CIVIL AND ENVIRONMENTAL ENGINEERING REPORTS

No. 5 2010

# DETERMINATION OF MOISTURE TRANSFER COEFFICIENT IN BUILDING MATERIALS ON THE BASIS OF KINETICS OF A ONE DIRECTION CAPILLARY RISE

Abdrahman ALSABRY
University of Zielona Góra, Faculty of Civil and Environmental Engineering,
Department of Building Structures
ul. prof. Z. Szafrana 1, 65-516 Zielona Góra, Poland
inzynier6@yahoo.com

The article presents an original mathematical presentation and calculation procedure to determine a moisture transfer coefficient in building materials on the basis of kinetics of a one direction capillary rise.

Keywords: moisture transport coefficient, capillary rise, mathematical model, capillary transport.

## 1. INTRODUCTION

An effect of moisture transport in porous building materials is a complex process, since apart from a molecular flow a surface diffusion, capillary transport and other forms of transport are present. The processes of sorption and desorption as well as phase transitions may occur in a condition of thermal nonequilibrium of thermal diffusion effect. The occurrence and intensity of the phenomena depend on the porous structure of the material and its thermal and moisture properties. The environment also affects the transport processes inside the space dividing elements by natural atmospheric conditions.

These phenomena influence the properties and durability of building materials.

Building materials feature a capillary-porous structure enabling absorption of water, penetrating the inside of the material and filling the pores either in full or in part. A moisture state of space dividers is defined as a whole of processes related to moistening and drying of the materials.

Currently, a significant scientific issue related to mass and energy flow through the space dividers is a need for theoretical analysis of the discussed processes on the basis of the principles of dynamics, diffusion and sorption theory.

To carry out thermal and technical calculations to determine transient temperature and humidity fields in capillary-porous building materials, data of heat and moisture transfer coefficients are required. These coefficients define a specific material and are used in heat transfer and mass flow equations. One of the coefficients is a moisture transfer coefficient (liquid moisture diffusion coefficient) [1, 2, 3, 4, and 5].

Articles [6, 7, 8, and 9] present a method to determine a moisture transport coefficient in building materials on the basis of kinetics of a one direction capillary rise. An experimental part of the method consists in the measurement of material sample weight variable in time. The sample is a rectangular prism and all sides, apart from the bottom one, are coated with waterproofing membrane. A bottom sample surface shall be in contact with water at $t = 0$ .

## 2. MATHEMATICAL PRESENTATION AND CALCULATION PROCEDURE

An equation describing moisture transfer and sample weight change in time:

$$\rho_0 \frac{\partial w}{\partial t} = \frac{\partial}{\partial x}\left( \beta(w) \frac{\partial w}{\partial x} \right), \tag{2.1}$$

where:

| | | |
|---|---|---|
| $\rho_0$ | - | material volume weight; |
| $w(x,t)$ | - | moisture content by weight (moisture weight to dry sample ratio), depending on coordinate x and time t, |
| $\beta(w)$ | - | moisture transfer coefficient of the material. |

The initial condition is an assumption that the sample at $t = 0$ was dry:

$$w(x,0) = 0, \cdots\cdots 0 < x \le L, \tag{2.2}$$

where $L$ is the sample length.

Boundary conditions show that for the bottom surface immersed in water, the sample maximum moisture content

$$w(0,t) = w_{\max}, \cdots t \ge 0, \tag{2.3}$$

where $w_{\max}$ is the maximum material moisture content by weight, with moisture flux density through the surface coated with waterproofing membrane is equal to zero:

$$\beta(w)\frac{\partial w}{\partial x}=0,\cdots\cdots x=L,\cdots\cdots t\geq 0. \tag{2.4}$$

Change of sample weight in time is defined as

$$m(t)=m_0+\rho_0\cdot S\int_0^L w(x,t)dx\,, \tag{2.5}$$

where $S$ is the surface area of sample cross-section.
Value of $\rho_0$ and $w_{\max}$ can be calculated by the following formula:

$$\rho_0=\frac{m_0}{S\cdot L},\cdots\cdots w_{\max}=\frac{m(T)-m(0)}{m(0)} \tag{2.6}$$

where $T$ is a specific moment, after which it is assumed that the sample is completely saturated with water, i.e. $m(t)=m(T)$, at $t\geq T$.
The task is to determine the moisture transfer coefficient as a function of moisture $\beta(w)$ on the basis of function $m(t)$ and sample dimensions $S$ and $L$.

To solve the problem, let us assume that the value $\beta$ depends on coordinates and time, and not considering *w*. The equation (2.1) is expressed as:

$$\rho_0\frac{\partial w(x,t)}{\partial t}=\frac{\partial}{\partial x}\left(\beta(x,t)\frac{\partial w(x,t)}{\partial x}\right) \tag{2.7}$$

At the monotony of a function *w* over *x* and *t*, a sufficient and necessary condition, that the value $\beta$ is a function of *w*, i.e. $\beta(x,t)=\beta(w(x,t))$ is the following equation:

$$\frac{\partial\beta(x,t)}{\partial x}\frac{\partial w(x,t)}{\partial t}=\frac{\partial\beta(x,t)}{\partial t}\frac{\partial w(x,t)}{\partial x}\,. \tag{2.8}$$

Equation (2.7) and (2.8) are a system of two equations with two unknown $\beta$ and *w*, dependent on coordinates and time. Apart from that, the conditions (2.2) to (2.5) shall be met.

Let us integrate the equation (2.7) with respect to $x$:

$$-\beta(x,t)\frac{\partial w(x,t)}{\partial x}=\rho_0\left(\int_0^L\frac{\partial w(\xi,t)}{\partial t}d\xi+C(t)\right), \tag{2.9}$$

where $C(t)$ is a specific time function.

For x = L, the right side of the integral (2.9) is equal to zero. A sufficient and necessary condition to meet the boundary conditions (2.4) is $C(t)=0$.

Value $\beta$ is determined by the following formula:

$$\beta(x,t)=-\rho_0 \frac{\int_0^L \frac{\partial w(\xi,t)}{\partial t} d\xi}{\frac{\partial w(\xi,t)}{\partial x}}. \tag{2.10}$$

Let us substitute (2.10) to (2.8) to obtain a single non-linear differential - integral equation for one of the unknowns $w$:

$$\left(\frac{\partial^2 w(x,t)}{\partial x^2}\cdot\frac{\partial w(x,t)}{\partial t}-\frac{\partial^2 w(x,t)}{\partial x \partial t}\cdot\frac{\partial w(x,t)}{\partial x}\right)\cdot\int_x^L \frac{\partial w(\xi,t)}{\partial t} d\xi + \\ +\frac{\partial w(x,t)^2}{\partial x}\cdot\int_x^L \frac{\partial^2 w(\xi,t)}{\partial t^2} d\xi+\left(\frac{\partial w(x,t)^2}{\partial t}\right)\cdot\frac{\partial w(x,t)}{\partial x}=0 \tag{2.11}$$

As boundary conditions of the equation, let us consider the correlation (2.2), (2.3) and (2.5) and that the sample is completely saturated with moisture in $T$.

$$w(x,t)=0 \qquad t=0, \qquad < x \le L \tag{2.12}$$

$$w(x,t)=w_{\max} \qquad 0 \le t = T, \qquad x=0 \tag{2.13}$$

$$\int_0^L w(\xi,t) d\xi=\frac{m(t)-m(0)}{\rho_0 S}. \qquad 0 \le t \le T, \tag{2.14}$$

$$w(x,t)=w_{\max} \qquad t=T, \qquad 0 \le x \le L. \tag{2.15}$$

Equation (2.11) considering the conditions (2.12) to (2.15) describes the change of sample moisture content in time and space, and for a specific change of sample weight $m(t)$, such as the diffusion equation (2.1) is satisfied, where the moisture transfer coefficient is a function of moisture content.

The equation can be linearised for a relatively small correction *Sw(x, t)* for *w(x, t)* by the Newton's method. Obtained linear equation can be solved for a rectangular grid of coordinates $(x,t)$. The matrix of linear system of equations for the grid is a block tridiagonal. The Thomas algorithm is applied.

As a simple approximation in the Newton's method, we can use the result obtained in article [2, 3, 4, 5] from the equation (2.1) with boundary conditions (2.2)-(2.4) and coefficient $\beta$ *(w)* chosen to satisfy the relation (2.5) in the most accurate way. A solution of an equation (2.11) for conditions (2.12) to (2.15) is substituted in the relation (2.10) and an unknown moisture transfer coefficient $\beta$ *(ω)* is calculated.

## REFERENCES

1. Mårten J.: *Methods of measuring the moisture diffusivity at high moisture levels*, Report TVBM-3076 Licentiate Thesis, Lund 1997.
2. Arfvidsson J.: *Isothermal moisture transport in porous materials, Calculation and evaluation of material data (in Swedish)*, Report TVBH-1007, Division of Building Physics, Lund Institute of Technology, 1994
3. Freitas V. P. De., Krus M., Künzel H., Quenard D.: *Determination of the water diffusivity of porous materials by gamma-ray attenuation and NMR, Proceeding of the International symposium on Moisture Problems in Building Walls*, 11-13 Sept, pp. 445-460, Porto 1995
4. Künzel H. M.: *Simultaneous Heat and Moisture Transport in Building Components – One- and two-dimensional calculation using simple parameters*, IRB Verlag, Stuttgart 1995
5. Wyrwała J., Świrska J.: *Moisture problems of building partitions. Polish Academy of Sciences, Committee of civil engineering*, Institute of Fundamental Technological, Warsaw 1998.
6. Alsabry A.: *Calculation of mass transfer coefficients in capillary-porous building materials.* Przeglad budowlany, 2, 2011 (in Polish)
7. Alsabry A.: *Predicting frost resistance in ceramic wall materials based on damp transmission parameters*. Przeglad budowlany, pp. 34-40, nr 12, 2010 (in Polish)
8. AlsabryA., Nikitin A.: *Determination of moisture penetration factor in the liquid chase of ceramic brick*, Przegląd Budowlany, pp. 32-36, nr 1, 2005 (in Polish)
9. Alsabry A., Nikitin V.: *Determination of moisture penetration factor in the liquid chase of lime-sand brick*. pp. 85-91, CWB, nr 2, 2005

# WYZNACZENIE WSPÓŁCZYNNIKA PRZENOSZENIA WILGOCI W MATERIAŁACH BUDOWLANYCH NA PODSTAWIE DANYCH O KINETYCE JEDNOKIERUNKOWEGO PODCIĄGANIA KAPILARNEGO

S t r e s z c z e n i e

Zjawiska transportu wilgoci zachodzące w porowatych materiałach budowlanych składają się na bardzo skomplikowany proces, gdyż oprócz przepływów molekularnych może występować w ich porach również dyfuzja powierzchniowa, transport kapilarny i inne rodzaje transportu. Równocześnie mogą zachodzić procesy sorpcji lub desorpcji, przemiany fazowe, w warunkach nierównowagi cieplnej zjawiska termodyfuzji. To, czy i z jaką intensywnością będą się ujawniać poszczególne zjawiska, zależy od struktury porowatości danego materiału i jego właściwości cieplno-wilgotnościowych.

Materiały budowlane w większości mają budowę kapilarno-porowatą, umożliwiającą pochłanianie wody, która może wnikać w głąb materiału wypełniająca całkowicie lub tylko częściowo jego pory. Przez stan wilgotnościowy przegród budowlanych rozumie się całokształt przebiegu procesów związanych z zawilgoceniem i wysychaniem materiałów w przegrodach budowlanych.

W artykule zaproponowano przez autora oryginalny opis matematyczny i procedura obliczeniowa wyznaczenie współczynnika przenoszenia wilgoci w materiałach budowlanych na podstawie danych o kinetyce jednokierunkowego podciągania kapilarnego.