathcal{V}^* = Z^*$, which provides the result of Proposition 10 in a straightforward way.

4.1. Nonlinear Systems with a Vector Relative Degree

The conditions of Theorem 1 are satisfied for nonlinear systems having a vector relative degree. A multivariable nonlinear system is said to have a vector relative degree $r = \{r_1, \cdots, r_p\}$ at a point x_o if

$$L_{g_j} L_f^k h_i(x) = 0 \tag{29}$$

for $j = 1, \ldots, m$ and $i = 1, \ldots, p$ for all $k < r_i - 1$, assuming that the matrix

$$A(x) := \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x) \\ \cdots & \cdots & \cdots \\ L_{g_1} L_f^{r_p-1} h_p(x) & \cdots & L_{g_m} L_f^{r_p-1} h_p(x) \end{bmatrix}$$

is nonsingular at $x = x_o$ or, equivalently,

$$\operatorname{rank} A(x_o) = m. \tag{30}$$

If condition (30) does not hold but there exist numbers r_i satisfying the property (29), then r_i are called the *relative orders* of the system (25).

Remark 5. It is easily seen that for linear systems represented in the form $\dot{x} = Ax + Bu$, $y = Cx$, the conditions (29) and (30) inherently include the condition (6) since, in this case, we write $f(x) = Ax$, $g(x) = B$, $h(x) = Cx$, which implies $L_f^k h(x) = CA^k x$ and therefore $L_g L_f^k h(x) = CA^k B$.

Lemma 1. *Let us suppose that the system (25) has a relative degree. Then the row vectors*

$$\mathrm{d}h_1(x_o), \ldots, \mathrm{d}L_f^{r_1-1} h_1(x_o), \ldots, \mathrm{d}h_p(x_o),$$
$$\ldots, \mathrm{d}L_f^{r_p-1} h_p(x_o)$$

are linearly independent.

Remark 6. From the proof of the lemma, see (Isidori, 1995), it is clear that (30) constitutes a necessary condition, i.e., the existence of the finite relative orders alone does not ensure linear independence of the entire system.

Remark 7. Since for any real valued function λ there holds $\mathrm{d}L_f \lambda(x) = L_f \mathrm{d}\lambda(x)$ and, by the algorithm CIcDA one has that all the codistributions $\mathrm{d}L_f^k h_i(x)$, satisfying the property $L_{g_j} L_f^k h_i(x) = 0$, are contained in Ω_*, i.e., in $\Delta^{*\perp}$, it follows that $\Delta^* \subset \operatorname{span}\{\mathrm{d}L_f^k h_i \,|\, k = 0, r_i - 1,\ i = 1, p\}^\perp$.

The conditions (26) and (27) can be interpreted as a special property of the *invertibility* of the system (25). Our interest in the determination of the output-zeroing manifold is motivated by the role played by these issues in the principle of invertibility and the construction of the reduced order inverse of linear and nonlinear controlled systems.

As has been already stated, if $\operatorname{rank} A(x) = m$, then

$$Z^* = \{x \mid L_f^k h_i = 0,\ i = 1,\dots,p \quad \text{and} \quad k = 0,\dots,r_i - 1\} \tag{31}$$

and

$$\Delta^{*\perp} = \ker \operatorname{span}\{\mathrm{d}L_f^k h_i,\ i = 1,\dots,p \quad \text{and} \quad k = 0,\dots,r_i - 1\}, \tag{32}$$

see also (Nijmeijer and van der Schaft, 1991). Moreover, the control feedback $u^*(x) = \alpha(x)$ is the solution of the equation

$$A(x)\alpha(x) + B(x) = 0 \tag{33}$$

by using the notation

$$B(x) := \begin{bmatrix} L_f^{r_1} h_1(x) \\ \vdots \\ L_f^{r_p} h_p(x) \end{bmatrix}.$$

Let $\xi = \Xi(x)$ denote the diffeomorphism formed by $(\xi^i)_{i=1,p}$ defined by $\xi^i = (L_f^k h_i(x))_{k=0,r_i-1}$. It is a standard computation that

$$\dot{\xi}^i = A^i \xi^i + B^i y_i^{(r_i)}, \tag{34}$$

where A^i and B^i are of the Brunowsky canonical form. Let us note that $\xi_1^i = y_i$.

Let us complete $\Xi(x)$ to be a diffeomorphism

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \Phi(x) := \begin{bmatrix} \Xi(x) \\ \Lambda(x) \end{bmatrix} \tag{35}$$

on X. Since $\partial_x \Xi = [\mathrm{d}L_f^k h_i]$, one has

$$\dot{\xi} = [\mathrm{d}L_f^k h_i] f|_{\Phi^{-1}} + [\mathrm{d}L_f^k h_i] g|_{\Phi^{-1}} u, \tag{36}$$

i.e., maintaining the nonzero rows

$$[\dot{\xi}^i_{r_i}] = B|_{\Phi^{-1}} + A|_{\Phi^{-1}} u \tag{37}$$

and

$$\dot{\eta} = \partial_x \Lambda f|_{\Phi^{-1}} + \partial_x \Lambda g|_{\Phi^{-1}} u. \tag{38}$$

The zero dynamics can be obtained by

$$\dot{\eta} = \partial_x \Lambda f|_{\Phi^{-1}} + \partial_x \Lambda g \alpha|_{\Phi^{-1}}, \tag{39}$$

putting $\xi = 0$. If g is involutive, then one can choose $\mathrm{d}\Lambda \subset g^\perp$, and then

$$\dot{\eta} = \partial_x \Lambda f|_{\Phi^{-1}}. \tag{40}$$

Example 2. Let us consider the following nonlinear input affine system subject to multiple faults:

$$\begin{aligned} \dot{x} &= f_o(x) + \sum_{i=1}^{m} f_i(x) u_i + \sum_{l=1}^{q} g_\ell(x) \nu_\ell, \\ y_j &= h_j(x), \quad j = 1,\dots,p. \end{aligned} \tag{41}$$

If one considers $f(x,u) = f_o(x) + \sum_{i=1}^m f_i(x) u_i$, then, by introducing time t as an auxiliary state, one may apply the results of the previous section to the augmented system.

The decoupling matrix A will also depend on the control inputs u and, similarly, on its derivatives, i.e., the condition for having a vector relative degree will also be dependent on the inputs. This is in contrast to the LTI case, where the inputs u do not play any role in the problem solvability.

Consider the system (41) determined by the functions

$$f_o(x) = f(x) = \begin{bmatrix} x_2 \\ 0 \\ x_1 x_4 \\ -1.2 x_3 \\ x_1 \end{bmatrix}, \quad g_1(x) = \begin{bmatrix} 1 \\ -x_2 \\ 0 \\ -x_4 \\ 1 \end{bmatrix}, \tag{42}$$

$$g_2(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ x_1 \\ -x_5 \end{bmatrix}, \quad h_1(x) = x_1, \quad h_2(x) = x_3, \tag{43}$$

i.e., for the sake of the greatest possible simplicity, we consider an autonomous system subject to failure modes ν_1 and ν_2. Then

$$A(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B(x) = \begin{bmatrix} x_2 \\ x_1 x_4 \end{bmatrix}. \tag{44}$$

Let us define the diffeomorphism

$$\Phi(x) = \begin{bmatrix} x_1 \\ x_3 \\ x_2 \\ x_4 \\ x_5 \end{bmatrix},$$

with

$$\partial_x \Phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{45}$$

It follows that

$$\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} = \partial_x \Phi(f + g\nu)|_{\Phi^{-1}}$$

$$= \begin{bmatrix} 0 \\ -1.2\xi_2 \\ \xi_1 \end{bmatrix} + \begin{bmatrix} -\eta_1 \\ -\eta_2 \\ 1 \end{bmatrix} \nu_1 + \begin{bmatrix} 0 \\ \xi_1 \\ -\eta_3 \end{bmatrix} \nu_2$$

and $y = \xi$. One can obtain the inverse system by using the relation

$$\nu = - \begin{bmatrix} \eta_1 \\ \xi_1 \eta_2 \end{bmatrix} + \dot{y}. \tag{46}$$

From $\xi = y$, one get the failure modes

$$\nu = \begin{bmatrix} \dot{y}_1 - \eta_1 \\ \dot{y}_2 - y_1 \eta_2 \end{bmatrix} \tag{47}$$

with

$$\dot{\eta} = \begin{bmatrix} 0 \\ -1.2y_2 \\ y_1 \end{bmatrix} + \begin{bmatrix} -\eta_1 \\ -\eta_2 \\ 1 \end{bmatrix} (\dot{y}_1 - \eta_1)$$

$$+ \begin{bmatrix} 0 \\ y_1 \\ -\eta_3 \end{bmatrix} (\dot{y}_2 - y_1 \eta_2).$$

For a general nonlinear system which cannot be represented in the form of (25), the question of the existence and computation of the codistribution Δ^* is far form being trivial. Moreover, the computation of the state transformation map that is necessary to determine the zero dynamics involves, in general, the integration of partial differential equations. Therefore, the general treatment of the problem in the framework of geometric nonlinear systems theory is not often computationally tractable and some useful progress requires an intermediate level of complexity.

Linear parameter varying (LPV) modelling techniques have proven to be useful in this application domain. The idea is that a lot of nonlinear systems can be converted into a quasi-linear form, obtaining the so-called quasi-linear parameter varying (qLPV) system models in which the state matrix depends affinely on a parameter vector. These classes of systems subjected to faults can be described as

$$\dot{x}(t) = A(\rho)x(t) + B(\rho)u(t) + \sum_{j=1}^{m} L_j(\rho)\nu_j(t), \tag{48}$$

$$y(t) = Cx(t),$$

with

$$A(\rho) = A_o + \rho_1 A_1 + \cdots + \rho_N A_N,$$

$$B(\rho) = B_o + \rho_1 B_1 + \cdots + \rho_N B_N,$$

$$L_j(\rho) = L_{j,o} + \rho_1 L_{j,1} + \cdots + \rho_N L_{j,N},$$

where the ρ_i's are time-varying parameters for the LPV case and parameters that depend on measurable outputs for the qLPV case, respectively (Bokor and Balas, 2004). It is assumed that each parameter ρ_i and its derivatives range between known extremal values. Let us denote by $\mathcal{P}$ this parameter set.

To apply the ideas presented in the previous sections to the systems (48), it is necessary to introduce the parameter varying counterpart of the invariant subspace $\mathcal{V}^*$.

Definition 3. Let $\mathcal{B}(\rho)$ denote Im $B(\rho)$. Then a subspace $\mathcal{V}$ is called a parameter-varying (A, B)-invariant subspace (or, briefly, the (A, B)-invariant subspace) if for all $\rho \in \mathcal{P}$ one has \$, $A(\rho)\mathcal{V} \subset \mathcal{V} + \mathcal{B}(\rho)$.\$,

The set of all parameter varying (A, B)-invariant subspaces containing a given subspace $\mathcal{C}$ is an upper semilattice with respect to the intersection of subspaces. This semilattice admits a maximum, denoted by

$$\mathcal{V}^* = \max \mathcal{V}(A(\rho), B(\rho), \mathcal{C}).$$

This subspace can be computed by a finite algorithm for systems of the type (48), see, e.g., (Balas *et al.*, 2003) and (Szabó *et al.*, 2003) for details. Using this subspace, the computation of the inverse system can be done following the same steps as in the LTI case.

5. Conclusions

In this paper the fault detection and isolation problem has been studied in view of the fault reconstruction process by means of dynamic system inversion. Along the discussion of this problem, linear time invariant as well as input affine nonlinear systems with stable zero dynamics were considered. It was shown that the detector relying on the inverse representation of the original system reconstructs the failure modes at its output on the basis of the standard input/output (sometimes state variable) measurements. The paper was devoted to the exposition of geometrical properties of the inverse and attempted to provide a better understanding of the conditions of the inversion procedure with a special focus on the aspects of fault detection and isolation. A procedure for the construction of the inverse system based on the concept of invariant subspaces and on the related coordinate transformations was given. It was shown that the solution methods obtained for nonlinear problems can be directly applied to the linear framework and the linear solutions can be viewed as special cases of the nonlinear ones. The procedure resulted in a minimal dimensional inverse system supposing that (i) it is given in the state space form, (ii) the representation has a relative degree and (iii) the representation is left invertible. The availability of state variable measurements (in certain cases, a direct access to derivatives) is assumed. Considering the recent progress of advanced measurements technology and the wide availability of sensors capable of providing the derivatives of a measured variable (see, e.g., some applications in aviation technology), this condition is not difficult to satisfy.

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