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H_-/H_∞ FAULT DETECTION OBSERVER DESIGN FOR A POLYTOPIC LPV SYSTEM USING THE RELATIVE DEGREE

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This paper proposes an H_-/H_∞ fault detection observer method by using generalized output for a class of polytopic linear parameter-varying (LPV) systems. As the main contribution, with the aid of the relative degree of output, a new output vector is generated by gathering the original output and its time derivative, and it is feasible to consider H_- actuator fault sensitivity in the entire frequency for the new system. In order to improve actuator and sensor fault sensitivity as well as guarantee robustness against disturbances, simultaneously, an H_-/H_∞ fault detection observer is designed for the new LPV polytopic system. Besides, the design conditions of the proposed observer are transformed into an optimization problem by solving a set of linear matrix inequalities (LMIs). Numerical simulations are provided to illustrate the effectiveness of the proposed method.

Keywords: H_-/H_∞ fault detection observer, polytopic LPV system, relative degree of output, actuator fault detection, sensor fault detection.

1. Introduction

In recent decades, with the increasing demands of safety and reliability for modern complex control systems, fault detection has become an important issue and received abundant results; see the works of Frank (1990), Chen and Patton (1999) or Ding (2008), and the references therein. In practice, most dynamic systems are nonlinear, as linear parameter varying (LPV) theory can offer an efficient paradigm to model nonlinear systems with online measurable parameters. Besides, it is convenient to extend the method and theory of linear systems into LPV systems, and fault detection for nonlinear systems based on the LPV method has gained a great deal of interest (Henry *et al.*, 2015a; Tanaka and Wang, 2001).

Among fault detection methods, the observer-based one has been investigated most deeply (Wang *et al.*, 2015a; Varga and Ossmann, 2014; Rodrigues *et al.*, 2015; Yin *et al.*, 2017). The basic idea of this method is to generate a residual between the real system and the observer, then compare the generated residual with a predefined threshold to determine whether a fault occurs. Balas *et al.* (2002), Bokor and Balas (2004), Bokor (2007) and Vanek *et al.* (2014) investigated the problem of fault detection for LPV systems based on a geometric approach, in which the directional LPV observer is designed by using parameter-varying (C, A)-invariant subspaces and a parameter-varying unobservable subspace. A nullspace approach is proposed by Varga (2008), which is a variant of the geometric method. Besides, a parameter-dependent state observer design method is studied by Bara *et al.* (2001), Millerioux *et al.* (2004) and Casavola *et al.* (2007).

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Due to the effect of an unknown disturbance on the residual, robustness against the disturbance has been one of the most important issues related to the observer-based fault detection method. One strategy is to decouple the disturbance from the residual, which comprises the eigenvalue assignment method and the unknown input observer method. However, the decoupled conditions are generally restrictive and hard to be satisfied for some cases. Another method is to attenuate the disturbance effect through H_∞ techniques. Furthermore, in order to distinguish the fault effect from the unknown disturbance, a robust fault detection observer needs to make the generated residual robust to the unknown disturbance and sensitive to faults simultaneously.

In order to achieve a proper trade-off between fault sensitivity and disturbance attenuation performance, a mixed H_-/H_∞ fault detection observer method was first proposed by Hou and Patton (1996), with the H_- index defined as the smallest nonzero singular value of the transfer function matrix at the particular frequency $\omega = 0$, which is also called the DC-gain. In the work of Armeni *et al*. (2009), a robust observer is designed for LPV systems with the highest H_∞ disturbance robustness performance and a predefined lower bounded on the DC-gain from fault to the residual. Liu *et al*. (2005) extended the definition of the H_- index extended as the minimum singular value of the transfer function matrix. In the work of Wei and Verhaegen (2008), the H_- index is defined on an infinite frequency domain for LPV systems.

In recent years, many researchers have paid attention to H_-/H_∞ fault detection observer design methods for nonlinear systems in infinite frequency domain (see, e.g., Cai and Wu, 2010; Wei and Verhaegen, 2011; Chadli *et al*., 2013; Estrada *et al*., 2015). Grenaille *et al*. (2008), Henry (2012) or Henry *et al*. (2015a) designed a robust fault detection filter with enhanced fault transmission H_- gain and large H_∞ nuisance attenuation for LPV systems in a finite frequency domain by using the weighting matrix method. Besides, Henry (2008) and Henry *et al.* (2014, 2015b) demonstrated that it is possible to cover a very large range of uncertainties while maintaining high fault sensitivity. Recently, the generalized Kalman–Yakubovich–Popov (KYP) lemma was proposed by Iwasaki *et al*. (2005), which gives an exact linear matrix inequality characterization of the H_- index in a finite frequency domain. Furthermore, H_-/H_∞ fault detection observer design method for nonlinear systems based on GKYP lemma was proposed by Chen *et al*. (2015) as well as Li and Yang (2014).

Note that the H_- index in an infinite frequency domain has to satisfy the full-rank column condition of the D-matrix between the faults and the measurement outputs. For strictly proper systems, the H_- index over $[0, \infty)$ is always zero (Wang and Yang, 2008). Besides, in most existing results on observer design in full frequency domain, the fault vector in the measurement equation is assumed to be the same with the actuator fault vector. However, in real systems, actuator faults are different from the sensor fault. Thus, it is infeasible for an actuator fault to consider the H_- index in infinite frequency, because it does not satisfy the full-rank column condition. In the work of Ichalal *et al*. (2016), an actuator fault diagnosis approach for linear systems is proposed based on the relative degree of output with respect to the fault. However, in the H_- index as well as the effect of unknown disturbance are not taken into consideration. Motivated by Ichalal *et al*. (2016), the use of the relative degree notion aims to generate new auxiliary outputs depending on the actuator faults. It is proved that with the aid of the relative degree, the actuator fault vector can be introduced into the output equation, such that it is feasible for the new system to consider fault sensitivity by using the H_- index. In this paper, disturbance robustness as well as actuator and sensor H_- fault sensitivity are considered in an infinite frequency domain.

The main contribution of this paper covers the following aspects. First, the fault diagnosis method based on the relative degree of output is extended into a polytopic LPV system. A new polytopic LPV system is generated by using the relative degree of output with respect to the actuator fault, such that it is feasible for the new system to consider actuator fault sensitivity with the H_- index in full-frequency domain. Then an H_-/H_∞ fault detection observer is designed to make the generated residual robust against disturbance, sensitive to actuator faults and sensor faults simultaneously. The proposed observer design conditions are transformed into an optimization problem by solving a set of LMIs. Besides, in order to reduce some conservatism, the observer is designed based on a parameter-dependent Lyapunov matrix method.

The rest of this paper is organized as follows. The preliminaries are given in Section 2. The problem statement is described in Section 3. In Section 4, a new system is first constructed with generalized output, then a mixed H_-/H_∞ observer is designed for the new polytopic LPV system. Simulation results are given in Section 5 to demonstrate the proposed approach. Finally, conclusions are given in Section 4.

Notation. Throughout the paper, $P > 0$ signifies that P is a positive-definite matrix and $Q < 0$ means that Q is a negative-definite matrix. The symbol $\star$ in a symmetric matrix denotes the transposed block in the symmetric position. For a matrix A, $\mathrm{He}(A)$ is used to denote $\mathrm{He}(A) := A + A^T$. The $\mathcal{L}_2$ norm of $x(t)$ is defined as $\|x(t)\|_2 = (\int_0^\infty x^T(t)x(t)\,\mathrm{d}t)^{1/2}$.

2. Preliminaries

Motivated by Li and Yang (2014), the following definition is given.

Definition 1. Consider an LPV system of the following form:

$$\begin{cases}\dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t),\\ y(t) = C(\rho(t))x(t) + D(\rho(t))u(t).\end{cases} \tag{1}$$

The system (1) is said to have an H_∞ performance index less than γ if under the zero initial condition the following inequality is satisfied:

$$\int_0^\infty y^T(t)y(t)\,\mathrm{d}t \le \gamma^2 \int_0^\infty u^T(t)u(t)\,\mathrm{d}t. \tag{2}$$

Assume that $D(\rho(t))$ in (1) has no column rank deficiency $\forall\rho(t)$, so that the H_- index is null. The system (1) is said to have an H_- performance index higher than $\beta \neq 0$ if under the zero initial condition the following inequality holds:

$$\int_0^\infty y^T(t)y(t)\,\mathrm{d}t \ge \beta^2 \int_0^\infty u^T(t)u(t)\,\mathrm{d}t. \tag{3}$$

Definition 2. (*Relative degree*) *(Isidori, 1995)*. Consider a linear system where matrix B is omitted to simplify the definition as

$$\begin{cases}\dot{x}(t) = Ax(t) + Ef(t),\\ y(t) = Cx(t),\end{cases} \tag{4}$$

where $x \in \mathbb{R}^{n_x}$, $f \in \mathbb{R}$ and $y \in \mathbb{R}$ are the state vector, actuator fault signal and measurement output, respectively. The relative degree of output $y(t)$ with respect to fault $f(t)$ is λ_f satisfying

$$\begin{cases}CA^{i-1}E = 0, \quad \forall i = 1,\dots,\lambda_f - 1\\ CA^{\lambda_f-1}E \neq 0.\end{cases} \tag{5}$$

The relative degree λ_f of output $y(t)$ with respect to fault $f(t)$ means that the λ_f-th time-derivative of output $y^{\lambda_f}(t)$ depends on the fault explicitly, while all lower order time-derivatives of output do not depend on the fault explicitly, i.e.,

$$y^{\lambda_f}(t) = CA^{\lambda_f}x(t) + \underbrace{CA^{(\lambda_f-1)}E}_{\neq 0} f(t). \tag{6}$$

Lemma 1. (Garcia and Bernusson, 1995) *The eigenvalues of a given matrix $A \in \mathbb{R}^{n\times n}$ belong to the closed circular region $\mathcal{D}(a,\tau)$ with the center $a + j0$ and radius τ if and only if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ such that*

$$\begin{bmatrix}-P & P(A-aI)\\ \star & -\tau^2 P\end{bmatrix} \le 0. \tag{7}$$

Lemma 2. (Wei and Verhaegen, 2011) *The LPV system in* (1) *is asymptotically stable and has a quadratic H_∞ performance index less than γ if there exists a matrix $P = P^T > 0$ such that*

$$\begin{bmatrix}\mathscr{F}_1 & PB(\rho(t)) + C^T(\rho(t))D(\rho(t))\\ \star & D^T(\rho(t))D(\rho(t)) - \gamma^2 I\end{bmatrix} \le 0, \tag{8}$$

where

$$\mathscr{F}_1 = \mathrm{He}\{A^T(\rho(t))P\} + C^T(\rho(t))C(\rho(t)).$$

Lemma 3. *The system in* (1) *is said to have a quadratic H_- performance index higher than β if there exists a matrix $Q = Q^T > 0$ such that the following LMI holds:*

$$\begin{bmatrix}\mathscr{F}_2 & Q\mathcal{B}(\rho) - \mathcal{C}^T(\rho)\mathcal{D}(\rho)\\ \star & -\mathcal{D}^T(\rho)\mathcal{D}(\rho) + \beta^2 I\end{bmatrix} \le 0, \tag{9}$$

where

$$\mathscr{F}_2 = \mathrm{He}\{A^T(\rho(t))Q\} - C^T(\rho(t))C(\rho(t)).$$

3. Problem formulation

Consider an LPV system such as

$$\begin{cases}\dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))(u(t) + f_a(t))\\ \qquad + D_w(\rho(t))w(t),\\ y(t) = Cx(t) + E_s f_s(t) + D_v v(t),\end{cases} \tag{10}$$

where $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$, $y(t) \in \mathbb{R}^{n_y}$ are the state, input and measurement output vectors, respectively. Here $f_a(t) \in \mathbb{R}^{n_a}$ is the actuator fault vector, $f_s(t) \in \mathbb{R}^{n_s}$ denotes the sensor fault vector, $w(t) \in \mathbb{R}^{n_w}$ represents the unknown disturbance and $v(t) \in \mathbb{R}^{n_v}$ represents the measurement noise. $A(\rho(t)) \in \mathbb{R}^{n_x\times n_x}$, $B(\rho(t)) \in \mathbb{R}^{n_x\times n_u}$, $D_w(\rho(t)) \in \mathbb{R}^{n_x\times n_w}$, $C \in \mathbb{R}^{n_y\times n_x}$, $E_s \in \mathbb{R}^{n_y\times n_s}$, $D_v \in \mathbb{R}^{n_y\times n_v}$ are system matrices. $\rho(t) = [\rho_1(t),\dots,\rho_s(t)]^T$ is the scheduling vector assumed to be measurable online and not affected by the faults, s is the number of the scheduling variables.

In this paper, as in the works of Gahinet *et al.* (1996) and Rodrigues *et al.* (2014), each element $\rho_i(t)$ in $\rho(t)$ ranges between known extreme values $\underline{\rho_i}$ and $\overline{\rho_i}$, i.e.,

$$\rho(t) \in \Gamma = \Big\{\rho_i \mid \underline{\rho}_i \le \rho_i(t) \le \overline{\rho}_i,\ i = 1,2,\dots,s\Big\}. \tag{11}$$

The system matrices $A(\rho(t))$, $B(\rho(t))$ and $D_x(\rho(t))$ are functions which depend affinely on the time-varying parameter vector $\rho(t)$, i.e.,

$$\mathcal{M}(\rho(t)) = \tilde{\mathcal{M}}_0 + \sum_{i=1}^{s} \rho_i(t)\tilde{\mathcal{M}}_i, \tag{12}$$

where $\mathcal{M}(\rho(t))$ stands for matrices $A(\rho(t))$, $B(\rho(t))$ and $D_x(\rho(t))$.

In this paper, the polytope is as a hyper-rectangle. The system (10) can be transformed into a convex interpolation of the vertices of Ω, with the vertices $\mathscr{M} = \begin{bmatrix} A_i & B_i & D_{wi} & C & E_s & D_v \end{bmatrix}, \forall i \in [1, \dots, N]$, where $N = 2^s$. Then, the polytopic coordinates are denoted by $h_i(\rho(t))$ and vary in the convex set Ω,

$$\Omega = \Big\{ h(\rho(t)) = [h_1(\rho(t)), \dots, h_N(\rho(t))]^T \mid \sum_{i=1}^{N} h_i(\rho(t)) = 1,\ 0 \le h_i(\rho(t)) \le 1 \Big\}. \tag{13}$$

Hence, the system (10) can be re-expressed by a polytopic representation as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} h_i(\rho(t))[A_i x(t) + B_i(u(t) + f_a(t)) \\ \qquad + D_{wi} w(t)], \\ y(t) = Cx(t) + E_s f_s(t) + D_v v(t), \end{cases} \tag{14}$$

where A_i, B_i, D_{wi} are system matrices of the i-th model.

The disturbance and measurement noise vectors can be treated as one vector, i.e.,

$$d(t) = \begin{bmatrix} w(t) \\ v(t) \end{bmatrix}. \tag{15}$$

Then, the system (14) is written as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} h_i(\rho(t))[A_i x(t) + B_i(u(t) + f_a(t)) \\ \qquad + D_{xi} d(t)], \\ y(t) = Cx(t) + E_s f_s(t) + D_y d(t), \end{cases} \tag{16}$$

where

$$D_x(\rho) = \begin{bmatrix} D_w(\rho) & 0 \end{bmatrix}, \quad D_y = \begin{bmatrix} 0 & D_v \end{bmatrix}.$$

In the following, $\mathcal{M}(\rho(t))$ is written as $\mathcal{M}(\rho)$ for the sake of brevity.

In the sequel, the following assumptions are used.

Assumption 1. The system $(A(\rho), C)$ is observable.

Assumption 2. The generalized disturbance vector $d(t)$ and its derivative $\dot{d}(t)$ as well as the sensor fault derivative $\dot{f}_s(t)$ are assumed to be energy bounded, i.e., $\|d(t)\|_2 \le \delta_1$, $\|\dot{d}(t)\|_2 \le \delta_2$, $\|\dot{f}_s(t)\|_2 \le \delta_3$, where $0 \le \delta_1, \delta_2, \delta_3 < \infty$.

Assumption 3. The relative degree of output $y(t)$ with respect to actuator fault $f_a(t)$ is assumed to guarantee that

$$\text{rank}(CB(\rho)) = \text{rank}(B(\rho)) = n_u. \tag{17}$$

Remark 1. According to Rodrigues *et al*. (2014), the structure of actuator fault vector $f_a(t)$ can be used to represent an additive or a multiplicative fault signal. Considering the faulty control input $u_f(t) = (I - \eta)u(t)$, it can be written as an external additive signal $u_f(t) = u(t) + f_a(t)$, where $f_a(t) = -\eta u(t)$ with

$$\eta = \text{diag}[\eta_2, \eta_2, \dots, \eta_{n_u}], \quad 0 \le \eta_i \le 1, \tag{18}$$

where $\eta_i = 1$ represents a total failure to the i-th actuator, $\eta_i = 0$ means i-th actuator is fault free and $0 < \eta_i < 1$ denotes that the loss of control effectiveness of the i-th actuator.

In order to detect actuator and sensor faults, an observer is designed with the following structure:

$$\begin{cases} \dot{\hat{x}}(t) = A(\rho)\hat{x}(t) + B(\rho)u(t) \\ \qquad + L(\rho)[y(t) - C\hat{x}(t)], \\ r(t) = M[y(t) - C\hat{x}(t)], \end{cases} \tag{19}$$

where $\hat{x}(t) \in \mathbb{R}^{n_x}$ is the state vector of the observer (19) to estimate the state vector $x(t)$ and $r(t) \in \mathbb{R}^{n_y}$ is the residual vector. The matrices $L(\rho) \in \mathbb{R}^{n_x \times n_y}$ and $M \in \mathbb{R}^{n_y \times n_y}$ are the gain matrices of a residual generator based on the observer (19) that have to be designed. Matrix $L(\rho)$ has the following form:

$$L(\rho) = \sum_{i=1}^{N} h_i(\rho(t)) L_i. \tag{20}$$

Define the state estimation error vector as

$$e(t) = x(t) - \hat{x}(t). \tag{21}$$

Subtracting (19) from (14), the dynamic system for the estimation error is obtained as

$$\begin{cases} \dot{e}(t) = [A(\rho) - L(\rho)C]e(t) + B(\rho)f_a(t) \\ \qquad - L(\rho)E_s f_s(t) + [D_x(\rho) - L(\rho)D_y]d(t), \\ r(t) = MCe(t) + ME_s f_s(t) + MD_y d(t). \end{cases} \tag{22}$$

The state estimation error system (22) can be divided into the following subsystems, cf. Wang *et al*. (2015b):

$$\begin{cases} \dot{e}_d(t) = [A(\rho) - L(\rho)C]e_d(t) \\ \qquad + [D_x(\rho) - L(\rho)D_y]d(t), \\ r_d(t) = MCe_d(t) + MD_y d(t), \end{cases} \tag{23}$$

$$\begin{cases} \dot{e}_{f_a}(t) = [A(\rho) - L(\rho)C]e_{f_a}(t) + B(\rho)f_a(t), \\ r_{f_a}(t) = MCe_{f_a}(t) \end{cases} \tag{24}$$

and

$$\begin{cases} \dot{e}_{f_s}(t) = [A(\rho) - L(\rho)C]e_{f_s}(t) - L(\rho)E_s f_s(t), \\ r_{f_s}(t) = MCe_{f_s}(t) + ME_s f_s(t), \end{cases} \tag{25}$$

where

$$\begin{cases} e(t) = e_d(t) + e_{f_a}(t) + e_{f_s}(t), \\ r(t) = r_d(t) + r_{f_a}(t) + r_{f_s}(t). \end{cases} \tag{26}$$

The observer (19) is called an H_-/H_∞ fault detection observer if the state estimation error system (22) is asymptotically stable and the following inequalities are satisfied:

$$\int_0^\infty r_d^T(t) r_d(t)\,\mathrm{d}t \le \gamma^2 \int_0^\infty d^T(t) d(t)\,\mathrm{d}t \tag{27}$$

$$\int_0^\infty r_{f_a}^T(t) r_{f_a}(t)\,\mathrm{d}t \ge \beta_1^2 \int_0^\infty f_a^T(t) f_a(t)\,\mathrm{d}t \tag{28}$$

$$\int_0^\infty r_{f_s}^T(t) r_{f_s}(t)\,\mathrm{d}t \ge \beta_2^2 \int_0^\infty f_s^T(t) f_s(t)\,\mathrm{d}t. \tag{29}$$

For the system (22), the sufficient conditions of the H_∞ disturbance attenuation performance (27) and the H_- index fault sensitivity performance (28) and (29) can be obtained easily based on Lemmas 2 and 3 separately.

For the system (24), note that the matrix $D(\rho)$ is zero, and it is infeasible for the system (22) to consider H_- actuator fault sensitive performance in a full frequency domain. If we consider the actuator fault $f_a(t)$ and sensor fault $f_s(t)$ as one fault vector,

$$\bar{f}(t) = \begin{bmatrix} f_a(t) \\ f_s(t) \end{bmatrix}, \tag{30}$$

then letting $d(t) = 0$, we can rewrite the system (22) as

$$\begin{cases} \dot{e}(t) = [A(\rho) - L(\rho)C]e(t) + \bar{B}(\rho)\bar{f}(t), \\ r(t) = MCe(t) + M\bar{E}_s\bar{f}(t), \end{cases} \tag{31}$$

where

$$\bar{B}(\rho) = \begin{bmatrix} B(\rho) & -L(\rho)E_s \end{bmatrix}, \quad \bar{E}_s = \begin{bmatrix} 0 & E_s \end{bmatrix}.$$

Note that the matrix of $\bar{E}_s$ is not full-column rank either, so that H_- actuator fault sensitivity and H_- sensor fault sensitivity cannot be considered by using the augmented technique. In order to overcome the problem, in this paper H_- actuator fault sensitivity and H_- sensor fault sensitivity are analyzed separately.

According to Ichalal *et al.* (2016), actuator fault also influences the output. The use of the notation of the relative degree aims to define new auxiliary outputs depending on actuator faults, and the actuator fault is transformed into the measurement output equation, so that the H_-/H_∞ fault detection observer is theoretically feasible. Based on the idea, in this paper we will design an H_-/H_∞ fault detection observer based on the relative degree of the output method for polytopic LPV systems.

4. Main results

In this section, an H_-/H_∞ fault detection observer is designed for polytopic LPV systems based on the relative degree of output. The scheme of the proposed fault detection method is depicted in Fig. 1.

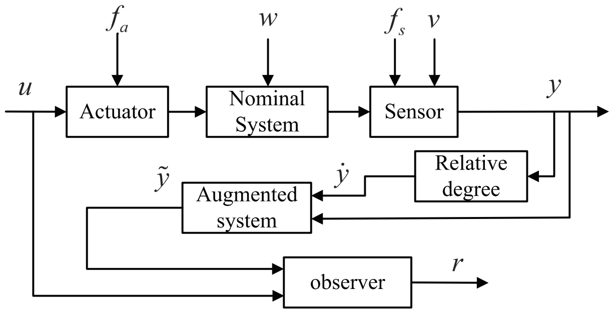

Fig. 1. Scheme of the proposed strategy.

4.1. New system generated by using the relative degree.

Consider first a single-input single-output system. Under Assumption 3 that the relative degree of the output with respect to the actuator fault is 1, the time derivative of the output vector $\dot{y}(t)$ is obtained as

$$\begin{aligned} \dot{y}(t) &= CA(\rho)x(t) + CB(\rho)[u(t) + f_a(t)] \\ &\quad + CD_x(\rho)d(t) + E_s\dot{f}_s(t) + D_y\dot{d}(t). \end{aligned} \tag{32}$$

Consider the disturbance and its derivative and the derivative of the sensor fault $\dot{f}_s(t)$ as a new generalized disturbance, i.e.,

$$\bar{d}(t) = \begin{bmatrix} d^T(t) & \dot{d}^T(t) & \dot{f}_s^T(t). \end{bmatrix}^T \tag{33}$$

By gathering the original output $y(t)$ and its time derivative $\dot{y}(t)$, a new generalized output $\tilde{y}(t)$ is generated as

$$\tilde{y}(t) = \tilde{C}(\rho)x(t) + R(\rho)f_a(t) + Sf_s(t) + \bar{D}_y(\rho)\bar{d}(t), \tag{34}$$

where

$$\tilde{y}(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) - CB(\rho)u(t) \end{bmatrix}, \qquad \tilde{C}(\rho) = \begin{bmatrix} C \\ CA(\rho) \end{bmatrix},$$
$$S = \begin{bmatrix} E_s \\ 0 \end{bmatrix}, \qquad R(\rho) = \begin{bmatrix} 0 \\ CB(\rho) \end{bmatrix},$$
$$\bar{D}_y(\rho) = \begin{bmatrix} D_y & 0 & 0 \\ CD_x(\rho) & D_y & E_s \end{bmatrix}. \tag{35}$$

Setting

$$\bar{D}_x(\rho) = \begin{bmatrix} D_x(\rho) & 0 & 0 \end{bmatrix}, \tag{36}$$

a new polytopic LPV system is obtained as

$$\begin{cases} \dot{x}(t) = A(\rho)x(t) + B(\rho)(u(t) + f_a(t)) \\ \qquad + \bar{D}_x(\rho)\bar{d}(t), \\ \tilde{y}(t) = \tilde{C}(\rho)x(t) + R(\rho)f_a(t) + Sf_s(t) \\ \qquad + \bar{D}_y(\rho)\bar{d}(t). \end{cases} \tag{37}$$

Now consider multiple-input and multiple-output systems. When there are more than one actuators, the system (10) can be written as

$$\begin{cases} \dot{x}(t) = A(\rho)x(t) + \sum_{j=1}^{n_f} B_j(\rho)(u_j(t) + f_{aj}(t)) \\ \qquad + D_w(\rho(t))w(t), \\ y(t) = Cx(t) + E_s f_s(t) + D_v v(t), \end{cases} \tag{38}$$

where $B_j(\rho)$ is the j-th column of the matrix $B(\rho)$.

Ichalal *et al.* (2016) proposed an analysis method for the MIMO situation. If the output has different relative degrees with respect to each actuator, each should be analyzed as the SISO case, and then all the new components $\tilde{y}_i(t)$ should be gathered as a whole to form the new output $\tilde{y}(t)$. However, in this paper it is assumed that the relative degree of each output with respect to each actuator fault $f_a(t)$ is 1, i.e., $\mathrm{rank}(CB(\rho)) = \mathrm{rank}(B(\rho)) = n_u$, the matrix $CB(\rho)$ satisfies the full-column condition. In this situation, the new generalized output can be obtained as in (34).

Then, for the system (37), an observer is designed as

$$\begin{cases} \dot{\hat{x}}(t) = A(\rho)\hat{x}(t) + B(\rho)u(t) + L(\rho)[\tilde{y}(t) - \hat{\tilde{y}}(t)], \\ \hat{\tilde{y}}(t) = \tilde{C}(\rho)\hat{x}(t), \\ r(t) = M[\tilde{y}(t) - \hat{\tilde{y}}(t)]. \end{cases} \tag{39}$$

With the definition of the state estimation error as (21), the estimation error system is obtained as

$$\begin{cases} \dot{e}(t) = [A(\rho) - L(\rho)\tilde{C}(\rho)]e(t) \\ \qquad + [B(\rho) - L(\rho)R(\rho)]f_a(t) - L(\rho)Sf_s(t) \\ \qquad + [\bar{D}_x(\rho) - L(\rho)\bar{D}_y(\rho)]\bar{d}(t), \\ r(t) = M\tilde{C}(\rho)e(t) + MR(\rho)f_a(t) + MSf_s(t) \\ \qquad + M\bar{D}_y(\rho)\bar{d}(t). \end{cases} \tag{40}$$

Remark 2. In this paper, the three assumptions are necessary. Assumption 1 is a normal condition for the observer design problem. Assumption 2 guarantees that the new generalized disturbance vector is energy bounded for the H_∞ performance condition. Besides, when the relative degree of output with respect to the actuator fault is 1, the new augmented system matrices (35) need the time-varying parameters $\rho(t)$, while if the relative degree is more than 1, the augmented system will contain high order terms of $\rho(t)$ and the derivatives of $\rho(t)$, which make the design conditions too complicated to solve. Moreover, Assumption 3 is required because H_- design requires the fault distribution matrix to be of full column rank.

4.2. Fault detection observer design. In this subsection, the design conditions are provided to synthesize the observer matrices L_i and M in (39).

Theorem 1. *Given a positive scalar $\gamma > 0$ and freedom scalar parameters θ_1, θ_2 and θ_3, the observer* (39) *is called an H_-/H_∞ observer if there exist symmetrical positive definite matrices $P_i \in \mathbb{R}^{n_x \times n_x}$, matrix $W_i \in \mathbb{R}^{n_x \times n_y}$ and symmetrical non-negative matrix $U \in \mathbb{R}^{n_y \times n_y}$, such that the following LMIs hold:*

$$\begin{aligned} &\Psi_{ii} \le 0, \quad i = 1, \dots, N, \\ &\Psi_{ij} + \Psi_{ji} \le 0, \quad 1 \le i < j \le N, \end{aligned} \tag{41}$$

$$\begin{aligned} &\Pi_{ii} \le 0, \quad i = 1, \dots, N, \\ &\Pi_{ij} + \Pi_{ji} \le 0, \quad 1 \le i < j \le N, \end{aligned} \tag{42}$$

$$\begin{aligned} &\Omega_{ii} \le 0, \quad i = 1, \dots, N, \\ &\Omega_{ij} + \Omega_{ji} \le 0, \quad 1 \le i < j \le N, \end{aligned} \tag{43}$$

$$\begin{aligned} &\Delta_{ii} \le 0, \quad i = 1, \dots, N, \\ &\Delta_{ij} + \Delta_{ji} \le 0, \quad 1 \le i < j \le N, \end{aligned} \tag{44}$$

where

$$\Psi_{ij} = \begin{bmatrix} \psi_{11} & \theta_1(P\bar{D}_{xj} - W_i\bar{D}_{yj}) + \tilde{C}_i^T U \bar{D}_{yj} \\ \star & \bar{D}_{yi}^T U \bar{D}_{yj} - \gamma^2 I \end{bmatrix},$$

$$\psi_{11} = \mathrm{He}\{\theta_1(PA_j - W_i\tilde{C}_j)\} + \tilde{C}_i^T U \tilde{C}_j,$$

$$\Pi_{ij} = \begin{bmatrix} \pi_{11} & \theta_2(PB_j - W_iR_j) - \tilde{C}_i^T U R_j \\ \star & -R_i^T U R_j + \beta_1^2 I, \end{bmatrix}$$

$$\pi_{11} = \mathrm{He}\{\theta_2(PA_j - W_i\tilde{C}_j)\} - \tilde{C}_i^T U \tilde{C}_j,$$

$$\Omega_{ij} = \begin{bmatrix} \omega_{11} & -\theta_3 W_i S_j - \tilde{C}_i^T U S_j \\ \star & -S_i^T U S_j + \beta_2^2 I \end{bmatrix},$$

$$\omega_{11} = \mathrm{He}\{\theta_3(PA_j - W_i\tilde{C}_j)\} - \tilde{C}_i^T U \tilde{C}_j,$$

$$\Delta_{ij} = \begin{bmatrix} -P & PA_j - W_iC_j - aP \\ \star & -\tau^2 P \end{bmatrix},$$

$$W_i = PL_i, \quad U = M^T M.$$

Given scalar κ, the proposed observer gain matrix L_i can be optimized through solving the optimization problem

$$\max \quad \kappa\beta_1^2 + (1-\kappa)\beta_2^2 \tag{45}$$

subject to (41)–(44).

Proof.
(i) *Disturbance attenuation condition.* The robustness of the residual signal $r(t)$ against disturbance $d(t)$ is first considered. Making $f_a(t) = 0$ and $f_s(t) = 0$ in (40), we have

$$\begin{cases} \dot{e}(t) = [A(\rho) - L(\rho)\tilde{C}(\rho)]e(t) \\ \qquad + [\bar{D}_x(\rho) - L(\rho)\bar{D}_y(\rho)]\bar{d}(t), & (46) \\ r(t) = M\tilde{C}(\rho)e(t) + M\bar{D}_y(\rho)\bar{d}(t). & (47) \end{cases}$$

According to Lemma 1, there exists a parameter-dependent Lyapunov matrix $P_1(\rho)$ such that

$$\Psi = \begin{bmatrix} \Upsilon_1 & \Upsilon_2 \\ \star & \bar{D}_y(\rho)^T M^T M \bar{D}_y(\rho) - \gamma^2 I \end{bmatrix} \le 0, \tag{48}$$

where $P_1 = P_1^T > 0$ and

$$\begin{aligned}\Upsilon_1 &= \mathrm{He}\{P_1[A(\rho) - L(\rho)\tilde{C}(\rho)]\} \\ &\quad + \tilde{C}^T(\rho) M^T M \tilde{C}(\rho), \\ \Upsilon_2 &= P_1[\bar{D}_x(\rho) - L(\rho)\bar{D}_y(\rho)] \\ &\quad + C^T M^T M \bar{D}_y(\rho).\end{aligned}$$

Setting $U = M^T M$ and $W_1(\rho) = P_1 L(\rho)$, according to Apkarian and Tuan (2000) as well as Tanaka and Wang (2001), it is not difficult to write (48) as

$$\begin{aligned}\Psi &= \sum_{j=1}^N \sum_{i=1}^N h_i(\rho(t)) h_j(\rho(t)) \tilde{\Psi}_{ij} \\ &= \sum_{i=1}^N h_i(\rho(t))^2 \tilde{\Psi}_{ii} \\ &\quad + \sum_{i=1}^N \sum_{i<j}^N h_i(\rho(t)) h_j(\rho(t)) (\tilde{\Psi}_{ij} + \tilde{\Psi}_{ji}) \le 0,\end{aligned} \tag{49}$$

where

$$\tilde{\Psi}_{ij} = \begin{bmatrix} \tilde{\psi}_{11} & (P_1 \bar{D}_{xj} - W_{1i}\bar{D}_{yj}) + \tilde{C}_i^T U \bar{D}_{yj} \\ \star & \bar{D}_{yi}^T U \bar{D}_{yj} - \gamma^2 I \end{bmatrix},$$

$$\tilde{\psi}_{11} = \mathrm{He}\{P_1 A_j - W_{1i}\tilde{C}_j\} + \tilde{C}_i^T U \tilde{C}_j.$$

Thus, the following condition is obtained:

$$\begin{cases} \tilde{\Psi}_{ii} \le 0, & i = 1, \dots, N, \\ \tilde{\Psi}_{ij} + \tilde{\Psi}_{ji} \le 0, & 1 \le i < j \le N. \end{cases} \tag{50}$$

(ii) *Actuator fault sensitivity condition.* In order to consider the actuator fault sensitivity of residual $r(t)$, letting $\bar{d}(t) = 0$ and $f_s(t) = 0$ in (40), we have

$$\begin{cases} \dot{e}(t) = [A(\rho) - L(\rho)\tilde{C}(\rho)]e(t) \\ \qquad\quad + [B(\rho) - L(\rho)R(\rho)]f_a(t), & (51) \\ r(t) = M\tilde{C}(\rho)e(t) + MR(\rho)f_a(t). & (52) \end{cases}$$

Based on Lemma 3, if there exists $P_2(\rho) = P_2(\rho)^T > 0$ such that

$$\Pi = \begin{bmatrix} \Upsilon_3 & \Upsilon_4 \\ \star & -R^T(\rho) M^T M R(\rho) + \beta_1^2 I \end{bmatrix} \le 0, \tag{53}$$

where

$$\begin{aligned}\Upsilon_3 &= \mathrm{He}\{P_2[A(\rho) - L(\rho)\tilde{C}(\rho)]\} \\ &\quad - \tilde{C}^T(\rho) M^T M \tilde{C}(\rho), \\ \Upsilon_4 &= P_2[B(\rho)) - L(\rho)R(\rho)] \\ &\quad - \tilde{C}^T(\rho) M^T M R(\rho),\end{aligned}$$

then H_- actuator fault sensitivity performance of the residual is assured.

Setting $U = M^T M$ and $W_2(\rho) = P_2 L(\rho)$, we have

$$\begin{aligned}\Pi &= \sum_{i=1}^N h_i(\rho(t))^2 \tilde{\Pi}_{ii} \\ &\quad + \sum_{i=1}^N \sum_{i<j}^N h_i(\rho(t)) h_j(\rho(t)) (\tilde{\Pi}_{ij} + \tilde{\Pi}_{ji}) \le 0,\end{aligned} \tag{54}$$

where

$$\tilde{\Pi}_{ij} = \begin{bmatrix} \tilde{\pi}_{11} & P_2 E_i - W_{2i} R_j - \tilde{C}_i^T U R_j \\ \star & -R_i^T U R_j + \beta_1^2 I \end{bmatrix},$$

$$\tilde{\pi}_{11} = \mathrm{He}\{P_2 A_j - W_{2i}\tilde{C}_j\} - \tilde{C}_i^T U \tilde{C}_j.$$

Then we obtain

$$\begin{cases} \tilde{\Pi}_{ii} \le 0, & i = 1, \dots, N, \\ \tilde{\Pi}_{ij} + \tilde{\Pi}_{ji} \le 0, & 1 \le i < j \le N. \end{cases} \tag{55}$$

(iii) *Sensor fault sensitivity condition.* The analysis of the sensor fault sensitivity condition is similar to that of the actuator fault sensitivity condition as mentioned above. Making $\bar{d}(t) = 0$ and $f_a(t) = 0$, (40) becomes

$$\begin{cases} \dot{e}(t) = [A(\rho) - L(\rho)\tilde{C}(\rho)]e(t) \\ \qquad\quad - L(\rho) S f_s(t), & (56) \\ r(t) = M\tilde{C}(\rho)e(t) + MSf_s(t). & (57) \end{cases}$$

Similarly, we can prove that there exists a positive definite symmetric matrix $P_3(\rho)$ such that the following LMI holds:

$$\Omega = \begin{bmatrix} \Upsilon_5 & \Upsilon_6 \\ \star & -S^T M^T M S + \beta_2^2 I \end{bmatrix} \le 0, \tag{58}$$

where

$$\begin{aligned}\Upsilon_5 &= \mathrm{He}\{P_3[A(\rho) - L(\rho)\tilde{C}(\rho)]\} \\ &\quad - \tilde{C}^T(\rho) M^T M \tilde{C}(\rho), \\ \Upsilon_6 &= -P_3 L(\rho) S - \tilde{C}^T(\rho) M^T M S.\end{aligned}$$

With $U = M^T M$ and $W_3(\rho) = P_3 L(\rho)$, we obtain

$$\begin{aligned}\Omega &= \sum_{i=1}^N h_i(\rho(t))^2 \tilde{\Omega}_{ii} \\ &\quad + \sum_{i=1}^N \sum_{i<j}^N h_i(\rho(t)) h_j(\rho(t)) (\tilde{\Omega}_{ij} + \tilde{\Omega}_{ji}) \le 0,\end{aligned} \tag{59}$$

where

$$\tilde{\Omega}_{ij} = \begin{bmatrix} \tilde{\omega}_{11} & -W_{3i} S - \tilde{C}_i^T U S \\ \star & -S^T U S + \beta_2^2 I \end{bmatrix},$$

$$\tilde{\pi}_{11} = \mathrm{He}\{P_3 A_j - W_{3i}\tilde{C}_j\} - \tilde{C}_i^T U \tilde{C}_j.$$

Then we have

$$\begin{cases}\tilde{\Omega}_{ii} \le 0, \quad i = 1, \dots, N, \\ \tilde{\Omega}_{ij} + \tilde{\Omega}_{ji} \le 0, \quad 1 \le i < j \le N. \end{cases} \tag{60}$$

Note that due to the existence of $P_1(\rho)L(\rho)$, $P_2(\rho)L(\rho)$, $P_3(\rho)L(\rho)$, there is a coupling between (50), (55) and (60). As $P_1(\rho)$, $P_2(\rho)$, $P_3(\rho)$ can be chosen different, in order to decrease some conservatism, setting

$$\begin{aligned} P_1 &= \theta_1 P, & P_2 &= \theta_2 P, \\ P_3 &= \theta_3 P, & W(\rho) &= PL(\rho) \end{aligned} \tag{61}$$

and substituting (61) into (50), (55) and (60), we can obtain (41), (42) and (43).

Moreover, in this paper, the performances of state error dynamics are specified via a regional pole constraint; based on Lemma 2, we have

$$\Delta = \begin{bmatrix} -P & P[A(\rho) - L(\rho)\tilde{C}(\rho) - aI] \\ \star & -\tau^2 P \end{bmatrix} < 0. \tag{62}$$

Then, with a similar process, (62) is equivalent to (44). This completes the proof. ■

The gain matrices L_i of the proposed observer are obtained based on $L_i = P^{-1}W_i$ and the fact that U is a non-negative symmetric matrix; the matrix M can be obtained as the square root of U.

Remark 3. The parameters θ_1, θ_2, θ_3 are determined beforehand, which can provide more degrees of design freedom. In the implementation, they can be chosen by a trial-and-error method.

5. Simulation results

In this section, numerical simulations are reported to demonstrate the effectiveness of the proposed H_-/H_∞ fault detection observer design method.

Consider a vertical takeoff and landing (VTOL) aircraft model in the vertical plane from the work of Jia *et al.* (2015), described in LPV form as in (10), where the state vector $x(t) = [V_h\ V_v\ q\ \theta]^T$ consists of horizontal velocity, vertical velocity, pitch rate and pitch angle, respectively, and the control input $u(t) = [u_c\ u_l]^T$ covers collective pitch control and longitudinal cyclic pitch control, respectively. Here $w(t)$ represents model uncertainties and unknown disturbances and $v(t)$ denotes the measurement noise.

The system matrices of the VTOL model are expressed as

$$A(\rho) = \begin{bmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 0.2632 & 5.0337 \\ 26.0922 & 2.6361 + \rho_1 & -4.1975 & -19.2774 + \rho_2 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B(\rho) = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 + \rho_2 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix},$$

$$E_s = I_4.$$

The disturbance distribution matrices are given as

$$D_w = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix},$$

$$D_y = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \\ 0 & 0 \\ 0.001 & 0 \end{bmatrix}.$$

As the relative degree of each output with respect to the actuator fault is 1, the new output and its corresponding matrices can be obtained based on (35). In this paper, it is assumed that $\rho(t) = [\rho_1(t)\ \ \rho_2(t)]^T$ are the scheduling vectors with $\rho_1(t) \in [-0.5,\ 0.5]$ and $\rho_2(t) \in [-2,\ 2]$. The scheduling variables $\rho_1(t)$ and $\rho_2(t)$ are shown in Figs. 2 and 3. Based on the vertex of $\rho_1(t)$ and $\rho_2(t)$, four local models are derived, and the weighting functions $h_i(\rho(t))$, $i = 1, \dots, N$, $i = 1, \dots, N = 4$, are described as

$$\begin{aligned} h_{11}(\rho(t)) &= \frac{\overline{\rho}_1 - \rho_1(t)}{\overline{\rho}_1 - \underline{\rho}_1} \cdot \frac{\overline{\rho}_2 - \rho_2(t)}{\overline{\rho}_2 - \underline{\rho}_2}, \\ h_{12}(\rho(t)) &= \frac{\overline{\rho}_1 - \rho_1(t)}{\overline{\rho}_1 - \underline{\rho}_1} \cdot \frac{\rho_2(t) - \underline{\rho}_2}{\overline{\rho}_2 - \underline{\rho}_2}, \\ h_{21}(\rho(t)) &= \frac{\rho_1(t) - \underline{\rho}_1}{\overline{\rho}_1 - \underline{\rho}_1} \cdot \frac{\overline{\rho}_2 - \rho_2(t)}{\overline{\rho}_2 - \underline{\rho}_2}, \\ h_{22}(\rho(t)) &= \frac{\rho_1(t) - \underline{\rho}_1}{\overline{\rho}_1 - \underline{\rho}_1} \cdot \frac{\rho_2(t) - \underline{\rho}_2}{\overline{\rho}_2 - \underline{\rho}_2}, \end{aligned} \tag{63}$$

where $h_{11}(\rho(t))$, $h_{12}(\rho(t))$, $h_{21}(\rho(t))$ and $h_{22}(\rho(t))$ correspond to the vertex defined by $(\rho_1 = -0.5, \rho_2 =$

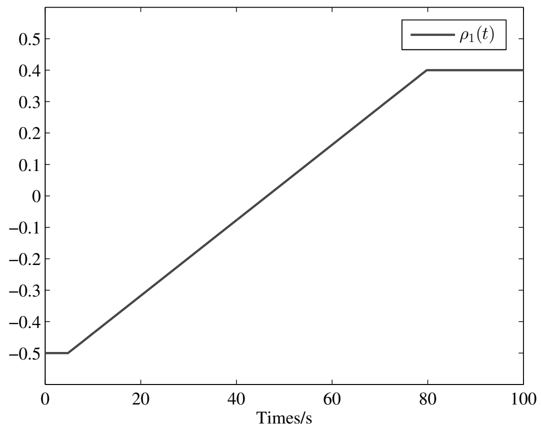

Fig. 2. First time-varying parameter $\rho_1(t)$.

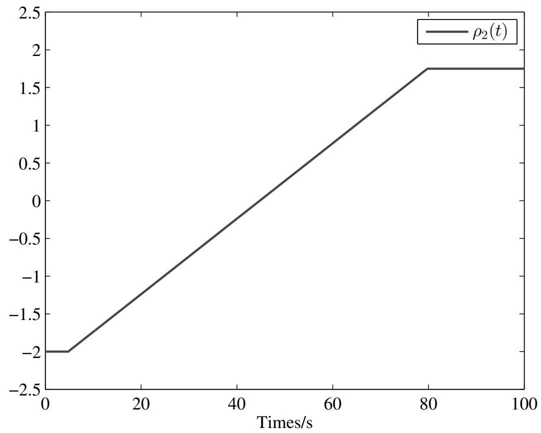

Fig. 3. Second time-varying parameter $\rho_2(t)$.

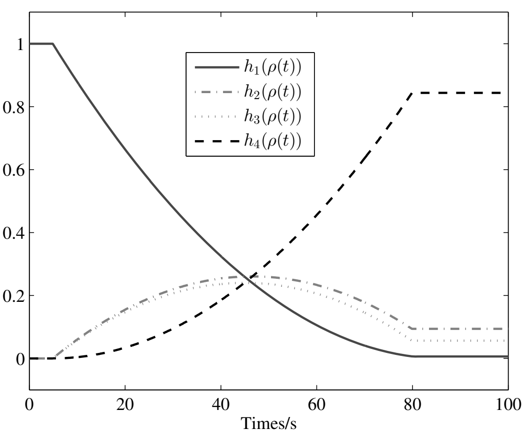

Fig. 4. Local weighting functions $h_i(\rho)$.

$-2)$, $(\rho_1 = -0.5, \rho_2 = 2)$, $(\rho_1 = 0.5, \rho_2 = -2)$ and $(\rho_1 = 0.5, \rho_2 = 2)$, respectively.

The local weighting function $h_i(\rho(t))$ of each local model is depicted in Fig. 4. In accordance with the separation principle of the controller and observer design, a controller has been synthesized with a state feedback structure so as to maintain the closed-loop system stable.

In this paper, the parameters in (45) are chosen as $\kappa = 0.4$, and other parameters are chosen as $\theta_1 = 2$, $\theta_2 = 1$, $\theta_3 = 0.5$. The disturbance attenuation performance is given by $\gamma = 0.05$, Besides, in the calculation, the matrix V is restricted as $\|V\| \leq 10$; then, by solving the optimization problem in (45) of Theorem 1 with the regional pole constraint $\mathcal{D}(-5, 4.9)$, H_- fault sensitivity performance is obtained as $\beta_1 = 0.5419$ and $\beta_2 = 0.1023$, and the gain matrices of the proposed observer are calculated as

$$\begin{aligned} L_{11} &= \begin{bmatrix} L_{11a} & L_{11b} \end{bmatrix}, \quad L_{12} = \begin{bmatrix} L_{12a} & L_{12b} \end{bmatrix}, \\ L_{21} &= \begin{bmatrix} L_{21a} & L_{21b} \end{bmatrix}, \quad L_{22} = \begin{bmatrix} L_{22a} & L_{22b} \end{bmatrix}. \end{aligned}$$

$$L_{11a} = \begin{bmatrix} 0.7752 & 0.2769 & 0.2485 & -0.3109 \\ -0.1075 & -4.8348 & -6.3840 & 5.9192 \\ -6.3197 & 2.7280 & 8.6263 & -2.2893 \\ -1.2230 & -4.0938 & -2.3330 & 3.9177 \end{bmatrix},$$

$$L_{11b} = \begin{bmatrix} -0.0825 & -0.0765 & -0.1019 & -0.0279 \\ 1.1443 & -0.4009 & 0.1428 & 0.4592 \\ 0.2670 & -0.0512 & 0.3010 & 0.3912 \\ 0.3627 & -0.2607 & 0.0728 & 0.0752 \end{bmatrix},$$

$$L_{12a} = \begin{bmatrix} 14.4268 & 5.9601 & 6.7832 & -5.6820 \\ -35.0021 & -21.3571 & -22.3228 & 20.7755 \\ -19.1608 & -0.4000 & -0.8557 & 0.7620 \\ -14.4270 & -9.5913 & -8.4461 & 8.7807 \end{bmatrix},$$

$$L_{12b} = \begin{bmatrix} -0.1330 & -0.1667 & -0.1026 & -0.0802 \\ 1.1057 & 0.0079 & 0.1885 & 0.7890 \\ 0.2103 & 0.5517 & -0.4382 & 0.3675 \\ 0.3251 & -0.0297 & -0.0533 & 0.1739 \end{bmatrix},$$

$$L_{21a} = \begin{bmatrix} 1.1953 & -0.1082 & -0.0024 & 0.0684 \\ 5.4471 & -1.1997 & -5.2597 & 2.0864 \\ -8.2455 & -0.0901 & 2.7796 & 0.2992 \\ 0.5972 & -3.5113 & -3.1147 & 3.0946 \end{bmatrix},$$

$$L_{21b} = \begin{bmatrix} -0.0876 & -0.0812 & -0.0955 & -0.0264 \\ 0.8548 & 0.0909 & -0.3803 & 0.4339 \\ 0.2769 & -0.0703 & 0.2884 & 0.3832 \\ 0.2823 & -0.0806 & -0.1268 & 0.0426 \end{bmatrix},$$

$$L_{22a} = \begin{bmatrix} 14.6124 & 6.3756 & 6.8226 & -5.8846 \\ -41.2032 & -24.7593 & -24.9648 & 22.4426 \\ -19.0451 & -3.2254 & -6.4556 & 4.0874 \\ -15.7429 & -10.8376 & -10.3446 & 9.5609 \end{bmatrix},$$

$$L_{22b} = \begin{bmatrix} -0.1400 & -0.1637 & -0.1125 & -0.0809 \\ 0.8614 & 0.1511 & 0.0484 & 0.7945 \\ 0.2389 & 0.3499 & -0.2381 & 0.3825 \\ 0.2750 & -0.0392 & -0.0548 & 0.1771 \end{bmatrix},$$

$$M = \begin{bmatrix} M_a & M_b \end{bmatrix}$$

$$M_a = \begin{bmatrix} 9.2197 & 0.6554 & -0.2725 & -0.8181 \\ 0.6554 & 4.9316 & 1.7524 & -3.5017 \\ -0.2725 & 1.7524 & 5.5968 & -1.5790 \\ -0.8181 & -3.5017 & -1.5790 & 3.1788 \\ 0.3395 & 0.2601 & 0.1404 & -0.0993 \\ -0.0708 & 0.0102 & -0.0709 & -0.1859 \\ -0.0164 & -0.1853 & 0.0951 & -0.1521 \\ -0.1273 & 0.0149 & -0.0016 & 0.2762 \end{bmatrix},$$

$$M_b = \begin{bmatrix} 0.3395 & -0.0708 & -0.0164 & -0.1273 \\ 0.2601 & 0.0102 & -0.1853 & 0.0149 \\ 0.1404 & -0.0709 & 0.0951 & -0.0016 \\ -0.0993 & -0.1859 & -0.1521 & 0.2762 \\ 0.0406 & -0.0320 & -0.0497 & 0.0400 \\ -0.0320 & 0.0554 & 0.0748 & -0.0791 \\ -0.0497 & 0.0748 & 0.1254 & -0.1180 \\ 0.0400 & -0.0791 & -0.1180 & 0.1226 \end{bmatrix}.$$

Besides, in order to demonstrate the effectiveness of the proposed method, it is compared with the classical H_∞ observer. For the system (10), design the observer (19) by solving the following LMIs:

$$\begin{bmatrix} \varsigma & PD_{xi} - W_i D_y + C^T U D_y \\ \star & D_y^T U D_y - \gamma^2 I \end{bmatrix} < 0,$$

$$\begin{bmatrix} -P & PA_j - W_i C - aP \\ \star & -\tau^2 P \end{bmatrix} < 0, \tag{64}$$

where

$$\varsigma = \mathrm{He}\{PA_j - W_i C\} + C^T U C.$$

The matrices of the H_∞ observer are obtained as

$$\tilde{L}_{11} = \begin{bmatrix} -4.4597 & -5.7813 & -4.7705 & 5.0337 \\ 52.1658 & 32.6016 & 29.9215 & -24.1270 \\ 26.0922 & 23.3596 & 22.5140 & -20.9823 \\ 0.0000 & -5.0850 & -4.0850 & 5.3800 \end{bmatrix},$$

$$\tilde{L}_{12} = \begin{bmatrix} -4.4597 & -5.7813 & -4.7705 & 5.0337 \\ 52.1658 & 32.6016 & 29.9215 & -24.1270 \\ 26.0922 & 19.3596 & 18.5140 & -16.9823 \\ 0.0000 & -5.0850 & -4.0850 & 5.3800 \end{bmatrix},$$

$$\tilde{L}_{21} = \begin{bmatrix} -4.4597 & -5.7813 & -4.7705 & 5.0337 \\ 52.1658 & 32.6016 & 29.9215 & -24.1270 \\ 26.0922 & 24.3596 & 22.5140 & -20.9823 \\ 0.0000 & -5.0850 & -4.0850 & 5.3800 \end{bmatrix},$$

$$\tilde{L}_{22} = \begin{bmatrix} -4.4597 & -5.7813 & -4.7705 & 5.0337 \\ 52.1658 & 32.6016 & 29.9215 & -24.1270 \\ 26.0922 & 20.3596 & 18.5140 & -16.9823 \\ 0.0000 & -5.0850 & -4.0850 & 5.3800 \end{bmatrix},$$

$$\tilde{M} = \begin{bmatrix} 0.3780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.4201 & 0.0421 & -0.0841 \\ 0.0000 & 0.0421 & 0.4201 & -0.0841 \\ 0.0000 & -0.0841 & -0.0841 & 0.2941 \end{bmatrix}.$$

Figure 5 displays the pole location of the estimation error system (22). It can be seen that the poles of the error system locate the given circle $(-5, 4.9)$.

In the simulations, the initial state is chosen as $[0\ \ 0\ \ 0\ \ 0]^T$, the input vector is given as $u(t) = [\sin(0.01t)\ \ \cos(0.05t + \pi/2)]^T$, the system uncertainty $w(t)$ and measurement noise $v(t)$ are set as random signals bounded by $[-1,\ 1]$. In this paper, residual evaluation is considered to be the root mean square (RMS) value of the generated residual as in the work of Huang *et al.* (2017). In this paper, two fault scenarios are assumed to be detected. First, an abrupt fault is assumed to occur in the first actuator with the 50 % loss of effectiveness at $t = 40$ s, i.e.,

$$\begin{aligned} f_a(t) &= -\eta u(t), \\ \eta &= \begin{cases} [0\ \ 0]^T, & 0\,\mathrm{s} < t \le 40\,\mathrm{s}, \\ [0.5\ \ 0]^T, & 40\,\mathrm{s} < t \le 100\,\mathrm{s}, \end{cases} \\ f_s(t) &= [0\,0\,0\ \ 0]^T. \end{aligned} \tag{65}$$

In the second fault scenario, we consider the sensor fault; it is assumed that a time-varying fault occurs in the first sensor with the formulation as

$$\begin{aligned} f_a(t) &= [0\ \ 0]^T, \\ f_s(t) &= \begin{cases} [0\,0\,0\ \ 0]^T, & 0\,\mathrm{s} < t \le 20\,\mathrm{s}, \\ [f_1(t)\ 0\ 0\ \ 0]^T, & 20\,\mathrm{s} < t \le 65\,\mathrm{s}, \\ [0\,0\,0\ \ 0]^T, & 65\,\mathrm{s} < t \le 100\,\mathrm{s}, \end{cases} \\ f_1(t) &= 0.03 + 0.01\sin(0.1\pi(t-20)). \end{aligned} \tag{66}$$

The simulation results are depicted in Figs. 6 and 7. The former shows residual evaluation generated by the

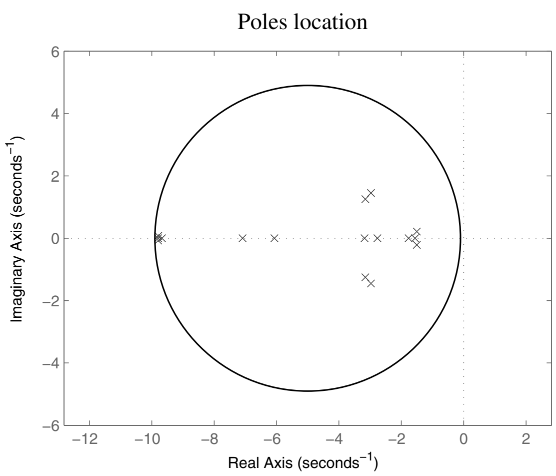

Fig. 5. Location of the poles for the system determining the estimation error.

proposed method and the H_∞ observer for the actuator fault (65). Therein, the dash-dotted line represents the residual evaluation generated by the proposed H_-/H_∞ observer and the dashed line is the residual evaluation generated by the H_∞ observer. The latter figure shows the residual evaluation for the sensor fault (66). It can be seen that the proposed method has more fault sensitivity than the H_∞ observer one.

6. Conclusions

In this paper, an H_-/H_∞ fault detection observer was designed for a class of polytopic LPV systems by using the relative degree of output. First, by considering the original output and its time derivative as a new output, a new augmented system was generated, such that the H_- index was feasible for the new system. Then the observer was designed to consider the H_∞ disturbance attenuation performance, H_- index of actuator fault sensitivity and sensor fault sensitivity simultaneously. Simulations results demonstrated the effectiveness of the proposed method.

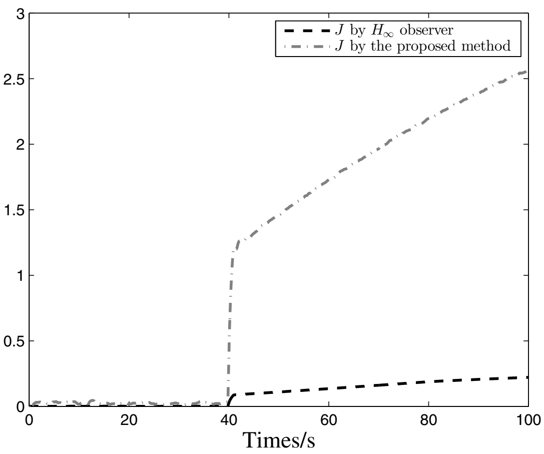

Fig. 6. Residual evaluation function of the first fault scenario (65).

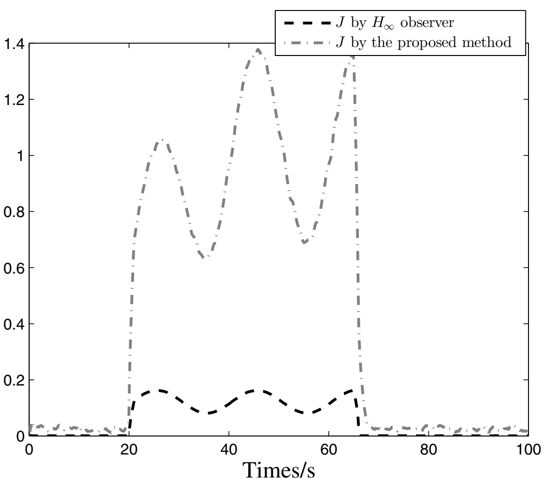

Fig. 7. Residual evaluation function of the second fault scenario (66).

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