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INTEGRATION OF MULTISENSOR MEASUREMENTS USING MODIFIED KALMAN FILTER†

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The problem of integrating measurement data that come from various navigation sources at different moments of time is considered. An algorithm is established which uses modified Kalman filter to process separately scalar data even in cases of vector input. This approach allows us to construct unified estimation algorithm for processing variable dimension measurement vectors and to decrease covariances of estimate errors if extra measurement appears at the filter input from a new source and with different sampling interval. A simple method is also proposed to avoid filter divergence resulting from computational errors. Statistical simulation of the algorithm proving its applicability is presented.

1. Introduction

The theoretical basis providing the way to develop algorithms for data processing in multisensor complex navigation systems is still in the process of development. Integration of multisensor measurements in many fields of engineering and especially in complex navigation systems that aim at detecting exact position location of moving objects still creates serious scientific and technical problems. This happens mainly because measurements come from various sources at different time moments.

There are numerous publications devoted to the analysis of approaches to position location estimation using one or several measurements from various sources that are in nonlinear relation with the moving object coordinates. One of the first publications devoted to the solution to the classic navigation problem, i.e. determination of an object position using measurements of two angles, employed maximum likelihood approach with some simplifications (Stansfield, 1947). In papers (Daniels, 1951) and (Marchand, 1964) this approach was generalized to difference–ranging systems, and Groginski (1959) considered position estimation problem employing range measurements.

The problem of combined processing of measurement data of various origin using generalized least squares approach was first considered by Fay (1976). The statistical analysis of the method for combined processing of information coming from multiple sources was presented by Wax (1983). The measurement stations location was considered to be not exactly defined, i.e. the spatial position of sensors was indefinite.

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The Kalman filter is considered as an alternative approach to determine the moving object location coordinates in conditions of indefiniteness in the sensors position. The most often used are: *linearized, extended* and so called *iterational extended* – Kalman filters (Anderson and Moor, 1984; Korbicz and Bidyuk, 1993; Leondes, 1980; Puzyrev and Gostukhina, 1981). The weighting coefficients of the linearized Kalman filter do not depend on the current state estimates and it is possible to compute them in advance using nominal steady–state solution. But on the other hand such an approach may result in a substantial deviation of state estimates from nominal values. The extended Kalman filter equations are linearized with respect to the last (in time) estimate of state. This means that weighting coefficients should be computed in real time. The iterational extended Kalman filter suggests existence of iterations for improving the estimates with each new measurement until change in the estimate value is nonsignificantly small.

Rao (1987) considered application of the Kalman filter to estimation of parameters characterizing Indian macroeconomy. He stated explicitly the problem of integrating the information coming from multiple sources but did not show an efficient way to solve the problem. Rauch *et al.* (1983) proposed a computer–based expert system for tactical data fusion. This system can be used to enhance the ability of decision makers in military command and control centers. The derivation of rules for the expert system requires extensive interaction between the computer scientist and human experts; formalizing the knowledge of such experts can be quite difficult.

A comparison of the Kalman filter based approach with nonlinear least squares one is given by Springarm (1987). But today there is no general theoretical substantiation for advantages of any of the approaches mentioned above for solving the problem of integrating multiple sensor data generated by various sources at different time moments. A particular method for data processing and corresponding estimation of states selected separately following each practical situation. Generally the convergence of the above–mentioned methods is also not guaranteed. According to examples given by Foy (1976) and Springarm (1987) convergence of estimates to the true values depends substantially on initial conditions.

In this paper we propose a modified computational algorithm for extended Kalman filter convenient for real–time realization. The filter convergence is considered on specific examples of the moving object coordinates estimation using measurements generated by multiple sources.

2. Moving Object Model and Modified Kalman Filter Algorithm

To characterize current state of a moving object we choose state vector $\boldsymbol{x}$ including three Cartesian coordinates x, y, z and velocity components along each of these coordinates, i.e.

$$\boldsymbol{x} = [x \; v_x \; y \; v_y \; z \; v_z]^T$$

It is supposed that measurement data are coming to the central processing station at different time moments from the following sources:

- radar system measuring spherical coordinates of a moving object such as range D, azimuth β and elevation angle ε (these coordinates are measured with mean square errors σ_D, σ_β and σ_ε, respectively),
- independent channel for azimuth measurements β (mean square error is σ_β),
- channel for radial velocity v_R measurements (mean square error is σ_v).

Let us introduce the following notation:

$t^{(1)}$ — a set of discrete time moments at which measurements of spherical coordinates are coming with sampling interval $T^{(1)}$,

$t^{(2)}$ — a set of time moments at which azimuth is measured independently with sampling period $T^{(2)}$,

$t^{(3)}$ — a set of time moments at which radial velocity is measured with sampling interval $T^{(3)}$.

The state and measurement vectors are in nonlinear relation which results in the following measurements model with a structure nonstationary in time:

$$z(k) = \begin{cases} z^{(1)}(k) = \begin{bmatrix} h_D[x(k)] + \eta_D(k) \\ h_\beta[x(k)] + \eta_\beta(k) \\ h_\varepsilon[x(k)] + \eta_\varepsilon(k) \end{bmatrix} & \text{at} \quad t_k \in t^{(1)}; \\ z^{(2)}(k) = h_\beta[x(k)] + \eta_\beta(k) & \text{at} \quad t_k \in t^{(2)}; \\ z^{(3)}(k) = h_v[x(k)] + \eta_v(k) & \text{at} \quad t_k \in t^{(3)}; \end{cases} \tag{1}$$

where

$z^{(1)}(k) = \begin{bmatrix} D^m(k) \\ \beta^m(k) \\ \varepsilon^m(k) \end{bmatrix}$ is the measurement vector for data source 1;

$\eta_D(k)$, $\eta_\beta(k)$, $\eta_\varepsilon(k)$ are measurement errors for data source 1;

$z^{(2)}(k) = \beta^m(k)$ are independent measurements of azimuth with error $\eta_\beta(k)$;

$z^{(3)}(k) = v_R^m(k)$ are radial velocity measurements with error $\eta_v^m(k)$.

Nonlinear functions relating state vector to measurements are

$$h_D[x(k)] = \left(x^2(k) + y^2(k) + z^2(k)\right)^{\frac{1}{2}}$$

$$h_\beta[x(k)] = \operatorname{arctg}\frac{x(k)}{y(k)}$$

$$h_\varepsilon[x(k)] = \arcsin \frac{x(k)}{\left(x^2(k) + y^2(k) + z^2(k)\right)^{\frac{1}{2}}}$$

$$h_v[x(k)] = \frac{x(k)v_x(k) + y(k)v_y(k) + z(k)v_z(k)}{\left(x^2(k) + y^2(k) + z^2(k)\right)^{\frac{1}{2}}}$$

The state equation is linear if Cartesian coordinates are used, i.e.

$$\boldsymbol{x}(k+1) = \boldsymbol{\phi}\boldsymbol{x}(k) + \boldsymbol{w}(k) \tag{2}$$

where $\boldsymbol{\phi}$ is state transition matrix, $\boldsymbol{w}(k)$ – a sequence of state noise with zero mean and covariance matrix $\boldsymbol{Q}(k)$.

Estimate of state vector $\boldsymbol{x}(k)$ can be found using equations of linear Kalman filter (Korbicz and Bidyuk, 1993). Prediction equations are as follows:

$$\widehat{\boldsymbol{x}}(k+1,k) = \boldsymbol{\phi}\widehat{\boldsymbol{x}}(k,k) \tag{3}$$

$$\boldsymbol{P}(k+1,k) = \boldsymbol{\phi}\boldsymbol{P}(k,k)\boldsymbol{\phi}^T + \boldsymbol{Q} \tag{4}$$

where $\boldsymbol{P}(k,k)$ is a covariance matrix for state estimate errors determined at previous recursion step of the filter. Equation for computing the state estimates can be created as a result of linearization of functions $h(\cdot)$ with respect to predictions $\widehat{\boldsymbol{x}}(k+1,k)$ and has the form:

$$\widehat{\boldsymbol{x}}(k+1,k+1) = \widehat{\boldsymbol{x}}(k+1,k) + \boldsymbol{K}(k+1))\left\{\boldsymbol{z}(k+1) - \boldsymbol{h}\left[\widehat{\boldsymbol{x}}(k+1,k)\right]\right\}$$

where $\boldsymbol{h}\left[\widehat{\boldsymbol{x}}(k+1,k)\right]$ is defined by equation (1).

The matrix of gain coefficients $\boldsymbol{K}(k+1)$ is computed using usual appropriate equation of linear filter:

$$\boldsymbol{K}(k+1) = \boldsymbol{P}(k+1,k)\boldsymbol{H}^T(k+1)\boldsymbol{R}^{-1}(k+1)$$

$$= \boldsymbol{P}(k+1,k)\boldsymbol{H}^T(k+1)\left[\boldsymbol{H}(k+1)\boldsymbol{P}(k+1,k)\boldsymbol{H}^T(k+1) + \boldsymbol{R}(k+1)\right]^{-1}$$

where $\boldsymbol{H}(k+1)$ is measurement matrix and $\boldsymbol{R}(\cdot)$ is covariance matrix for measurement errors. Finally, covariance matrix for errors of filtering is defined by equation:

$$\boldsymbol{P}(k+1,k+1) = \left[\boldsymbol{I} - \boldsymbol{K}(k+1)\boldsymbol{H}(k+1)\right]\boldsymbol{P}(k+1,k)$$

Matrices $\boldsymbol{H}$ and $\boldsymbol{R}$ may have different structures that depend upon specific measurements being processed currently, i.e.

$$\boldsymbol{H}(k+1)=\begin{cases} \dfrac{\partial \begin{bmatrix} h_D \\ h_\beta \\ h_\varepsilon \end{bmatrix}}{\partial \boldsymbol{x}}\Bigg|_{x=\widehat{x}(k+1,k)} = \begin{bmatrix} \dfrac{\widehat{x}}{\widehat{D}} & 0 & \dfrac{\widehat{y}}{\widehat{D}} & 0 & \dfrac{\widehat{z}}{\widehat{D}} & 0 \\ \dfrac{\widehat{y}}{\widehat{d}^2} & 0 & -\dfrac{\widehat{x}}{\widehat{d}^2} & 0 & 0 & 0 \\ -\dfrac{\widehat{x}\widehat{z}}{\widehat{D}^2\widehat{d}} & 0 & -\dfrac{\widehat{y}\widehat{z}}{\widehat{D}^2\widehat{d}} & 0 & \dfrac{\widehat{d}}{\widehat{D}^2} & 0 \end{bmatrix}, & t_k \in t^{(1)} \\ \dfrac{\partial h_\beta}{\partial \boldsymbol{x}}\Bigg|_{x=\widehat{x}(k+1,k)} = \begin{bmatrix} \dfrac{\widehat{y}}{\widehat{d}^2} & 0 & -\dfrac{\widehat{x}}{\widehat{d}^2} & 0 & 0 & 0 \end{bmatrix}, & t_k \in t^{(2)} \\ \dfrac{\partial h_v}{\partial \boldsymbol{x}}\Bigg|_{x=\widehat{x}(k+1,k)} = \begin{bmatrix} \dfrac{\widehat{x}\widehat{v}_R - \widehat{D}\widehat{v}_x}{\widehat{D}^2} & \dfrac{\widehat{x}}{\widehat{D}} & \dfrac{\widehat{y}\widehat{v}_R - \widehat{D}\widehat{v}_y}{\widehat{D}^2} & \dfrac{\widehat{y}}{\widehat{D}} & \dfrac{\widehat{z}\widehat{v}_R - \widehat{D}\widehat{v}_z}{\widehat{D}^2} & \dfrac{\widehat{z}}{\widehat{D}} \end{bmatrix}, & t_k \in t^{(3)} \end{cases} \tag{5}$$

where $\widehat{D} = (\widehat{x}^2 + \widehat{y}^2 + \widehat{z}^2)^{1/2}$; $\widehat{d} = (\widehat{x}^2 + \widehat{y}^2)^{1/2}$;

$$\boldsymbol{R}(k+1) = \begin{cases} \begin{bmatrix} \sigma_D^2 & 0 & 0 \\ 0 & \sigma_\beta^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{bmatrix}, & t_k \in t^{(1)} \\ \left(\sigma'_\beta\right)^2, & t_k \in t^{(2)} \\ \sigma_v^2, & t_k \in t^{(3)} \end{cases} \tag{6}$$

3. Algorithm for Computing the State Vector Estimates

The variable structure of matrices $\boldsymbol{H}$ and $\boldsymbol{R}$, used in the computations process, complicates substantially realization of the filter algorithm given above. To simplify the process of computing the estimates we propose to replace one multivariable measurement by a sequence of scalar measurements appearing at the filter input with zero time intervals. Such an approach to processing of measurements allows us to avoid processing of measurement vector having variable dimension as well as computing of gain matrix, innovation vector and other parameters with dimensions that may change from one iteration to another. It should be noted that the same method of Kalman filter realization is used in cases when dimension of measurement vector is high so that to avoid matrix inverse calculation for the matrix $(\boldsymbol{HPH}^T + \boldsymbol{R})$ used to compute filter gains (Marchuk, 1980).

To create the filter algorithm of the type under consideration we propose to introduce n–dimensional vector of indicators showing what kind of navigation information is available at the beginning of each main loop of the filter. Dimension n

is determined by a number of parameters of various nature generated by all available navigation means (on the sea, on the ground and in the air). All these measurements are supposed to improve estimates of a moving object state.

For our case $n = 4$, because four parameters are measured: range, azimuth, elevation angle and radial velocity. The vector $\boldsymbol{i}(4)$ accepts values given in Table 1.

Tab. 1. Values of vector $\boldsymbol{i}$.

n	Parameter, measured at arbitrary moment of time	Value of vector $\boldsymbol{i}$
1	range D	$\boldsymbol{i} = [1\,0\,0\,0]$
2	azimuth β	$\boldsymbol{i} = [0\,1\,0\,0]$
3	angle ε	$\boldsymbol{i} = [0\,0\,1\,0]$
4	velocity v_R	$\boldsymbol{i} = [0\,0\,0\,1]$ (+ combinations)

This approach allows a simplification of the measurement equation structure, i.e.

$$\boldsymbol{z}(k) = \begin{cases} h_D[\boldsymbol{x}(k)] + \eta_D(k), & i = 1\,0\,0\,0 \\ h_\beta[\boldsymbol{x}(k)] + \eta_\beta(k), & i = 0\,1\,0\,0 \\ h_\varepsilon[\boldsymbol{x}(k)] + \eta_\varepsilon(k), & i = 0\,0\,1\,0 \\ h_v[\boldsymbol{x}(k)] + \eta_v(k), & i = 0\,0\,0\,1 \end{cases} \tag{7}$$

Measurement matrix $\boldsymbol{H}$ becomes a row matrix; measurement error covariance matrix $\boldsymbol{R}$ and innovation vector $\boldsymbol{\nu}(k) = \{\boldsymbol{z}(k) - \boldsymbol{h}[\boldsymbol{x}(k)]\}$ become scalars, i.e.:

1. $i = 1\,0\,0\,0: \qquad R = \sigma_D^2, \qquad \nu = \nu_D = D^m - h_D[\widehat{\boldsymbol{x}}]$

$$\boldsymbol{H}_1 = \boldsymbol{H}_D = \begin{bmatrix} \frac{\widehat{x}}{\widehat{D}} & 0 & \frac{\widehat{y}}{\widehat{D}} & 0 & \frac{\widehat{z}}{\widehat{D}} & 0 \end{bmatrix} \tag{8}$$

2. $i = 0\,1\,0\,0: \qquad R = \sigma_\beta^2, \qquad \nu = \nu_\beta = \beta^m - h_\beta[\widehat{\boldsymbol{x}}]$

$$\boldsymbol{H}_2 = \boldsymbol{H}_\beta = \begin{bmatrix} \frac{\widehat{y}}{\widehat{d}^2} & 0 & -\frac{\widehat{x}}{\widehat{d}^2} & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

3. $i = 0\,0\,1\,0: \qquad R = \sigma_\varepsilon^2, \qquad \nu = \nu_\varepsilon = \varepsilon^m - h_\varepsilon[\widehat{\boldsymbol{x}}]$

$$\boldsymbol{H}_3 = \boldsymbol{H}_\varepsilon = \begin{bmatrix} -\frac{\widehat{x}\widehat{z}}{\widehat{D}^2\widehat{d}} & 0 & -\frac{\widehat{y}\widehat{z}}{\widehat{D}^2\widehat{d}} & 0 & \frac{\widehat{d}}{\widehat{D}^2} & 0 \end{bmatrix} \tag{10}$$

4. $i = 0\,0\,0\,1: \qquad R = \sigma_v^2, \qquad \nu = \nu_v = v_r^m - h_v[\widehat{\boldsymbol{x}}]$

$$\boldsymbol{H}_4 = \boldsymbol{H}_v = \begin{bmatrix} \frac{\widehat{x}\widehat{v}_r - \widehat{D}\widehat{v}_x}{\widehat{D}^2} & \frac{\widehat{x}}{\widehat{D}} & \frac{\widehat{y}\widehat{v}_r - \widehat{D}\widehat{v}_y}{\widehat{D}^2} & \frac{\widehat{y}}{\widehat{D}} & \frac{\widehat{z}\widehat{v}_r - \widehat{D}\widehat{v}_z}{\widehat{D}^2} & \frac{\widehat{z}}{\widehat{D}} \end{bmatrix} \tag{11}$$

Thus, the Kalman filter is modified for processing measurements separately though the same equations of filter are used. At the same time two or more measurements may come simultaneously from sensors. Measurements that appear at the filter input simultaneously are considered as ones received with zero time interval, and the extrapolation equations used are as follows:

$$\hat{\boldsymbol{x}}(k+1,k) = \hat{\boldsymbol{x}}(k,k)$$
$$\boldsymbol{P}(k+1,k) = \boldsymbol{P}(k,k)$$

The flowchart of the modified filter is presented in Figure 1.

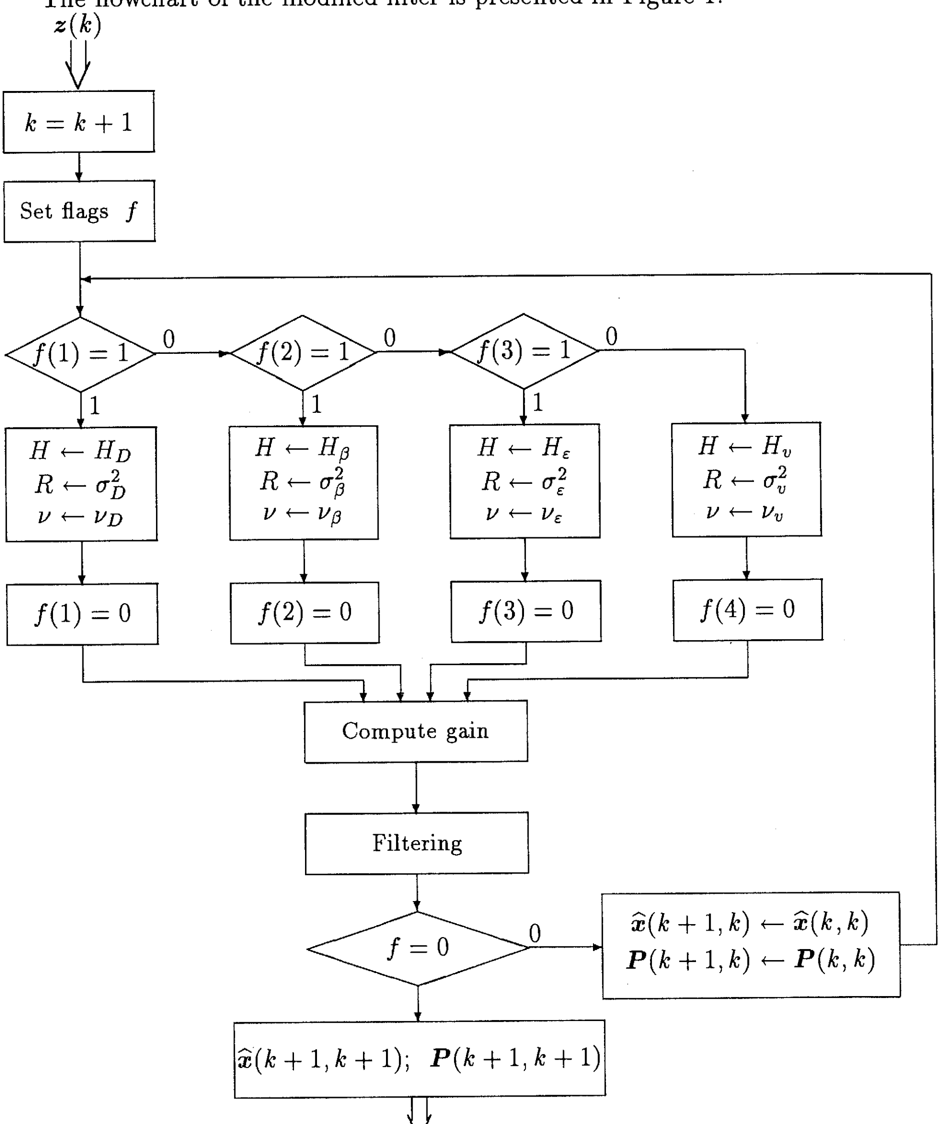

Fig. 1. Flow–chart of the modified estimation algorithm.

When new measurement vector (or scalar) enters the filter input at moment t_{k+1}, vector $\boldsymbol{i}$ is set to an appropriate value that corresponds to specific measurements received. Then flags contained in vector $\boldsymbol{i}$ are tested to determine what kind of parameter is to be processed currently, and to select appropriate values for $\boldsymbol{H}$, $\boldsymbol{R}$, and $\boldsymbol{\nu}$ according to (8)–(11). As soon as values for $\boldsymbol{H}$, $\boldsymbol{R}$ and $\boldsymbol{\nu}$ are selected respective flag of vector $\boldsymbol{i}$ is set to zero, and filtering is performed for the measured parameter according to the unified procedure considered above. Then vector $\boldsymbol{i}$ is tested again to determine if there are some unprocessed parameters left that come at the moment t_{k+1}. If there are some unprocessed parameters $(\boldsymbol{i} \neq 0)$ extrapolation is performed for a zero time interval and next measured parameter is processed. Otherwise, if $(\boldsymbol{i} = 0)$ extrapolation is performed for $\widehat{\boldsymbol{x}}(k+1,k)$ and $\boldsymbol{P}(k+1,k)$ using equations (3), (4) and state estimates are read for further usage.

4. Initial Conditions and Covariances

Transition period of the Kalman filter can be substantially decreased and its performance made more stable if initial state vector estimate and covariance of its error are known.

Consider a case when spherical coordinates and radial velocity are measured. If information characterizing initial state of moving object is not available (it is usually the case), its estimate can be found using results of the first measurement as follws:

$$\widehat{\boldsymbol{x}}(1,1) = [x^m(1)\ v_x^m(1)\ y^m(1)\ v_y^m(1)\ z^m(1)\ v_z^m(1)]^T \tag{12}$$

where

$$x^m(1) = D^m(1)\sin[\beta^m(1)]\cos[\varepsilon^m(1)], \qquad y^m(1) = D^m(1)\cos[\beta^m(1)]\cos[\varepsilon^m(1)]$$

$$z^m(1) = D^m(1)\sin[\varepsilon^m(1)]$$

are pseudomeasurements of Cartesian coordinates, and $v_x^m(1)$, $v_y^m(1)$, $v_z^m(1)$ are pseudomeasurements of velocity components along Cartesian coordinates, that are determined as follows:

$$v_x^m(1) = v_r^m(1)\sin[\beta^m(1)]\cos[\varepsilon^m(1)], \qquad v_y^m(1) = v_r^m(1)\cos[\beta^m(1)]\cos[\varepsilon^m(1)]$$

$$v_z^m(1) = v_r^m(1)\sin[\varepsilon^m(1)]$$

The last expressions are true when the object is moving towards observation point.

Let us determine the covariance matrix for the initial state estimate error:

$$\boldsymbol{P}(1,1) = E\Big\{[\boldsymbol{x}(1) - \widehat{\boldsymbol{x}}(1,1)][\boldsymbol{x}(1) - \widehat{\boldsymbol{x}}(1,1)]^T\Big\}$$

$$= \begin{bmatrix} \sigma_x^2 & r_{xv_x} & r_{xy} & r_{xv_y} & r_{xz} & r_{xv_z} \\ r_{xv_x} & \sigma_{v_x}^2 & r_{v_xy} & r_{v_xv_y} & r_{v_xz} & r_{v_xv_z} \\ r_{xy} & r_{xv_y} & \sigma_y^2 & r_{yv_y} & r_{yz} & r_{yv_z} \\ r_{xv_y} & r_{v_xv_y} & r_{yv_y} & \sigma_{v_y}^2 & r_{v_yz} & r_{v_yv_z} \\ r_{xz} & r_{v_xz} & r_{yz} & r_{v_y} & \sigma_z^2 & r_{zv_z} \\ r_{xv_z} & r_{v_xv_z} & r_{yv_z} & r_{v_yv_z} & r_{zv_z} & \sigma_{v_z}^2 \end{bmatrix} \tag{13}$$

Diagonal elements of this matrix are variances of pseudomeasurement errors for Cartesian coordinates and respective velocity components, and non–diagonal elements are mutual correlations for these pseudomeasurements.

The elements of matrix (13) can be determined by making use of complete differentials for pseudomeasurements of Cartesian coordinates and velocity components as follows:

$$\mathrm{d}x = \frac{\partial x}{\partial D}\mathrm{d}D + \frac{\partial x}{\partial \beta}\mathrm{d}\beta + \frac{\partial x}{\partial \varepsilon}\mathrm{d}\varepsilon \tag{14}$$

$$\mathrm{d}v_x = \frac{\partial v_x}{\partial D}\mathrm{d}D + \frac{\partial v_x}{\partial \beta}\mathrm{d}\beta + \frac{\partial v_x}{\partial \varepsilon}\mathrm{d}\varepsilon \tag{15}$$

Taking into consideration that measurement errors of spherical coordinates and radial velocity are relatively small it is possible to replace differentials (14), (15) by their increments:

$$\begin{aligned}\Delta x(1) &= \sin[\beta(1)]\cos[\varepsilon(1)]\Delta D + D(1)\cos[\beta(1)]\cos[\varepsilon(1)]\Delta\beta \\ &\quad - D(1)\sin[\beta(1)]\sin[\varepsilon(1)]\Delta\varepsilon \\ \Delta v_x(1) &= \sin[\beta(1)]\cos[\varepsilon(1)]\Delta v_r + v_r(1)\cos[\beta(1)]\cos[\varepsilon(1)]\Delta\beta \\ &\quad - v_r(1)\sin[\beta(1)]\sin[\varepsilon(1)]\Delta\varepsilon\end{aligned}$$

Hence

$$\begin{aligned}\sigma_x^2 &= E[\Delta x^2] = \sin^2[\beta(1)]\cos^2[\varepsilon(1)]\sigma_D^2 + D^2(1)\cos^2[\beta(1)]\cos^2[\varepsilon(1)]\sigma_\beta^2 \\ &\quad + D^2(1)\sin^2[\beta(1)]\sin^2[\varepsilon(1)]\sigma_\varepsilon^2 \\ \sigma_{v_x}^2 &= E[\Delta v_x^2] = \sin^2[\beta(1)]\cos^2[\varepsilon(1)]\sigma_{v_r}^2 + v_r^2(1)\cos^2[\beta(1)]\cos^2[\varepsilon(1)]\sigma_\beta^2 \\ &\quad + v_r^2(1)\sin^2[\beta(1)]\sin^2[\varepsilon(1)]\sigma_\varepsilon^2 \\ r_{xv} &= E[\Delta x \Delta v_x] = D(1)v_r(1)\left\{\cos^2[\beta(1)]\cos^2[\varepsilon(1)] + \sin^2[\beta(1)]\sin^2[\varepsilon(1)]\right\}\end{aligned}$$

Expressions for all other elements of matrix $\boldsymbol{P}(1,1)$ can be found in a similar way. As far as true values of $D(1)$, $\beta(1)$, $\varepsilon(1)$ and $v_r(1)$ are unknown numeric values for the elements of matrix $\boldsymbol{P}(1,1)$ they are computed by making use of respective measurements.

5. Computer Simulation

Computer simulation of the algorithm considered above was carried out to estimate the moving object position. The simulation was performed statistically and with initial conditions determined by (12) and (13). It was supposed that measurements of spherical coordinates and radial velocity enter the filter input with sampling interval $T_{S1} = \varphi_S$. The mean square measurement errors were chosen as follows:

$$\sigma_D = 60\,m, \qquad \sigma_\beta = 0.008\,rad, \qquad \sigma_\varepsilon = 0.006\,rad, \qquad \sigma_{v_R} = 6\,m/s$$

Figures 2 and 3 illustrate (curve 1) time history of mean square errors of filtering for angle coordinates (azimuth and elevation angle). It can be seen that the generalized Kalman filter under consideration does not provide convergence of estimates to their true values and clearly exhibits divergence. Analysis of some intermediate simulation results showed that with time "k" growing and number of processed measurements increased the covariance matrix of estimate errors $\boldsymbol{P}(k,k)$ looses its positive definiteness. It happens because at each step of filtering the covariance matrix is updated according to recursive equation:

$$\boldsymbol{P}(k,k) = \boldsymbol{P}(k,k-1) - \boldsymbol{K}(k)\boldsymbol{H}(k)\boldsymbol{P}(k,k-1)$$

Computational errors become substantial when nonlinear operations on (6×6) dimension matrices are performed.

The method of square–root filtering usually allows to improve the computational properties of estimation algorithm (Leondes, 1980) but simpler approaches exist. It can be easily shown that a simple method of decreasing the computational conditioning of the problem is in adding to the covariance matrix some diagonal matrix $\alpha\boldsymbol{I}$ ($\alpha > 0$, $\boldsymbol{I}$ is unity matrix) (Marchuk, 1980), as

$$\operatorname{cond}\big[\boldsymbol{P}(k,k) + \alpha I\big] < \operatorname{cond}\big[\boldsymbol{P}(k,k)\big]$$

With parameter α increasing (its value is usually tuned in the process of modeling) the computational conditioning of the problem is decreased but optimality of estimates decreases, too.

In the process of simulation we used a simple method of decreasing conditioning of matrix $\boldsymbol{P}(k,k)$ that results in an improvement of computational properties of the filter. The method is based on the choice of diagonal matrix for initial covariances:

$$\boldsymbol{P}(1,1) = \begin{bmatrix} \sigma_x^2 & & & & & \\ & \sigma_{v_x}^2 & & & 0 & \\ & & \sigma_y^2 & & & \\ & & & \sigma_{v_y}^2 & & \\ & 0 & & & \sigma_z^2 & \\ & & & & & \sigma_{v_z}^2 \end{bmatrix} \qquad (16)$$

Such a choice leads to a decrease in optimality of the algorithm at the first several steps of filtering.

The simulation results with this choice of initial estimate error covariance matrix are represented by curve 2 in Figures 2 and 3, and by Figure 4. As it can be seen from the figures diagonalization of covariance matrix of initial estimate error provides a substantial improvement of computational stability of the filter and prevents divergence of estimates.

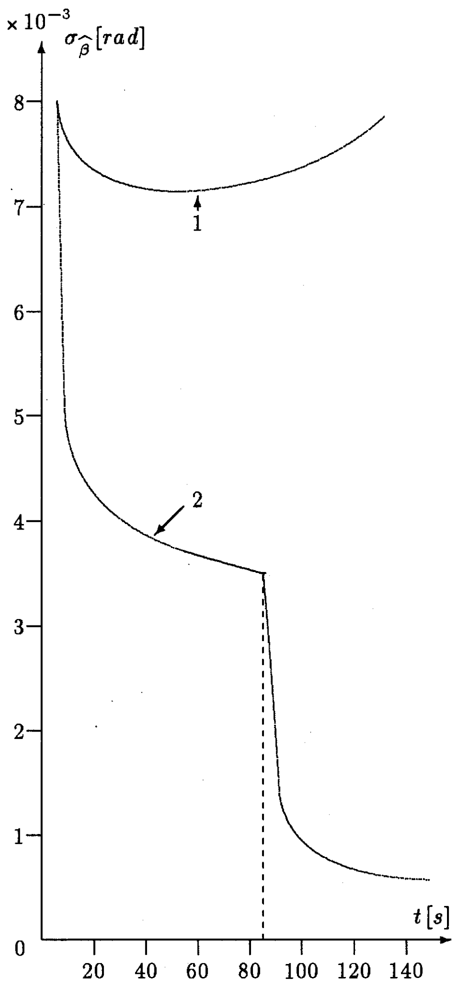

Fig. 2. Mean square error of azimuth filtering.

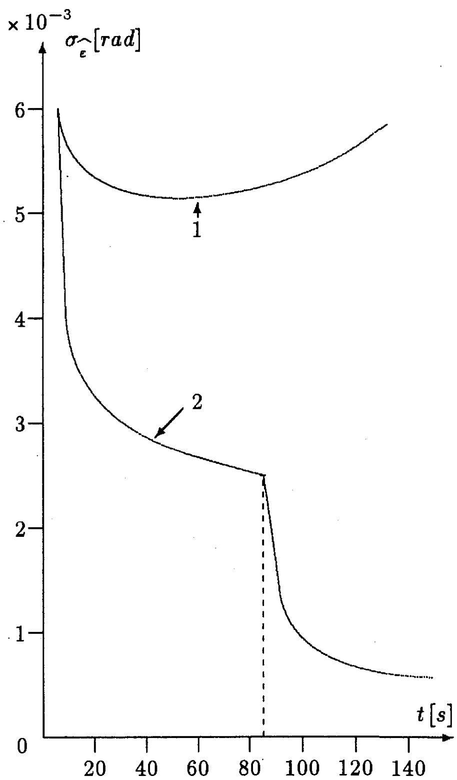

Fig. 3. Mean square error of elevation angle filtering.

At the time moment $t = 85\,s$ the second source of data begins to work and generates extra measurements of angle coordinates (β and ε) with sampling period $T_{S_2} = 2\,s$, and two times lower measurement errors. This additional information of angle coordinates does not affect errors of range measurement filtering (c.f. Fig. 4) but it makes it possible to decrease errors of filtering for the angle coordinates more than two times.

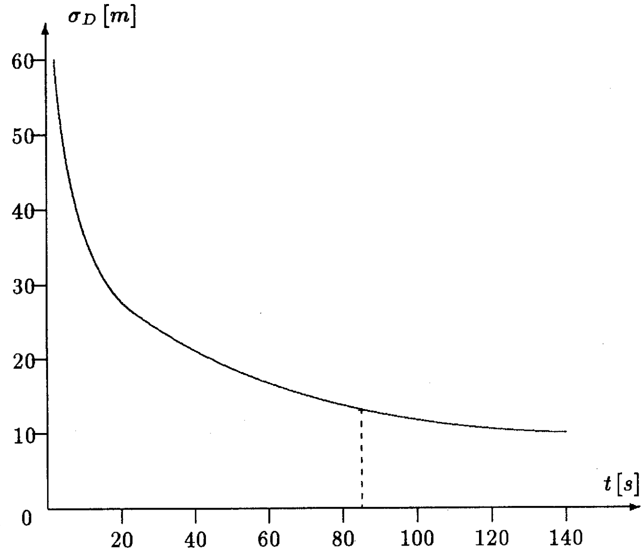

Fig. 4. Mean square error of range filtering.

6. Conclusions

A version of modified linearized Kalman filter was considered to integrate of multisensor data characterizing current position of a moving object like an aircraft. The filter represents a unified computing algorithm for integrating measurements of various origin with the purpose of improving observed object state estimates. A simple method for decreasing conditioning of the filter was proposed that ensures substantial enhancement of estimates quality and computational stability of the algorithm. Statistical simulation of the algorithm should result in substantial improvement of estimates when a new source of measurement information enters the filter. It was also shown that the filter successfully processes measurements that have different sampling periods.

It can be suggested that this kind of Kalman filter may be applied in other areas not considered here, e.g. for integrating economic data, processing multiple origin information on satellites and space stations, processing measurements in seismology and ecology.

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