# WEIGHT MINIMIZATION OF STRUCTURAL COMPONENTS REINFORCED WITH FIBER-MAT 

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#### Abstract

The results of investigation in the area of designing of lightweight composite structures are presented. The thin, two-dimensional and linearly elastic disk reinforced with fibermat and subjected to service loading is considered. The problem of optimal layout of the fiber-mat in the disk domain in order to obtain the minimal weight and the assumed mechanical properties of this structure is discussed. The adequate model of this structure and relevant optimality conditions for this type of design problem are derived. To solve of the problem, the optimization procedure based on the evolutionary algorithm is proposed. The problem of weight minimization of structural components reinforced with the fiber-mat is illustrated by simple numerical example.


Keywords: composite disk, fiber-mat, optimal design, evolutionary algorithm

## 1. INTRODUCTION

During the last decades there has been a growing interest in using fibrous composite materials for products used in many areas of technical applications. This group of modern materials is different from more conventional ones. The fibrous composites can consist of unidirectional fibers, fiber-mat or woven fibers suspended in a matrix. The fibers are the principal reinforcing or loadcarrying agent. They are typically strong and stiff. The functions of the light matrix is to support and protect the fibers and to provide a means of distributing load among and transmitting load between the fibers. Thus, the fibrous composites are characterized by very good mechanical properties associated with their small weight and they are ideal for many structural components in which these properties are required.

The optimal design of the composite structure is a very complex process. To fulfill the small weight and assumed mechanical properties of the composite structure we can modify some its structural parameters, such as properties of the matrix and the reinforcing fibers, percentage participation of the fibers in the matrix or fiber shape and orientation. Thus, the correct design of such structures requires adequate analysis and, in particular, adequate and accurate models for numerical simulation. In the present paper, the results of investigation in the area of designing of the lightweight disks reinforced with fiber-mat are presented. These results can be treated as a starting point for optimal design of real composite structures subjected to service load. It will allow for avoiding expensive experimental testing, which can be reduced to the final phase of structural design.

## 2. PROBLEM FORMULATION

Let us consider a thin, two-dimensional and linearly elastic disk (Fig.1) supported on the boundary portion $S_{U}$ with prescribed displacement $\mathbf{u}^{0}=\left\{u_{x}{ }^{0}, u_{y}{ }^{0}\right\}^{\mathrm{T}}$ and loaded by body forces $\mathbf{f}^{0}=\left\{f_{x}^{0}, f_{y}^{0}\right\}^{\mathrm{T}}$ with domain $A$ as well as by external traction $\mathbf{T}^{0}=\left\{T_{x}^{0}, T_{y}^{0}\right\}^{\mathrm{T}}$ acting along the boundary portion $S_{T}$.


Fig. 1. Two-dimensional composite disk subjected to service loading
The material of the disk is a composite made of a matrix reinforced with a fiber-mat about lower mechanical properties compared to the matrix. This reinforcement consists of two orthogonal families of long and straight fibers. Let us assume that:

- The matrix is homogeneous, isotropic and linearly elastic. The mass density of the matrix is $\gamma_{m}$, and the mechanical properties of matrix are characterized by Young's modulus $E_{m}$ and Poisson's ratio $v_{m}$.
- The fibers are homogeneous, isotropic and linearly elastic. They are regularly spaced and perfectly aligned in the matrix. The mass density of the
fibers is $\gamma_{m}$, and Young's modulus and Poisson's ratio are denoted by $E_{w}$ and $\nu_{w}$, respectively.
- The fiber volume fraction in the composite material is $\rho_{w}$ and it is the sum of volume of fiber $\rho_{w 1}$ in the 1 -direction and volume of fiber $\rho_{w 2}$ in the 2 direction of reinforcement.
- The fibers layout at any point of the composite is defined by the fiber orientation $\theta$, which is the angle between the fiber line in the 1 -direction and the $x$-axis of the global coordinate system.
- The fibrous composite is macroscopically homogeneous and linearly elastic material with orthotropic material properties.

The general idea of the design of composite materials regards the modification of the structure parameters of the composite, such as properties of the reinforcing fibers and the matrix, percentage participation of the fibers in the material or fiber orientation. Each of these parameters influences the properties of composite material and can be treated as the design variable $\mathbf{b}$ during the optimal design of structural components made of the composite.

The problem discussed in this paper concerns the optimal design of the composite structure in order to obtain the minimal weight of the disk with the imposed requirements in the range of its mechanical properties. This optimization problem can be written in general as the minimization of the mass density of the composite material, expressed in form as follows:

$$
\begin{equation*}
\min . F_{c}=\left(\rho_{w 1}+\rho_{w 2}\right)\left(\gamma_{w}-\gamma_{m}\right)+\gamma_{m} \tag{2.1}
\end{equation*}
$$

subjected to the global or local behavioral constraints:

$$
\begin{equation*}
\left(\int_{A} \Gamma(\boldsymbol{\sigma}, \mathbf{e}, \mathbf{u}, \mathbf{b}) d A+\int_{S_{T}} \Psi\left(\mathbf{T}^{0}, \mathbf{u}\right) d S_{T}\right)-G_{0} \leq 0 \tag{2.2}
\end{equation*}
$$

where $\Gamma$ and $\Psi$ are continuous functions depending on the displacement $\mathbf{u}=\left\{u_{x}, u_{y}\right\}^{\mathrm{T}}$, strain $\mathbf{e}=\left\{e_{x}, e_{y}, \gamma_{x y}\right\}^{\mathrm{T}}$ and stress $\boldsymbol{\sigma}=\left\{\sigma_{x}, \sigma_{y}, \tau_{x y}\right\}^{\mathrm{T}}$ fields induced in the deformed disk for the configuration of composite structure described by design vector $\mathbf{b}$.

## 3. ANALYSIS OF STRUCTURAL BEHAVIOR

The behavior of the composite disk shown in Fig. 1 can be described by the equilibrium equation given in form [6]:

$$
\begin{equation*}
\operatorname{div} \boldsymbol{\sigma}+\mathbf{f}^{0}=0 \tag{3.1}
\end{equation*}
$$

as well a kinematical relation between the strain and the displacement fields [6]:

$$
\begin{equation*}
\mathbf{e}=\mathbf{B} \cdot \mathbf{u} \tag{3.2}
\end{equation*}
$$

where $\mathbf{B}$ is a linear differential operator relating the displacement field with the strain field. A linear stress-strain relation is assumed in the form of generalized Hooke's law [6]:

$$
\begin{equation*}
\boldsymbol{\sigma}=\mathbf{D} \cdot \mathbf{e} \tag{3.3}
\end{equation*}
$$

where $\mathbf{D}$ denotes the extensional stiffness matrix for the model of the composite material reinforced with the fiber-mat. Besides, the disk is subjected to the boundary conditions expressed as follows [6]:

$$
\begin{cases}\boldsymbol{\sigma} \cdot \mathbf{n}=\mathbf{T}^{0} & \text { on } S_{T}  \tag{3.4}\\ \mathbf{u}=\mathbf{u}^{0} & \text { on } S_{U}\end{cases}
$$

where $\mathbf{n}=\left\{n_{x}, n_{y}\right\}^{\mathrm{T}}$ is the unit normal vector on the external boundary $S$ of the disk.

The set of equations (3.1)-(3.4) constitutes the boundary problem for the disk. This problem can be solved, for instance, with the aid of the finite element method (FEM). The detailed description of this method is presented in [1].


Fig. 2. Material reinforced with fiber-mat: a) real composite, b) model of composite
In analysis step of structural behavior, the microscopically non-homogeneous composite material is modeled by a plane, homogeneous, orthotropic and linearly elastic material (Fig.2). The purpose of the modeling process is to determine the extensional stiffness matrix $\mathbf{D}$ for that model in the global coordinate system and to express its components in terms of the mechanical properties of the fibers and the matrix, as well in terms of the fiber volume fraction and the fiber orientation.

The extensional stiffness matrix $\mathbf{D}$, appearing in (3.3), for the assumed model of composite in the global coordinate system $x-y$ is expressed by [4]:

$$
\begin{equation*}
\mathbf{D}=\mathbf{T}^{-1} \cdot \mathbf{C} \cdot \mathbf{T}^{-\top} \tag{3.5}
\end{equation*}
$$

The matrix $\mathbf{C}$ denotes here the stiffness matrix for the composite with respect to the material axes 1-2, coinciding with the fiber directions and it has the following form [4]:

$$
\mathbf{C}=\left[\begin{array}{ccc}
\frac{E_{1}}{1-v_{12} v_{21}} & \frac{E_{1} v_{21}}{1-v_{12} v_{21}} & 0  \tag{3.6}\\
\frac{E_{2} v_{12}}{1-V_{12} v_{21}} & \frac{E_{2}}{1-v_{12} v_{21}} & 0 \\
0 & 0 & G_{12}
\end{array}\right]
$$

where $E_{1}$ and $E_{2}$ are the apparent Young's moduli in the 1-direction and the 2-direction, respectively, while $\nu_{12}$ is the major and $\nu_{21}$ is the minor Poisson's' ratio, and $G_{12}$ denotes the in-plane shear modulus for the composite. Using the model of lamina, presented in [2], these so-called engineering constants are obtained in two steps of modeling process (Fig.3).


Fig. 3. Two steps of modeling process for material reinforced with fiber-mat

First, the engineering constants for the material reinforced with one family of long fibers with the fiber volume fraction $\rho_{w 1}$ are calculated as follows [2]:

$$
\begin{align*}
& E_{1}^{(1)}=E_{w} \rho_{w 1}+E_{m}\left(1-\rho_{w 1}\right) \\
& E_{2}{ }^{(1)}=\frac{E_{w}\left[1+\left(k_{E}-1\right) \rho_{w 1}\right]}{\left[\rho_{w 1}+k_{E}\left(1-\rho_{w 1}\right)\right]\left[1+\left(k_{E}-1\right) \rho_{w 1}\right]-\left(k_{E} v_{m}-v_{w}\right)^{2} \rho_{w 1}\left(1-\rho_{w 1}\right)} \\
& v_{12}{ }^{(1)}=v_{w} \rho_{w 1}+v_{m}\left(1-\rho_{w 1}\right) \quad \text { and } \quad v_{21}{ }^{(1)}=v_{12}{ }^{(1)} \frac{E_{2}{ }^{(1)}}{E_{1}{ }^{(1)}}  \tag{3.7}\\
& G_{12}{ }^{(1)}=\frac{E_{m}\left[k_{E}\left(1+v_{m}\right)\left(1+\rho_{w 1}\right)+\left(1+v_{w}\right)\left(1-\rho_{w 1}\right]\right.}{2\left(1+v_{m}\right)\left[k_{E}\left(1+v_{m}\right)\left(1-\rho_{w 1}\right)+\left(1+v_{w}\right)\left(1+\rho_{w 1}\right)\right]} \\
& \text { where } k_{E}=E_{w} / E_{m}
\end{align*}
$$

Such unidirectionally fiber-reinforced composite is treated as an orthotropic matrix and next, it is reinforced with the second family of fibers with the fiber volume fraction $\rho_{w 2}$ in the direction perpendicular to the 1 -direction. Finally, the engineering constants for the bidirectionally fiber-reinforced composite can be expressed in the following form [2]:

$$
\begin{align*}
& E_{1}=E_{w} \rho_{w 1}+E^{(*)}\left(1-\rho_{w 1}\right) \text { where } E^{(*)}=E_{2}{ }^{(1)} \text { for } \rho_{w}=\rho_{w 2} \\
& E_{2}=E_{w} \rho_{w 2}+E_{2}{ }^{(1)}\left(1-\rho_{w 2}\right) \\
& v_{21}=v_{w} \rho_{w 2}+v_{21}{ }^{(1)}\left(1-\rho_{w 2}\right) \quad \text { and } \quad v_{12}=v_{21} \frac{E_{1}}{E_{2}}  \tag{3.8}\\
& G_{12}=\frac{k_{G}\left(1+\rho_{w 2}\right)+\left(1-\rho_{w 2}\right)}{k_{G}\left(1-\rho_{w 2}\right)+\left(1+\rho_{w 2}\right)} G_{12}{ }^{(1)} \quad \text { where } k_{G}=\frac{E_{w}}{2\left(1+v_{w}\right) G_{12}{ }^{(1)}}
\end{align*}
$$

The matrix $\mathbf{T}$, appearing in equation (3.5), denotes the transformation matrix from the global coordinate system $x-y$ to the material axes 1-2 and it has the form [4]:

$$
\mathbf{T}=\left[\begin{array}{ccc}
\cos ^{2} \theta & \sin ^{2} \theta & 2 \sin \theta \cos \theta  \tag{3.8}\\
\sin ^{2} \theta & \cos ^{2} \theta & -2 \sin \theta \cos \theta \\
-\sin \theta \cos \theta & \sin \theta \cos \theta & \cos ^{2} \theta-\sin ^{2} \theta
\end{array}\right]
$$

This matrix is considered as the matrix function of fiber orientation angle $\theta$.

## 4. OPTIMIZATION PROCEDURE

To perform the optimization task, defined by equations (2.1)-(2.2), the evolutionary algorithm is proposed (Fig.4). This method based on the imitation of the evolution processes occurring in the nature still finds the growing interest in engineering design problems. The evolutionary algorithm is a simple, powerful and effective tool used for finding the best solution in a complicated space of design parameters and it is not limited by a restrictive assumption about the search space. This method needs only the information based on the value of objective function and constraints, which is its main advantage in comparison to the optimization methods based on the gradient information of the objective function and constraints. Besides, in contrast to deterministic methods, which often fall into a local optimum, the evolutionary algorithm always finds the global optimum or the solution close to this optimum.


Fig. 4. Flow chart of evolutionary algorithm
It should be added that the evolutionary algorithm is generally suited for unconstrained optimization problems. Thus, in the case of the optimization problem with the constraints, the penalty function approach [3] is applied in the proposed algorithm. Using this approach, the constrained problem (2.1)-(2.2) is transformed to an unconstrained one as follows:

$$
\begin{equation*}
\min . Z(\mathbf{b}, \boldsymbol{\alpha})=\min .\left[F_{c}(\mathbf{b})+\frac{1}{2} \sum_{i=1}^{n_{g}} \alpha_{i}\left[\max .\left(0 ; G_{i}(\mathbf{b})\right)\right]^{2}\right] \tag{4.1}
\end{equation*}
$$

where $\boldsymbol{\alpha}$ is a vector of positive coefficients of penalty functions, and $n_{g}$ is a number of inequality constraints in the constrained problem.

As it is shown in Fig.4, the evolutionary algorithm starts from random selection of the initial population of $N$ chromosomes. Each chromosome is a coded vector of design parameters $\mathbf{b}$, and it represents a point in design space and describes one possible solution to the given problem. The floating point representation [5] is applied in this step of the evolutionary algorithm:

$$
\begin{equation*}
c h_{j} \rightarrow \mathbf{b}_{j}=\left[b_{1}, b_{2}, \ldots, b_{p}\right] \quad \text { where } \Lambda_{i=1 \ldots p} b_{i}=b_{i(\min )}+r\left(b_{i(\max )}-b_{i(\min )}\right) \tag{4.2}
\end{equation*}
$$

The notations $b_{i(\min )}$ and $b_{i(\max )}$ are the variable bounds for the $i$-th design parameter of the vector $\mathbf{b}$, while $r$ is a random number in range $\langle 0,1\rangle$.

Thereafter, all chromosomes in the current population are evaluated using the objective functional $F_{c}$ :

$$
\begin{equation*}
\Lambda_{j=1 . . . N} \quad v\left(c h_{j}\right) \equiv F_{c}\left(\mathbf{b}_{j}\right) \tag{4.3}
\end{equation*}
$$

where $v\left(c h_{j}\right)$ denotes a fitness value of the $j$-th chromosome. This value is related to the value of the objective functional $F_{c}$ for the $j$-th vector of design parameters $\mathbf{b}_{j}$. The analysis of structural behavior is performed using the finite element method (FEM) in this step of the evolutionary algorithm.

The current population is processed by three main operators of the evolutionary algorithm. They are deterministic selection, heuristic crossover and non-uniform mutation. Before these processes, the power law scaling [5] is applied in the algorithm. It is mainly responsible for the better search aspect of the evolutionary algorithm. At this point, the functional $Z$ is transformed to the new form, so-called the fitness functional $F_{p}$, according to the rule:

$$
\begin{equation*}
F_{p}(\mathbf{b})=e^{\left(-a \frac{Z(\mathbf{b})-Z_{\min }(\mathbf{b})}{Z_{\max }(\mathbf{b})-Z_{\min }(\mathbf{b})}\right)} \tag{4.4}
\end{equation*}
$$

where $Z_{\min }(\mathbf{b})$ and $Z_{\max }(\mathbf{b})$ denote minimal and maximal value of the functional $Z$ in the current population, respectively, while $a$ is a positive scaling parameter.

The main idea of the selection operator is that ,good" chromosomes are picked from the current population and multiple copies of them are created. As a result of this, ,,bad" chromosomes are eliminated from the population and do not undergo any further modification. The deterministic selection [5] is used in the
proposed version of the evolutionary algorithm. First, the selection probability for each chromosome $p_{j}$ is calculated as follows:

$$
\begin{equation*}
\Lambda_{j=1 . . N} p_{j}=\frac{F_{p}\left(\mathbf{b}_{j}\right)}{\sum_{j=1}^{N} F_{p}\left(\mathbf{b}_{j}\right)} \tag{4.5}
\end{equation*}
$$

and next the number of its expected copies $o l k_{(c h)}$ is obtained:

$$
\begin{equation*}
\Lambda_{j=1 . . . N} \operatorname{olk}_{\left(c h_{j}\right)}=p_{j} * N \tag{4.6}
\end{equation*}
$$

The number of copies of each chromosome is equal to integer part of $o l k_{(c h)}$. Finally, the chromosomes are placed in the population according to the fractional part of $o l k_{(c h)}$ and the empty places in this population are filled with the copies of chromosomes from the beginning of the population.

The heuristic crossover [5] operator recombines randomly chosen two parents chromosomes $c h_{1}$ and $c h_{2}$ to form a better child chromosome $c h$ ', according to the following scheme:

$$
\begin{equation*}
c h^{\prime}=r\left(c h_{2}-c h_{1}\right)+c h_{2} \quad \text { where : } \quad F_{p}\left(c h_{2}\right) \geq F_{p}\left(c h_{1}\right) \tag{4.7}
\end{equation*}
$$

where $r$ is a random number in range $\langle 0,1\rangle$. This operation is carried out with a crossover probability $p_{c}$. It must be added that the child chromosome can be an infeasible solution. In this case, the crossover operator is repeated.

As the last operator, the non-uniform mutation [5] is applied. This operator alters a chromosome locally:

$$
\begin{equation*}
c h_{j}=\left[b_{1}, \ldots, b_{i}, \ldots, b_{n}\right] \rightarrow c h_{j}^{\prime}=\left[b_{1}, \ldots, b_{i}^{\prime}, \ldots, b_{n}\right] \tag{4.8}
\end{equation*}
$$

The component of the chromosome $b_{i}$ is chosen with a mutation probability $p_{m}$. Its new value is calculated from the following relationship:

$$
b_{i}^{\prime}= \begin{cases}b_{i}+\left(1-r^{(1-k / L P) s}\right)\left(b_{i(\max )}-b_{i}\right) & \text { if } l=0  \tag{4.9}\\ b_{i}-\left(1-r^{(1-k / L P) s}\right)\left(b_{i}-b_{i(\text { min })}\right) & \text { if } l=1\end{cases}
$$

where $b_{i(\min )}$ and $b_{i(\max )}$ are the variable bounds for the $i$-th design parameter $b_{i}, r$ denotes a random number in range $\langle 0,1\rangle, k$ is a iteration number and $L P$ is a maximal number of generations, while $s$ defines a degree of non-uniform.

Applying these three operators, a new population of solutions is created and the single cycle of the evolutionary algorithm, which is known as a generation, comes to the end. Each successive generation contains better ,,partial solutions" than in the previous generations, and converges towards the global
optimum. This procedure is continued until no substantial improvement of the best or average population statistics for a few consecutive generations.

## 5. NUMERICAL EXAMPLE

To illustrate the problem of optimal design of lightweight composite structure, simple numerical example is presented in this Section.

Let us consider a thin disk supported along its left boundary and uniformly loaded by traction on the upper boundary, as it is shown in Fig.5. The material of the disk is a composite made of epoxy matrix reinforced with glassmat. The material data of the components of the composite material are given in Table 1.
It is also assumed that the stiffness of reinforced disk corresponding to work done by external forces should be equal to 2.20 [J].


Fig. 5. Composite disk subjected to load and boundary conditions

Table 1. Material data of components of composite material

|  | $\gamma\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ | $E[\mathrm{GPa}]$ | $v$ |
| :--- | :---: | :---: | :---: |
| fibers (glass E) | $2.49 * 10^{3}$ | 75 | 0.25 |
| matrix (epoxy) | $1.15 * 10^{3}$ | 3.5 | 0.38 |

The problem discussed in this example concerns the optimal orientation of the fiber-mat in the composite material so that the disk should be as light as possible.

First, the reference solution for the composite disk reinforced with the glass-mat with the standard fiber orientation $\left(0^{\circ} ; 90^{\circ}\right)$ is presented in Fig.9. However, one can note that this solution corresponds to the assumed stiffness of the disk, but it does not guarantee a minimal weight of the disk. To fulfill the assumed requirements for the disk we can modify volume participation of the
reinforcing fibers and fiber orientation in the optimization process under actual loading conditions of this composite structure.

The optimization problem, defined generally by equations (2.1)-(2.2), was written in the following form:

$$
\begin{equation*}
\min . F_{c}=\left(\rho_{w 1}+\rho_{w 2}\right)\left(\gamma_{w}-\gamma_{m}\right)+\gamma_{m} \tag{5.1}
\end{equation*}
$$

subjected to the global behavioral constraint:

$$
\begin{equation*}
\int_{S_{T}} \mathbf{u}^{\mathrm{T}} \cdot \mathbf{T}^{0} d S_{T}-S T I F F_{0} \leq 0 \tag{5.2}
\end{equation*}
$$

and the geometrical constraints:

$$
\left\{\begin{array}{l}
0 \leq \rho_{w 1}+\rho_{w 2} \leq 1  \tag{5.3}\\
0^{0} \leq \theta \leq 90^{0}
\end{array}\right.
$$

where $\rho_{w 1}$ and $\rho_{w 2}$ are the fiber volume fractions in the 1 -direction and 2 direction of reinforcement, respectively and $\theta$ denotes the angle of fiber orientation between fiber line in the 1-direction and the $x$-axis of the global coordinate system $x-y$. These variables define the layout as well the volume participation of the fiber-mat in the composite and they were treated as the vector of design variables $\mathbf{b}$, i.e. $\mathbf{b}=\left\{\rho_{w 1}, \rho_{w 2}, \theta\right\}$, which was determined in the optimization process.

To solve the optimization task, the evolutionary algorithm, presented in Section 4 was used. The data of this algorithm are given in Table 2. To analyze the behavior of the disk, its domain was discretized into $15 \times 10$ two-dimensional four-node quadrilateral elements (Fig.6).
Table 2. Data of evolutionary algorithm

| type of data | value |
| :--- | :---: |
| number of chromosomes in population | $N=30$ |
| positive scaling parameter | $a=0.1$ |
| crossover probability | $p_{c}=0.95$ |
| replication number of crossover process | $r c=10$ |
| mutation probability | $p_{m}=0.20$ |
| degree of non-uniform | $s=0.9$ |
| replication number of mutation process | $r m=10$ |
| stop criterion | $w s=1 \mathrm{e}-4$ |
| number of testing generations in stop criterion | $L G=25$ |
| maximal number of generations | $L P_{\max }=1000$ |



Fig. 6. Discretization of disk domain using finite elements
The design problem was considered for two cases of the fiber volume fraction in the directions of reinforcement. First, the same fiber volume fraction, and next different fiber volume fraction in the 1-direction and 2-direction was discussed. The results of the optimization process are given in Table 3, and the optimal layouts of the reinforcing fiber-mat in the disk domain are shown in Fig. 7 and 8.

Finally, these optimal solutions were compared with the reference solution (Fig.7) for the disk reinforced with the fiber-mat with the standard fiber orientation $\left(0^{0} ; 90^{\circ}\right)$ in order to qualify the results of the optimization.

Table 3. Results of optimization process and reference solution

|  | fiber <br> orientation | fiber volume <br> fraction | mass density <br> of disk | stiffness <br> of disk |
| :--- | :---: | :---: | :---: | :---: |
| optimal <br> solution 1 | $57.81^{0} ; 147.81^{0}$ | $\rho_{w 1}=0.28$ <br> $\rho_{w 2}=0.28$ | $\gamma=1.90^{*} 10^{3}$ <br> $\left[\mathrm{~kg} / \mathrm{m}^{3}\right]$ | $2.20[\mathrm{~J}]$ |
| optimal <br> solution 2 | $64.35^{0} ; 154.35^{0}$ | $\rho_{w 1}=0.14$ <br> $\rho_{w 2}=0.38$ | $\gamma=1.85^{*} 10^{3}$ <br> $\left[\mathrm{~kg} / \mathrm{m}^{3}\right]$ | $2.20[\mathrm{~J}]$ |
| reference <br> solution | $0^{0} ; 90^{0}$ | $\rho_{w 1}=0.35$ <br> $\rho_{w 2}=0.35$ | $\gamma=2.09 * 10^{3}$ <br> $\left[\mathrm{~kg} / \mathrm{m}^{3}\right]$ | $2.20[\mathrm{~J}]$ |



Fig. 7. Optimal disk reinforced with fiber mat - case 1


Fig. 8. Optimal disk reinforced with fiber mat - case 2


Fig. 9. Reference disk reinforced with fiber mat

It can be easily observed from results presented in Table 3 as well Fig.7-9 that each of these solutions satisfies assumed mean stiffness of the disk. However, the optimal layout of the reinforcing fiber-mat in the disk domain with the same fiber volume fraction in the 1-direction and 2-direction decreases the weight of the disk by $10 \%$, while in the case of different percentage participation of fibers in the directions of reinforcement, this weight decrease by $12 \%$, when compared to the reference disk.

## 6. CONCLUDING REMARKS

The results of investigation in the area of designing of lightweight structural components reinforced with fiber-mat are presented in the paper. The results allow us to state that the minimal weight and the assumed requirements in the range of the mechanical properties for these composite structures can be obtained when the fibers are optimally distributed and oriented in the structure with respect to the assumed measure of structural behavior. For finding this optimal layout of the reinforcing fiber-mat in the structural components domain, the evolutionary algorithm was proposed. This algorithm can constitute an alternative technique for classical methods applied in optimization of composite structures, or can supplement them.

The presented analysis can be also treated as a starting point for computer-oriented optimal design procedures of real structural components made of composite materials. Such a procedure can allow for avoiding expensive experimental testing, which can be reduced to the final phase of structural design.

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# OPTYMALNE PROJEKTOWANIE LEKKICH ELEMENTÓW KONSTRUKCYJNYCH WZMACNIONYCH MATAMI WŁÓKIENNICZYMI 

Streszczenie

W pracy przedstawiono wyniki badań dotyczące optymalnego projektowania lekkich struktur kompozytowych. Obiektem badań były płaskie, dwuwymiarowe i liniowosprężyste elementy konstrukcyjne wykonane z materiału kompozytowego wzmocnionego matą włókienniczą i obciążone statycznie siłami działającymi w ich płaszczyźnie. Projektowanie takich struktur rozpatrzono z uwagi na optymalne ułożenie maty włókienniczej w materiale kompozytowym, tak aby element konstrukcyjny wykonany z tego materiału uzyskiwał możliwie najmniejszą mase właściwą przy jednoczesnym spełnieniu stawianych wymagań w zakresie określonych własności mechanicznych. W pracy przedstawiono odpowiedni model struktury kompozytowej, warunki optymalnego rozwiązywania tego typu problemu, a do poszukiwania optymalnych rozwiązań zaproponowano metodę optymalizacyjną opartą na algorytmie ewolucyjnym. Rozpatrywany problem zilustrowano przykładem numerycznym. Uzyskane wyniki moga stanowić punkt wyjścia do projektowania optymalnej struktury materiałów kompozytowych wzmacnianych matami włókienniczymi i będących tworzywem konkretnej konstrukcji pracującej pod zadanym obciążeniem, pozwalając tym samym uniknąć kosztownych i pracochłonnych badań doświadczalnych, które można ograniczyć do końcowych badań eksperymentalnych gotowej konstrukcji.

