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THE ONSET OF SORET DRIVEN FERROTHERMOCONVECTIVE INSTABILITY IN THE PRESENCE OF DARCY POROUS MEDIUM WITH ANISOTROPY EFFECT AND MFD VISCOSITY

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The effect of magnetic field dependent (MFD) viscosity on Soret driven ferrothermohaline convection in a densely packed anisotropic porous medium has been studied. The Soret effect is focused on the system. A linear stability analysis is carried out using a normal mode technique and a perturbation method is applied. It is found that a stationary mode is favorable for the Darcy model. Vertical anisotropy tends to destabilize the system and the magnetization effect is found to stabilize the system. It is also found that the MFD viscosity delays the onset of convection. Numerical computations are made and illustrated graphically.

Key words: anisotropy effect, Darcy model, MFD viscosity, porous medium, Soret coefficient.

1. Introduction

A fluid (liquid or gas or plasma) is a substance that continuously deforms (flows) under an applied shear stress. Generally, fluids are classified into four categories: real and ideal fluids, Newtonian and non-Newtonian fluids. The flow of real fluids exhibits viscous effect, that is they tend to stick to solid surfaces and have stresses within their body. Examples of real fluids are heavy oils (motor oil), syrup, etc. Ideal fluids are those which are incapable of sustaining any tangential force (shearing stresses) or action in the form of pressure acting between the adjoining layer, which means that an ideal fluid offers no internal resistance to change its shape. Ideal fluids are known as inviscid fluids (zero viscosity) or frictionless fluids or perfect fluids. Examples of ideal fluids are gasoline (low viscosity and faster flows), air, water, etc.

Ferro fluids are suspensions of magnetic particles of diameter approximately *10 nm* stabilized by surfactants in carrier liquids. The large magnetic susceptibility of ferrofluids allows the mobilization of ferrofluid through permeable rock and soil by the application of strong external magnetic fields. Suspensions of magnetic nano-particles exhibit normal liquid behaviour coupled with super paramagnetic properties. This leads to the possibility of controlling the properties and the flow of these liquids with moderate magnetic fields. The magnetic control enables the design of various applications as well as basic experiments in hydrodynamics. Ferro fluids and their general properties will be introduced and as an example the control of their viscous properties by means of magnetic fields will be discussed to show the potential of magnetic fluid control.

The effect of uniform distribution of heat source on the onset of stationary ferroconvection was investigated by Rudraiah *et al.* [1]. The effects of a magnetic field and non-uniform temperature gradient on Marangoni convection was analysed by Rudraiah *et al.* [2]. The effect of a magnetic field dependent (MFD) viscosity on ferroconvection in an anisotropic porous medium was carried out by Ramanathan and Suresh [3]. Vaidyanathan *et al.* [4] discussed the effect of a magnetic field dependent viscosity on ferroconvection in a sparsely distributed porous medium. Paras Ram *et al.* [5-6] discussed the ferrofluid flow with a magnetic field

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dependent viscosity due to a rotating disk with and without a porous medium. The effect of a magnetic field dependent (MFD) viscosity on ferroconvection in a rotating disc with and without a porous medium was studied by Vaidyanathan *et al.* [7-8]. The effect of a magnetic field dependent viscosity on the onset of convection in a ferromagnetic fluid layer heated from below and cooled from above in the presence of a vertical magnetic field with constant heat flux was investigated by Nanjundappa *et al.* [9].

Hemalatha [10] analysed the effect of a magnetic field dependent viscosity on a Soret driven ferrothermohaline convection in a rotating porous medium. The comparison of theoretical and computational ferroconvection induced by a magnetic field dependent viscosity in an anisotropic porous medium was analyzed by Suresh *et al.* [11]. A nonlinear stability analysis for a thermoconvective and duble-diffusive magnetized ferrofluid with MFD viscosity was investigated by Sunil *et al.* [12-13]. Sunil *et al.* [14] studied theoretically the effect of a magnetic field dependent viscosity on the thermal convection in a ferromagnetic fluid layer with or without dust particles. Vaidyanathan *et al.* [15] investigated the effect of a horizontal thermal gradient on ferroconvection. Vasanthakumari *et al.* [16] studied differential equations in stability analysis of ferrofluids. Gaikwad *et al.* [17] analysed the effect linear stability on double diffusive convection in a fluid saturated anisotropic porous layer with the Soret effect. Selvaraj *et al.* [18] investigated convective instability of strongly magnetized ferrofluids. Sekar *et al.* [19] carried out the stability analysis of the Soret effect on thermohaline convection in a dusty ferrofluid saturating a Darcy porous medium.

Anitha *et al.* [20] investigated the application of differential equation in stability analysis of dependent viscosity of thermohaline convection in a ferromagnetic fluid in a densely packed porous medium. Ravisha *et al.* [21] studied the thermomagnetic convection in porous media with the effect of anisotropy and local thermal nonequilibrium (LTNE). The weakly nonlinear oscillatory convection in a viscoelastic fluid saturated porous medium with through flow and temperature modulation was studied by Kiran *et al.* [22]. The combined effects of Soret and Dufour on MHD flow of a power-law fluid over a flat plate in slip flow regime was investigated by Saritha *et al.* [23]. Raju [24] investigated the effect of a temperature dependent viscosity on ferrothermohaline convection saturating an anisotropic porous medium with the Soret effect using the Galerkin technique. Sekar *et al.* [25] carried out the stability analysis of ferrothermohaline convection in a Darcy porous medium with Soret and MFD viscosity effects. Sekar *et al.* [26] studied the linear stability effect of densely distributed porous medium and Coriolis force on the Soret driven ferrothermohaline convection. Arunkumar *et al.* [27] investigated the effect of MFD viscosity on Benard-Marangoni ferroconvection in a rotating ferrofluid layer. Prakash *et al.* [28] investigated the ferromagnetic convection in a sparsely distributed porous medium with a magnetic field dependent viscosity. More recently, Sekar *et al.* [29] made a linear analytical study of Coriolis force on the Soret driven ferrothermohaline convection in a Darcy anisotropic porous medium with MFD viscosity. Most recently, Prakash *et al.* [30] derived the effect of a magnetic field dependent viscosity on ferromagnetic convection in a rotating sparsely distributed porous medium.

2. Mathematical formulation

We consider an infinite, horizontal layer of incompressible Boussinesq ferromagnetic fluid of thickness ‘ d ’ saturating a densely packed anisotropic porous medium heated from below and salted from above. Further, the whole system is assumed anisotropic along the vertical direction which is taken as the z axis (Fig.1). The fluid viscosity is assumed to be magnetic dependent in the form $\mu = \mu_1 \left(1 + \boldsymbol{\delta} \cdot \boldsymbol{B}\right)$, where μ_1 is viscosity of the fluids when the applied magnetic field is absent. The temperature and salinity at the bottom and top surfaces are $z = \pm d / 2$ are $T_0 \pm \Delta T / 2$ and $S_0 \pm \Delta S / 2$, respectively. Both the boundaries are taken to be free and perfect conductors of heat and solute. The Soret effect on the temperature gradient is considered.

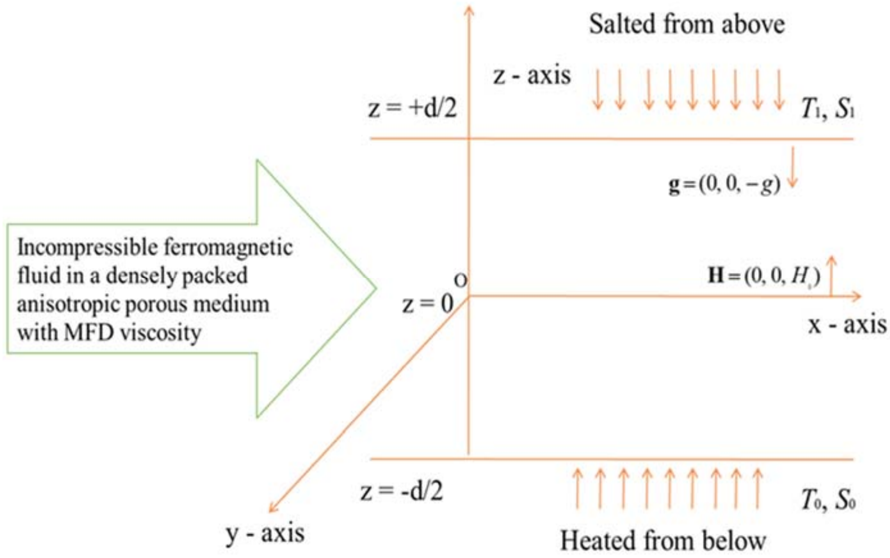

Fig.1. Geometrical configuration.

The variation in the coefficient of the magnetic field dependent viscosity δ has been taken to be isotropic, that is, $\delta = \delta_1 = \delta_2 = \delta_3$. Hence the component μ_1 can be written as

$$\mu_x = \mu_1(1+\delta B_1),\ \mu_y = \mu_1(1+\delta B_2)\ \text{and}\ \mu_z = \mu_1(1+\delta B_3).$$

The continuity equation is

$$\nabla .\, \boldsymbol{q} = 0. \tag{2.1}$$

The modified Navier-Stokes equation is

$$\rho_o \frac{D\boldsymbol{q}}{Dt} = -\nabla p + \rho \boldsymbol{g} + \nabla .(\boldsymbol{HB}) - \frac{\mu_1(1+\boldsymbol{\delta}\ .\ \boldsymbol{B})}{k}\boldsymbol{q}. \tag{2.2}$$

The modified thermal diffusivity equation is

$$\left[\rho_o C_{V,H} - \mu_o \boldsymbol{H}.\left(\frac{\partial \boldsymbol{M}}{\partial T}\right)_{V,H}\right]\frac{dT}{dt} + \mu_o T\left(\frac{\partial \boldsymbol{M}}{\partial T}\right)_{V,H} .\frac{d\boldsymbol{H}}{dt} = K_1 \nabla^2 T + \phi. \tag{2.3}$$

Fick's diffusion equation is

$$\frac{\partial S}{\partial t} + (\boldsymbol{q}.\nabla)S = K_s \nabla^2 S + S_T \nabla^2 T. \tag{2.4}$$

Maxwell's equations are

$$\nabla .\boldsymbol{B} = 0, \nabla \times \boldsymbol{H} = 0. \tag{2.5a,b}$$

Further, $\boldsymbol{B}$, $\boldsymbol{M}$ and $\boldsymbol{H}$ are related by

$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{M} + \boldsymbol{H} \right). \tag{2.6}$$

Combining Eqs (5a) and (6), we get

$$\nabla . \left(\boldsymbol{M} + \boldsymbol{H} \right) = 0 . \tag{2.7}$$

The magnetization is aligned with the magnetic field and depends on the magnitude of the magnetic field, temperature and salinity, so

$$\boldsymbol{M} = \frac{\boldsymbol{H}}{H} M \left(H, T, S \right). \tag{2.8}$$

The magnetic equation of state is

$$M = M_0 + \chi \left(H - H_0 \right) - K \left(T - T_0 \right) + K_2 \left(S - S_0 \right) \tag{2.9}$$

where $\chi = (\partial M / \partial H)_{H_0, T_0}$, $K = -(\partial M / \partial T)_{H_0, T_0}$ and $K_2 = (\partial M / \partial S)_{H_0, S_0}$.
The density equation of state for an incompressible two-component fluid is

$$\rho = \rho_0 \left[1 - \alpha_t \left(T - T_0 \right) + \alpha_s \left(S - S_0 \right) \right] \tag{2.10}$$

where $\alpha_t = -\left(1/\rho \right) \left(\partial \rho / \partial T \right)$ and $\alpha_s = \left(1/\rho \right) \left(\partial \rho / \partial S \right)$.
The basic state is assumed to be the quiescent state

$$\begin{gathered} \boldsymbol{q} = \boldsymbol{q}_b = 0, \quad \mathrm{p} = \mathrm{p}_b(\mathrm{z}), \quad \frac{\partial T}{\partial z} = -\beta_t \Rightarrow T_b = T_0 - \beta_t z, \\ \frac{\partial S}{\partial z} = \beta_S \Rightarrow S_b = S_0 + \beta_s z, \\ \boldsymbol{H}_b \left(Z \right) = \left[H_0 + \frac{\mathrm{K} \left(T_b - T_0 \right)}{1 + \chi} - \frac{K_2 \left(S_b - S_0 \right)}{1 + \chi} \right] \hat{\boldsymbol{k}}, \\ \boldsymbol{M}_b \left(Z \right) = \left[M_0 - \frac{\mathrm{K} \left(T_b - T_0 \right)}{1 + \chi} + \frac{K_2 \left(S_b - S_0 \right)}{1 + \chi} \right] \hat{\boldsymbol{k}}. \end{gathered} \tag{2.11}$$

3. Linear stability theory

The basic state is disturbed by a small thermal perturbation. Consider a perturbed state such that $\boldsymbol{q} = \boldsymbol{q}'$, $p = p_b \left(z \right) + p'$, $\mu = \mu_b \left(z \right) + \mu'$, $T = T_b \left(z \right) + T'$, $\boldsymbol{H} = \boldsymbol{H}_b \left(z \right) + \boldsymbol{H}'$, $\boldsymbol{M} = \boldsymbol{M}_b \left(z \right) + \boldsymbol{M}' a$ where $\boldsymbol{q}'$, p', μ', T', $\boldsymbol{H}'$ and $\boldsymbol{M}'$ are perturbed variables and are assumed to be small

$$H_i^{'}+M_i^{'}=\left(1+\frac{M_0}{H_0}\right)H_i^{'},\qquad (i=1,2), \tag{3.1}$$

$$H_3^{'}+M_3^{'}=(1+\chi)H_3^{'}-KT^{'}+K_2S^{'}+S_TKT^{'}. \tag{3.2}$$

Let $\boldsymbol{B}=(B_1,B_2,B_3)$ be the magnetic induction, using Eq.(2.6), one gets the result $B_i=\mu_0\left(M_i^{'}+H_i^{'}\right)$ and Eqs (3.1) and (3.2) become

$$B_i=\mu_0\left(1+\frac{M_0}{H_0}\right)H_i' \ ,\qquad (i=1,2), \tag{3.3}$$

$$B_3=\mu_0\left[(1+\chi)H_3' \ -KT'+K_2S'+S_{\mathrm{T}}KT'+M_0+H_0\right], \tag{3.4}$$

when Eq.(2.5) is used in Eq.(2.1) and resulting equation are linearized with $B_i\left(i=1,\ 2,\ 3\right)$ given by Eqs (3.3) and (3.4), we obtain in the following components

$$\rho_0\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\mu_0(M_0+H_0)\frac{\partial H_1'}{\partial z}-\frac{\mu_1}{k_1}u\ , \tag{3.5}$$

$$\rho_0\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}+\mu_0(M_0+H_0)\frac{\partial H_2'}{\partial z}-\frac{\mu_1}{k_1}v\ , \tag{3.6}$$

$$\begin{aligned}\rho_0\frac{\partial w}{\partial t}=&-\frac{\partial p}{\partial z}+\mu_0(M_0+H_0)\frac{\partial H_3'}{\partial z}+\mu_0H_3'\ K_2\beta_s-\mu_0H_3'\ K\beta_t+\frac{\mu_0K^2\beta_tT'}{1+\chi}(1-S_{\mathrm{T}})+\\&-\frac{\mu_0KK_2\beta_sT'}{1+\chi}(1-S_{\mathrm{T}})-\frac{\mu_0KK_2\beta_tS'}{1+\chi}+\frac{\mu_0K_2^2\beta_sS'}{1+\chi}+\rho_0g\alpha_tT'-\rho_0g\alpha_sS'-\frac{\mu_1}{k_2}w+\\&-\frac{\mu_1}{k_2}\delta\mu_0(M_0+H_0)w.\end{aligned} \tag{3.7}$$

Differentiating Eqs (3.5)-(3.7) with respect to *x, y* and *z*, respectively, and adding, the following equation is obtained upon using Eq.(2.1):

$$\begin{aligned}\nabla^2p=&\mu_0(M_0+H_0)\frac{\partial}{\partial z}(\nabla.\boldsymbol{H'})+\mu_0K_2\beta_s\frac{\partial H_3'}{\partial z}+\frac{\mu_0K^2\beta_t}{1+\chi}(1-S_T)\frac{\partial T'}{\partial z}+\\&-\frac{\mu_0KK_2\beta_s}{1+\chi}(1-S_T)\frac{\partial T'}{\partial z}+\frac{\mu_1}{k_1}\left(\frac{\partial w}{\partial z}\right)+\frac{\mu_0K_2^2\beta_s}{1+\chi}\frac{\partial S'}{\partial z}-\mu_0K\beta_t\frac{\partial H_3^{'}}{\partial z}+\\&-\frac{\mu_0KK_2\beta_t}{1+\chi}\frac{\partial S'}{\partial z}-\frac{\mu_1}{k_2}\left(\frac{\partial w}{\partial z}\right)+\rho_0g\alpha_t\frac{\partial T'}{\partial z}-\rho_0g\alpha_s\frac{\partial S'}{\partial z}-\frac{\mu_1}{k_2}\delta\mu_0(M_0+H_0)\frac{\partial w}{\partial z}\end{aligned} \tag{3.8}$$

where $\boldsymbol{H'}$ has the components (H_1',H_2',H_3').

From Eq.(2.5b), $\boldsymbol{H'} = \nabla\phi$ where ϕ is a scalar potential. Upon elimination of p from Eqs (3.5)-(3.7) and using Eq.(3.8), we get

$$\begin{aligned}
&\rho_0 \frac{\partial}{\partial t}\left(\nabla^2 w\right) = \mu_0 K_2 \beta_s \frac{\partial}{\partial z}\left(\nabla_1^2 \phi\right) - \rho_0 g \alpha_s \nabla_1^2 S' + \rho_0 g \alpha_t \nabla_1^2 T' - \mu_0 K \beta_t \frac{\partial}{\partial z}\left(\nabla_1^2 \phi\right) + \\
&+ \frac{\mu_0 K^2 \beta_t}{1+\chi}\left(1 - S_T\right)\nabla_1^2 T' - \frac{\mu_1}{k_2}\nabla_1^2 w - \frac{\mu_0 K K_2 \beta_s}{1+\chi}\left(1 - S_T\right)\nabla_1^2 T' + \\
&- \frac{\mu_1}{k_2}\delta\mu_0\left(M_0 + H_0\right)\nabla_1^2 w + \frac{\mu_0 K_2^2 \beta_s}{1+\chi}\nabla_1^2 S' - \frac{\mu_0 K K_2 \beta_t}{1+\chi}\nabla_1^2 S' - \frac{\mu_1}{k_1}\left(\frac{\partial^2 w}{\partial z^2}\right)
\end{aligned} \tag{3.9}$$

where $\nabla_1^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ and $\nabla^2 = \nabla_1^2 + \dfrac{\partial^2}{\partial z^2}$.

4. Normal mode analysis

The normal mode solution of all dynamical variables can be written as

$$\begin{aligned}
&f(x, y, z, t) = f(z, t)e^{i(k_x x + k_y y)}, \qquad \phi = \phi(z, t)e^{i(k_x x + k_y y)}, \\
&w = w(z, t)e^{i(k_x x + k_y y)}, \qquad T' = \theta(z, t)e^{i(k_x x + k_y y)}, \qquad S' = S(z, t)e^{i(k_x x + k_y y)}.
\end{aligned} \tag{4.1}$$

The wave number k_0 is given by

$$k_0^2 = k_x^2 + k_y^2. \tag{4.2}$$

Using Eqs (4.1) and (4.2) in Eq.(3.9), one gets the vertical component of the momentum equation which can be written as

$$\begin{aligned}
&\rho_0 \frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial z^2} - k_0^2\right) w = \frac{\mu_0 K_2 \beta_s}{1+\chi}\left[\left(1+\chi\right)\frac{\partial \phi}{\partial z} + K_2 S\right] k_0^2 + \frac{\mu_0 K \beta_t}{1+\chi}\left[\left(1+\chi\right)\frac{\partial \phi}{\partial z} + \right. \\
&\left. - K\theta\left(1 - S_T\right)\right] k_0^2 - \rho_0 g \alpha_t k_0^2 \theta + \rho_0 g \alpha_s k_0^2 S - \frac{\mu_0 K K_2}{1+\chi}\left[\beta_s\left(1 - S_T\right)\theta - \beta_t S\right] k_0^2 + \\
&+ \frac{\mu_1}{k_2} k_0^2 w - \frac{\mu_1}{k_1}\left(\frac{\partial^2 w}{\partial z^2}\right) w + \frac{\mu_1}{k_2} k_0^2 \delta\mu_0\left(M_0 + H_0\right) w.
\end{aligned} \tag{4.3}$$

The linearized perturbed temperature equation is

$$\rho_0 C_{V,H}\frac{\partial\theta}{\partial t}-\mu_0 K T_0\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right)=K_1\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)\theta+$$
$$+\left[\rho_0 C_{V,H}\beta_t-\frac{\mu_0 K^2 T_0^2\beta_t}{1+\chi}+\frac{\mu_0 K K_2 T_0\beta_s}{1+\chi}\right]w \tag{4.4}$$

where $\rho_0 C=\rho_0 C_{V,H}+\mu_0 K H_0$.

The salinity equation is

$$\frac{\partial S}{\partial t}+\beta_s w=K_s\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)S+S_T\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)\theta\,. \tag{4.5}$$

The magnetic potential equation is

$$(1+\chi)\frac{\partial^2\phi}{\partial z^2}-\left(1+\frac{M_0}{H_0}\right)k_0^2\phi-K\frac{\partial\theta}{\partial z}+K_2\frac{\partial S}{\partial z}+S_T K\frac{\partial\theta}{\partial z}=0\,. \tag{4.6}$$

The above equations can be written in dimensionless form using

$$t^*=\frac{\nu t}{d^2},\qquad w^*=\frac{wd}{\nu},\qquad T^*=\left(\frac{K_1 a R^{1/2}}{\rho_0 C_{V,H}\beta_t\nu d}\right)\theta,\qquad a=k_0 d,\qquad z^*=\frac{z}{d},$$

$$\phi^*=\left(\frac{(1+\chi)K_1 a R^{1/2}}{K\rho_0 C_{V,H}\beta_t\nu d^2}\right)\phi,\qquad \gamma=\frac{\mu}{\rho_0},\qquad k_1{}^*=\frac{k_1}{d^2},\qquad k_2{}^*=\frac{k_2}{d^2},$$

$$D=\frac{\partial}{\partial z^*},\qquad S^*=\left(\frac{K_s a R_s^{1/2}}{\rho_0 C_{V,H}\beta_s\nu d}\right)S,\qquad \delta^*=\mu_0\delta H_0(1+\chi).$$

Following the normal mode analysis, the linearized perturbation dimensionless equations for the thermosolutal convection due to the Soret effect in a ferrofluid are

$$\frac{\partial}{\partial t^*}\left(D^2-a^2\right)w^*=a\,R^{1/2}M_1M_5D\phi^*+\frac{a^2}{k_2{}^*}M_3\delta^*w^*+a\,R_S^{1/2}\left[1+M_4+\frac{M_4}{M_5}\right]S^*+$$
$$-a\,R^{1/2}M_1M_5(1-S_T)T^*+a\,R^{1/2}\left[M_1D\phi^*-\left(1+M_1(1-S_T)\right)T^*\right]-\left(\frac{D^2}{k_1{}^*}-\frac{a^2}{k_2{}^*}\right)w^*, \tag{4.7}$$

$$P_r\left[\frac{\partial T^*}{\partial t^*}-M_2\frac{\partial}{\partial t^*}(D\phi^*)\right]=\left(D^2-a^2\right)T^*+aR^{1/2}(1-M_2-M_2M_5)w^*, \tag{4.8}$$

$$P_r \frac{\partial S^*}{\partial t^*} = \tau(D^2 - a^2)S^* - aR_S^{1/2} M_6 w^* + S_T \left(\frac{M_5}{M_6}\right)\left(\frac{R_S}{R}\right)^{1/2} (D^2 - a^2)T^*, \tag{4.9}$$

$$D^2\phi^* - M_3 a^2 \phi^* - (1 - S_T) DT^* + \frac{M_5}{M_6}\left(\frac{R}{R_S}\right)^{1/2} DS^* = 0 \tag{4.10}$$

where the non-dimensional parameters used are

$$M_1 = \frac{\mu_0 K^2 \beta_t}{(1+\chi)\rho_0 g \alpha_t}, \quad M_2 = \frac{\mu_0 K^2 T_0}{(1+\chi)\rho_0 C_{v,H}}, \quad M_5 = \frac{K_2 \beta_s}{K \beta_t}, \quad M_3 = \frac{1 + M_0 / H_0}{1+\chi},$$

$$M_4 = \frac{\mu_0 K^2 \beta_s}{(1+\chi)\rho_0 g \alpha_s}, \quad M_6 = \frac{K_S}{K_1}, \quad \tau = \rho_0 C_{v,H}\left(\frac{K_S}{K_1}\right), \quad P_r = \frac{\mu C_{v,H}}{K_1},$$

$$R_S = \frac{\rho_0 C_{v,H} \beta_s \alpha_s g d^4}{\nu K_S}, \quad R = \frac{\rho_0 C_{v,H} \beta_t \alpha_t g d^4}{\nu K_1}$$

where R_S is the salinity Rayleigh number, R is the thermal Rayleigh number, P_r is the Prandtl number and other parameters describe non-dimensional parameters.

5. Mathematical Analysis

The boundary conditions on velocity, temperature and salinity are

$$w^* = D^2 w^* = T^* = D\phi^* = S^* = 0 \quad \text{at} \quad z^* = \pm 1/2. \tag{5.1}$$

The exact solutions satisfying above Eq.(5.1) are

$$w^* = Ae^{\sigma t^*} \cos \pi z^*, \quad T^* = Be^{\sigma t^*} \cos \pi z^*, \quad S^* = Ce^{\sigma t^*} \cos \pi z^*,$$
$$D\phi^* = Ee^{\sigma t^*} \cos \pi z^*, \quad \phi^* = \frac{E}{\pi} e^{\sigma t^*} \sin \pi z^* \tag{5.2}$$

where A, B, C, E are constants and $k_2^* = \varepsilon k_1^*$.
In this part, all the partial derivatives and asterisks are removed with the use of exact solutions to find the solution of the system of homogeneous equations in (5.3) to (5.6). Using Eqs (5.2) in Eqs (4.7) to (4.10), we get

$$\left[\sigma\left(\pi^2 + a^2\right) + \left(\frac{\pi^2 \varepsilon + a^2}{k_1 \varepsilon}\right) + \frac{1}{k_1 \varepsilon} a^2 M_3 \delta\right] A - aR^{1/2}\left[1 + M_1 (1 - S_T) + \right.$$
$$\left. + M_1 M_5 (1 - S_T)\right] B + aR_S^{1/2}\left(1 + M_4 + M_4 M_5^{-1}\right) C + aR^{1/2} M_1 (1 + M_5) E = 0, \tag{5.3}$$

$$aR^{1/2}(1 - M_2 - M_2 M_5) A - \left(\pi^2 + a^2 + P_r \sigma\right) B + P_r \sigma M_2 E = 0, \tag{5.4}$$

$$aR_S^{1/2}M_6A+S_T\left(\frac{M_5}{M_6}\right)\left(\frac{R_S}{R}\right)^{1/2}\left(\pi^2+a^2\right)B+\left[\tau\left(\pi^2+a^2\right)+\sigma P_r\right]C=0, \tag{5.5}$$

$$-R_S^{1/2}\pi^2\left(1-S_T\right)B+R^{1/2}\pi^2M_5M_6^{-1}C+R_S^{1/2}\left(\pi^2+a^2M_3\right)E=0. \tag{5.6}$$

The determinant of coefficients $A,\ B,\ C$ and E vanish for the existence of non-trivial Eigen functions. Equations (5.3)-(5.6) lead to

$$U\sigma^3+V\sigma^2+W\sigma+X=0,$$

$$U=\left(\pi^2+a^2\right)\left(\pi^2+a^2M_3\right)P_r^2,$$

$$V=\left(\pi^2+a^2M_3\right)\left[\left(\pi^2+a^2\right)^2(1+\tau)+P_r\left(\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1}+\frac{1}{\varepsilon k_1}a^2M_3\delta\right)\right]P_r,$$

$$\begin{aligned}W=&\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)\left[\tau\left(\pi^2+a^2\right)^2+P_r(1+\tau)\left(\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1}+\frac{1}{\varepsilon k_1}a^2M_3\delta\right)\right]+\\&+a^2RP_r\left(\pi^2+a^2M_3\right)\left[1+M_1(1+M_5)(1-S_T)\right]+\\&-a^2RP_r\pi^2M_1(1+M_5)\left[(1-S_T)+M_5\right]+a^2R_sP_r\left(\pi^2+a^2M_3\right)\left(1+M_4+\frac{M_4}{M_5}\right)M_6,\end{aligned} \tag{5.7}$$

$$\begin{aligned}X=&\tau\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)^2\left(\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1}+\frac{1}{\varepsilon k_1}a^2M_3\delta\right)+\\&-a^2R\tau\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)\left[1+(1-S_T)M_1(1+M_5)\right]+\\&+a^2R\left(\pi^2+a^2\right)M_1(1+M_5)\pi^2\left[S_T\left(\frac{M_5}{M_6}\right)^2+\tau(1-S_T)+M_5\right]+\\&-a^2R_s\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)\left(1+M_4+\frac{M_4}{M_5}\right)\left[S_T\left(\frac{M_5}{M_6}\right)+M_6\right].\end{aligned}$$

6. Stationary Convection

For the steady state (i.e., the validity of the principle of exchange of stability), we have $\sigma=0$ at the margin of stability. Then Eq.(5.7) helps one to obtain Eigen value R_{SC} for which a solution exists;

$$R_{SC}=\frac{N_r}{D_r}$$

where

$$\mathrm{N}_r = \left(\pi^2 + a^2\right)\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k_1} + \frac{1}{\varepsilon k_1} a^2 M_3 \delta\right) - a^2 R_s \tau^{-1}\left(1 + \mathrm{M}_4 + \frac{M_4}{M_5}\right)\left[S_\mathrm{T}\left(\frac{\mathrm{M}_5}{\mathrm{M}_6}\right) + \mathrm{M}_6\right]$$

and

$$D_r = a^2\left[1 + (1 - S_\mathrm{T})\mathrm{M}_1(1 + \mathrm{M}_5)\right] +$$
$$-\pi^2\left[\frac{a^2 \mathrm{M}_1 (1 + \mathrm{M}_5)}{\pi^2 + a^2 \mathrm{M}_3}\right]\left[S_\mathrm{T}\left(\frac{\mathrm{M}_5}{\mathrm{M}_6}\right)^2 \tau^{-1} + (1 - S_\mathrm{T}) + \mathrm{M}_5 \tau^{-1}\right].$$

For M_1 very large, the critical magnetic thermal Rayleigh number $N_{SC} = R_{SC} M_1$ for stationary mode could be simplified as

$$N_{SC} = R_{SC} M_1 = \frac{N_r}{D_r}$$

where

$$\mathrm{N}_r = \left(\pi^2 + a^2\right)\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k_1} + \frac{1}{\varepsilon k_1} a^2 M_3 \delta\right) - a^2 R_s \tau^{-1}\left(1 + \mathrm{M}_4 + \frac{M_4}{M_5}\right)\left[S_\mathrm{T}\left(\frac{\mathrm{M}_5}{\mathrm{M}_6}\right) + \mathrm{M}_6\right]$$

and

$$D_r = a^2\left[(1 - S_\mathrm{T})(1 + \mathrm{M}_5)\right] - \pi^2\left[\frac{a^2 (1 + \mathrm{M}_5)}{\pi^2 + a^2 \mathrm{M}_3}\right]\left[S_\mathrm{T}\left(\frac{\mathrm{M}_5}{\mathrm{M}_6}\right)^2 \tau^{-1} + (1 - S_\mathrm{T}) + \mathrm{M}_5 \tau^{-1}\right].$$

7. Overstability

Taking $\sigma = i\sigma$ and $\sigma^2 > 0$, in Eq.(5.7), one gets the real value of the Rayleigh number because the Rayleigh number is not a complex number (i.e., $\operatorname{Im} R_{oc} = 0$), implies that R_{oc} is a real number. Therefore, the critical Rayleigh number for oscillatory mode has been calculated using

$$R_{OC} = \frac{U_3 V_1 \sigma^4 + (U_1 U_2 - V_1 V_3)\sigma^2 - U_1 V_2}{U_1^2 + V_1^2 \sigma^2}$$

where

$$U_1 = a^2\left(\pi^2 + a^2\right)\mathrm{M}_1(1 + \mathrm{M}_5)\pi^2\left[S_\mathrm{T}\left(\frac{\mathrm{M}_5}{\mathrm{M}_6}\right)^2 + \tau(1 - S_\mathrm{T}) + \mathrm{M}_5\right] +$$
$$-a^2\tau\left(\pi^2 + a^2\mathrm{M}_3\right)\left(\pi^2 + a^2\right)\left[1 + (1 - S_\mathrm{T})\mathrm{M}_1(1 + \mathrm{M}_5)\right],$$

$$U_2 = \left(\pi^2 + a^2\mathrm{M}_3\right)P_r\left[\left(\pi^2 + a^2\right)^2(1 + \tau) + P_r\left(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k_1} + \frac{1}{\varepsilon k_1} a^2 M_3 \delta\right)\right],$$

$$U_3 = \left(\pi^2 + a^2\mathrm{M}_3\right)\left(\pi^2 + a^2\right)P_r^2,$$

$$V_1 = a^2\pi^2 M_1(1+M_5)P_r\left[(1-S_T)+M_5\right] - a^2 P_r\left(\pi^2+a^2M_3\right)\left[1+M_1(1+M_5)(1-S_T)\right],$$

$$V_2 = \tau\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)^2\left(\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1}+\frac{1}{\varepsilon k_1}a^2M_3\delta\right)+$$
$$-a^2R_s\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)\left(1+M_4+\frac{M_4}{M_5}\right)\left[S_T\left(\frac{M_5}{M_6}\right)+M_6\right],$$

$$V_3 = \left(\pi^2+a^2M_3\right)\left[\tau\left(\pi^2+a^2\right)^3+\left(\pi^2+a^2\right)(1+\tau)P_r\left(\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1}+\frac{1}{\varepsilon k_1}a^2M_3\delta\right)+\right.$$
$$\left.-a^2R_s\left(1+M_4+\frac{M_4}{M_5}\right)M_6P_r\right],$$

$$\sigma^2 = \frac{U_1V_3-V_1V_2}{U_1U_3-U_2V_1}.$$

If oscillatory instability exists, the time factor $\sigma = i\sigma$. Since U, V, W and X are real, Eq.(5.7) could be satisfied for $\sigma = i\sigma$ if and only if $\sigma = 0$. R_{OC} and R_{SC} are critical Rayleigh numbers for the oscillatory and stationary convection system.

8. Method of Solution

The Soret-driven thermoconvective instability of a ferromagnetic fluid layer heated from below and salted from above saturating a densely packed anisotropic porous medium with a magnetic field dependent (MFD) viscosity has been analyzed using the Darcy model. The perturbation method is applied and normal mode analysis is adopted. In the perturbation method, due to the application of a magnetic field, the system is perturbed from the basic state (quiescent state). The governing and other equations are modified. Linear stability analysis is considered. Then normal mode analysis is taken, non-dimensional analysis is carried out and the exact solutions satisfying the appropriate boundary conditions are taken yielding algebraic equations. For getting a non-trivial solution for the system of linear homogeneous equations, the coefficients of the dynamic variables are equated to zero and on simplification, the expression for R_{SC} is obtained. Varying the values of the parameters in the allowable range and getting the corresponding R_{SC} values, we get the stability pattern.

9. Results and Discussion

Before discussing the significant results of the convective system, we turn our attention to the possible range of values of various parameters arising in the study. The anisotropic parameter ε, takes the values from *10* to *70*. The value of the Prandtl number is P_r is *0.01*. The values S_T starts from -*0.002* to *0.002*, R_s is varied from - *500* to *500*. The values of τ are assumed to be *0.03, 0.05, 0.07, 0.09* and *0.11*. The coefficient of MFD viscosity δ is assumed from *0.01* to *0.09*. The magnetization parameter M_1 is 1000; for a very large value of M_1, the effect of magnetic mechanism is very large, when compared to the buoyancy effect. For such fluids, M_2 is assumed to have a negligible value and hence taken to be zero. M_3 is varied from *1* to *25* because M_3 cannot take a value less than one. M_6 is taken to be *0.1*. M_4 is the effect of magnetization due to salinity. This is allowed to vary from *0.1* to *0.5* taking values less than the magnetization parameter M_3. M_5 represents the ratio of the salinity effect on the magnetic field and pyromagnetic coefficient. This is varied between *0.1* and *0.5*. The permeability of porous medium k is assumed to take the values *0.001*, *0.003*, *0.005*, *0.007*, *0.009*.

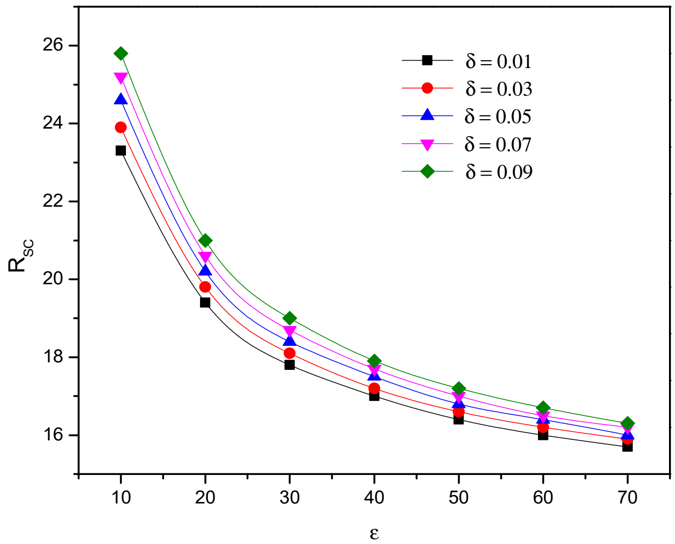

Fig.2. Critical thermal Rayleigh number R_{SC} versus anisotropic parameter ε for various δ, $\tau = 0.03$, $S_T = -0.002$, $k = 0.001$, $R_S = -500$ and $M_3 = 5$.

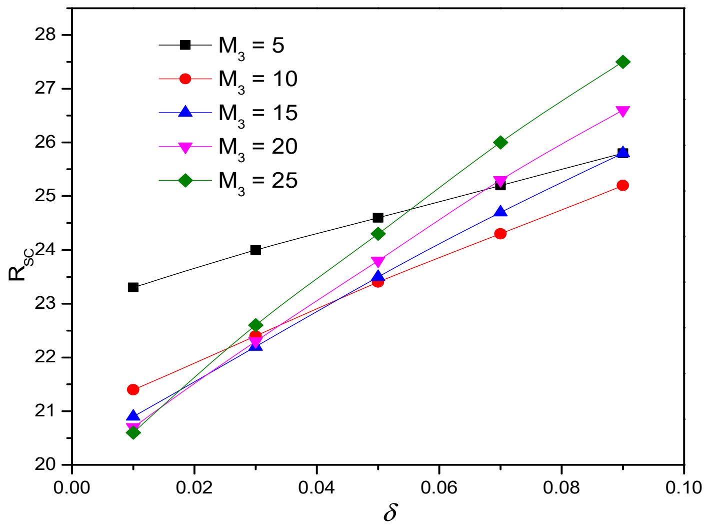

Fig.3. Critical thermal Rayleigh number R_{SC} versus coefficient of MFD viscosity δ for various M_3, $R_S = -500$, $\tau = 0.03$, $S_T = -0.002$, $\varepsilon = 10$ and $k = 0.001$.

Figure 2 shows that the vertical anisotropy of permeability of the porous medium destabilizes the system. This is because of the decrease in R_{SC} when ε is increased. As far as the MFD viscosity δ is concerned, the increase in $\delta,(0.001,0.003,0.005,0.007,0.009)$, increases R_{SC} for a fixed ε. The same effect is found when ε is increased from *10* to *70*. This indicates the stabilizing nature of the system.

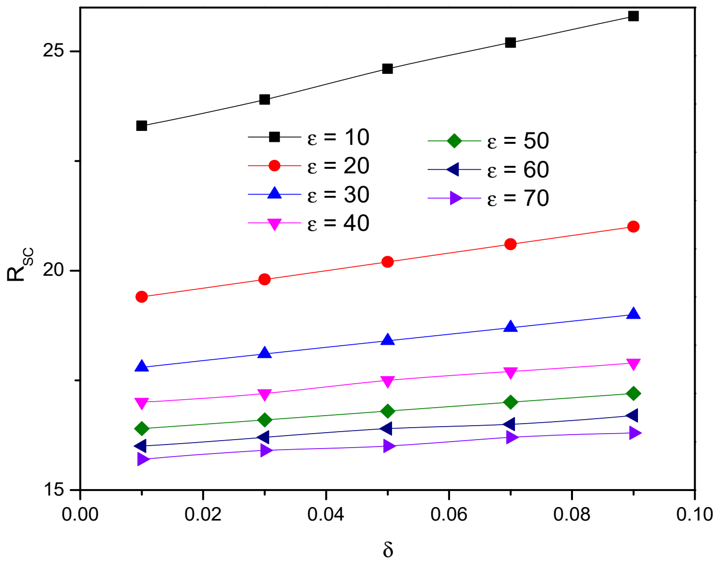

Fig.4. Critical thermal Rayleigh number R_{SC} versus coefficient of MFD viscosity δ for various ε, $R_S=-500$, $S_T=-0.002$, $k=0.001$, $\tau=0.03$ and $M_3=5$.

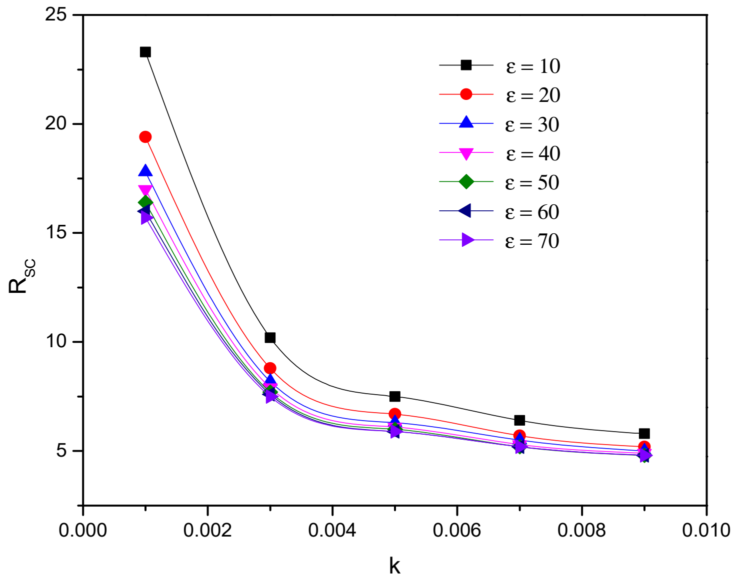

Fig.5. Critical thermal Rayleigh number R_{SC} versus permeability of porous medium k for various ε, $R_S=-500$, $S_T=-0.002$, $\delta=0.01$, $\tau=0.03$ and $M_3=5$.

Figures 3 and 4 illustrate the variation of R_{SC} versus δ for different values of M_3 and ε. From Figs 3-4, one can find that as the coefficient of a magnetic field dependent viscosity is increased from *0.01* to *0.09*, the critical thermal Rayleigh number increases. This means that the system is stabilized through viscosity variation with respect to the magnetic field. This leads to the conclusion that the magnetic field dependent viscosity delays the onset of convection for a ferrofluid in a densely distributed porous medium. Figure 3 illustrates that as M_3 increases, the values of R_{SC} decrease for small values of δ, whereas for higher values of δ, R_{SC} decreases for lower values of M_3, and then increases for higher values of M_3.

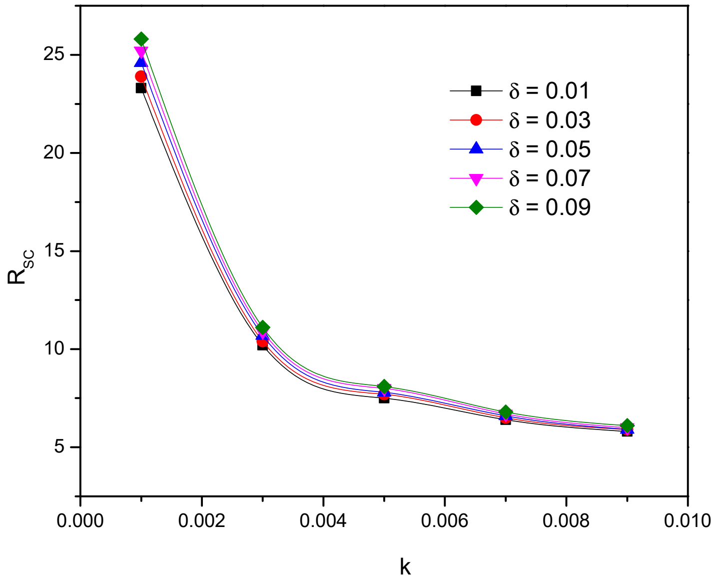

Fig.6. Critical thermal Rayleigh number R_{SC} versus permeability of porous medium k for various δ, $R_S = -500$, $\tau = 0.03$, $S_T = -0.002$, $\varepsilon = 10$ and $M_3 = 5$.

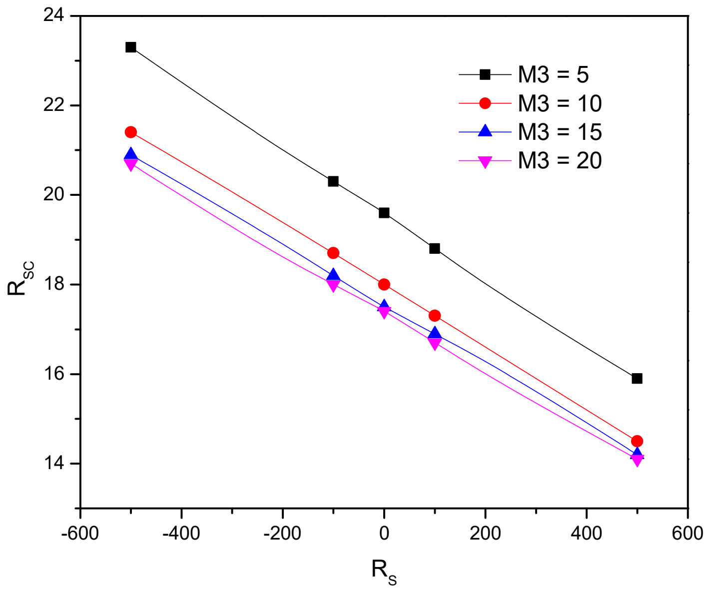

Fig.7. Critical thermal Rayleigh number R_{SC} versus salinity Rayleigh number R_S for various M_3, $\delta = 0.01$, $\tau = 0.03$, $S_T = -0.002$, $\varepsilon = 10$ and $k = 0.001$.

Figures 5 and 6 indicate the variation of the critical Rayleigh number R_{SC} with respect to permeability of the porous medium k for different ε and δ. It is clear that the system destabilizes as permeability of the porous medium k increases. This is indicated by a decrease in R_{SC} values. The reason is that as the pore size increases, it becomes easier for the flow to destabilize the system. It is observed from the figures that the anisotropic parameter ε is to destabilize the system and the dependent viscosity δ is found to stabilize the system.

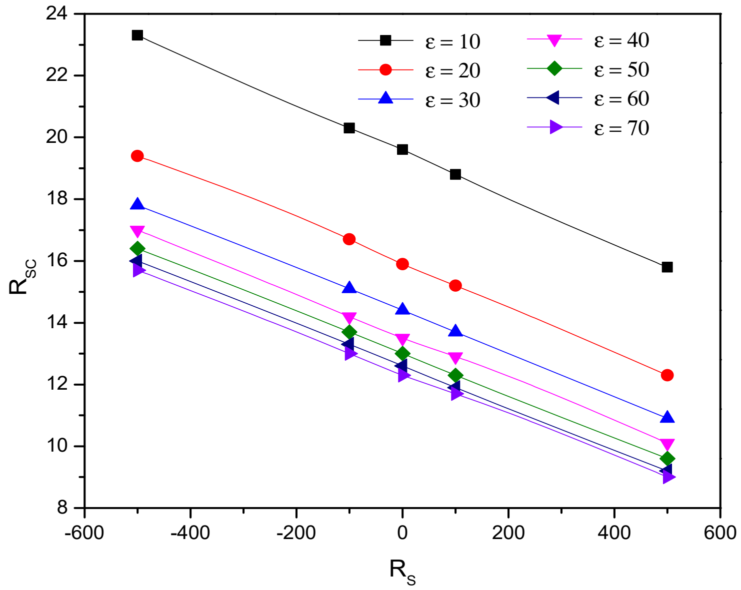

Fig.8. Critical thermal Rayleigh number R_{SC} versus salinity Rayleigh number R_S for various ε, $\delta = 0.01$, $S_T = -0.002$, $k = 0.001$, $\tau = 0.03$ and $M_3 = 5$.

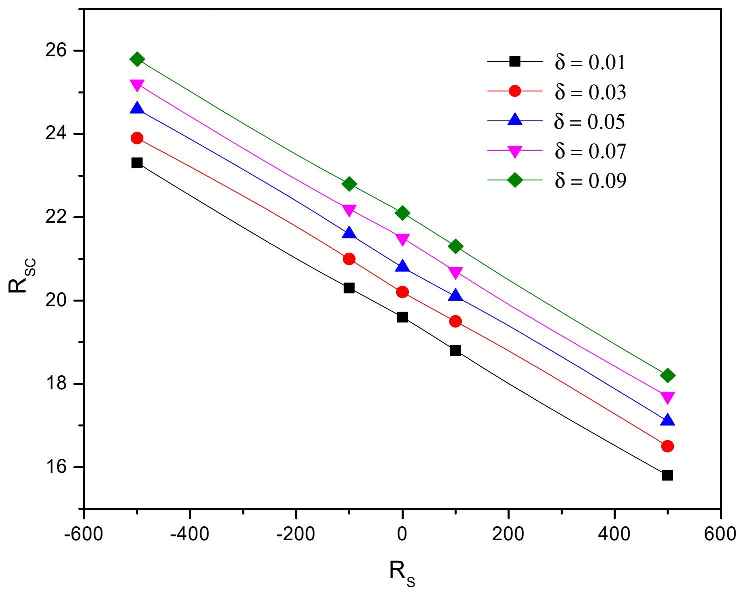

Fig.9. Critical thermal Rayleigh number R_{SC} versus salinity Rayleigh number R_s for various δ, $\tau = 0.03$, $S_T = -0.002$, $k = 0.001$, $\varepsilon = 10$ and $M_3 = 5$.

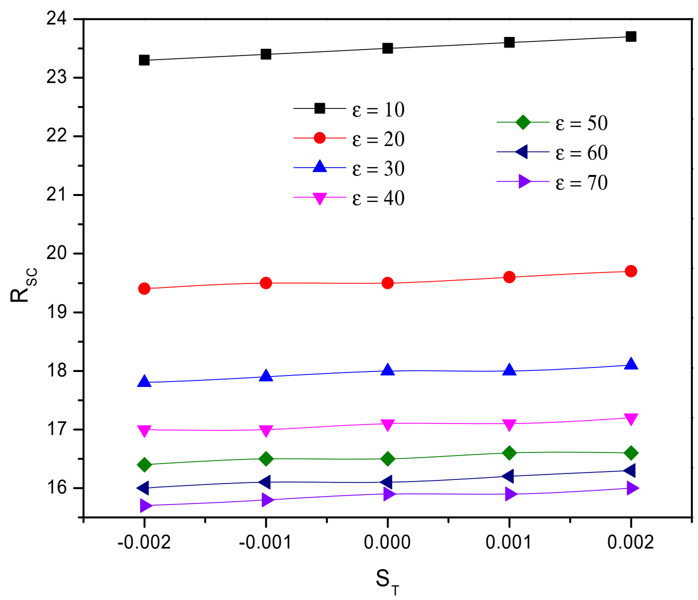

Fig.10. Critical thermal Rayleigh number R_{SC} versus Soret parameter S_T for various ε, $\delta = 0.01$, $R_S = -500$, $k = 0.001$, $\tau = 0.03$ and $M_3 = 5$.

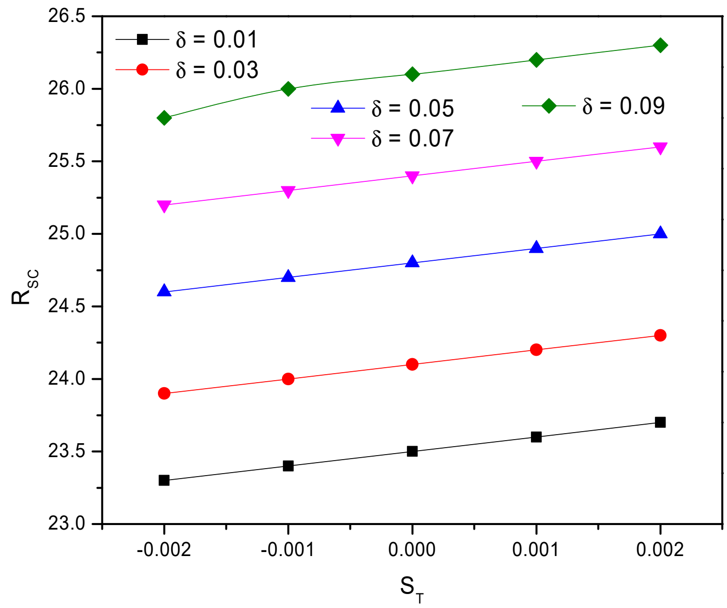

Fig.11. Critical thermal Rayleigh number R_{SC} versus Soret parameter S_T for various δ, $R_S = -500$, $\tau = 0.03$, $k = 0.001$, $\varepsilon = 10$ and $M_3 = 5$.

Figures 7, 8 and 9 represent the variation of R_{SC} versus R_S for different values of M_3, ε and δ. When the salinity Rayleigh number R_S increases from *-500* to *500*, the critical thermal Rayleigh number R_{SC} decreases. Therefore the system shows a destabilizing behaviour. It is observed from Figs 7 and 8 that the magnetization parameter M_3 and anisotropic parameter ε are found to destabilize the system. Also, the stabilizing trend of MFD viscosity δ is seen in Fig.9.

Figures 10 and 11 indicate the variation of the critical Rayleigh number R_{SC} with respect to the Soret parameter S_T for various ε and δ. It is found that the increase in the Soret effect stabilizes the system, thereby delaying the onset of convection. Both figures exhibit a stabilizing trend. This is due to the fact that the modulation of the salinity gradient by temperature gradient promotes stabilization. Positive values of S_T stabilize the system more. The destabilizing trend of ε is seen from Fig.10 and stabilizing behaviour of δ is seen from Fig.11, as would mean adding salt from top.

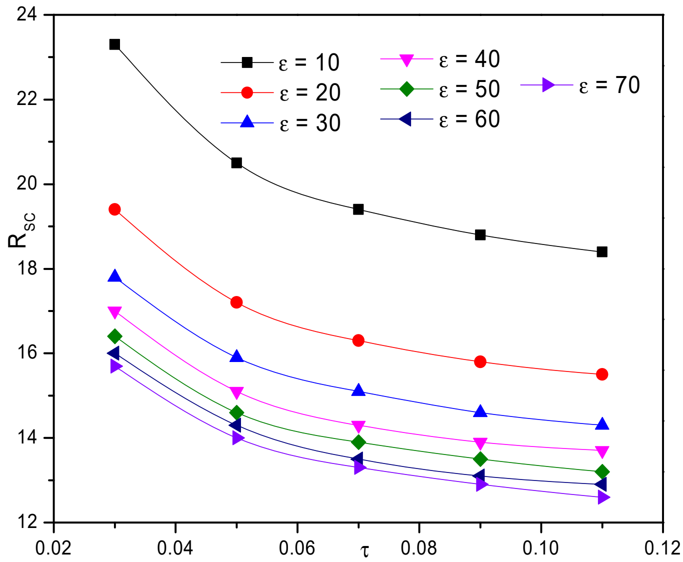

Fig.12. Critical thermal Rayleigh number R_{SC} versus ratio of the mass transport to heat transport τ for various ε, $\delta = 0.01$, $R_S = -500$, $k = 0.001$, $S_T = -0.002$ and $M_3 = 5$.

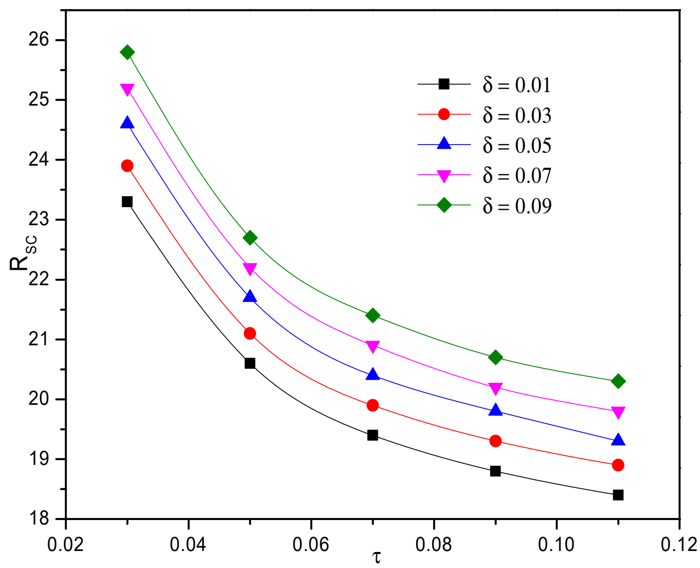

Fig.13. Critical thermal Rayleigh number R_{SC} versus ratio of mass transport to heat transport τ for various δ, $R_S = -500$, $S_T = -0.002$, $k = 0.001$, $\varepsilon = 10$ and $M_3 = 5$.

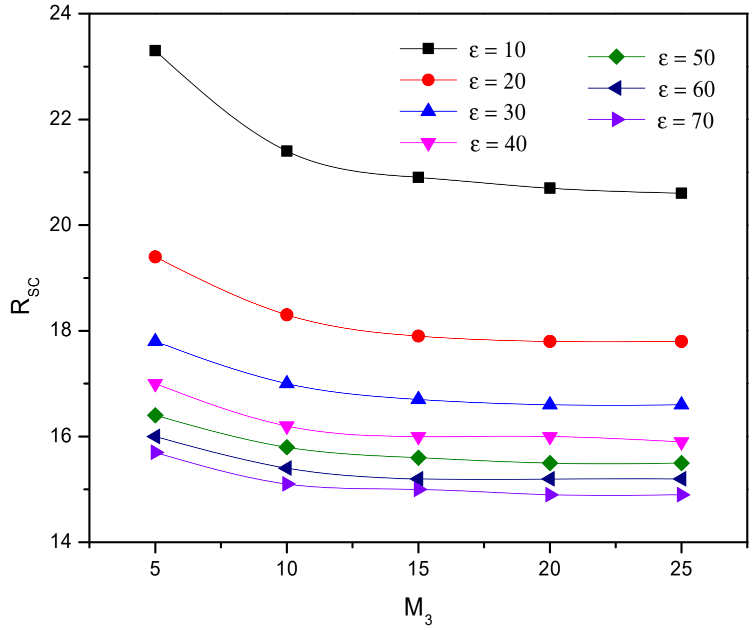

Fig.14. Critical thermal Rayleigh number R_{SC} versus magnetization M_3 for various ε, $\delta = 0.01$, $R_S = -500$, $k = 0.001$, $S_T = -0.002$ and $\tau = 0.03$.

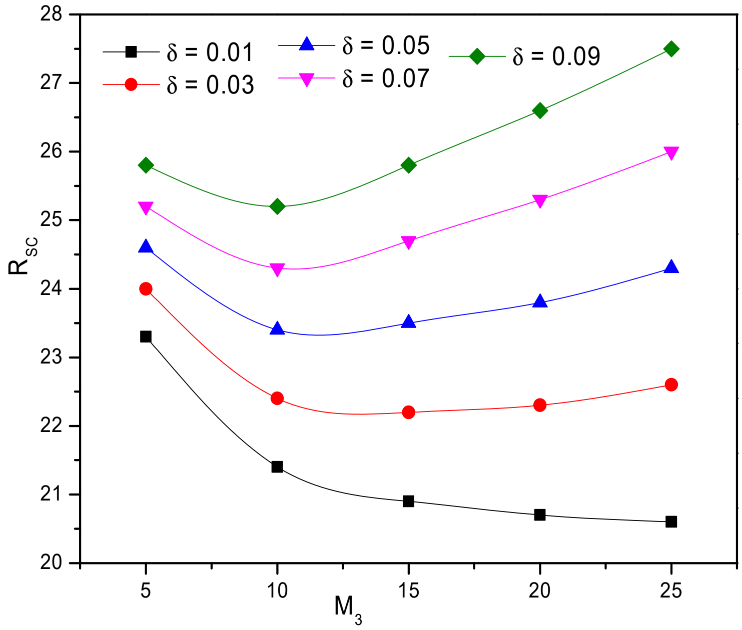

Fig.15. Critical thermal Rayleigh number R_{SC} versus magnetization M_3 for various δ, $R_S = -500$, $S_T = -0.002$, $k = 0.001$, $\tau = 0.03$ and $\varepsilon = 10$.

Figures 12 and 13 show the variation of the critical Rayleigh number R_{SC} versus the ratio of mass transport to heat transport τ for different ε and δ. It is seen from the figures that the system destabilizes as the ratio of mass transport to heat transport τ increases. This is shown by a fall in R_{SC} values. It is observed from the figures that the anisotropic parameter ε is found to destabilize the system and the magnetic field dependent viscosity δ is found to stabilize the system.

Figures 14 and 15 give the variation of the critical Rayleigh number R_{SC} versus the non-buoyancy magnetization parameter M_3 for different anisotropic parameter ε and magnetic field dependent viscosity parameter δ. It is seen from Fig.15 that as the value of M_3 increases from *5* to *25*, the value of R_{SC} degreases for a small value $\delta = 0.01$, thus destabilizes the system for $\delta = 0.01$. whereas for higher values of δ (*0.05, 0.07* and *0.09*). R_{SC} gets increasing values. In this situation, the system shows a stabilizing behavior which is increasing slowly. The destabilizing trend of the anisotropic parameter ε is also seen from Fig.14, when M_3 increases R_{SC} decreases indicating the onset of instability. This is because high magnetization tends to release large energy to the system causing instability to set in earlier.

10. Conclusions

The critical thermal Rayleigh number is calculated for both stationary and oscillatory modes. When $Ta = 0$ the thermal Rayleigh number is identical to the results obtained by Sekar *et al.* [29]. When $\delta=0, \varepsilon=0, Ta=0$ and $k_1 \to \infty$ this tends to critical Rayleigh number obtained by Vaidyanathan *et al.* [31]. When $\delta=0, \varepsilon=1$ and $Ta = 0$ the thermal Rayleigh number is identical to the results obtained by Sekar *et al.* [32]. When $\delta=0$, one gets the critical Rayleigh number calculated in Sekar *et al.* [33].

For the stationary convection, the coefficient of MFD viscosity δ has a destabilizing behavior for various values of, R_S, τ, ε, M_3 and k which are studied in Figs 2-10, 12-15. But, the convective system has a stabilizing effect which is analyzed in Fig.11 for the Soret parameter S_T. It is evident from Fig.3 that lower values of R_{SC} are needed for the onset of convection with an increase in M_3 for smaller values of δ, whereas higher values of R_{SC} are needed for the onset of convection with an increase in M_3 for smaller values of δ, hence justifying the competition between the destabilizing effect of the magnetization M_3 and the stabilizing effect of the MFD viscosity δ. It is very clear from Fig.10 that the Soret coefficient S_T for different values of the anisotropy parameter ε has a destabilizing effect on the system. But, due to the effect of the Soret coefficient S_T and salinity Rayleigh number R_S, the system shows a stabilizing behavior which is plotted in Figs 9 and 11. Thus, the system is dominated by the Soret coefficient.

The MFD viscosity always has a stabilizing effect, whereas the permeability of the porous medium always has a destabilizing effect on the onset of convection. In the absence of MFD viscosity $(\delta = 0)$ (which means the viscosity is constant), magnetization always has a destabilizing effect. In the presence of MFD viscosity, nothing specific can be said, since there is a competition between the destabilizing role of the magnetization M_3 and the stabilizing role of the MFD viscosity δ. Thus magnetization destabilizes the system and the coefficient of field dependent viscosity stabilizes the system for both modes. This leads to the conclusion that the MFD viscosity delays the onset of convection for a ferrofluid saturating a densely distributed anisotropic porous medium.

Nomenclature

$\boldsymbol{B}$ – magnetic induction
$C_{v,H}$ – effective heat capacity at constant volume and magnetic field (kJ/m^3K)
D/Dt – convective derivative $s^{-1}\left[D/Dt = \partial/\partial t + \boldsymbol{q}.\nabla\right]$
d – thickness of the fluid layer m
$\boldsymbol{g}$ – gravitational acceleration $(0, 0, -g)$ ms^{-2}
$\boldsymbol{H}$ – magnetic field amp/m

K – mass diffusivity
K – pyromagnetic coefficient $\left[\equiv -(\partial \mathrm{M} / \partial T)_{H_0,T_0}\right]$
K_1 – thermal diffusivity *W/mK*
K_2 – salinity magnetic coefficient $\left[\equiv (\partial \mathrm{M} / \partial S)_{H_0,T_0}\right]$
K_s – concentration diffusivity *W/mkg*
k – permeability of the porous medium
k_0 – resultant wave number $\left(k_0 = \sqrt{k_x^2 + k_y^2}\right)$ m^{-1}
kx, k_y – wave number in the x and y direction m^{-1}
$\boldsymbol{M}$ – magnetization $Ampm^{-1}$
M_0 – mean value of the magnetization at $H = H_0$ and $T = T_0$
p – hydrodynamic pressure (N/m^2)
$\boldsymbol{q}$ – velocity of the ferrofluid (u, v, w) ms^{-1}
S – solute concentration *kg*
S_T – Soret coefficient
T – temperature K
t – time s
α_t – coefficient of thermal expansion K^{-1}
α_s – analogous solvent coefficient of expansion K^{-1}
β_t – uniform temperature gradient Km^{-1}
β_s – uniform concentration gradient kgm^{-1}
θ – perturbation in temperature (K)
μ – dynamic viscosity $kgm^{-1}s^{-2}$
μ_0 – magnetic permeability of vacuum
ρ – density of the fluid kgm^{-3}
ρ_0 – mean density of the clean fluid kgm^{-3}
σ – growth rate s^{-1}
φ – viscous dissipation factor containing second order terms in velocity
ϕ – magnetic scalar potential *Amp*
χ – magnetic susceptibility $\left[\equiv (\partial \mathrm{M} / \partial H)_{H_0,T_0}\right]$
δ – MFD viscosity
∇ – Hamilton operator $\left[\equiv \boldsymbol{i}(\partial / \partial x) + \boldsymbol{j}(\partial / \partial y) + \boldsymbol{k}(\partial / \partial z)\right]$

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