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OBSERVER–BASED FAULT–TOLERANT CONTROL AGAINST SENSOR FAILURES FOR FUZZY SYSTEMS WITH TIME DELAYS

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This paper addresses the problems of robust fault estimation and fault-tolerant control for Takagi–Sugeno (T–S) fuzzy systems with time delays and unknown sensor faults. A fuzzy augmented state and fault observer is designed to achieve the system state and sensor fault estimates simultaneously. Furthermore, based on the information of on-line fault estimates, an observer-based dynamic output feedback fault-tolerant controller is developed to compensate for the effect of faults by stabilizing the resulting closed-loop system. Sufficient conditions for the existence of both a state observer and a fault-tolerant controller are given in terms of linear matrix inequalities. A simulation example is given to illustrate the effectiveness of the proposed approach.

Keywords: fuzzy time-delay systems, sensor faults, state observer, fault-tolerant control, linear matrix inequalities, stability analysis.

1. Introduction

During the past years, the Takagi–Sugeno (T–S) fuzzy model has attracted a lot of attention since it is a universal approximation of any smooth nonlinear system (Takagi *et al.*, 1985; Boukezzoula *et al.*, 2007). A common practice is as follows: First, this fuzzy model is described by a family of fuzzy IF-THEN rules which represent local linear input-output relations of a nonlinear system. The overall fuzzy model of the system is achieved by smoothly blending these local linear models together through membership functions. Based on the T–S fuzzy models and by taking full advantage of modern linear theory, extensive results have been presented for investigating uncertain nonlinear systems (Zhou *et al.*, 2002; Tanaka *et al.*, 1992; 2001; 1998; Miguel *et al.*, 2005; Dong *et al.*, 2008) or uncertain nonlinear systems with time delays (Liu *et al.*, 2003; Cao *et al.*, 2000; 2001; Lin *et al.*, 2006; Chen *et al.*, 2005).

Although great developments have been observed for fuzzy controller design based on fuzzy models, the above-mentioned control approaches all assume that all components are in good operating conditions. As we know, some actuator or sensor faults often occur in the real process, which can degrade the control performances and even result in the instability of control systems. It is thus important to develop a reliable control scheme against actuator or sensor failures.

To handle the problem of fuzzy systems with actuator faults, several robust reliable fuzzy control design approaches have been developed (Wu *et al.*, 2007; Gassara *et al.*, 2008; Chen *et al.*, 2004). The actuator faults addressed in these approaches are assumed to be bounded and without fault detection or estimation. Based on the passive FTC idea, fuzzy fault-tolerant controllers against actuator faults are proposed by Wu *et al.* (2004; 2010) and Tong *et al.* (2008). However, the issues of fault detection and estimation are not involved either.

Recently, dynamic output feedback fault-tolerant controllers have been developed by Shi *et al.* (2009), Gao *et al.* (2010) and Zhang *et al.* (2010) for T–S fuzzy systems with actuator faults in which a fuzzy augmented fault observer is proposed to yield fault estimates and, based on the information of on-line fault estimates, observer-based output feedback fault-tolerant controllers are designed. However, the proposed fuzzy fault-tolerant control approaches do not consider fuzzy systems with sensor faults, and with a restrictive assumption on the faults, i.e., $f(t) \in L_2[0.\infty)$. Mao *et al.* (2007), Gao *et al.* (2008) and Nguang *et al.* (2007) investigate the problem of sensor fault estimation for T–S fuzzy models via designing a descriptor augmented state observer. The design approaches discuss only sensor fault estimation or detection without considering the problem of fault-tolerant controller design or time delays. It should be mentioned that time delays often exist in real engineering systems, such as chemical reactors, recycled storage tanks, wind tunnels, cold rolling mills, robotic systems, etc. A time delay may destroy the stability of a control system or degrade its performance, and therefore stability analysis and robust control design for fuzzy systems with time delays are important in theory and applications.

Based on the above works, this paper further investigates the issues of fault estimation and fault-tolerant controllers for T–S fuzzy time-delay systems with unbounded sensor faults or output disturbances. A fuzzy augmented state and fault estimation observer is designed to achieve state and fault estimates simultaneously. Furthermore, based on the information of on-line fault estimates, an observer-based dynamic output feedback fault-tolerant controller is developed to compensate for the effect of faults by stabilizing the closed-loop system. Moreover, sufficient conditions for the existence of both a state observer and a fault tolerant controller are given in terms of Linear Matrix Inequalities (LMIs), and the stability of the resulting control system is proved by the Lyapunov function method.

2. Problem statement

The T–S fuzzy model is described by the following fuzzy IF-THEN rules, which can characterize a class of the nonlinear systems. The i-th rule of the T–S fuzzy model is of the following form:

Plant Rule i:

IF $z_1(t)$ is M_{i1} and $z_2(t)$ is M_{i2} and $\cdots$ and $z_q(t)$ is M_{iq}, THEN

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[A_i x(t) \\ \qquad + A_{1i}x(t-\tau) + B_i u(t)\Big], \\ y(t) = \sum_{i=1}^{r} h_i(z(t))[C_i x(t) + w(t)], \end{cases} \tag{1}$$

where τ is a constant time-delay, $z(t) = [z_1(t) \cdots z_q(t)]$ is a premise variable vector, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $y(t) \in \mathbb{R}^p$ is the measurement output vector, $w(t) \in \mathbb{R}^p$ is the sensor fault vector (it may be unbounded). Here $\phi(t) \in \mathbb{R}^n$ is the initial state vector with $t \in [-\tau, 0]$. A_i , B_i and C_i are matrices of appropriate dimensions r is the number of IF-THEN rules and M_{ij} are fuzzy sets.

Assumption 1. Suppose that (A_i, B_i) is locally controllable and (A_i, C_i) is locally observable. The overall fuzzy model achieved by fuzzy blending of each individual plant rule (local model) is given by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[A_i x(t) \\ \qquad + A_{1i}x(t-\tau) + B_i u(t)\Big], \\ y(t) = \sum_{i=1}^{r} h_i(z(t))[C_i x(t) + w(t)], \end{cases} \tag{2}$$

where

$$\begin{aligned} h_i(z(t)) &= \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}, \\ w_i(z(t)) &= \prod_{j=1}^{q} M_{ij}(z_j(t)). \end{aligned} \tag{3}$$

It is assumed that

$$h_i(z(t)) \geq 0, \quad \sum_{i=1}^{r} h_i(z(t)) = 1. \tag{4}$$

3. Observer design and sensor fault estimation

This paper assumes that the state vector $x(t)$ is unavailable for measurement, and the sensor fault vector $w(t)$ is unknown. This section will construct a state and fault observer. Write (2) in the following modified form:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[A_i x(t) \\ \qquad + A_{1i}x(t-\tau) + B_i u(t)\Big], \\ y(t) = Cx(t) \\ \qquad + \sum_{i=1}^{r} h_i(z(t))(C_i - C)x(t) + w(t), \end{cases} \tag{5}$$

where C is any output matrix chosen from among $C_1, C_2, \ldots, C_r$.

By letting

$$w_0 = \sum_{i=1}^{r} h_i(z(t))(C_i - C)x(t) + w(t), \tag{6}$$

(5) can be expressed as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[A_i x(t) \\ \qquad + A_{1i}x(t-\tau) + B_i u(t)\Big], \\ y(t) = Cx(t) + w_0(t), \end{cases} \tag{7}$$

Construct the fuzzy augmented descriptor system as follows:

$$\begin{cases} \bar{E}\dot{\bar{x}}_0(t) = \sum_{i=1}^{r} h_i(z)\Big[\bar{A}_i\bar{x}_0(t) \\ \qquad + \bar{A}_{1i}\bar{x}_0(t-\tau) + \bar{B}_i u(t) + \bar{N}w_0(t)\Big], \\ y(t) = \bar{C}\bar{x}_0(t) = C^0\bar{x}_0(t) + w_0(t), \end{cases} \tag{8}$$

where

$$\begin{aligned} \bar{x}_0(t) &= \begin{pmatrix} x(t) \\ w_0(t) \end{pmatrix}, & \bar{E} &= \begin{pmatrix} I_n & 0 \\ 0 & 0 \end{pmatrix}, \\ \bar{A}_i &= \begin{pmatrix} A_i & 0 \\ 0 & -I_p \end{pmatrix}, & \bar{A}_{1i} &= \begin{pmatrix} A_{1i} & 0 \\ 0 & 0 \end{pmatrix}, \\ \bar{B}_i &= \begin{pmatrix} B_i \\ 0 \end{pmatrix}, & \bar{N} &= \begin{pmatrix} 0 \\ I_p \end{pmatrix}, \\ \bar{C} &= \begin{pmatrix} C & I_p \end{pmatrix}, & C^0 &= \begin{pmatrix} C & 0 \end{pmatrix}. \end{aligned} \tag{9}$$

The state and fault observer is designed as follows:

$$E_n\dot{\xi} = \sum_{i=1}^{r} h_i(z(t))\Big[A_{ni}\xi(t) + A_{1ni}\xi(t-\tau) + \bar{B}_i u(t)\Big], \tag{10}$$

$$\hat{\bar{x}}_0 = \xi(t) + K_n y, \tag{11}$$

$$\begin{bmatrix} \hat{x}(t) \\ \hat{w}(t) \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ \Delta(t) & I_p \end{bmatrix}^{-1} \hat{\bar{x}}_0(t), \tag{12}$$

where $\xi(t) \in \mathbb{R}^{n+p}$ is an auxiliary state vector, and $\hat{\bar{x}}_0(t) \in \mathbb{R}^{n+p}$ is the estimate of

$$\bar{x}_0(t) = \begin{bmatrix} x(t) \\ w_0(t) \end{bmatrix} \in \mathbb{R}^{n+p},$$

E_n, $A_{ni} \in \mathbb{R}^{(n+p)\times(n+p)}$ and $K_n \in \mathbb{R}^{(n+p)\times p}$ are design matrices, and $\Delta(t) = \sum_{i=1}^{r} h_i(t)(C_i - C)$.

By substituting $\xi(t) = \hat{\bar{x}}_0(t) - K_n y(t)$ into (10), we obtain

$$\begin{aligned} &E_n\dot{\hat{\bar{x}}}_0(t) - E_nK_n\bar{C}\dot{\bar{x}}_0(t) \\ &= \sum_{i=1}^{r} h(z(t))\Big\{A_{ni}[\hat{\bar{x}}_0(t) - K_n\bar{C}\bar{x}_0(t) - K_n w(t)]\Big\} \\ &+ \sum_{i=1}^{r} h(z(t))\Big\{A_{1ni}[\hat{\bar{x}}_0(t-\tau) \\ &- K_n\bar{C}\bar{x}_0(t-\tau)] + \bar{B}_i u(t)\Big\}. \end{aligned} \tag{13}$$

Subtracting (13) from (8) yields

$$\begin{aligned} &(\bar{E} + E_nK_n\bar{C})\dot{\bar{x}}_0(t) - E_n\dot{\hat{\bar{x}}}_0(t) \\ &= \sum_{i=1}^{r} h(z(t))\Big[\bar{A}_i + A_{ni}K_nC^0)\bar{x}_0(t) \\ &- A_{ni}\bar{x}_0(t)\Big] \\ &+ \sum_{i=1}^{r} h(z(t))\Big[\bar{A}_{1i} + A_{1ni}K_n\bar{C})\bar{x}_0(t-\tau) \\ &- \bar{A}_{1ni}\bar{x}_0(t-\tau)\Big] \\ &+ \sum_{i=1}^{r} h(z(t))\Big[\bar{N}w(t) + A_{ni}K_n w(t)\Big]. \end{aligned} \tag{14}$$

Let $\bar{e}(t) = \bar{x}_0(t) - \hat{\bar{x}}_0(t)$, and suppose that

$$\begin{cases} \overline{A_i} + A_{ni}K_nC^0 = A_{ni}, \\ \bar{N} = -A_{ni}K_n, \\ \overline{E} + E_nK_n\bar{C} = E_n, \\ \overline{A_{1i}} + A_{1ni}K_n\bar{C} = A_{1ni}. \end{cases} \tag{15}$$

From (15), we get the error dynamics

$$E_n\dot{\bar{e}}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[A_{ni}\bar{e}(t) + A_{1ni}\bar{e}(t-\tau)\Big]. \tag{16}$$

From (15), we conclude that

$$\begin{aligned} A_{ni} &= \begin{pmatrix} A_i & 0 \\ -C & -I_p \end{pmatrix}, & A_{1ni} &= \begin{pmatrix} A_{1i} & 0 \\ FC & F \end{pmatrix}, \\ K_n &= \begin{pmatrix} 0 \\ I_p \end{pmatrix}, & E_n &= \begin{pmatrix} I & 0 \\ MC & M \end{pmatrix}, \end{aligned} \tag{17}$$

where F is a full-rank matrix.

By using (17), the error dynamics (16) can be written as

$$\begin{aligned} \dot{\bar{e}}(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[E_n^{-1}A_{ni}\bar{e}(t) + E_n^{-1}A_{1ni}\bar{e}(t-\tau)\Big] \\ &= \sum_{i=1}^{r} h_i(z(t))\Big[A_{i*}\bar{e}(t) + A_{1i*}\bar{e}(t-\tau)\Big], \end{aligned} \tag{18}$$

where

$$\begin{aligned} A_{i*} &= \begin{pmatrix} A_i & 0 \\ -CA_i - M^{-1}C & -M^{-1} \end{pmatrix}, \\ A_{1i*} &= \begin{pmatrix} A_{1i} & 0 \\ -CA_{1i} + M^{-1}FC & M^{-1}F \end{pmatrix}. \end{aligned} \tag{19}$$

Theorem 1. *If there exist matrices $P > 0$ and $R > 0$, such that*

$$\Gamma = \begin{bmatrix} A_{i*}^T P + PA_{i*} + R & PA_{1i*} \\ * & -R \end{bmatrix} < 0, \tag{20}$$

then the state observer in the form of (10)–(12) can asymptotically estimate the states and sensor faults.

Proof. Consider the Lyapunov function candidate as

$$V(\bar{e}(t)) = \bar{e}^T(t)P\bar{e}(t) + \int_{t-\tau}^{t} \bar{e}^T(\sigma)R\bar{e}(\sigma)\,\mathrm{d}\sigma. \quad (21)$$

The time derivative of $V(\bar{e}(t))$ along the trajectory of (18) is

$$\begin{aligned}\dot{V}(\bar{e}(t)) &= \sum_{i=1}^{r} h(z(t))[\bar{e}^T(t)(A_{i*}^T P + RA_{i*})\bar{e}(t) \\ &\quad + 2\bar{e}^T(t)PA_{1i*}\bar{e}(t-\tau) \\ &\quad + \sum_{i=1}^{r} h(z(t))[\bar{e}^T(t)R\bar{e}(t) \\ &\quad - \bar{e}^T(t-\tau)R\bar{e}(t-\tau) \\ &= \sum_{i=1}^{r} h(z(t))\xi^T(t)\Gamma\xi(t),\end{aligned} \quad (22)$$

where

$$\xi^T(t) = \begin{bmatrix}\bar{e}^T(t) & \bar{e}^T(t-\tau)\end{bmatrix}^T,$$
$$\Gamma = \begin{bmatrix} A_{i*}^T P + PA_{i*} + R & PA_{1i*} \\ * & -R \end{bmatrix}.$$

■

Based on Theorem 1, if $\Gamma < 0$, we see that $\dot{V}(\bar{e}(t)) < 0$. Therefore, there exists a fuzzy state observer in the form of (10)–(12) to estimate the state and sensor faults asymptotically. On the other hand, we can obtain simultaneous estimates of $x(t)$, $w_0(t)$ and

$$\lim_{t\to\infty}\left(\begin{bmatrix} x(t) \\ w_0(t)\end{bmatrix} - \hat{\bar{x}}_0(t)\right) = 0.$$

Since (4) indicates that $h_i(z(t))$ is bounded for $i = 1,\dots,r$, it follows that

$$\lim_{t\to\infty}\begin{bmatrix} I & 0 \\ \Delta(t) & I\end{bmatrix}^{-1}\left(\begin{bmatrix} x(t) \\ w_0(t)\end{bmatrix} - \hat{\bar{x}}_0(t)\right) = 0 \quad (23)$$

By using (6) and the third equation of (12), we deduce that

$$\begin{aligned}&\begin{bmatrix} x(t) \\ w(t)\end{bmatrix} - \begin{bmatrix} \hat{x}(t) \\ \hat{w}(t)\end{bmatrix} \\ &= \begin{bmatrix} x(t) \\ w(t)\end{bmatrix} - \begin{bmatrix} I & 0 \\ \Delta(t) & I\end{bmatrix}^{-1}\hat{\bar{x}}_0(t) \\ &= \begin{bmatrix} I & 0 \\ \Delta(t) & I\end{bmatrix}^{-1}\begin{bmatrix} I & 0 \\ \Delta(t) & I\end{bmatrix}\begin{bmatrix} x(t) \\ w(t)\end{bmatrix} \\ &\quad - \begin{bmatrix} I & 0 \\ \Delta(t) & I\end{bmatrix}^{-1}\hat{\bar{x}}_0(t) \\ &= \begin{bmatrix} I & 0 \\ \Delta(t) & I\end{bmatrix}^{-1}\left\{\begin{bmatrix} x(t) \\ w(t)\end{bmatrix} - \hat{\bar{x}}_0(t)\right\}.\end{aligned} \quad (24)$$

By using (23) and (24), we immediately get

$$\lim_{t\to\infty}\left(\begin{bmatrix} x(t) \\ w(t)\end{bmatrix} - \begin{bmatrix} \hat{x}(t) \\ \hat{w}(t)\end{bmatrix}\right) = \lim_{t\to\infty}[\bar{x}(t) - \hat{\bar{x}}(t)] = 0, \quad (25)$$

i.e., the proposed state and fault observer (10)–(12) can asymptotically estimate the states and sensor faults simultaneously. In the sequel, as an application, the state observer (10)–(12) is applied to fault estimation.

Case 1: Sensor fault estimation.

Consider the following fuzzy system with sensor faults:

$$\begin{cases}\dot{x}(t) = \sum_{i=1}^{r} h_i(z)[A_i x(t) \\ \qquad + A_{1i}x(t-\tau) + B_i u(t)], \\ y(t) = \sum_{i=1}^{r} h_i(z)C_i x(t) + D_s f_s(t),\end{cases} \quad (26)$$

where $f_s(t) \in \mathbb{R}^k$ is the sensor fault, $D_s \in \mathbb{R}^{p\times k}$ is a full column matrix, and the other symbols are defined as before. By letting

$$w(t) = D_s f_s(t), \quad (27)$$

using the state observer (10)–(12), we can obtain the estimate of $w(t)$. Furthermore, the estimate of $f_s(t)$ can be obtained as follows:

$$\begin{aligned}\hat{f}_s(t) &= (D_s^T D_s)^{-1}D_s^T\begin{bmatrix} 0_{p\times n} & I_p\end{bmatrix} \\ &\quad \times \begin{bmatrix} I & 0 \\ -\Delta(t) & I\end{bmatrix}\hat{\bar{x}}_0(t),\end{aligned} \quad (28)$$

where $\hat{\bar{x}}_0(t)$ is the augmented state estimate vector defined in (10).

Case 2: Sensor disturbance and fault estimation.

Consider the following fuzzy system both with sensor faults and sensor disturbances:

$$\begin{cases}\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[A_i x(t) \\ \qquad + A_{1i}x(t-\tau) + B_i u(t)\Big], \\ y(t) = \sum_{i=1}^{r} h_i(z(t))C_i x(t) \\ \qquad + D_d d(t) + D_s f_s(t),\end{cases} \quad (29)$$

where $f_s(t) \in \mathbb{R}^k$ is the measurement fault, $d(t) \in \mathbb{R}^l$ is the measurement noise, and the other symbols are defined as before. By letting

$$\begin{aligned} w(t) &= D_d d(t) + D_s f_s(t) \\ &= \begin{bmatrix} D_d & D_s \end{bmatrix} \begin{bmatrix} d(t) \\ f_s(t) \end{bmatrix}, \end{aligned} \tag{30}$$

if $D_{ds} = \begin{bmatrix} D_d & D_s \end{bmatrix} \in \mathbb{R}^{p\times(k+l)}$ is full column, the estimate of the sensor fault $w(t)$ via the observer (10) can be obtained as follows:

$$\begin{aligned} \hat{f}_s(t) &= \begin{bmatrix} 0_{l\times k} & I_k \end{bmatrix} (D_{ds}^T D_s)^{-1} D_{ds}^T \\ &\quad \times \begin{bmatrix} 0_{p\times n} & I_p \end{bmatrix} \begin{bmatrix} I & 0 \\ -\Delta(t) & I \end{bmatrix} \hat{\bar{x}}_0(t), \end{aligned} \tag{31}$$

where $\hat{\bar{x}}_0(t)$ is the augmented state estimate defined by (11). In the same way, the estimate of the sensor disturbance $d(t)$ via the state observer (10)–(12) can be obtained as

$$\begin{aligned} \hat{d}(t) &= \begin{bmatrix} I_l & 0_{l\times k} \end{bmatrix} (D_{ds}^T D_s)^{-1} D_{ds}^T \\ &\quad \times \begin{bmatrix} 0_{p\times n} & I_p \end{bmatrix} \begin{bmatrix} I & 0 \\ -\Delta(t) & I \end{bmatrix} \hat{\bar{x}}_0(t). \end{aligned} \tag{32}$$

4. Observer-based fault-tolerant controller design

In this section, a fault-tolerant fuzzy output feedback control design will be developed by using the state and fault observer (10)–(12), and sufficient conditions to guarantee the stability of the resulting closed-loop system will be given in the form of LMIs. Based on the Parallel Distributed Compensation (PDC) (Zhou *et al.*, 2002; Tanaka *et al.*, 1992), a fuzzy output feedback controller based on the state observer is designed as follows:

Control Rule i:

IF $z_1(t)$ is M_{i1} and $z_2(t)$ is M_{i2} and $\cdots$ and $z_q(t)$ is M_{iq}, THEN

$$u(t) = \bar{K}_i \hat{\bar{x}}(t). \tag{33}$$

The overall fuzzy controller is represented by

$$u(t) = \sum_{i=1}^{r} h_i(z(t)) \bar{K}_i \hat{\bar{x}}(t), \tag{34}$$

where $\bar{K}_i = [K_i \quad 0]$.

By using (33) and (34), (11) and (13) can be expressed as

$$\begin{cases} \bar{E}\dot{x}(t) = \displaystyle\sum_{i=1}^{r} h_i(z(t)) \Big[\tilde{A}_i \bar{x}(t) \\ \qquad + \bar{A}_{1i} x(t-\tau) + \bar{B}_i u(t) \Big], \\ y(t) = \bar{C}\bar{x}(t) = C^0 \bar{x}(t) + w(t), \end{cases} \tag{35}$$

where

$$\tilde{A}_i = \begin{pmatrix} A_i & 0 \\ 0 & 0 \end{pmatrix}.$$

Substituting (34) into (35), we conclude that

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) [G_{ij} x(t) \\ &\quad + A_{1i} x(t-\tau) - B_i \bar{K}_j \bar{e}(t)] \end{aligned} \tag{36}$$

where $G_{ij} = A_i + B_i K_j$.

Theorem 2. *Assume that there exist common matrices*

$$X > 0, \quad X_1 > 0, \quad \tilde{R} > 0, \quad \tilde{\bar{R}}_1 > 0, \quad \tilde{\bar{R}}_2 > 0$$

and some matrices M_i , $i = 1, 2, \ldots, r$ *such that.*

$$\Omega^1 = \begin{pmatrix} \tilde{\Theta}_{ii} & A_{1i}X & -B_iM_i & 0 & 0 & 0 \\ XA_{1i}^T & -\tilde{R} & 0 & 0 & 0 & 0 \\ -M_i^T B_i^T & 0 & XA_i^T + A_iX + \tilde{\bar{R}}_1 & -X(A_i+I)^T C^T & A_{1i}X & 0 \\ 0 & 0 & -C(A_i+I)X & -2X_1 + \tilde{\bar{R}}_2 & -C(A_{1i}-I)X & X_1 \\ 0 & 0 & XA_{1i}^T & -X(A_{1i}-I)^T C^T & -\tilde{\bar{R}}_1 & 0 \\ 0 & 0 & 0 & X_1 & 0 & -\tilde{\bar{R}}_2 \end{pmatrix}, \tag{37}$$

$$\Omega^2 = \begin{pmatrix} \tilde{\Theta}_{ij} + \tilde{\Theta}_{ji} & (A_{1i}+A_{1j})X & -(B_iM_j + B_jM_i) & 0 & 0 & 0 \\ * & -2\tilde{R} & 0 & 0 & 0 & 0 \\ * & * & \tilde{\Omega}_{33}^2 & \tilde{\Omega}_{34}^2 & (A_{1i}+A_{1j})X & 0 \\ * & * & * & -4X_1 + 2\tilde{\bar{R}}_2 & \tilde{\Omega}_{45}^2 & 2X_1 \\ * & * & * & * & -2\tilde{\bar{R}}_1 & 0 \\ * & * & * & * & * & -2\tilde{\bar{R}}_2 \end{pmatrix}, \tag{38}$$

where

$$\tilde{\Theta}_{ij} = XA_i^T + A_iX + M_j^T B_i^T + B_iM_j + \tilde{R},$$

$$\tilde{\Omega}_{33}^2 = X(A_i + A_j)^T + (A_i + A_j)X + 2\tilde{\bar{R}}_1,$$

$$\tilde{\Omega}_{34}^2 = -X(A_i + A_j + 2I)^T C^T,$$

$$\tilde{\Omega}_{45}^2 = -C(A_{1i} + A_{1j} - 2I)X,$$

$$X = P^{-1}, \quad X_1 = P_1^{-1}, \quad M_i = K_iX, \quad \tilde{R} = XRX,$$

$$\tilde{\bar{R}}_1 = X\bar{R}_1X, \quad \tilde{\bar{R}}_2 = X_1\bar{R}_2X_1.$$

Then the fuzzy system (36) is asymptotically stable.

Proof. Consider the Lyapunov function as

$$V = x^T P x + \int_{t-\tau}^{t} x^T R x \,\mathrm{d}\sigma + \bar{e}^T \bar{P} \bar{e} + \int_{t-\tau}^{t} \bar{e}^T \bar{R} \bar{e} \,\mathrm{d}\sigma. \tag{39}$$

The time derivative of V along the trajectories (35) and (36) is

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))[x^T(t)\\ &\quad \times (G_{ij}^T P + P G_{ij})x(t) + 2x^T(t) P A_{1i} x(t-\tau)\\ &\quad - 2x^T(t) P B_i \bar{K}_j \bar{e}(t)\\ &\quad + x^T R x - x^T(t-\tau) R x(t-\tau)\\ &\quad + \bar{e}^T(t)(A_{i*}\bar{P} + \bar{P}A_{i*})\bar{e}(t)\\ &\quad + 2\bar{e}^T(t)\bar{P}A_{1i*}\bar{e}(t-\tau)\\ &\quad + \bar{e}^T(t)\bar{R}\bar{e}(t) - \bar{e}^T(t-\tau)\bar{R}\bar{e}(t-\tau)]\\ &= \sum_{i=1}^{r} h_i^2(z(t))\eta^T \Gamma^1 \eta\\ &\quad + \sum_{i=1, i<j}^{r} h_i(z(t))h_j(z(t))\eta^T\Gamma^2\eta,\end{aligned} \tag{40}$$

where

$$\eta = [x^T(t) \quad x^T(t-\tau) \quad \bar{e}^T(t) \quad \bar{e}^T(t-\tau)]^T,$$

and

$$\Gamma^1 = \begin{pmatrix} \Theta_{ii} & PA_{1i} & -PB_i\bar{K}_i & 0\\ * & -R & 0 & 0\\ * & * & \Psi_i & \bar{P}A_{1i*}\\ * & * & * & -\bar{R}\end{pmatrix}, \tag{41}$$

$$\Gamma^2 = \begin{pmatrix} \Theta_{ij}+\Theta_{ji} & P(A_{1i}+A_{1j}) & -P(B_i\bar{K}_j + B_j\bar{K}_i) & 0\\ * & -2R & 0 & 0\\ * & * & \Psi_i+\Psi_j & \bar{P}(A_{1i*}+A_{1j*})\\ * & * & * & -2\bar{R}\end{pmatrix}, \tag{42}$$

with

$$\begin{aligned}\Theta_{ij} &= (A_i + B_iK_j)^T P + P(A_i + B_iK_j) + R,\\ \Psi_i &= A_{i*}^T\bar{P} + \bar{P}A_{i*} + \bar{R}.\end{aligned}$$

■

Suppose that

$$\bar{P} = \begin{pmatrix} P & \\ & P_1\end{pmatrix}, \quad \bar{R} = \begin{pmatrix} \bar{R}_1 & \\ & \bar{R}_2\end{pmatrix}.$$

By substituting $\bar{P}$, $\bar{R}$, A_{i*} and A_{1i*} into (41), (42), respectively, Γ^1 and Γ^2 can be expressed as

$$\Gamma^1 = \begin{pmatrix} \Theta_{ii} & PA_{1i} & -PB_iK_i & 0 & 0 & 0\\ * & -R & 0 & 0 & 0 & 0\\ * & * & A_i^TP + PA_i + \bar{R}_1 & -(A_i+I)^TC^TP_1 & PA_{1i} & 0\\ * & * & * & -2P_1 + \bar{R}_2 & -P_1C(A_{1i}-I) & P_1\\ * & * & * & * & -\bar{R}_1 & 0\\ * & * & * & * & * & -\bar{R}_2\end{pmatrix}, \tag{43}$$

$$\Gamma^2 = \begin{pmatrix} \Theta_{ij}+\Theta_{ji} & P(A_{1i}+A_{1j}) & -P(B_iK_j+B_jK_i) & 0 & 0 & 0\\ * & -2R & 0 & 0 & 0 & 0\\ * & * & \Omega_{33}^2 & \Omega_{34}^2 & P(A_{1i}+A_{1j}) & 0\\ * & * & * & -4P_1 + 2\bar{R}_2 & \Omega_{45}^2 & 2P_1\\ * & * & * & * & -2\bar{R}_1 & 0\\ * & * & * & * & * & -2\bar{R}_2\end{pmatrix}, \tag{44}$$

where

$$\begin{aligned}\Theta_{ij} &= (A_i + B_iK_j)^TP + P(A_i + B_iK_j) + R,\\ \Omega_{33}^2 &= (A_i + A_j)^TP + P(A_i + A_j) + 2\bar{R}_1,\\ \Omega_{34}^2 &= -(A_i + A_j + 2I)^TC^TP_1,\\ \Omega_{45}^2 &= -P_1C(A_{1i} + A_{1j} - 2I).\end{aligned}$$

If we suppose that

$$\Gamma^1 < 0, \quad \Gamma^2 < 0, \tag{45}$$

the conditions (45) sufficiently ensure that $\dot{V}(\bar{x}(t)) < 0$. Therefore, we can conclude that the fuzzy system (36) is asymptotically stable. Note that since the matrix in equalities $\Gamma^1 < 0$ and $\Gamma^2 < 0$ are not LMIs, from (45) we cannot find common stable matrices $X > 0$, $X_1 > 0$, $\tilde{R} > 0$, $\tilde{R}_1 > 0$, $\tilde{R}_2 > 0$ and control gain matrices K_i. Therefore, pre and post-multiplying $\Gamma^1 < 0$ and $\Gamma^2 < 0$ by $\mathrm{diag}\{\, P^{-1} \quad P^{-1} \quad P^{-1} \quad P_1^{-1} \quad P^{-1} \quad P_1^{-1} \,\}$, respectively, and using the Schur complement, we can obtain the LMIs (37) and (38) in Theorem 2, which are equivalent to $\Gamma^1 < 0$ and $\Gamma^2 < 0$, respectively. Therefore, by solving (37) and (38), we can obtain common matrices $X > 0$, $X_1 > 0$, $\tilde{R} > 0$, $\tilde{R}_1 > 0$, $\tilde{R}_2 > 0$ and control gain matrices K_i.

5. Simulation example

Consider a nonlinear system characterized by the following T–S fuzzy system:

Rule 1: If $y_1^2(t)$ is M_1 (small), then

$$\begin{cases} \dot{x}(t) = aA_1x(t) + (1-a)A_{11}x(t-\tau) \\ \qquad + B_1u(t), \\ y(t) = C_1x(t) + w. \end{cases} \tag{46}$$

Rule 2: If $y_1^2(t)$ is M_2 (big), then

$$\begin{cases} \dot{x}(t) = aA_2x(t) + (1-a)A_{12}x(t-\tau) \\ \qquad + B_2u(t), \\ y(t) = C_2x(t) + w. \end{cases} \tag{47}$$

In the afore-mentioned equations, $y_1(t) \in [0,1]$, and $a \in [0,1]$,

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -a & -2a & 0 \\ 2a & -a & 0 \\ a & 0 & -3a \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$A_{11} = \begin{bmatrix} -(1-a) & -2(1-a) & 0 \\ 2(1-a) & -(1-a) & 0 \\ 1-a & 0 & -3(1-a) \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2a & a & 0 \\ 0 & -0.5a & -a \\ a & 0 & -a \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} -2(1-a) & 1-a & 0 \\ 0 & -0.5(1-a) & -(1-a) \\ 1-a & 0 & -(1-a) \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

According to Gao *et al.* (2008) and from the maximal and minimal values, $y_1^2(t)$ can be represented by

$$y_1^2(t) = M_1 \times 0 + M_2 \times 1,$$

where the membership functions M_1 and M_2 satisfy $M_1 + M_2 = 1$. As a result, we have $M_1 = 1 - y_1^2(t)$ and $M_2 = y_1^2(t)$. From (3), we get

$$h_1 = \frac{w_1(t)}{\sum_{i=1}^{2} w_i(t)} = 1 - y_1^2(t),$$

$$h_2 = \frac{w_2(t)}{\sum_{i=1}^{2} w_i(t)} = y_1^2(t).$$

The final fuzzy system is then inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} h_i \Big[A_ix(t) + A_{1i}x(t-\tau) \\ \qquad + B_iu(t)\Big], \\ y(t) = \sum_{i=1}^{2} h_iC_ix(t) + w(t), \end{cases} \tag{48}$$

where $w(t)$ represents the sensor faults.

Note that if $a = 1$ in (48), the fuzzy system (48) becomes the fuzzy system discussed by Zhang *et al.* (2010).

Case 1: State and sensor fault estimation.

Given input $u(t) = \sin(t)$ and sensor faults as follows:

$$w(t) = \begin{bmatrix} f_{s1}(t) & f_{s2}(t) & f_{s3}(t) \end{bmatrix}^T,$$

in which

$$f_{s1}(t) = \sin t,$$

$$f_{s2}(t) = \begin{cases} 0.1\sin[5(t-3)], & t \geq 3, \\ 0, & t < 3, \end{cases}$$

$$f_{s3}(t) = \begin{cases} 0.01(t-4) + 0.2, & t \geq 4, \\ 0, & t < 4, \end{cases}$$

and choosing $M = I$ and $F = I$, construct the state observer (10)–(12) to estimate the state vector $x(t)$ and the sensor fault vector $w(t)$. In the simulation, choosing $a = 0.8$ and $\tau = 0.5$, the initial values of the state $x(t)$ and $\xi(t)$ are respectively chosen as

$$x(0) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$$

and

$$\xi(0) = \begin{bmatrix} 1 & 1 & 1 & -1 & 0 & -1 \end{bmatrix}^T.$$

The simulation results are shown in Figs.1–4, where Fig. 1 displays the trajectories of the observer errors $e_1 = x_1 - \hat{x}_1$, $e_2 = x_2 - \hat{x}_2$ and $e_3 = x_3 - \hat{x}_3$, while Figs. 2–4 show the trajectories of the sensor faults f_{s1}, f_{s2}, f_{s3} and their estimates $\hat{f}_{s1}$, $\hat{f}_{s2}$ and $\hat{f}_{s3}$, respectively. The simulation results in Figs. 1–4 clearly demonstrate that the accurate estimates of the state and sensor fault signals are achieved via the proposed state observer. The afore-mentioned accurate nonzero fault estimates automatically imply fault detection and diagnosis.

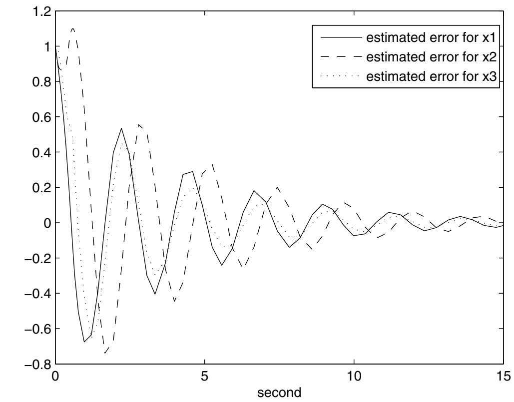

Fig. 1. State observer errors $e_1 = x_1 - \hat{x}_1$, $e_2 = x_2 - \hat{x}_2$ and $e_3 = x_3 - \hat{x}_3$.

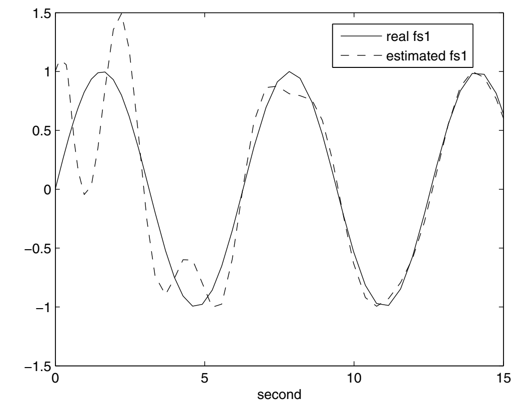

Fig. 2. Trajectories of the fault f_{s1} and its estimate $\hat{f}_{s1}$.

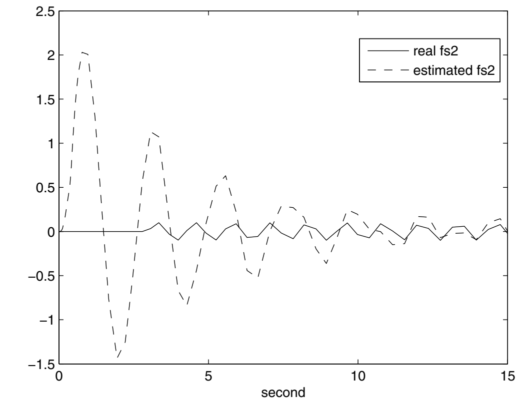

Fig. 3. Trajectories of the fault f_{s2} and its estimate $\hat{f}_{s2}$.

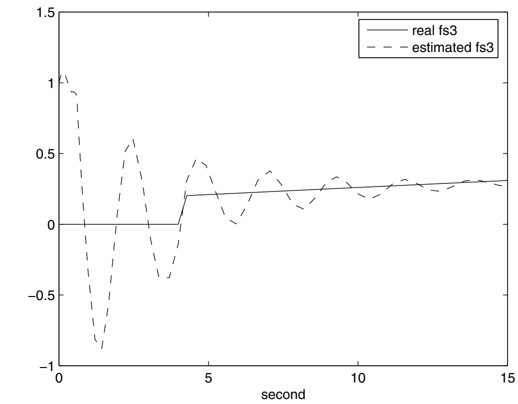

Fig. 4. Trajectories of the fault f_{s3} and its estimate $\hat{f}_{s3}$.

Case 2: Fault-tolerant control based on state observer.

According to Theorem 2 and by solving the LMIs (37) and (38), the common stable and control gain matrices are obtained as follows:

$$X = \begin{bmatrix} 0.1634 & 0.0368 & 0.0799 \\ 0.0368 & 0.6322 & 0.1129 \\ 0.0799 & 0.1129 & 0.4610 \end{bmatrix},$$

$$X_1 = \begin{bmatrix} 7.3110 & -0.1949 & -0.1112 \\ -0.1949 & 9.9525 & -0.2355 \\ -0.1112 & -0.2355 & 4.4435 \end{bmatrix},$$

$$\tilde{R} = \begin{bmatrix} 0.7044 & 0.5367 & 0.0395 \\ 0.5367 & 0.8470 & 0.1951 \\ 0.0395 & 0.1951 & 0.8415 \end{bmatrix},$$

$$\tilde{\tilde{R}}_1 = \begin{bmatrix} 0.0163 & -0.0129 & 0.0077 \\ -0.0129 & 0.0345 & 0.0351 \\ 0.0077 & 0.0351 & 0.1302 \end{bmatrix},$$

$$\tilde{\tilde{R}}_2 = \begin{bmatrix} 7.3110 & -0.1949 & -0.1112 \\ -0.1949 & 9.9525 & -0.2355 \\ -0.1112 & -0.2355 & 4.4435 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -13.0403 & -3.6426 & 2.5770 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -13.1169 & -3.6225 & 2.5894 \end{bmatrix}.$$

Choose the sensor faults and the initial values of the states $x(t)$ and $x_i(t)$ the same as in Case 1. The simulation results are shown in Figs. 5–9, where Fig. 5 displays the trajectories of the states x_1, x_2 and x_3, and Fig. 6 shows the state observer errors $e_1 = x_1 - \hat{x}_1$, $e_2 = x_2 - \hat{x}_2$ and $e_3 = x_3 - \hat{x}_3$. Figures 7–9 display the trajectories of the sensor faults f_{s1} , f_{s2} , f_{s3} and their estimates, and Fig. 10 shows the control u.

The simulation results show that the proposed observer-based fault-tolerant control approach can guarantee that the closed-loop system is asymptotically stable and the observer errors asymptotically converge to zero even though in the controlled fuzzy system there exist sensor faults and time delays.

6. Conclusions

In this paper, a robust fault-tolerant control method for T–S fuzzy systems with time delays and unknown sensor faults was given. First, a fuzzy augmented state and fault estimation observer was designed to produce system state and sensor fault estimates simultaneously. Furthermore, utilizing the information of on-line fault estimates, an observer-based dynamic output feedback fault tolerant controller was developed to compensate for the effect of faults by stabilizing the closed-loop system. Sufficient conditions for the existence of both a state observer and a fault tolerant controller were given in terms of linear matrix inequalities. A simulation example was presented to illustrate the effectiveness of the proposed approach.

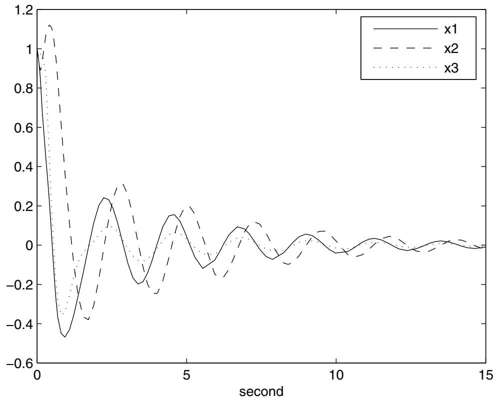

Fig. 5. Trajectories of the states x_1 , x_2 and x_3.

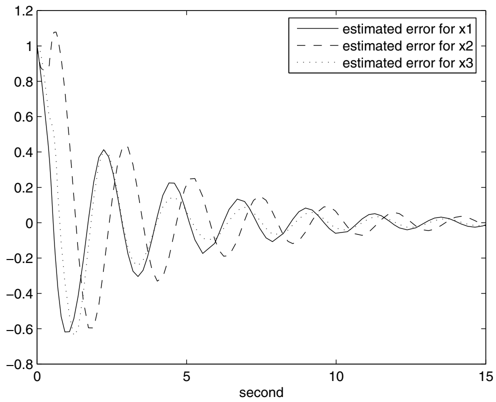

Fig. 6. Trajectories of the observer errors $e_1 = x_1 - \hat{x}_1$, $e_2 = x_2 - \hat{x}_2$ and $e_3 = x_3 - \hat{x}_3$.

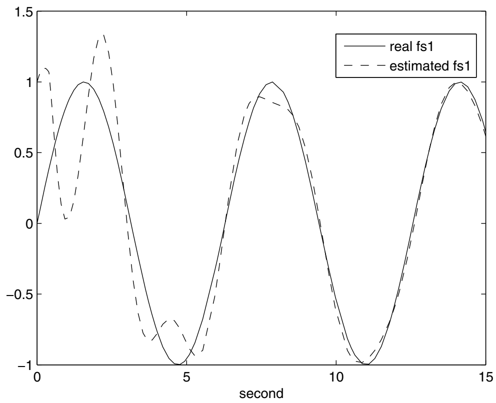

Fig. 7. Trajectories of the fault f_{s1} and its estimate $\hat{f}_{s1}$.

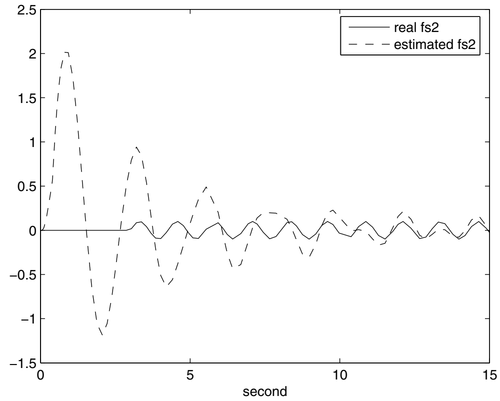

Fig. 8. Trajectories of the fault f_{s2} and its estimate $\hat{f}_{s2}$.

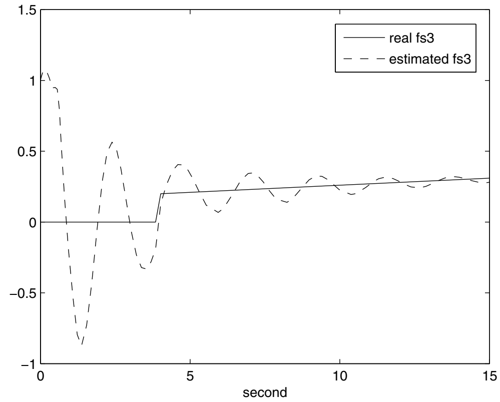

Fig. 9. Trajectories of the fault f_{s3} and its estimate $\hat{f}_{s3}$.

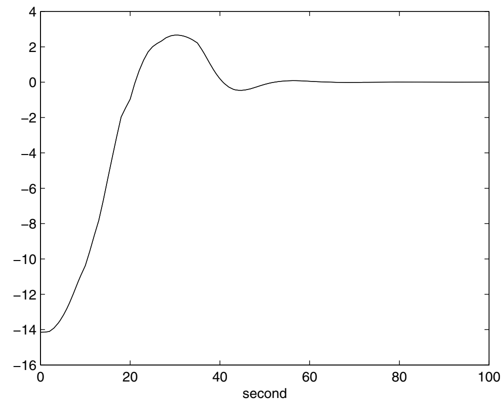

Fig. 10. Control u.

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References

Boukezzoula, R., Galichet, S. and Foulloy, L. (2007). Fuzzy feedback linearizing controller and its equivalence with the fuzzy nonlinear internal model control structure, *International Journal of Applied Mathematics and Computer Science* **17**(2): 233–248, DOI: 10.2478/v10006-007-0021-4.

Cao, Y.Y. and Frank, P.M. (2000). Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach, *IEEE Transactions on Fuzzy Systems* **8**(2): 200–211.

Cao, Y.Y. and Frank, P.M.(2001). Stability analysis and synthesis of nonlinear time-delay system via linear Takagi–Sugeno fuzzy models, *Fuzzy Sets and Systems* **124**(2): 213–229.

Chen, B. and Liu, X.P. (2005). Delay-dependent robust H^{∞} control for T–S fuzzy systems with time delay, *IEEE Transactions on Fuzzy Systems* **13**(4): 543–555.

Chen, B. and Liu, X.P.(2004). Reliable control design of fuzzy dynamic systems with time-varying delay, *Fuzzy Sets and Systems* **146**(3): 349–374.

Dong, J.X. and Yang, G.H. (2008). Dynamic output feedback control synthesis for continuous-time T–S fuzzy systems via a switched fuzzy control scheme, *IEEE Transactions on Systems, Man, and Cybernetics, Part B* **38**(4): 1166–1175.

Gao, Z.F., Jiang, B., Qi, R.Y., Xu, Y.F. and Cheng, Y.H. (2010). Fuzzy observer design for near space vehicle with application to sensor fault estimation, *ICIC Express Letters* **4**(1): 177–182.

Gao, Z., Shi, X. and Ding, S.X. (2008). Fuzzy state/disturbance observer design for T–S fuzzy systems with application to sensor fault estimation, *IEEE Transactions on Systems, Man, and Cybernetics* **38**(3): 875–880.

Gassara, H., Hajjaji, A.E. and Chaabane, M. (2008). Robust H^{∞} control for T–S fuzzy systems with time-varying delay, *International Journal of Systems Science* **41**(12): 1481–1491.

Lin, C., Wang, Q.G. and Lee, T.H. (2006). Stability and stabilization of a class of fuzzy time-delay descriptor systems, *IEEE Transactions on Fuzzy Systems* **14**(4): 542–551.

Liu, X.D. and Zhang, Q.L. (2003). New approaches to H^{∞} controller designs based on fuzzy observers for T–S fuzzy systems via LMI, *Automatica* **39**(9): 1571–1582.

Mao, Z.H. and Jiang, B. (2007). Fault identification and fault-tolerant control for a class of networked control systems, *International Journal of Innovative Computing, Information and Control* **3**(5): 1121–1130.

Miguel, B. and Hušek, P. (2005). Non-quadratic performance design for Takagi–Sugeno fuzzy systems, *International Journal of Applied Mathematics and Computer Science* **15**(3): 383–391.

Nguang, S.K., Shi, P. and Ding, S.X. (2007). Fault detection for uncertain fuzzy systems: An LMI approach, *IEEE Transactions on Fuzzy Systems* **15**(6): 1251–1262.

Shi, S.H., Yuan, Z.H. and Zhang, Q.L. (2009). Fault-tolerant H_{∞} filter design of a class of switched systems with sensor failures, *International Journal of Innovative Computing, Information and Control* **5**(11(A)): 3827–3838.

Tanaka, K.and Sugeno, M. (1992). Stability analysis and design of fuzzy control systems, *Fuzzy Sets and Systems* **45**(2): 135–156.

Tanaka, K. and Wang, H.O.(2001). *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, Wiley, New York, NY.

Tanaka, K., Ikeda, T.and Wang, H.O. (1998). Fuzzy regulators and fuzzy observers: Relaxed stability conditions and LMI-based designs, *IEEE Transactions on Fuzzy Systems* **6**(2): 250–265.

Takagi, T. and Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control, *IEEE Transactions on Systems, Man, and Cybernetics, Part B* **15**(1): 116–132.

Tong, S., Wang, T.C. and Zhang, W. (2008). Fault tolerant control for uncertain fuzzy systems with actuator failures, *International Journal of Innovative, Computing, Information and Control* **4**(10): 2461–2474.

Wu, H.N. and Bai, M.Z. (2010). Stochastic stability analysis and synthesis for nonlinear fault tolerant control systems based on the T–S fuzzy model, *International Journal of Innovative Computing, Information and Control* **6**(9): 3989–4000.

Wu, H.N. and Li, H.X. (2007). New approach to delay-dependent stability analysis and stabilization for continuous-time fuzzy systems with time-varying delay, *IEEE Transactions on Fuzzy Systems* **15**(3): 482–493.

Wu, H.N. (2004). Reliable LQ fuzzy control for continuous-time nonlinear systems with actuator faults, *IEEE Transactions on Systems, Man and Cybernetics, Part B* **34**(4): 1743-1752.

Zhang, K. and Jiang, B. (2010). Dynamic output feedback fault tolerant controller design for Takagi–Sugeno fuzzy systems with actuator faults, *IEEE Transactions on Fuzzy Systems* **18**(1): 194–201.

Zhou, C. (2002). Fuzzy-arithmetic-based Lyapunov synthesis in the design of stable fuzzy controllers: A computing-with-words approach, *International Journal of Applied Mathematics and Computer Science* **12**(3): 411–421.

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