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PERFORMANCE OF FOUR DIFFERENT NANOPARTICLES IN BOUNDARY LAYER FLOW OVER A STRETCHING SHEET IN POROUS MEDIUM DRIVEN BY BUOYANCY FORCE

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This contemporary work explores the theoretical analysis of energy transfer performance of distinct nanoparticles (silver, copper, aluminium oxide and titanium oxide) adjacent to a moving surface under the influence of a porous medium which is driven by the buoyancy force. A mathematical model is presented which is converted to similarity equations by employing similarity transformation. The condensed nonlinear equations were approximated by the iterative method called RKF 45th-order. The flow and energy transference characteristics are explained through graphs and tabulated values. The notable findings are: silver- water is an appropriate nanofluid for enhancing the thermal conductivity of the base fluid. Titanium oxide – water shows a lower fluid flow movement due to porosity.

Key words: natural convection, porous parameter, nanofluid, numerical solutions, volume fraction.

1. Introduction

The development of nanotechnology depends on a proper choice of nanoparticles asit possessessufficient or more chemical and physical properties to face the challenges in industries;it offers cushion to solve the problems they encounter on a day to day basis. Nanofluids are being used in the fields of nuclear waste management, food industries, paper industries, cooling systems, etc., asnanoparticles have a unique property of high thermal conductivity.Choi *et al*. [1], a pioneer in nanofluids, mentioned in their theoretical work that the thermal conductivity increased by two-fold when the nanoparticles were suspended into a normal fluid. Masuda *et al*. [2] and Minsta *et al*. [3] confirmed that the addition of a minor amount of nanoparticles to a regular fluid yielded a substantial improvement (*10* to *50* %) in the thermal conductivity of ordinary fluids.

The metallurgy field, chemical industries, textile industry, paper industry, etc…, require the knowledge of flow and energy transference characteristics adjacent to moving surface. These engineering processes undergo cooling of strings by drawing them through a quiescent fluid. Sakiadis [4] instigated a theoreticalstudy on Blasius flow, later experimentally proven by Tsou *et al.* [5]. An extension work of [4] was carried out by Cran [6] and confirmed that the velocity of the moving surface was straightaway related to the distance from the slit. Chen [7] explored a similarity solution for two different cases like flow over the linear moving surface with linear surface temperature distribution and flow over the isothermal sheet.

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Furthermore, many mathematicians have studied this theory of flow over a moving surface in the different physical situations [8-21].

Many industrial revolutions have been stimulated by the process of free convection in a porous medium. For instance, production of heat in the storage of farming crops, extraction of crude oil, design of pebble-bed nuclear reactors etc. Elbashbeshy and Aldawody [22] analyzed the importance of porosity in the fluid field over a moving surface. Gireesha *et al*. [23] explored the effect of suspended particles in porous media. Furthermore, many researchers analyzed different factors that influence fluid flow through porous media by considering normal fluids and nanoparticles [24-32]. Eid [33] conducted a study of a chemical reaction and heat generation or absorption effects due to an exponentially stretching sheet on an MHD mixed convective boundary layer flow of a nanofluid through a porous medium. Characteristics of heat transfer of gold nanoparticles (Au-NPs) in flow past a power-law stretching surface were discussed by Mohamed *et al.* [34], by considering a Sisko bio-nanofluid flow (with blood as a base fluid) in the presence of non-linear thermal radiation. Eid [35] analyzed the effects of slip velocity and heat generation/absorption on the time-dependent stagnation-point flow and heat transfer of a nanofluid over a stretching sheet in a porous medium. Mohamed [36] reported that the mathematical model of the heat and mass transfer in a non-Newtonian fluid flow through a permeable nonlinear stretching vertical wall in the presence of effects such as, heat generation/absorption, thermal radiation, and heat and mass fluxesThe impact of the magnetic field and nanoparticles on the two-phase flow of a generalized non-Newtonian Carreau fluid over a permeable non-linearly stretching surface was analyzed in the existence of suction/injection and thermal radiation by Mohamed *et al.* [37]. Al-Hossainy *et al.* [38] studied the impact of yield stress and convective conditions on a 3D mixed convection magneto-hydrodynamic boundary layer flow of a two-phase Casson nanofluid past a stretching plate in a porous medium.

The above reviews havenot addressedwhich nanoparticlesare suitable for excellent transfer of heat. The proper choice of nanoparticles will help to improve the effectiveness of fluid flow and thermal conductivity. Hence, a sincerer attempt is made to find the appropriate nanoparticle that increases the thermal conductivity of the base fluid. At this point, an examination of thermal characteristics of four different nanoparticles such as Ag, Cu, Al_2O_3 and TiO_2 suspended in the base fluid water has been done. The fluid movement is considered in the porous media under the influence of buoyant force.

2. Mathematical model

A two- dimensional flow of a nanofluid adjacent to a moving surface through a permeable medium driven by buoyant force isconsidered. The viscosity of the base fluid is assumed to be varying with temperature.The a_1-axis is taken along the direction of the sheet and the b_1 – axis is normal to it. The wall is assumed to be impermeable as shown in Fig.1. Under the aforesaid hypothesis, the mathematical model takes the following form

$$\frac{\partial a_1}{\partial x_1}+\frac{\partial b_1}{\partial x_2}=0, \tag{2.1}$$

$$\left(a_1\frac{\partial a_1}{\partial x_1}+b_1\frac{\partial b_1}{\partial x_2}\right)\rho_{nf}=\mu_{nf}\frac{\partial^2 a_1}{\partial x_2^2}-\frac{\mu_{nf}}{K}a_1+g\rho_{nf}\beta_{nf}(T_2-T_\infty), \tag{2.2}$$

$$a_1\frac{\partial T_2}{\partial x_1}+b_1\frac{\partial T_2}{\partial x_2}=\frac{K_{nf}}{(\rho Cp)_{nf}}\frac{\partial^2 T_2}{\partial x_2^2}. \tag{2.3}$$

Subjected to the boundary conditions

$$a_1=U_w=cx_1,\quad b_1=0,\quad T_2=T_w\quad \text{at}\quad x_2=0,$$

$$a_1=0,\quad T_2=T_\infty\quad \text{as}\quad x_2\to\infty. \tag{2.4}$$

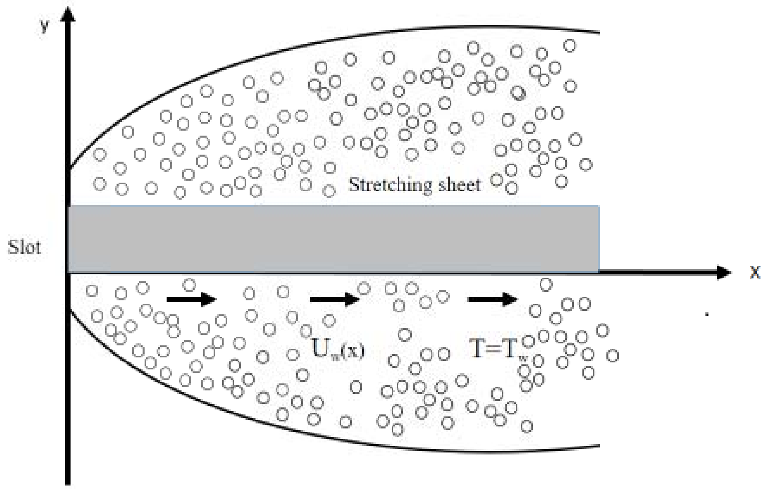

Fig.1. Physical geometry of the problem.

Table 1. Thermo- physical properties ofnanoparticles[39, 40].

Property	TiO_2	**Ag**	**Cu**	Al_2O_3	H_2O
Density $\left(kg.m^{-3}\right)$	*4250*	*5610*	*8933*	*3970*	*997.1*
Thermal conductivity $\left(W.K^{-1}.m^{-1}\right)$	*6.69*	*60*	*400*	*40*	*0.6071*
Thermal expansion coefficient $\left(K^{-1}\right)$	*.0000157*	*.00009*	*.000076*	*.000051*	*.000256*
Heat capacitance $\left(JK^{-1}\right)$	*686.2*	*41.086*	*385*	*765*	*4179*

Table 2. Thermo physical model.

Properties	Nanofluid
Density $\left(kg.m^{-3}\right)$	$\rho_{nf}=\left(1-\phi_2\right)\rho_f+\phi_2\rho_s$
Heat capacity $\left(JK^{-1}\right)$	$(\rho Cp)_{nf}=\left(1-\phi_2\right)(\rho Cp)_f+\phi_2(\rho Cp)_s$
Viscosity $\left(Ns.m^{-2}\right)$	$\mu_{nf}=\dfrac{\mu_f}{\left(1-\phi_2\right)^{2.5}}$
Thermal conductivity $\left(W.K^{-1}.m^{-1}\right)$	$\dfrac{K_{nf}}{K_f}=\dfrac{K_s+\left(n-1\right)K_f-\left(n-1\right)\phi_2(K_f-K_s)}{K_s+\left(n-1\right)K_f+\phi_2(K_f-K_s)}$
Thermal expansion coefficient $\left(K^{-1}\right)$	$\beta_{nf}=\left(1-\phi_2\right)\beta_f+\phi_2\beta_s$

The fundamental thermo-physical properties of nanofluids at $25^0 C$ (remove this) were taken from various standard studiesand aregiven in Tabs 1 and 2.

The derived Eqs (2.1)-(2.3) are reduced into a pair of highly non-linear ordinary differential equations by employing the following similarity transformations

$$a_1 = cx_1 f'(\eta), \quad b_1 = -\sqrt{c\upsilon_f}\, f(\eta), \quad \eta = -\sqrt{c/\upsilon_f}\, x_2, \quad \theta(\eta) = \frac{T_2 - T_\infty}{T_w - T_\infty}. \tag{2.5}$$

The resultant equations take the following form

$$\begin{aligned} &\left[(1-\phi_2)+\phi_2\frac{\rho_{s2}}{\rho_f}\right]*(1-\phi_2)^{2.5}*\left[-f'(\eta)^2+f''(\eta)f(\eta)+\right.\\ &\left.-\left[(1-\phi_2)+\phi_2\frac{\rho_{s2}}{\rho_f}\right]*\lambda\theta(\eta)\right]+f'''(\eta)+Da*f'(\eta)=0, \end{aligned} \tag{2.6}$$

$$\left[(1-\phi_2)+\phi_2\frac{(\rho Cp)_{s2}}{(\rho Cp)_f}\right]*k_f \Pr f(\eta)\theta'(\eta)+K_{nf}\theta''(\eta)=0. \tag{2.7}$$

The boundary condition takes the following form by applying (2.5)

$$f(0)=0, \quad f'(0)=1, \quad \theta(0)=1, \quad f'(\infty)=0, \quad \theta(\infty)=0. \tag{2.8}$$

The local skin friction coefficient C_f and local Nusselt number Nu_x are given by

$$C_f = \frac{\mu_{nf}}{\rho_f u_w^2}\left(\frac{\partial a_1}{\partial x_2}\right)_{x_2=0}, \tag{2.9}$$

$$\mathrm{Nu}_x = \frac{x_1 K_{nf}}{k_f(T_w - T_\infty)}\left(-\frac{\partial T_2}{\partial x_2}\right)_{x_2=0}. \tag{2.10}$$

Further, Eqs (2.9) and (2.10) get reduced to

$$\mathrm{Re}_x^{1/2} C_f = \frac{1}{(1-\phi_2)^{2.5}} f''(0),$$

$$\mathrm{Re}_x^{-1/2} Nu_x = -\frac{K_{nf}}{k_f}\theta'(0).$$

3. Numerical solution

The Runge-Kutta-Fehlsberg 45th-order scheme is employed to solve the highly nonlinear differential Eqs (2.6) and (2.7) with the prescribed boundary condition (2.8). The acquired numerical results are compared with the previous results of Wang [41], Khan and Pop [42], and Gorla and Sidwai [43]. The results are in excellent agreement as shown in Tab.3.

Table 3. Comparison results for the temperature gradient $-\theta'(0)$ for the parameter Pr when $\phi_2 = \mathrm{Da} = \lambda = 0$.

Pr	Present Study	Wang[41]	Khan and Pop[42]	Gorla and Sidwai [43]
2.0	*0.91135*	*0.9114*	*0.9113*	*0.9114*
6.13	*1.75968*	-	-	-
7.0	*1.89540*	*1.8954*	*1.8954*	*1.8954*
20.0	*1.35390*	*1.3539*	*1.3539*	*1.3539*

4. Results and discussion

This section provides an insight intothe effect of the volume fraction of all the four nanoparticles on the free convection flow of nanofluids over a stretching sheet through a porous medium. Figures 2-12 are employed to interpret the results of the current research work regarding velocity $f'(\eta)$, temperature $\theta(\eta)$ and the rate of temperature $-\theta'(0)$. The following values are assumed for various parameters such as: volume fraction $0 \le \phi_2 \le 0.3$, buoyancy $0 \le \lambda \le 3$, porosity $0 \le \mathrm{Da} \le 3$ for the computation of numerical values.

Figures 2-5 are drawn to explore the effect of ϕ_2 on $f'(\eta)$ in the case of Cu-water, Ag water, Al_2O_3 water and TiO_2 water. It is noted that $f'(\eta)$ along the stretching sheet acclerated with a rise in ϕ_2 in both the cases (i.e.,$Cu+H_2O$ and $Ag+H_2O$). Furthermore, it is observed that $f'(\eta)$ for $Cu+H_2O$ is pridominantly higher than that of $Ag+H_2O$. The velocity of the fluid with the suspension of Al_2O_3 and TiO_2 particles increased with the increase in ϕ_2 which is less compared to the Cu-water and Ag -water. The impact of the ϕ_2 on the thermal distributionis shown in Figs 6 and 7. One can infer from these figures that escalation of ϕ_2 from *0.1* and *0.3* lead to scattering of nanoparticles in the base fluid. As a result, the heat capacitance of the fluid increased; hence, the corresponding layer increased. Virtually, it is established that Ag nanofluid has more heat conducting capacitance than the other nanofluids which is due to the bulk thermal conductivity of Ag nanoparticles.

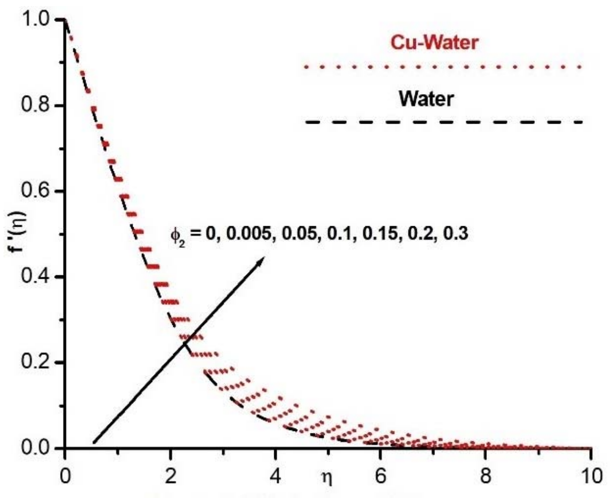

Fig.2. Effect of ϕ_2 on $f'(\eta)$.

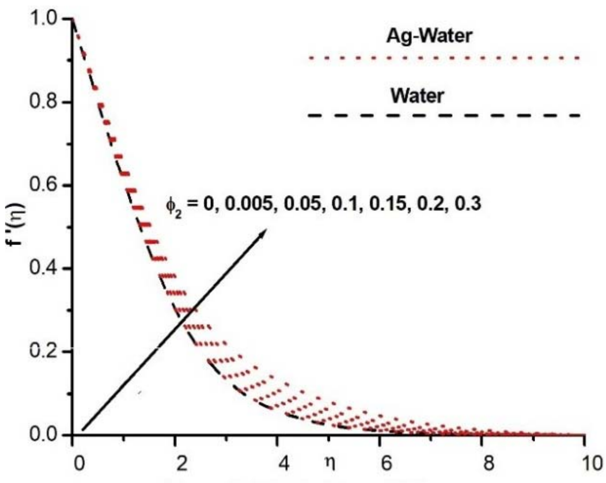

Fig.3. Effect of ϕ_2 on $f'(\eta)$.

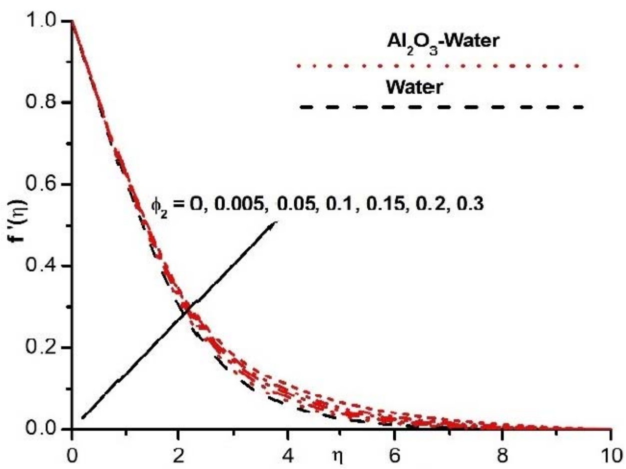

Fig.4. Effect of ϕ_2 on $f'(\eta)$.

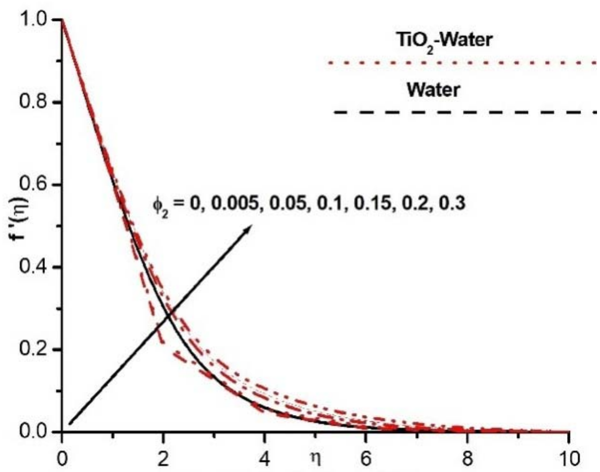

Fig.5. Effect of ϕ_2 on $f'(\eta)$.

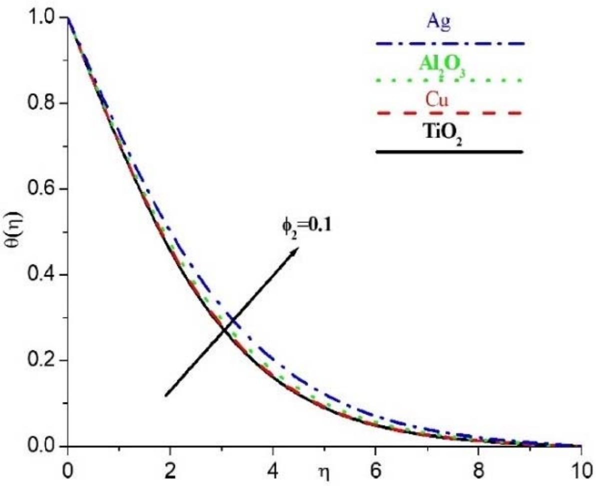

Fig.6. Effect of ϕ_2 on $\theta(\eta)$ for various nanoparticles.

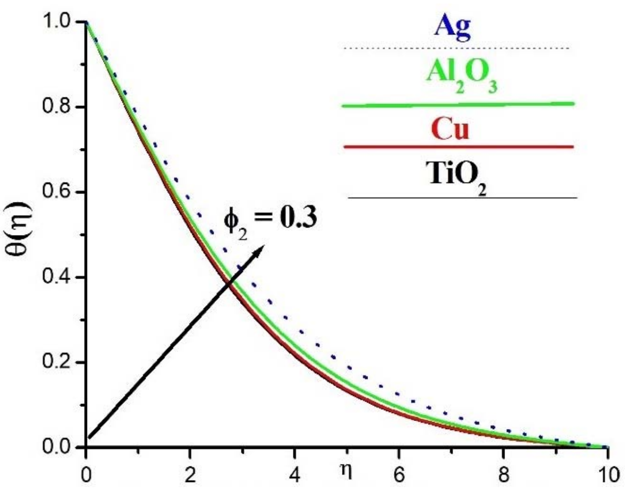

Fig.7. Effect of ϕ_2 on $\theta(\eta)$ for various nanoparticles.

Figure 8 illustrates the nature of $f'(\eta)$ for various nanofluids when $\mathrm{Da} = 2$. TiO_2-water nanofluid showsa lower velocity profile than other nanofluids.Figure 9 discloses the impact of Da on $\theta(\eta)$ for different nanofluids. Ag -water showsa greater thermal conductivity compared to other nanofluids when $\mathrm{Da} = 2$. The buoyancy effect (λ) on velocity $f'(\eta)$ and temperature $\theta(\eta)$ is depicted in Figs 10 and 11. From Fig.10, it is inferred that Ag nanoparticles has more fluid flow than other nanoparticles. Physically, this indicates the expansion of convection currents in Ag nanoparticles is higher than for other nanoparticles. Figure 11 shows that $\theta(\eta)$ is more for Ag nanofluids.

Figure 12 portrays the nature of the rate of heat transfer $-\theta'(0)$ over different values of volume fraction ϕ_2 for the four different nanofluids. Silver (Ag) nanoparticles have abetter rate of heat transfer and titanium oxide hasthe least when compared to other nanoparticles.

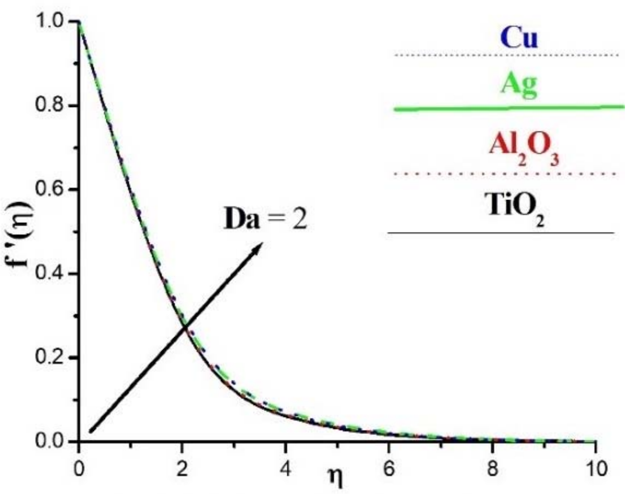

Fig.8. Effect of Da on $f'(\eta)$ for various nanoparticles.

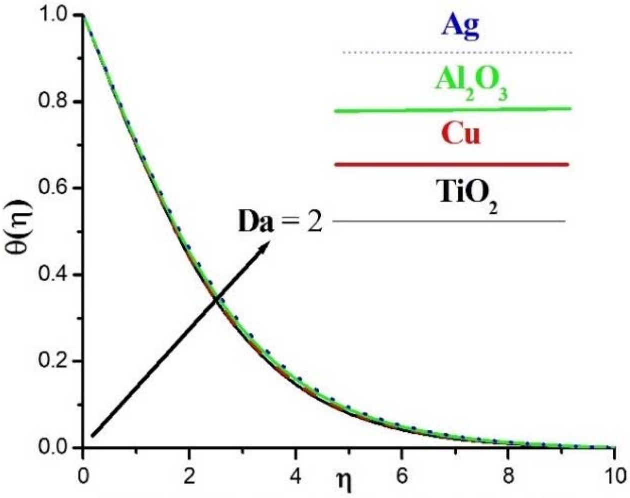

Fig.9. Effect of Da on $\theta(\eta)$ for various nanoparticles.

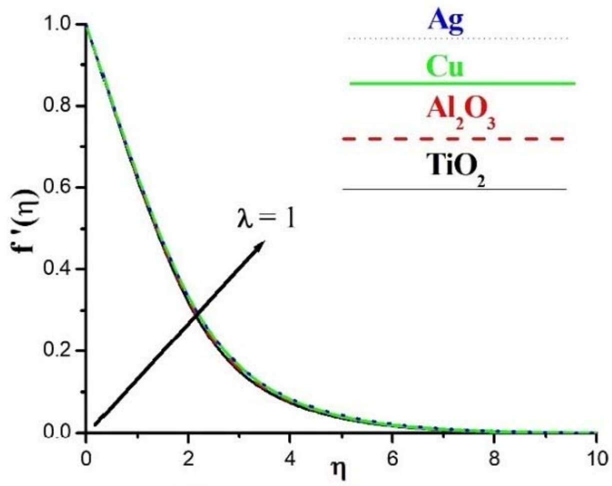

Fig.10. Effect of λ on $f'(\eta)$ for various nanoparticles. nanoparticles.

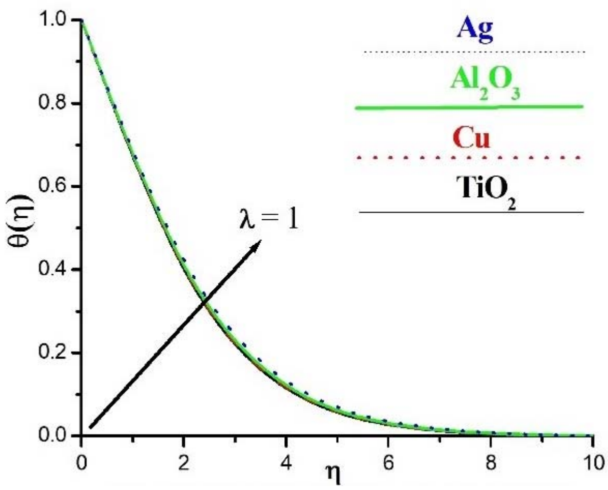

Fig.11. Effect of λ on $\theta(\eta)$ for various

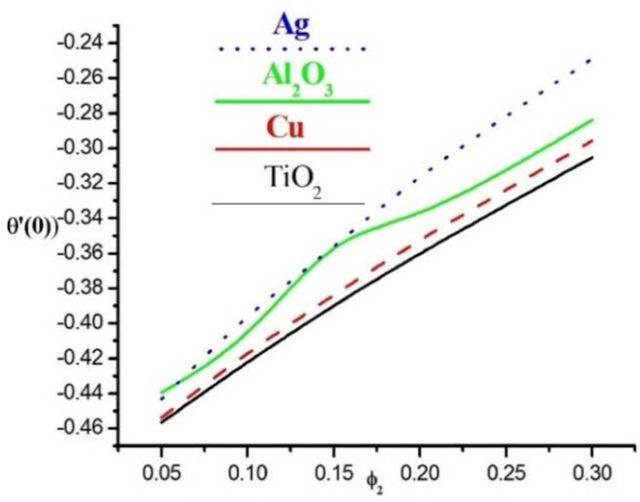

Fig.12. Effect of ϕ_2 on $\theta'(0)$.

5. Conclusion

The impact of various nanoparticles on the boundary layer flow through a porous medium over a stretching sheet in the presence of buoyant force is theoretically studied by using graphs. An increase in the volume fraction increased the thermal conductivity as well as the rate of heat transfer of $Ag\text{-}H_2O$ nanofluids and a reverse effect is observed for $TiO_2\text{-}H_2O$. Hence, it is concluded that $Ag\text{-}H_2O$ is the appropriate nanofluid for enhancing the thermal conductivity of the base fluid (water). It is also observed that $TiO_2\text{-}H_2O$. has a lower fluid movement due to porosity of the medium.

Nomenclature

$a_1\left(m.s^{-1}\right)$ − velocity component along x_1 the axis
$b_1\left(m.s^{-1}\right)$ − velocity component along x_2 the axis
$c_s\left(J.K^{-1}\right)$ − heat capacity of solid surface
$Da=\dfrac{\nu_f}{Kc}$ − porous medium parameter
$g\left(m.s^{-2}\right)$ − acceleration due to gravity
$K_{nf}\left(w.K^{-1}.m^{-1}\right)$ − effective thermal conductivity of nanofluid
$k_f\left(w.K^{-1}.m^{-1}\right)$ − thermal conductivity of the fluid
$k_s\left(w.K^{-1}.m^{-1}\right)$ − thermal conductivity of the solid
$T_2(K)$ − temperature of the nanofluid
$T_\infty(K)$ − temperature of the ambient fluid
$U_w\left(m.s^{-1}\right)$ − stretching velocity of sheet
$\alpha_{nf}\left(m^2.s^{-1}\right)$ − thermal diffusivity of nanofluid
$\beta_{nf}\left(K^{-1}\right)$ − coefficient of thermal expansion of nanofluid
$\lambda=\dfrac{g\beta_f(T_w-T_\infty)}{c^2x}$ − convection parameter
$\mu_f\left(Ns.m^{-2}\right)$ − viscosity of the fluid
$\mu_{nf}\left(Ns.m^{-2}\right)$ − effective viscosity of nanofluid
ϕ_2 − solid volume fraction of nanoparticle
$\rho_f\left(kg.m^{-3}\right)$ − reference density of fluid fraction
$\rho_{nf}\left(kg.m^{-3}\right)$ − effective density of the nanofluid
$\rho_s\left(kg.m^{-3}\right)$ − reference density of water

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