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EFFECTS OF PRESSURE WORK OVER A SEMI-INFINITE VERTICAL OSCILLATING CYLINDER

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An arithmetical methodology is used to study natural convection with properties of pressure work over a semi-infinite vertical oscillating cylinder. The governing partial differential equations are set up and the resulting equations are changed into a non-dimensional form using the proper non-dimensional quantities. The set of non-dimensional partial differential equations is solved arithmetically using a well-organized method known as the Crank-Nicolson method. The velocity, as well as temperature profiles for different values of parameters are studied with the assistance of graphs.

Key words: cylinder, finite difference, pressure work, oscillating, skin friction.

1. Introduction

Heat transfer by free convection frequently occurs in our environment and is used in engineering devices. Two dimensional free convection flows past a semi-infinite vertical oscillating cylinder have received the attention of many researchers because of their extensive application in industry and engineering fields.

Dring and Gebhart [1] analysed the transient natural convection in association with the thin wires in liquids. In their study, the quasi static solution predictions are given only for transients in air. Velusamy and Garg [2] obtained the numerical solution for the transient natural convection adjacent to heat generating, vertical cylindrical fuel rods with constant surface heat flux. In particular, they paid special attention the rate of propagation of the leading edge. Lee *et al.* [3] considered the natural convection along a slender vertical cylinder with varying surface temperature. The solution of steady governing equations was achieved through a spline interpolation system. Collins and Dennis [4] considered a numerical calculation of the system of development in the powers of time for an impulsively started circular cylinder by a numerical integration technique. The acquired outcomes agree very well with earlier numerical as well as experimental work. Collins and Dennis [5] studied a symmetrical flow along over a uniformly accelerated circular cylinder.

Alam *et al*. [6] studied the convective flow over a permeable cone with variable surface temperature with pressure work term. The non-similarity solution is found for the governing equations of the flow. Rashad [7] discussed the effect of thermal radiation and pressure stress work on free convective flows of a Newtonian fluid-saturated porous medium. Elbashbeshy *et al*. [8] analyzed pressure work and heat generation along a cone with variable surface heat flux. The equations were analysed and numerical results were presented for the local skin friction and Nusselt number.

Alam *et al*. [9] considered the Joule heating, heat conduction and stress work on convective flow over a vertical plate. They found a similarity solution of the problem, the developed equations were made dimensionless using suitable transformations. Asma Begum *et al*. [10] focused on natural convective flow past an inclined plate with pressure effect. The boundary layer equations were solved approximately by means of the Nachtsheim-Swigert shooting technique along with Runge-Kutta method of sixth order. They consider two cases of motion of the flow, one with impulsive force and the other one was the uniformly accelerated motion.

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A transient free convective flow over a semi-infinite vertical oscillating cylinder with pressure work has not received the attention of any researcher. Henceforth, now we offer to study a free convective flow over a semi-infinite isothermal vertical oscillating cylinder with effects of pressure work.

2. Mathematical Modelling

The basic equations used to understand as well as examine natural convection are the partial differential equation of motion of the boundary layer which are the conservation of momentum and energy. In order to acquire these equations on the physical grounds, we implement the ensuing idea:

1. here the x-axis is taken along the surface of the cylinder and r symbolizes the radius distance perpendicular to cylinder;
2. initially, the fluid and cylinder are of the same temperature;
3. as $t' > 0$ the cylinder starts to oscillate in the vertical direction with velocity $u_0 \cos\omega' t'$;
4. it is also presumed that the radiative heat flux in the x-direction is negligible as compared with the radial direction;
5. the viscous dissipative heat is considered in the energy equation;
6. all the fluid properties are assumed to be constant except the density variation in the body force term.

With these assumptions and Boussinesq's approximation, the boundary layer equations are written as:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = g\beta(T' - T_\infty{}') + \frac{v}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{\sigma B_0^2}{\rho}u, \tag{2.2}$$

$$\frac{\partial T'}{\partial t'} + u\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial r} = \frac{k}{\rho C_\rho}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T'}{\partial r}\right) + \frac{T'\beta u}{\rho C_\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho C_\rho}\frac{1}{r}\left(\frac{\partial u}{\partial r}\right)^2. \tag{2.3}$$

Boundary as well as initial conditions are as follows:

$$\begin{aligned}
&t' \le 0: \quad u = 0, \quad v = 0, \quad T' = T'_\infty \quad \text{for all} \quad x \text{ and } r \ge r_0, \\
&t' > 0: \quad u = u_0 \cos\omega' t', \quad v = 0, \quad T' = T'_w \quad \text{at} \quad r = r_0, \\
&u = 0, \quad T' = T_\infty{}', \quad \text{at} \quad x = 0 \quad \text{and} \quad r \ge r_0, \\
&u \to 0, \quad T' \to T_\infty{}', \quad \text{as} \quad r \to \infty.
\end{aligned} \tag{2.4}$$

The local values of the shear stress, heat transfer rate and Sherwood number are given below:

$$\tau_x = -\mu\left(\frac{\partial u}{\partial r}\right)_{r=r_0}, \tag{2.5}$$

$$Nu_x = \frac{x}{T'_\omega - T'_\infty}\left(-\frac{\partial T'}{\partial r}\right)_{r=r_0} . \tag{2.6}$$

The time dependence values of the shear stress, heat transfer rate and Sherwood number are given below:

$$\overline{\tau}_L = \frac{1}{L}\int_0^L \mu\left(\frac{\partial u}{\partial r}\right)_{r=r_0} dx, \tag{2.7}$$

$$\overline{Nu_L} = \int_0^L \frac{1}{T'_\omega - T'_\infty}\left(-\frac{\partial T'}{\partial r}\right)_{r=r_0} dx. \tag{2.8}$$

Using the ensuing non-dimensional quantities:

$$X = \frac{xv}{u_0 r_0^2}, \qquad R = \frac{r}{r_0}, \qquad U = \frac{u}{u_0 \nu}, \qquad V = \frac{vr_0}{\nu}, \qquad t = \frac{vt'}{r_0^2},$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \qquad \mathrm{Gr} = \frac{g\beta\left(T'_w - T'_\infty\right)r_0^2}{\nu u_0}, \qquad \mathrm{Pr} = \frac{\mu C_\rho}{k}, \qquad \varepsilon = \frac{g\beta r_0^2}{C_\rho}, \tag{2.9}$$

$$\frac{\partial P}{\partial x} = \rho g, \qquad \omega = \frac{r_0^2\omega'}{v}, \qquad \lambda = \frac{T'_w}{T'_\infty} - 1, \qquad \mathrm{Ec} = \frac{u_0^2}{C_\rho\left(T'_w - T'_\infty\right)}.$$

Equations (2.1), (2.2) and (2.3) are compressed to the succeeding dimensionless form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial R} + \frac{V}{R} = 0, \tag{2.10}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial R} = GrT + \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right) - MU, \tag{2.11}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial R} = \frac{1}{\mathrm{Pr}}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) + \varepsilon\left[T + \frac{1}{\lambda}\right]U + \mathrm{Ec}\left(\frac{\partial U}{\partial R}\right)^2 . \tag{2.12}$$

The corresponding dimensionless initial and boundary conditions are:

$$\begin{aligned} &t \le 0: \quad U = 0, \quad V = 0, \quad T = 0, \quad \text{for all } X \text{ and } R, \\ &t > 0: \quad U = \cos\omega t, \quad V = 0, \quad T = 1, \quad \text{at } R = 1, \\ &U = 0, \quad T = 0, \quad \text{at } X = 0, \\ &U \to 0, \quad T \to 0, \quad \text{as } R \to \infty. \end{aligned} \tag{2.13}$$

The local shearing stress, rate of heat transfers as well as the rate of mass transfer in a non-dimensional form are:

$$\tau_X = -\left(\frac{\partial U}{\partial R}\right)_{R=1}, \tag{2.14}$$

$$Nu_X = \frac{-X\left(\frac{\partial T}{\partial R}\right)_{R=1}}{T_{R=1}}. \tag{2.15}$$

A dimensionless form of the average shearing stress, rate of heat transfer as well as rate of mass transfer are:

$$\overline{\tau} = -\int_0^1 X\left(\frac{\partial U}{\partial R}\right)_{R=1} dX, \tag{2.16}$$

$$\overline{Nu} = -\int_0^1 \frac{\left(\frac{\partial T}{\partial R}\right)_{R=1}}{T_{R=1}} dX. \tag{2.17}$$

3. Computational Procedures

The transient, non-linear coupled partial differential equations (2.10)-(2.12) with (2.13) are solved by using the well-known Crank-Nicholson method. The integral area is considered as a rectangle with sides $X_{max}(=1)$ and $R_{max}(=21)$. The value of R_{max} corresponds to $R=\infty$. The extreme of R was picked as [21] when roughly primary inquiries were carried out, subsequently the latter two of the boundary conditions (2.13) remain fulfilled.

To get a reasonable and consistent mesh scheme for the calculations, a grid independence is executed. Henceforth the mesh scheme of $50x105$ remains nominated for the entire ensuing analysis by way of $\Delta X = 0.02$ and $\Delta R = 0.2$. Besides the time-step dimension dependence is implemented, which produces $\Delta t = 0.01$ for consistent outcomes.

Henceforth, the above mentioned sizes have been chosen as suitable mesh sizes for calculation. Calculations continued until the stable state is reached. The stable state solutions are assumed to have been reached, at time the absolute variance among the quantities of U, as well as temperature T at two succeeding iterations are fewer than 10^{-5} at all meshes.

The derivatives involved in Eqs (2.14)-(2.17) are assessed by means of a five-point approximation formulae, after that Newton-Cotes closed integration formulae are used to evaluate integrals.

4. Discussion of fallouts

Figures 1 and 2 indicate velocity and temperature outlines for distinct values of Gr and Pr at $X = 1.0$. For the incremental values of Pr, viscosity nature of the result in a reduction of thermal diffusivity and velocity. The time required to achieve the stable state position becomes larger and thus the momentum boundary layer grows to a denser form.

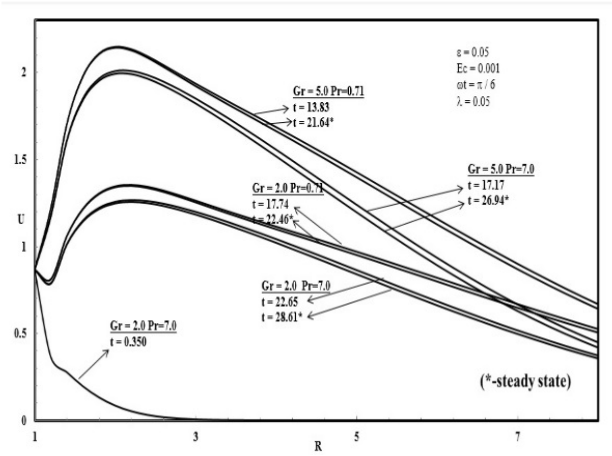

Fig.1. Unsteady outlines of velocity in numerous values of Gr and Pr.

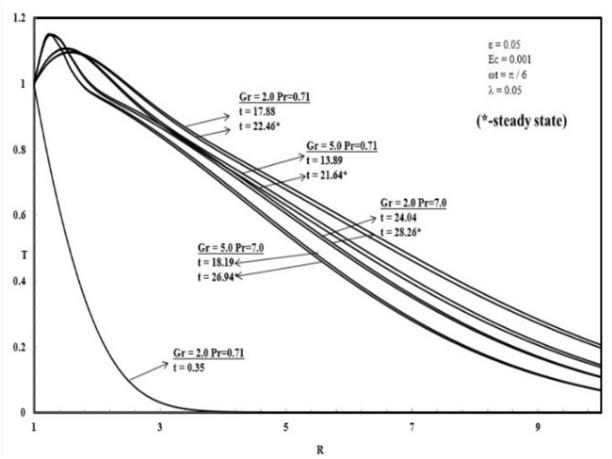

Fig.2. Unsteady outlines of temperature in numerous values of Gr and Pr.

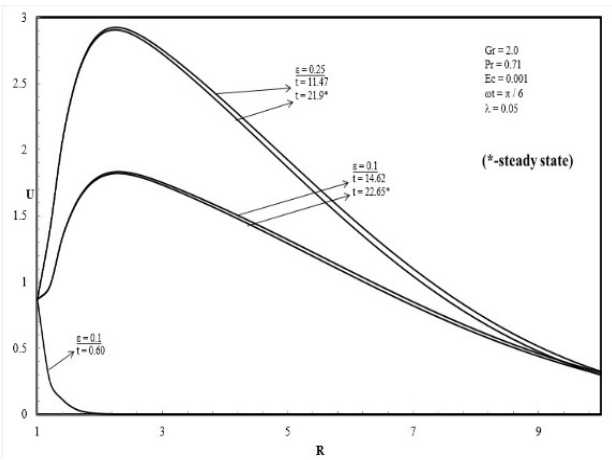

Fig.3. Unsteady outlines of velocity in numerous values of ε .

For fluids similar to air, the velocity gradually upsurges through time, reaches a time-based maximum *13.84* and *17.74* for numerous values of Gr then reaches the stable position at *21.64* and *22.46*. Similarly, for the case of water, it reaches the time-based maximum at *17.17* and *22.65*, then reaches a stable position in time *26.94* and *28.61*. Also, we observe from the calculated approximate values that the temperatures fall as the Grashof number grows.

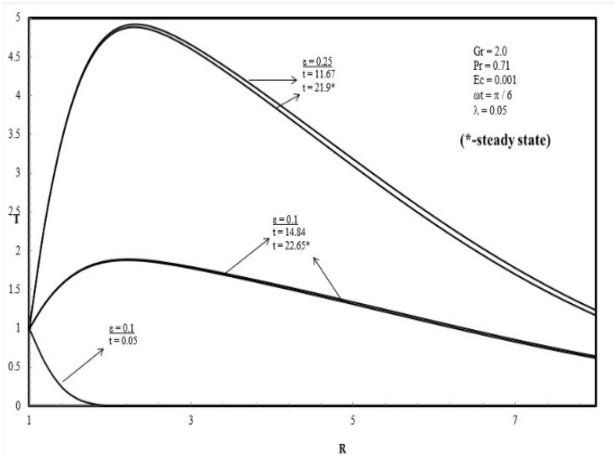

Fig.4. Unsteady outlines of temperature in numerous values of ε .

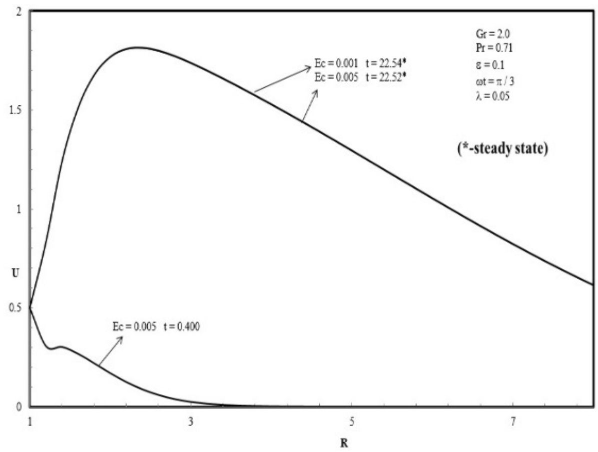

Fig.5. Unsteady outlines of velocity in numerous values of Ec .

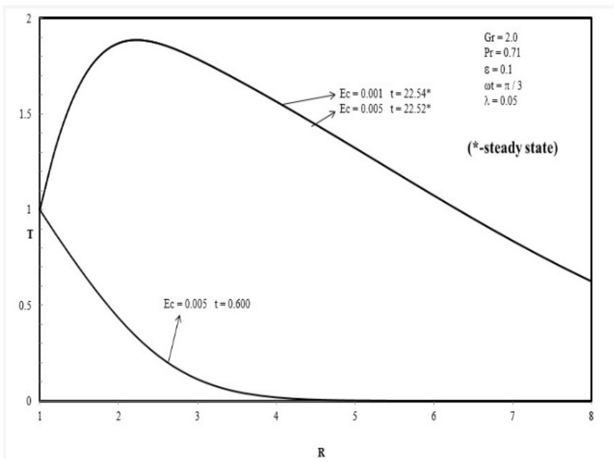

Fig.6. Unsteady outlines of temperature in numerous values of Ec .

Figures 3 and 4 show the impact of the pressure work factor ε on velocity and temperature distribution at $\Pr = 0.71$. From Fig. 3, it follows that the velocity profile changes to some extent upward and by way of the upsurge of ε. From Fig.4, we understood that the temperature outline is influenced by a growing value of ε. Furthermore, we perceived that extra time is needed to reach the stable position for smaller values of ε.

Figures 5 and 6 show the impact of several values of the Eckert number Ec on velocity and temperature at the foremost edge $X = 1.0$. From the approximate values, we conclude that the influence of the Eckert number on velocity and temperature is nil.

The impact of dissimilar values of the oscillation factor ωt on velocity and temperature distribution is displayed in Figs. 7 and 8. It is experiential that the velocity upsurges with a growing value of the phase angle ωt. Additional time is essential to reach the stable position at the lower value of $\omega t = 0$. Temperature sketches upswing with the growing ωt.

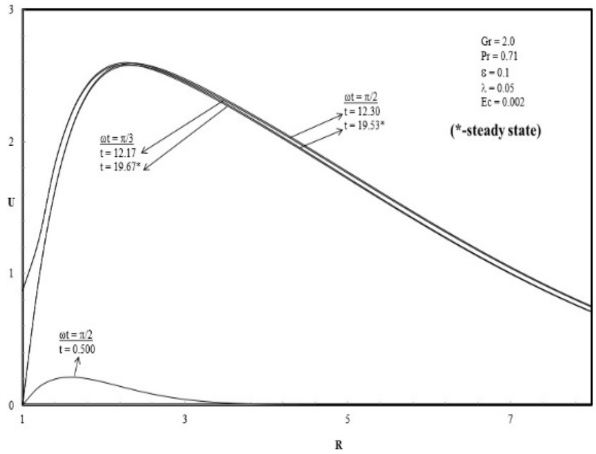

Fig.7. Unsteady outlines of velocity in numerous values of ωt.

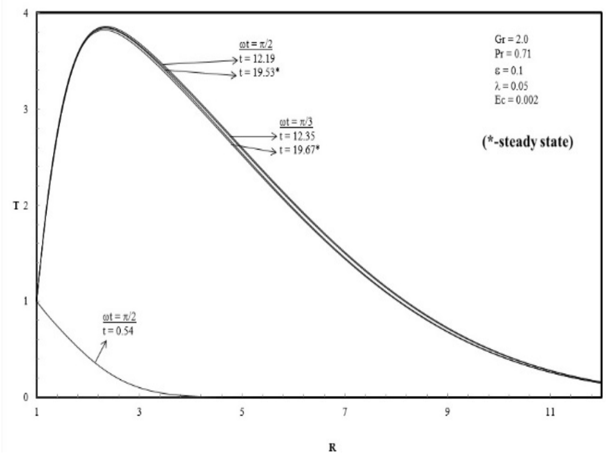

Fig.8. Unsteady outlines of temperature in numerous values of ωt.

In Fig. 9, values of the shear stress are designed for numerous values of the pressure work factor ε and Eckert number Ec. The local values of the shear stress rise by way of X escalations. The local wall shear stress declines with growing work pressure.

The local rate of heat transfers for dissimilar values of the pressure work factor ε and Eckert number Ec is shown in Fig 10. The rate of heat transfer falls with a growing value of work pressure.

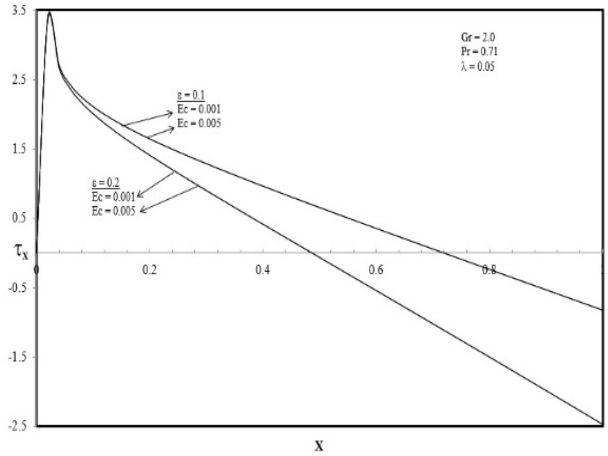

Fig.9. Local skin friction.

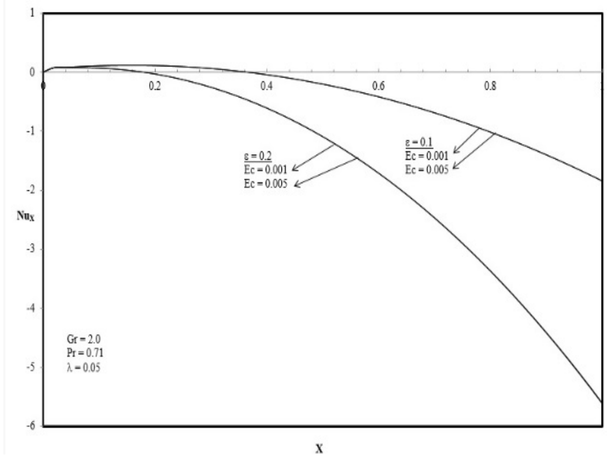

Fig.10. Local Nusselt number.

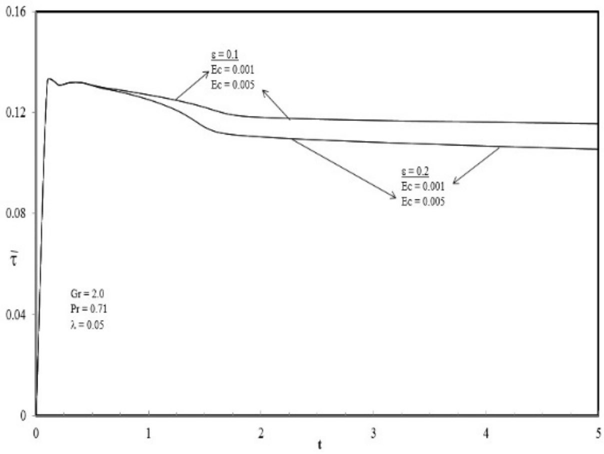

Fig.11. Average skin friction.

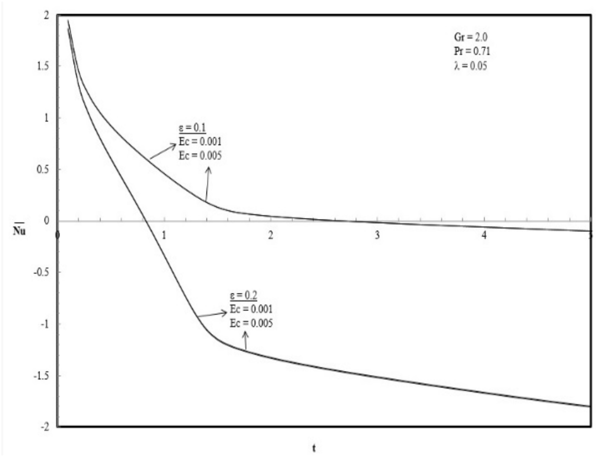

Fig.12. Average Nusselt number.

The effect of the pressure work factor ε and Eckert number Ec on the average skin friction and the Nusselt number is shown in Figs 11 and 12 correspondingly. In Fig.11, it is observed that the skin friction increases with time and becomes steady after some time. The average skin friction upsurges with the pressure work parameter throughout the transitory period and at stable position. The average Nusselt number is found to upsurge with reducing the value of the pressure work parameter.

5. Concluding remarks

An exhaustive numerical investigation has been carried out for a transient free convective flow past a semi-infinite vertical oscillating cylinder under the influence of pressure work. The non-dimensional form of governing equations is solved through a well-organized implicit finite-difference system. Conclusions are as follows:

1. the time required to achieve the stable state position becomes larger and thus the momentum boundary layer grows to a denser form;
2. we understood that the temperature profile is influenced by the growing value of ε ;
3. from the approximate values, we conclude that the influence of the Eckert number on velocity and temperature is zero;
4. the local wall shear stress declines with a growing work pressure;
5. the average Nusselt number is found to upsurge with reducing the value of the pressure work parameter.

Nomenclature:

C_p – specific heat at constant pressure
Ec – Eckert number
g – acceleration due to gravity
Gr – thermal Grashof number
k – thermal conductivity
Nu_X – dimensionless local Nusselt number
Nu_x – local Nusselt number
$\bar{N}u_L$ – average Nusselt number
$\bar{N}u$ – non-dimensional average Nusselt number
Pr – Prandtl number
t – dimensionless time

t' – time
T – dimensionless temperature
T' – temperature
T'_∞ – temperature of liquid distant from the cylinder
T'_w – cylinder temperature
u – velocity component in the x- direction
U – dimensionless velocity component in the X- direction
v – velocity component in the r- direction
V – dimensionless velocity component in the R- direction
X – dimensionless spatial coordinate along the cylinder
x – spatial coordinate along the cylinder
r – spatial coordinate normal to the cylinder
R – dimensionless spatial coordinate normal to the cylinder surface
α – thermal diffusivity
β – volumetric coefficient of thermal expansion
ε – pressure term parameter
μ – dynamic viscosity
ν – kinematic viscosity
ρ – density
τ_x – local skin friction
τ_X – non-dimensional local skin friction
$\overline{\tau}_L$ – average skin friction
$\overline{\tau}$ – non-dimensional average skin friction
w – conditions on the wall
∞ – free stream conditions

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