

LINKS AND ACTUATORS KINEMATICS OF IRb-6 MANIPULATOR

TADEUSZ SZKODNY*

A fundamental problem in industrial robots control concerns algorithms generating reference trajectories. In the papers (Gerke, 1985; Gousenes, 1984; Jacak, 1989; Lozano-Perez, 1983) some generating algorithms based on an arbitrary discretization of the manipulators internal coordinates are suggested. Each point of discretization in the external space approximating a reference trajectory is represented by known discretized internal coordinates of the manipulator. On the other hand, in papers (Duleba and Lysakowska, 1990; Jacak and Lysakowska, 1990; Taylor, 1983) iteration methods of determining the internal coordinates corresponding to external coordinates of the reference trajectory point have been suggested. By applying this technique, the point of the reference trajectory is approached in successive steps of an iterative procedure. In the paper of Duleba and Lysakowska (1990) a modified iterative method of generating a straight segment of the reference trajectory has been presented.

Analytic formulae which are the solution to an inverse problem of manipulator kinematics enable us to design trajectory generating algorithms which compute in one step only internal coordinates of points lying exactly on the reference trajectory, with the accuracy resulting from the computer register length.

The paper presents equations of links and actuators kinematics of IRb-6 manipulator in a matrix form. Also solution to equations of link kinematics as well as formulae joining link and actuator natural coordinates of the manipulator have been presented.

1. Introduction

To describe the manipulators kinematics, dekstrorotary coordinate system associated with particular components of the manipulator will be used. A homogenous transformation (Craig, 1989; Paul, 1983; Ranky and Ho, 1985; Szkodny, 1985) will be used to describe the position and orientation of the systems. Due to this transformation a joint description of the position and orientation is possible, which is essential while describing kinematics, and particularly dynamics of manipulators.

The term *natural coordinates* of links will be used for those describing relative motion of adjacent links (Niederliński, 1983; Szkodny, 1985; Wojnarowski and Nowak, 1992).

* Institute of Automation, Silesian Technical University, 44-101 Gliwice, ul. Akademicka 16, Poland

External space of the manipulator is described by its *external coordinates* of position x, y, z and orientation Φ, Θ, Ψ (Euler angles). These coordinates describe the manipulator effector in relation to a selected relative system regardless of the manipulator kinematic structure. The manipulator *internal space* is described by its *internal coordinates*. These are natural coordinates of links and actuators (Szkodny, 1985).

Industrial robots are most often equipped with manipulators with V class kinematic pairs and only such are regarded in the paper.

In algorithms based on an arbitrary discretization of the manipulators internal coordinates (Gerke, 1985; Gousenes, 1984; Jacak, 1989; Lozano-Perez, 1983) a discretized description of the manipulator external space results from the discretized description of its internal space. A reference trajectory in the external space is approximated using such discretized description of the manipulator external space. The disadvantage of these algorithms is that they demand large memory, big sets, and being a discretized description of the internal space, have to be searched, and there is no possibility to reduce the arbitrary discretization of the manipulator internal space.

In iteration methods (Dulęba and Lysakowska, 1990; Jacak and Lysakowska, 1990; Taylor, 1983) discretization step of the internal coordinates in successive steps of iterative computation depends on the error of external coordinates in the preceding iteration step. Large memories are not demanded in iteration methods as computation is carried out only for reference trajectory approximating points. In these methods a reference trajectory approximation error may be reduced through a number of iterative computation steps. In a modified iterative method (Dulęba and Lysakowska, 1990) the modification is based on an arbitrary assumption of error distribution in the external space, thus reducing the number of iterative computation steps. Nevertheless, the assumed error distribution is right with only short segments.

A disadvantage of iterative methods is the necessity of multiple iterative computation. Whereas the advantage of methods based on arbitrary discretization of internal spaces as well as iterative methods is the simplicity of computation which lies in using only the equations of the manipulator forward kinematics. This advantage, however may be a catch for those computer programmers who have not considered kinematic singularities of the manipulator (Szkodny, 1985).

Analytic formulae which are the solution to an inverse problem of manipulator kinematics enable render it possible to design a trajectory generating algorithms which compute in one step only internal coordinates of points lying exactly on the reference trajectory, with the accuracy resulting from the computer register length. These formulae make the programmers look for alternative solutions to manipulator kinematic singularities. Analytic formulae as a solution to an inverse problem of kinematics of the six degrees-of-freedom manipulators have been presented in the papers (Craig, 1989; Knapczyk and Kisiel, 1987; Paul, 1983; Szkodny, 1990). The same formulae for the $N < 6$ degrees-of-freedom manipulators have been presented in the papers (Craig, 1989; Knapczyk and Stepniewski, 1985). However a constraint equation of an effector link (Szkodny, 1985) has not been presented in the paper (Knapczyk and Stepniewski, 1985). It suggests that the link is able to realize the reference trajectories with six degrees of freedom, which is not possible.

It follows from the foregoing review that the kinematics models as presented in papers (Duleba and Lysakowska, 1990; Gerke, 1985; Gousenes, 1984; Jacak, 1989; Jacak and Lysakowska, 1990; Knapczyk and Stepniewski, 1985; Lozano-Perez, 1983; Taylor, 1983) do not allow us to design accurate and at the same time fast reference trajectory generating algorithms with a defined kinematics for the manipulators with less than six degrees of freedom .

In the next section equations of forward kinematics of IRb-6 manipulator links have been presented. The third section contains formulae being the solution of inverse problem for links kinematics. Then equations of actuators kinematics follow which are crucial for dynamic analysis of IRb-6 manipulator. The fifth section presents an example illustrating the usage of the formulae presented in Section 3. The last section contains conclusions.

2. Equations of Links Kinematics

IRb-6 manipulator (see Fig. 1) has six links joined by rotational kinematic pairs. Figure 2 shows a homogenous transformation graph describing the manipulator kinematics. Numbers of links in Figure 1 have been circled. Coordinate systems have been associated with links following Hartenberg-Denavit notation. Hartenberg-Denavit parameters describing this manipulator are shown in Table 1 (Szkodny, 1985).

Tab. 1. Link parameters for IRb-6 manipulator.

Link number	α_i [°]	l_i [m]	λ_i [m]	Θ_i [°]
1	90	0	0.70	$90 \div 430$
2	0	0.45	0	$50 \div 130$
3	0	0.67	0	$-130 \div -50$
4	90	0	0	$-25 \div -220$
5	0	0	0.095	$\Delta\Theta_5 = 360$

The following modification of angles will be introduced to facilitate solving inverse problem of kinematics (Szkodny, 1985):

$$\Theta'_1 = \Theta_1 - 90^\circ, \quad \Theta'_2 = \Theta_2 - 90^\circ, \quad \Theta'_3 = \Theta_3 + 90^\circ, \quad \Theta'_4 = \Theta_4 - 90^\circ, \quad \Theta'_5 = \Theta_5 \quad (1)$$

Ranges of change of these angles are as follows (Szkodny, 1985):

$$\begin{aligned} 0^\circ &\leq \Theta'_1 \leq 340^\circ \\ -40^\circ &\leq \Theta'_2 \leq 40^\circ \\ -40^\circ - \Theta'_2 &\leq \Theta'_3 \leq 40^\circ && \text{for } -40^\circ \leq \Theta'_2 \leq -15^\circ \\ -40^\circ - \Theta'_2 &\leq \Theta'_3 \leq 25^\circ - \Theta'_2 && \text{for } -15^\circ \leq \Theta'_2 \leq 0^\circ \\ -40^\circ &\leq \Theta'_3 \leq 25^\circ - \Theta'_2 && \text{for } 0^\circ \leq \Theta'_2 \leq 40^\circ \\ -90^\circ - \Theta'_2 - \Theta'_3 &\leq \Theta'_4 \leq 90^\circ - \Theta'_2 - \Theta'_3 \\ -270^\circ + k_5^{-1}(\Theta'_2 + \Theta'_3 + \Theta'_4) &\leq \Theta'_5 \leq 90^\circ + k_5^{-1}(\Theta'_2 + \Theta'_3 + \Theta'_4), \quad k_5^{-1} = 32/19 \quad (2) \end{aligned}$$

To simplify the notation, the following designation will be used:

$$\sin \Theta'_i = S_i, \quad \cos \Theta'_i = C_i, \quad \sin(\Theta'_i + \Theta'_j) = S_{ij}, \quad \cos(\Theta'_i + \Theta'_j) = C_{ij} \quad \text{etc.}$$

In further considerations it will be assumed that the angles $\Theta'_1 \div \Theta'_5$ are natural coordinates of links.

Transformation matrices $A_1 \div A_5$ and E have the following form (Szkodny, 1985):

$$\begin{aligned} A_1 &= \begin{bmatrix} -S_1 & 0 & C_1 & 0 \\ C_1 & 0 & S_1 & 0 \\ 0 & 1 & 0 & \lambda_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, & A_2 &= \begin{bmatrix} -S_2 & -C_2 & 0 & -l_2 S_2 \\ C_2 & -S_2 & 0 & l_2 C_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ A_3 &= \begin{bmatrix} S_3 & C_3 & 0 & l_3 S_3 \\ -C_3 & S_3 & 0 & -l_3 C_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, & A_4 &= \begin{bmatrix} -S_4 & 0 & C_4 & 0 \\ C_4 & 0 & S_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ A_5 &= \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & \lambda_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, & E &= \begin{bmatrix} 1 & 0 & 0 & l_6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (3)$$

Matrices T_5 and X describing the effector link and task as shown in Figure 1 have the following form (Szkodny, 1985):

$$\begin{aligned} T_5 &= \begin{bmatrix} S_1 S_{234} C_5 + C_1 S_5 & -S_1 S_{234} S_5 + C_1 C_5 & -S_1 C_{234} \\ -C_1 S_{234} C_5 + S_1 S_5 & C_1 S_{234} S_5 + S_1 C_5 & C_1 C_{234} \\ C_{234} C_5 & -C_{234} S_5 & S_{234} \\ 0 & 0 & 0 \end{bmatrix} \\ &\quad \begin{bmatrix} l_2 S_1 S_2 - l_3 S_1 C_{23} - \lambda_5 S_1 C_{234} \\ -l_2 C_1 S_2 + l_3 C_1 C_{23} + \lambda_5 C_1 C_{234} \\ \lambda_1 + l_2 C_2 + l_3 S_{23} + \lambda_5 S_{234} \\ 1 \end{bmatrix} \end{aligned} \quad (4)$$

$$X = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

where $n_x \div n_z$, $o_x \div o_z$, $a_x \div a_z$ are identical with the elements of matrix T_5 . Elements in the last column are as follows:

$$p_x = l_2 S_1 S_2 - l_3 S_1 C_{23} - \lambda_5 S_1 C_{234} + l_6 (S_1 S_{234} C_5 + C_1 S_5) - \lambda_6 S_1 C_{234}$$

$$p_y = -l_2 C_1 S_2 + l_3 C_1 C_{23} + \lambda_5 C_1 C_{234} + l_6 (-C_1 S_{234} C_5 + S_1 S_5) + \lambda_6 C_1 C_{234}$$

$$p_z = \lambda_1 + l_2 C_2 + l_3 S_{23} + \lambda_5 S_{234} + l_6 C_{234} C_5 + \lambda_6 S_{234}$$

Matrices T_5 and X enable us both to solve the forward problem of kinematics of the manipulator and to determine of the work space (Kucharski, 1989).

3. Solution of Link Kinematics Equations

The solution to the inverse problem of IRb-6 manipulator kinematics will be expressed by means of elements of matrix $T_{5\text{ref}} = X_{\text{ref}} E^{-1}$ whose form is as follows:

$$T_{5\text{ref}} = X_{\text{ref}} E^{-1} = \text{Trans}(x_{\text{ref}}, y_{\text{ref}}, z_{\text{ref}}) \text{Euler}(\Phi_{\text{ref}}, \Theta_{\text{ref}}, \Psi_{\text{ref}}) E^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 & x_{\text{ref}} \\ 0 & 1 & 0 & y_{\text{ref}} \\ 0 & 0 & 1 & z_{\text{ref}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \Phi_{\text{ref}} \cos \Theta_{\text{ref}} \cos \Psi_{\text{ref}} - \sin \Phi_{\text{ref}} \sin \Psi_{\text{ref}} \\ \sin \Phi_{\text{ref}} \cos \Theta_{\text{ref}} \cos \Psi_{\text{ref}} + \cos \Phi_{\text{ref}} \sin \Psi_{\text{ref}} \\ -\sin \Theta_{\text{ref}} \cos \Psi_{\text{ref}} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\cos \Phi_{\text{ref}} \cos \Theta_{\text{ref}} \sin \Psi_{\text{ref}} - \sin \Phi_{\text{ref}} \cos \Psi_{\text{ref}} & \cos \Phi_{\text{ref}} \sin \Theta_{\text{ref}} & 0 \\ -\sin \Phi_{\text{ref}} \cos \Theta_{\text{ref}} \sin \Psi_{\text{ref}} + \cos \Phi_{\text{ref}} \cos \Psi_{\text{ref}} & \sin \Phi_{\text{ref}} \sin \Theta_{\text{ref}} & 0 \\ \sin \Theta_{\text{ref}} \sin \Psi_{\text{ref}} & \cos \Theta_{\text{ref}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 & 0 & -l_6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\lambda_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$n_x = \cos \Phi_{\text{ref}} \cos \Theta_{\text{ref}} \cos \Psi_{\text{ref}} - \sin \Phi_{\text{ref}} \sin \Psi_{\text{ref}}$$

$$n_y = \sin \Phi_{\text{ref}} \cos \Theta_{\text{ref}} \cos \Psi_{\text{ref}} + \cos \Phi_{\text{ref}} \sin \Psi_{\text{ref}}$$

$$n_z = -\sin \Theta_{\text{ref}} \cos \Psi_{\text{ref}}$$

$$o_x = -\cos \Phi_{\text{ref}} \cos \Theta_{\text{ref}} \sin \Psi_{\text{ref}} - \sin \Phi_{\text{ref}} \cos \Psi_{\text{ref}}$$

$$o_y = -\sin \Phi_{\text{ref}} \cos \Theta_{\text{ref}} \sin \Psi_{\text{ref}} + \cos \Phi_{\text{ref}} \cos \Psi_{\text{ref}}$$

$$o_z = \sin \Theta_{\text{ref}} \sin \Psi_{\text{ref}}$$

$$\begin{aligned}
a_x &= \cos \Phi_{\text{ref}} \sin \Theta_{\text{ref}} \\
a_y &= \sin \Phi_{\text{ref}} \sin \Theta_{\text{ref}} \\
a_z &= \cos \Theta_{\text{ref}} \\
p_x &= x_{\text{ref}} - n_x l_6 - a_x \lambda_6 \\
p_y &= y_{\text{ref}} - n_y l_6 - a_y \lambda_6 \\
p_z &= z_{\text{ref}} - n_z l_6 - a_z \lambda_6
\end{aligned} \tag{6}$$

where x_{ref} , y_{ref} , z_{ref} , Φ_{ref} , Θ_{ref} , Ψ_{ref} are the task required external coordinates and l_6 , λ_6 are the task kinematic parameters.

The IRb-6 manipulator has five degrees of freedom and this implies one constraint equation of the effector link wrist. The kinematic structure of the manipulator hinders the rotation of effector link around the z_0 -axis of the base coordinate system (Szkodny, 1985), and this is represented by the equation

$$a_x p_y - a_y p_x = 0 \tag{7}$$

which must be satisfied by elements of matrix $T_{5\text{ref}}$ in each point of the reference trajectory. It is one of the necessary conditions for the reference trajectory to be realized. The formulae being the solution of the inverse problem of kinematics for the matrix $T_{5\text{ref}}$ in form (6) are as follows (Szkodny, 1985)

$$\Theta'_1 = \begin{cases} \Theta_1^* & \text{for } p_x \leq 0 \text{ and } p_y \geq 0 \\ \Theta_1^* + 180^\circ & \text{for } p_y < 0 \\ \Theta_1^* + 360^\circ & \text{for } p_x > 0 \text{ and } p_y \geq 0 \end{cases}$$

$$\Theta_1^* = \arctg \left(\frac{-p_x}{p_y} \right) \tag{8}$$

$$\Theta'_3 = \arctg \frac{S_3}{C_3}, \quad S_3 = \frac{w_1^2 + w_2^2 - (l_2^2 + l_3^2)}{2l_2 l_3}, \quad C_3 = (1 - S_3^2) \tag{9}$$

$$\Theta'_2 = \arctg \frac{S_2}{C_2}, \quad S_2 = \frac{w_2 l_3 C_3 - w_1 (l_3 S_3 + l_2)}{l_3^2 C_3^2 + (l_3 S_3 + l_2)^2}, \quad C_2 = \frac{w_1 l_3 C_3 + w_2 (l_3 S_3 + l_2)}{l_3^2 C_3^2 + (l_3 S_3 + l_2)^2}$$

$$w_1 = -S_1 p_x + C_1 p_y + \lambda_5 S_1 a_x - \lambda_5 C_1 a_y$$

$$w_2 = p_z - \lambda_1 - \lambda_5 a_z \tag{10}$$

$$\Theta'_{34} = \begin{cases} \Theta_{34}^* & \text{for } \lambda_5 C_{34} \geq 0 \\ \Theta_{34}^* + 180^\circ & \text{for } \lambda_5 S_{34} > 0 \text{ and } \lambda_5 C_{34} < 0 \\ \Theta_{34}^* - 180^\circ & \text{for } \lambda_5 S_{34} < 0 \text{ and } \lambda_5 C_{34} < 0 \end{cases}$$

$$\lambda_5 S_{34} = S_1 S_2 p_x - C_1 S_2 p_y + C_2 p_z - \lambda_1 C_2 - l_2 - l_3 S_3$$

$$\lambda_5 C_{34} = -S_1 C_2 p_x + C_1 C_2 p_y + S_2 p_z - \lambda_1 S_2 - l_3 C_3$$

$$\Theta_{34}^* = \arctg \frac{\lambda_5 S_{34}}{\lambda_5 C_{34}} \quad (11)$$

$$\Theta'_4 = \Theta'_{34} - \Theta'_3 \quad (12)$$

$$S_5 = C_1 n_x + S_1 n_y, \quad C_5 = C_1 o_x + S_1 o_y$$

$$\Theta_5^* = \arctg \frac{S_5}{C_5} \quad (13)$$

Boundary angles $\Theta'_{5\min}$ and $\Theta'_{5\max}$ depend on $\Theta'_2 \div \Theta'_4$ as well as angles $\Theta'_{5\max} - \Theta'_{5\min} = 360^\circ$ (see formulae (2)) and signs of S_5 and C_5 must be examined in order to determine angle Θ'_5 . The analysis of the above formulae makes it clear that for explicitly determined elements of the matrix $T_{5\text{ref}}$, there may be two solutions for $\Theta'_5 = \Theta'_{5\min}$ or $\Theta'_5 = \Theta'_{5\max}$. This is the kinematic singularity of the first kind. There is no singularity of the second kind of the IRb-6 manipulator (Szkodny, 1985).

4. Equations of Actuators Kinematics

Driving kinematics has been illustrated in Figures 3-6. The transformation matrices illustrated in these figures have been taken from paper (Kucharski, 1989). The $x_{bi}y_{bi}z_{bi}$ coordinates are associated with the body of the i -th actuator. The $x_{ai}y_{ai}z_{ai}$ coordinates are associated with the rotor of the i -th actuator. The angle of rotation Θ_{ai} of the $x_{ai}y_{ai}z_{ai}$ system coordinates around the z_{bi} -axis is a natural coordinate of the i -th actuator.

Formulae binding the $\Theta'_1 \div \Theta'_5$ natural coordinates of links to the natural coordinates $\Theta_{a1} \div \Theta_{a5}$ of actuators are as follows (Kucharski, 1989; Szkodny, 1985):

$$\begin{aligned} \Theta'_1 &= k_1^{-1} \Theta_{a1} \\ \Theta'_2 &= -\arccos \frac{AB^2 + BC^2 - [A_0 C - (h_2/2\pi)\Theta_{a2}]^2}{2 \cdot AB \cdot BC} + \alpha \\ \Theta'_3 &= -\arccos \frac{DE^2 + EF^2 - [D_0 F - (h_3/2\pi)\Theta_{a3}]^2}{2 \cdot DE \cdot EF} + \beta - \Theta'_2 \\ \Theta'_4 &= k_4^{-1} \Theta_{a4} - (\Theta'_2 + \Theta'_3) \\ \Theta'_5 &= k_4^{-1} k_5^{-1} (\Theta_{a4} - \Theta_{a5}) \end{aligned} \quad (14)$$

These formulae correspond to the following equations:

$$\begin{aligned}
 \Theta_{a1} &= k_1 \Theta'_1 \\
 \Theta_{a2} &= (2\pi/h_2) \left\{ -[AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos(\alpha - \Theta'_2)]^{\frac{1}{2}y} + A_0 C \right\} \\
 \Theta_{a3} &= (2\pi/h_3) \left\{ -[DE^2 + EF^2 - 2 \cdot DE \cdot EF \cdot \cos(\beta - \Theta'_2 - \Theta'_3)]^{\frac{1}{2}} + D_0 F \right\} \\
 \Theta_{a4} &= k_4 (\Theta'_2 + \Theta'_3 + \Theta'_4) \\
 \Theta_{a5} &= k_4 (\Theta'_2 + \Theta'_3 + \Theta'_4) - k_4 k_5 \Theta'_5 \\
 \alpha &= \arccos \frac{AB^2 + BC^2 - A_0 C^2}{2 \cdot AB \cdot BC} \\
 \beta &= \arccos \frac{DE^2 + EF^2 - D_0 F^2}{2 \cdot DE \cdot EF}
 \end{aligned} \tag{15}$$

The matrices $T_{a1} \div T_{a5}$ describing the manipulator driving kinematics are of the form (Kucharski, 1989):

$$T_{a1} = \begin{bmatrix} -S_\varphi & C_\varphi & 0 & 0 \\ 0 & 0 & 1 & -\lambda_{11} \\ C_\varphi & S_\varphi & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

where $S_\varphi = \sin \varphi$, $C_\varphi = \cos \varphi$, $\varphi = \Theta_1 - \Theta_{a1}$,

$$T_{a2} = \begin{bmatrix} C_a C_\varphi & -S_a C_\varphi & S_\varphi & -l_{22} - \lambda_{a2} S_\varphi \\ C_a S_\varphi & -S_a S_\varphi & -C_\varphi & l_{21} + \lambda_{a2} C_\varphi \\ S_a & C_a & 0 & -\lambda_{21} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

where $S_a = \sin \Theta_{a2}$, $C_a = \cos \Theta_{a2}$, $S_\varphi = \sin \varphi$, $C_\varphi = \cos \varphi$, $\varphi = \varphi_2 - \Theta'_2$

$$\varphi_2 = \arccos \frac{b_2 + a_2 C_2 - l_{21} S_2}{d_2 - l_{21} C_2 - a_2 S_2}$$

$$\lambda_{a2} = \left[(b_2 + a_2 C_2 - l_{21} S_2)^2 + (d_2 - l_{21} C_2 - a_2 S_2)^2 \right]^{\frac{1}{2}}$$

$$T_{a3} = \begin{bmatrix} C_a S_\varphi & -S_a S_\varphi & C_\varphi & -l_{32} - l_{31} S_3 - \lambda_{a3} C_\varphi \\ C_a C_\varphi & -S_a C_\varphi & -S_\varphi & -l_{31} C_3 + \lambda_{a3} S_\varphi \\ S_a & C_a & 0 & -(\lambda_{31} + \lambda_{32}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

where $S_\varphi = \sin \varphi$, $C_\varphi = \cos \varphi$, $\varphi = \Theta'_2 + \Theta'_3 - \varphi_3$,

$$\varphi_3 = \arctg \frac{b_3 - a_3 S_{23}}{d_3 - a_3 C_{23}}$$

$$\lambda_{a3} = \left[(d_3 - a_3 C_{23})^2 + (b_3 - a_3 S_{23})^2 \right]^{\frac{1}{2}}$$

$$T_{a4} = \begin{bmatrix} -S_\varphi & C_\varphi & 0 & l_{43}S_4 - l_{42}C_{34} \\ 0 & 0 & 1 & -(\lambda_{41} + \lambda_{42}) \\ C_\varphi & S_\varphi & 0 & -l_{43}C_4 - l_{42}S_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (19)$$

where $S_\varphi = \sin \varphi$, $C_\varphi = \cos \varphi$, $\varphi = \Theta_2 + \Theta_3 + \Theta_4 - \Theta_{a4}$, $l_{41} = l_{44}$

$$T_{a5} = \begin{bmatrix} -C_5 S_\varphi & C_5 C_\varphi & S_5 & l_{53}S_4 C_5 - l_{52}C_{34}C_5 - (\lambda_{51} + \lambda_{52})S_5 \\ S_5 S_\varphi & -S_5 C_\varphi & C_5 & -l_{53}S_4 S_5 + l_{52}C_{34}S_5 - (\lambda_{51} + \lambda_{52})C_5 \\ C_\varphi & S_\varphi & 0 & -l_{53}C_4 - l_{52}S_{34} - \lambda_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (20)$$

where $S_\varphi = \sin \varphi$, $C_\varphi = \cos \varphi$, $\varphi = \Theta'_2 + \Theta'_3 + \Theta'_4 - \Theta_{a5}$, $l_{51} = l_{54}$.

Figures 4 and 5 illustrate the $x_{310}y_{310}z_{310}$ -coordinates describing an element equilibrating, the third link. Kinematics of this element is illustrated in Figure 7. The matrix T_{031} describing this coordinates in relation to the base coordinate system has the form

$$T_{031} = \begin{bmatrix} -S_1 C_{23} & S_1 S_{23} & C_1 & 0 \\ C_1 C_{23} & -C_1 S_{23} & S_1 & 0 \\ S_{23} & C_{23} & 0 & \lambda_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (21)$$

5. Example

The formulae presented in Section 3 are the solution to inverse problem of IRb-6 manipulator kinematics. On the basis of these formulae algorithms generating natural coordinates of actuators are designed. The coordinates correspond to the reference trajectory of a manipulation object as described in the manipulator external space. The algorithms computing the actuators natural coordinates form a reference trajectory generating a tier which is a functional structure element of the adaptation robots control system. These algorithms are indispensable program means interconnecting vision tier and control drives tier (Niederliński, 1983).

A computer algorithm PLAN2 generating task trajectories of the IRb-6 manipulator has been worked out. Reference external coordinates of the points of a generated trajectory will be called *main fulcrums*. No such generation is possible without an

introductory description of the trajectory in the form of external coordinates values of, at least, two main fulcrums optionally distant from each other. Algorithm PLAN2 generates additional fulcrums between consecutive main fulcrums.

For the defined reference external coordinates $x_{\text{ref}}, y_{\text{ref}}, z_{\text{ref}}, \Phi_{\text{ref}}, \Theta_{\text{ref}}, \Psi_{\text{ref}}$ and the reference time T – describing consecutive main fulcrums the algorithm determines matrix $T_{5\text{ref}}$, checks if constraint equation (7) is satisfied and computes natural coordinates $\Theta'_1 \div \Theta'_5$ from formulae (8)–(13). Then the algorithm asks about a coordinate system describing the shape of trajectory segment between the consecutive main fulcrums. For a straight segment Cartesian coordinates should be set, whereas for a curvilinear segment either cylindrical or spherical coordinate system should be chosen. Once external coordinates of all the main fulcrums have been set, the algorithm asks about parameters defining the accuracy of generating the reference trajectory (Szynawa, 1991) and parameters l_6 and λ_6 describing the task. After input parameters have been read in, the algorithm determines additional fulcrums accepting a linear change of external coordinates $x, y, z, \Phi, \Theta, \Psi$ along the length of the segment joining consecutive fulcrums. Angle Φ is a result of the altered external coordinates and the constraint equation (7).

Figure 8 shows a task rectilinear reference trajectory which is limited by the initial fulcrums P and the final point K . The coordinates of these fulcrums are:

$$\begin{array}{lll} x_{P\text{ref}} = -0.60 \text{ m}, & y_{P\text{ref}} = 0.60 \text{ m}, & z_{P\text{ref}} = 1.0 \text{ m} \\ \Phi_{P\text{ref}} = 135^\circ, & \Theta_{P\text{ref}} = 179^\circ, & \Psi_{P\text{ref}} = 359^\circ \\ x_{K\text{ref}} = -0.65 \text{ m}, & y_{K\text{ref}} = 0.60 \text{ m}, & z_{K\text{ref}} = 1.0 \text{ m} \\ \Phi_{K\text{ref}} = 137.29^\circ, & \Theta_{K\text{ref}} = 1^\circ, & \Psi_{K\text{ref}} = 180^\circ \end{array}$$

The following input parameters of the PLAN2 algorithm, defining kinematics of the trajectory as in Figure 8, were adopted: the task parameters $l_6 = 0$ and $\lambda_6 = 0.16 \text{ m}$; time assigned to point P : $T_P = 0$; time assigned to point K $T_K = 1.0 \text{ sec}$; trajectory shape –straight line.

1283 additional fulcrums shown in Figure 9 and Figure 10, result from such generation. It follows from Figure 9 and Figure 10 that natural coordinates graphs of the links and actuators are similar. Θ'_5 coordinate undergoes a sudden alteration from the minimum to the maximum boundary value, determined by inequalities (2). It follows from these inequalities that Θ'_5 angle jump at time $t \cong 0.5 \text{ sec}$ equals 360° . There is a similar change in the Θ_{a5} coordinate. Other coordinate graphs are smooth.

The main fulcrum P corresponds to the following natural coordinates:

$$\begin{array}{lllll} \Theta'_1 = 45^\circ, & \Theta'_2 = -25^\circ, & \Theta'_3 = 37.7^\circ, & \Theta'_4 = -102^\circ, & \Theta'_5 = -181^\circ \\ \Theta_{a1} = -7110^\circ, & \Theta_{a2} = -2567.5^\circ, & \Theta_{a3} = 1360.5^\circ, & \Theta_{a4} = 11392^\circ, & \Theta_{a5} = -2364^\circ \end{array}$$

The main fulcrum K corresponds to the following natural coordinates:

$$\begin{array}{lllll} \Theta'_1 = 47.3^\circ, & \Theta'_2 = -39^\circ, & \Theta'_3 = 12^\circ, & \Theta'_4 = 116^\circ, & \Theta'_5 = 0^\circ \\ \Theta_{a1} = -7472^\circ, & \Theta_{a2} = -3818.8^\circ, & \Theta_{a3} = -2921.3^\circ, & \Theta_{a4} = -11392^\circ, & \Theta_{a5} = -11392^\circ \end{array}$$

6. Conclusions

The kinematics models presented above render it possible to:

- a) analyse reference trajectories graphs in the internal space of IRb-6 manipulator;
- b) determine analytic description of the IRb-6 manipulator work space (Kucharski, 1989);
- c) design some algorithms generating the reference trajectory for the IRb-6 manipulator (e.g. algorithm PLAN2 (Szynawa, 1991));
- d) determine instantaneous position and angular velocities of links and other elements of IRb-6 manipulator which are essential for their dynamics, stress and strain analysis.

As the IRb-6, IRb-60, IRp-6, IRp-60 manipulators have similar kinematic structure, the equations of kinematics of Section 2 may be used to describe each of them as well.

References

- Craig J.J. (1989): *Introduction to Robotics. Mechanics and Control*. — New York: Addison-Wesley Publ. Comp.
- Duleba I. and Lysakowska B. (1990): *R.M. Taylor's modified method for parametrization of straight IRb-6 robot movement*. — Scientific Papers of the Inst. of Tech. Cybernetics of the Tech. Univ. of Wrocław, Poland, III National Conf. Robotics, v.2, pp.61, (in Polish).
- Gerke W. (1985): *Collision-free and shortest path for industrial robots found by dynamic programming*. — Robotosysteme, No.1.
- Gousenes L. (1984): *Strategies for solving collision-free trajectories. Problem for mobile and manipulator robots*. — Int. J. Robotics Res., v.3, No.4.
- Jacak W. (1989): *Strategies of searching for collision-free manipulators motions: automata theory approach*. — Robotica, v.7, No.2.
- Jacak W. and Lysakowska B. (1990): *Discrete methods of a robot's kinematics modelling*. — Scientific Papers of the Inst. of Technical Cybernetics of the Tech. Univ. of Wrocław, Poland, III National Conf. Robotics, v.2, pp.176, (in Polish).
- Knapczyk J. and Kisiel I. (1987): *Vector method for determination of manipulator links movement (inverse problem of kinematics)*. — Silesian Techn. Univ. Publ., Mechanics, No.86, (in Polish).
- Knapczyk J. and Stepniewski A. (1985): *Kinematic and dynamic analysis of manipulator using matrix method for the reference trajectory*. — Silesian Techn. Univ. Publ., Mechanics, No.86, (in Polish).
- Kucharski D. (1989): *Microcomputer Planning of IRb-6 Manipulator Trajectory*. — M. Sc. Thesis, Automation Inst. of Silesian Techn. Univ., (in Polish).
- Lozano-Perez T. (1983): *Spatial planning: configuration space approach*. — IEEE Trans. Computers, v.32, No.2.
- Niederliński A. (1983): *Industrial Robots*. — Warsaw: School and Pedagogical Publ., (in Polish).

- Paul R.P. (1983): *Robot Manipulators: Mathematics, Programing and Control*. — Cambridge: MIT Press.
- Ranky P.G. and Ho C.Y. (1985): *Robot Modelling Control and Applications with Software IFS*. — Berlin: Springer-Verlag.
- Szkodny T. (1985): *Industrial Robot Manipulators -Mathematical Models*. — Silesian Techn. Univ. Publ., No.1507, (in Polish).
- Szkodny T. (1990): *Kinematic decomposition of movement of IRb-60 robot of the effector*. — Scientific Papers of the Inst. of Tech. Cybernetics of the Tech. Univ. of Wrocław, Poland, III National Conf. Robotics, v.2, pp.247, (in Polish).
- Szynawa T. (1991): *Computer Planning of Minitimes Trajectories with Reference Kinematics for IRb-6 Manipulator*. — M. Sc. Thesis, Automation Inst. of Silesian Techn. Univ., (in Polish).
- Taylor R.M. (1983): *Planning and Execution of Straight-line Manipulator Trajectories*. — In: M. Brady (Ed.), *Robotmotion: Planning and Control*, Cambridge: MIT Press.
- Wojnarowski J. and Nowak A. (1992): *Manipulators - Robots Mechanics in Motors*. — Silesian Techn. Univ. Publ., (in Polish).

Symbols

- A_i — homogeneous transformation describing the relation between the $i-1$ -st link and i -th link
- $\alpha_i, l_i, \lambda_i, \Theta_i$ — Denavit-Hartenberg parameters
- E — homogeneous transformation describing the relation between the task and the (5-th) working link
- Φ, Θ, Ψ — external coordinates of orientation (Euler angles)
- h_2, h_3, k_1, k_4, k_5 — kinematic parameters of driving units of IRb-6 manipulator
- T_5 — homogenous transformation describing the relation between the 5-th link and base link
- T_{ai} — homogenous transformation describing the relation between the i -th actuator effector and the i -th link
- Θ'_i — natural coordinate of the i -th link
- Θ_{ai} — natural coordinate of the i -th actuator
- x, y, z — external coordinates of position
- $x_{ai}y_{ai}z_{ai}$ — coordinate system associated with the i -th actuator effector
- $x_{bi}y_{bi}z_{bi}$ — coordinate system associated with the body of the i -th actuator

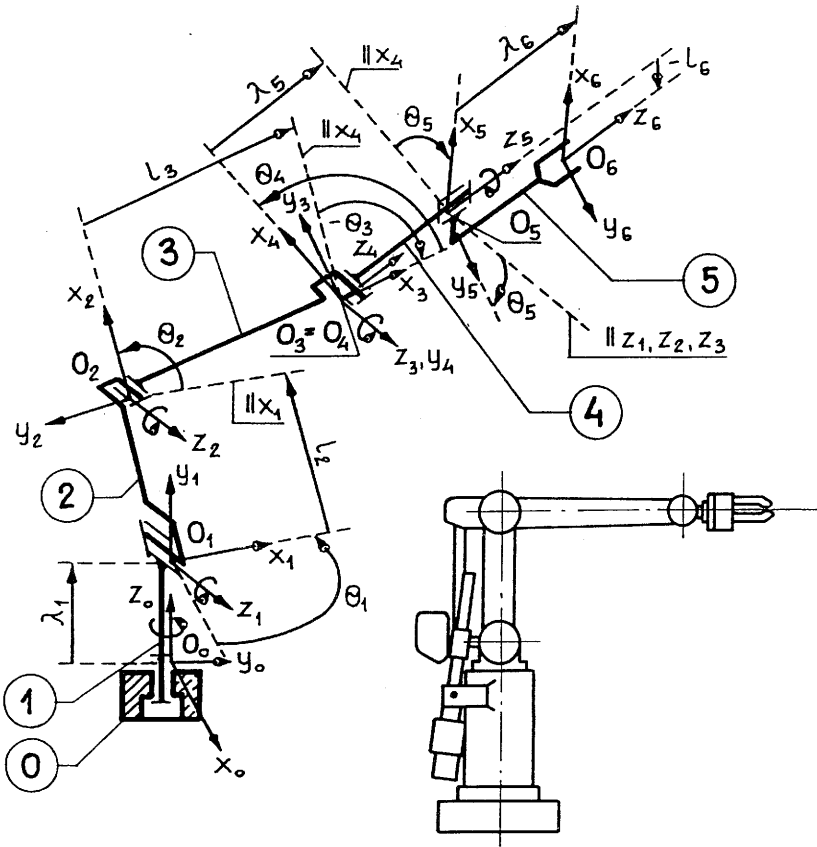


Fig. 1. IRb-6 robot manipulator.

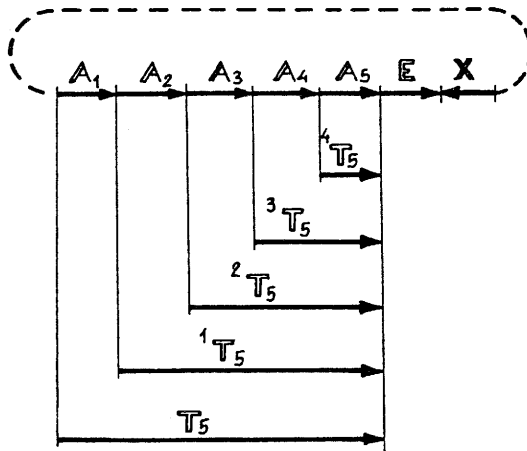
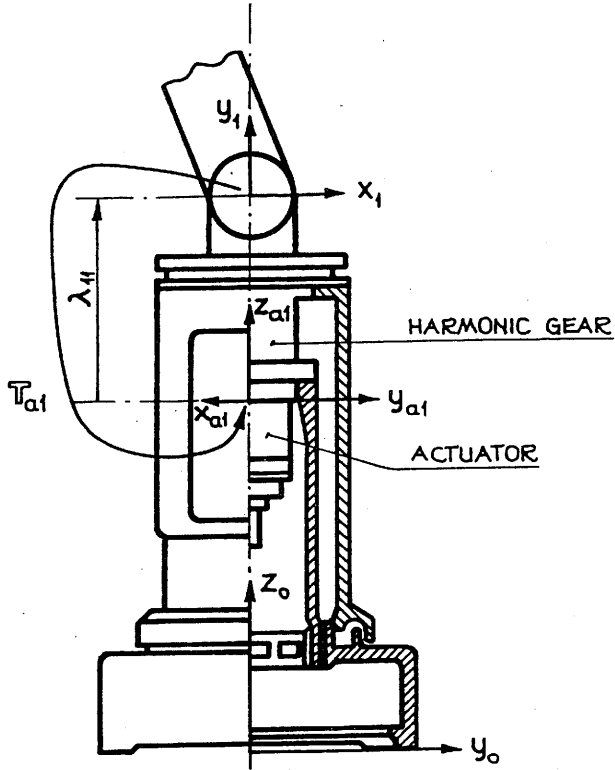
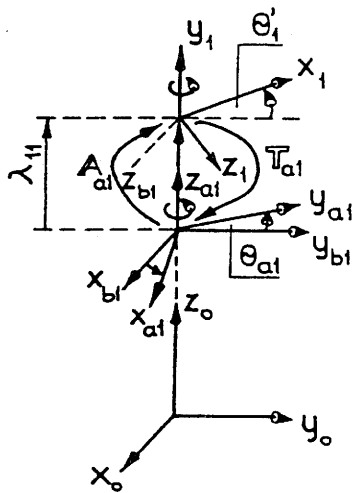


Fig. 2. Homogenous transformation graph of IRb-6 manipulator.

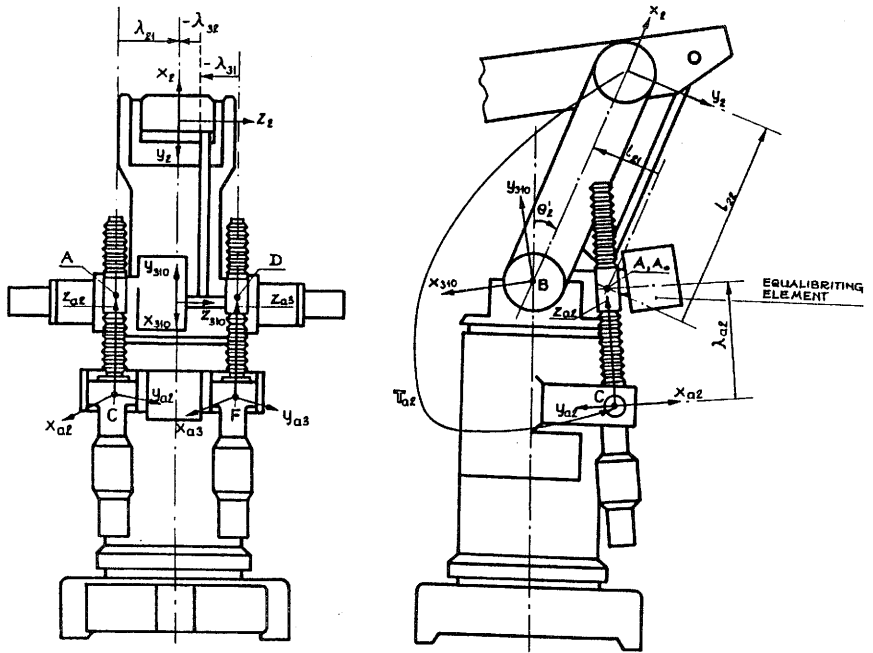


a)

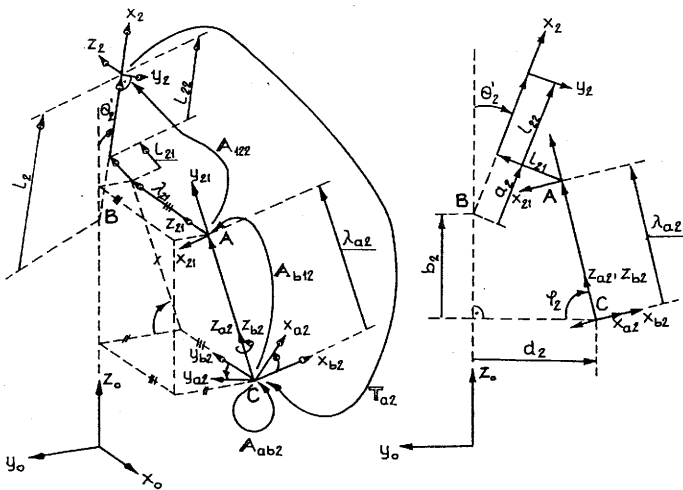


b) $A_{a1} = \text{Rot}(z, -\theta_{a1} + \theta'_1 + 90^\circ) \text{Trans}(0, 0, \lambda_{11}) \text{Rot}(x, 90^\circ)$, $T_{a1} = A_{a1}^{-1}$

Fig. 3. The first degree-of-freedom drive unit.

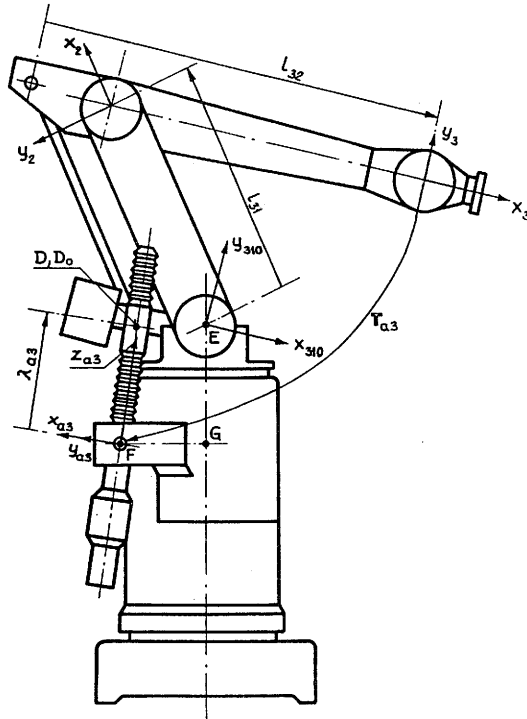


a)

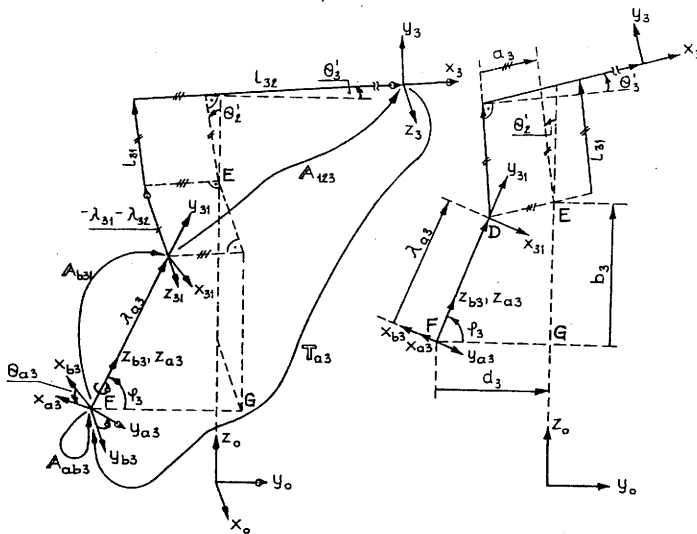


$$\begin{aligned}
 \text{b) } & \mathbf{A}_{ab2} = \text{Rot}(z, -\Theta_{a2}), \quad \mathbf{A}_{ab12} = \text{Rot}(z, 180^\circ) \text{Trans}(0, 0, \lambda_{a2}) \text{Rot}(x, 90^\circ) \\
 & \mathbf{A}_{122} = \text{Rot}(z, 90^\circ - \varphi_2 + \Theta_2') \text{Trans}(0, 0, \lambda_{z1}) \text{Trans}(l_{z1}, 0, 0) \text{Rot}(z, 90^\circ) \\
 & \text{Trans}(l_{z2}, 0, 0), \quad \mathbf{T}_{a2} = (\mathbf{A}_{ab2} \mathbf{A}_{b12} \mathbf{A}_{122})^{-1}
 \end{aligned}$$

Fig. 4. The second degree-of-freedom drive unit.



a)



$$\begin{aligned}
 \text{b) } & A_{ab3} = \text{Rot}(z, -\Theta_{a3}), \quad A_{b31} = \text{Rot}(z, 180^\circ) \text{Trans}(0, 0, \lambda_{a3}) \text{Rot}(x, 90^\circ) \\
 & A_{123} = \text{Rot}(z, 180^\circ - \varphi_3 + \Theta_2') \text{Trans}(0, 0, \lambda_{32} + \lambda_{31}) \text{Trans}(l_{31}, 0, 0) \text{Rot}(z, -90^\circ + \Theta_3') \\
 & \text{Trans}(l_{32}, 0, 0), \quad T_{a3} = (A_{ab3} A_{ab31} A_{123})^{-1}
 \end{aligned}$$

Fig. 5b. The third degree-of-freedom drive unit.

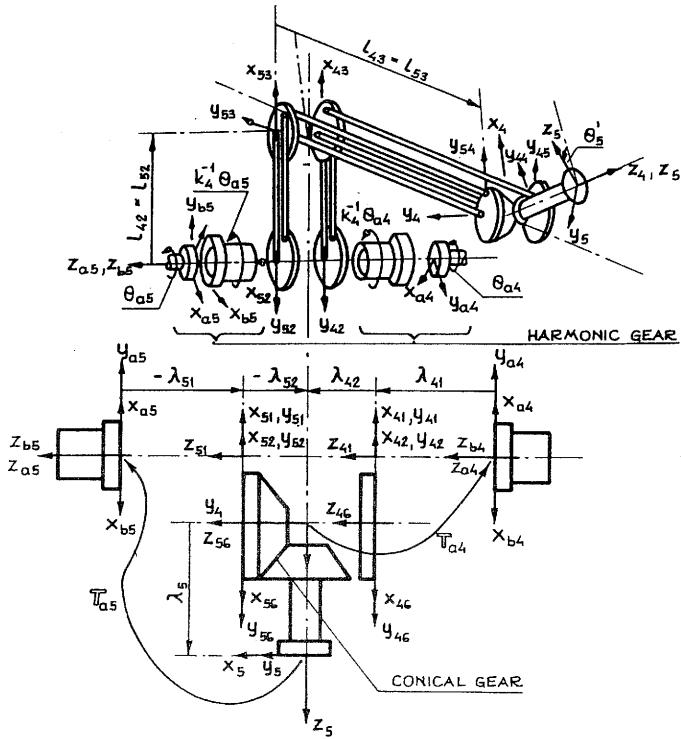


Fig. 6a. The fourth and fifth degree-of-freedom drive unit.

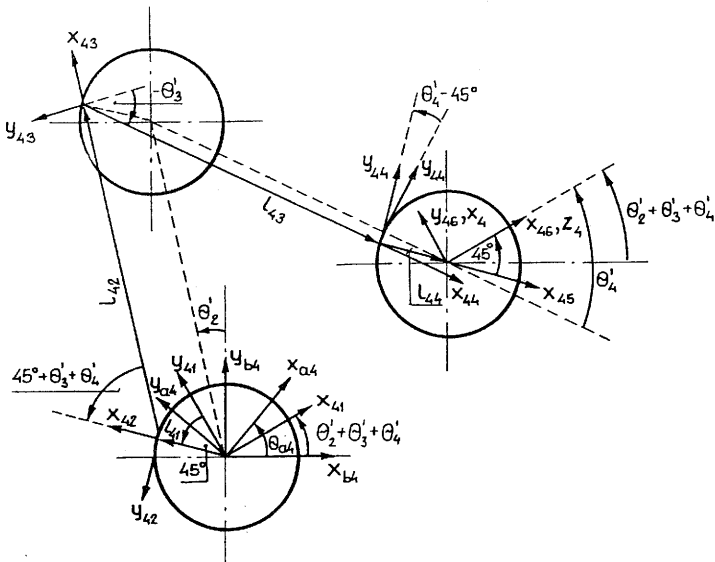
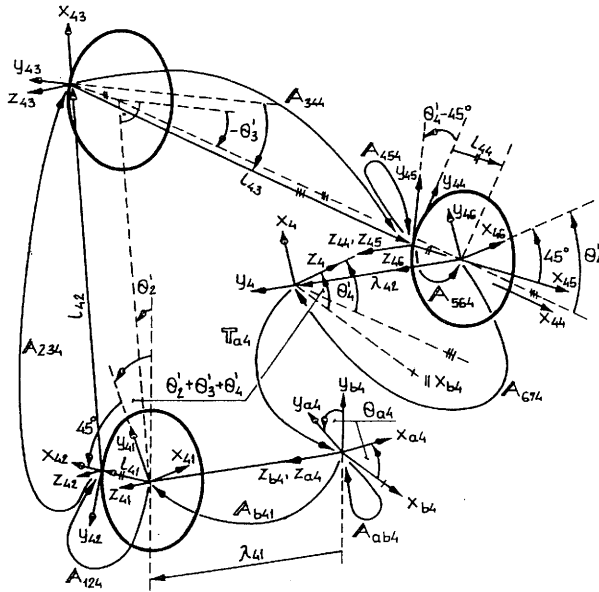


Fig. 6b. The fourth degree-of-freedom drive unit.



$$\begin{aligned}
 \mathbf{A}_{ab4} &= \text{Rot}(z, -\Theta_{a4}), \quad \mathbf{A}_{b14} = \text{Rot}(z, \Theta'_2 + \Theta'_3 + \Theta'_4) \text{Trans}(0, 0, \lambda_{41}) \\
 \mathbf{A}_{124} &= \text{Rot}(z, 90^\circ + 45^\circ) \text{Trans}(l_{41}, 0, 0) \\
 \mathbf{A}_{234} &= \text{Rot}(z, -45^\circ - \Theta'_3 - \Theta'_4) \text{Trans}(l_{42}, 0, 0) \\
 \mathbf{A}_{344} &= \text{Rot}(z, \Theta'_3 - 90^\circ) \text{Trans}(l_{43}, 0, 0), \quad \mathbf{A}_{454} = \text{Rot}(z, -45^\circ + \Theta'_4) \\
 \mathbf{A}_{564} &= \text{Trans}(l_{44}, 0, 0) \text{Rot}(z, 45^\circ), \quad \mathbf{A}_{674} = \text{Rot}(z, 90^\circ) \text{Trans}(0, 0, \lambda_{42}) \text{Rot}(x, 90^\circ) \\
 \mathbf{T}_{a4} &= (\mathbf{A}_{ab4} \mathbf{A}_{b14} \mathbf{A}_{124} \mathbf{A}_{234} \mathbf{A}_{344} \mathbf{A}_{454} \mathbf{A}_{564} \mathbf{A}_{674})^{-1}
 \end{aligned}$$

Fig. 6c. The fourth degree-of-freedom drive unit.

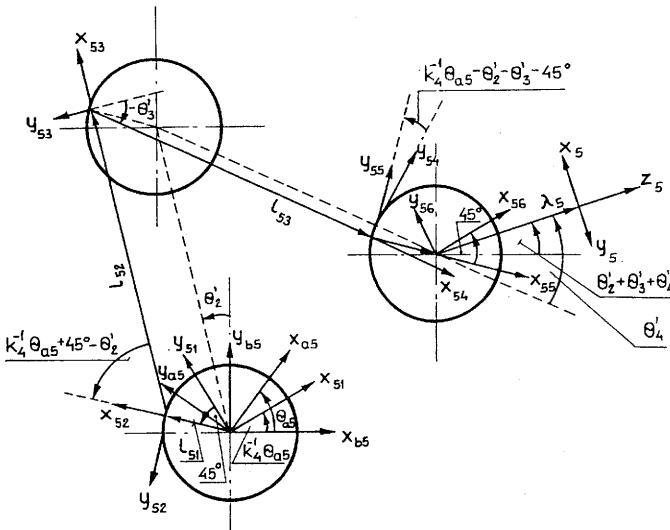
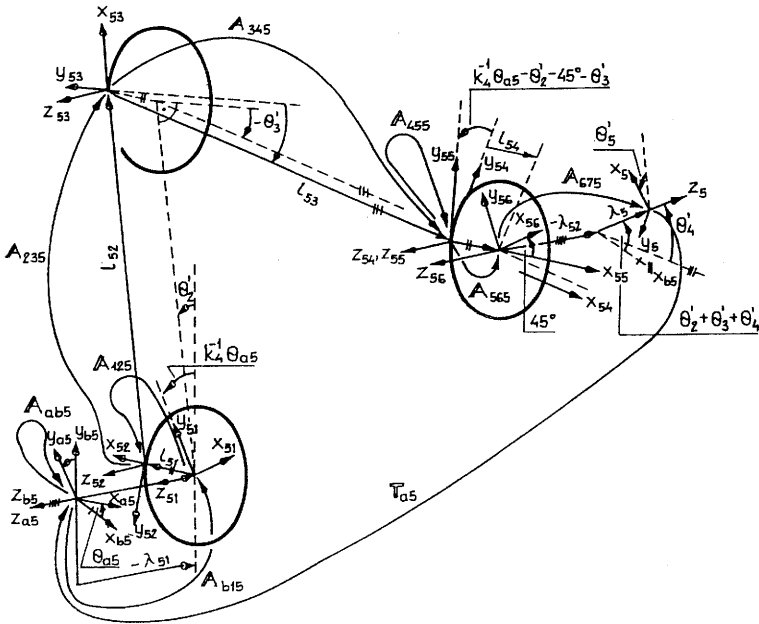


Fig. 6d. The fifth degree-of-freedom drive unit.



$$\begin{aligned}
 A_{ab5} &= \text{Rot}(z, -\Theta_{a5}), & A_{b15} &= \text{Rot}(z, k_4^{-1}\Theta_{a5})\text{Trans}(0, 0, \lambda_{51}) \\
 A_{125} &= \text{Rot}(z, 90^\circ + 45^\circ)\text{Trans}(l_{51}, 0, 0) \\
 A_{235} &= \text{Rot}(z, \Theta'_2 - k_4^{-1}\Theta_{a5} - 45^\circ)\text{Trans}(l_{52}, 0, 0) \\
 A_{345} &= \text{Rot}(z, \Theta'_3 - 90^\circ)\text{Trans}(l_{53}, 0, 0), & A_{455} &= \text{Rot}(z, k_4^{-1}\Theta_{a5} - \Theta'_2 - \Theta'_3 - 45^\circ) \\
 A_{565} &= \text{Trans}(l_{54}, 0, 0)\text{Rot}(z, 45^\circ) \\
 A_{675} &= \text{Rot}(z, 90^\circ - k_4^{-1}\Theta_{a5} + \Theta'_2 + \Theta'_3 + \Theta'_4)\text{Trans}(0, 0, \lambda_{52})\text{Rot}(x, 90^\circ) \\
 &\quad \text{Trans}(0, 0, \lambda_5)\text{Rot}(z, \Theta'_5) \\
 T_{a5} &= (A_{ab5}A_{b15}A_{125}A_{235}A_{345}A_{455}A_{565}A_{675})^{-1}
 \end{aligned}$$

Fig. 6e. The fifth degree of freedom drive unit.

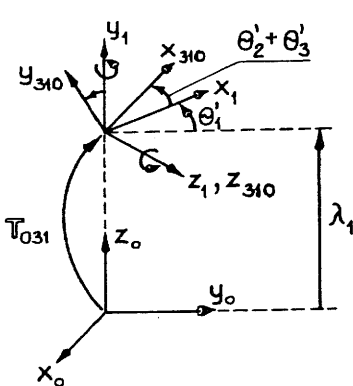


Fig. 7. Description of the third link equilibrator.

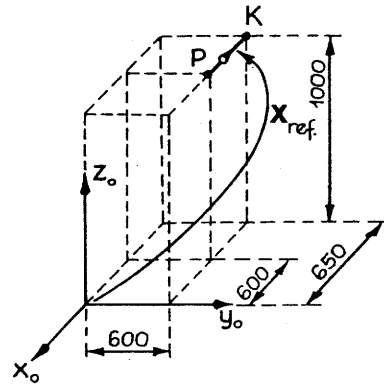


Fig. 8. Reference trajectory X_{ref} .

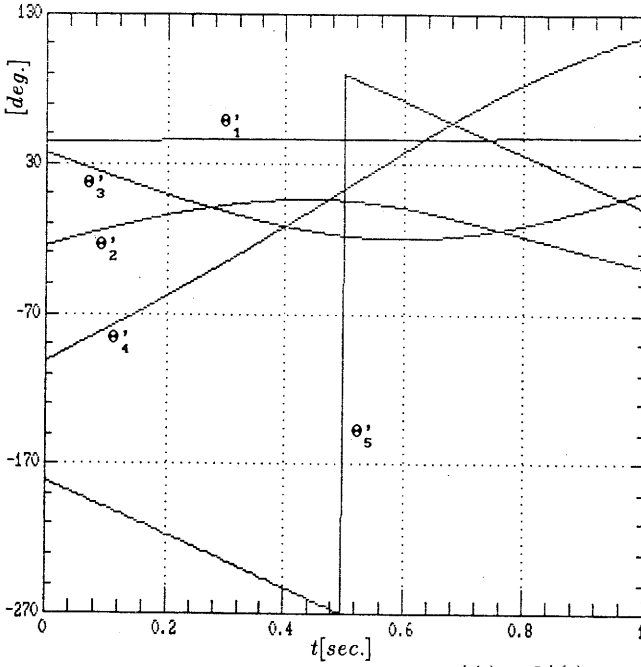


Fig. 9. Link natural coordinates $\Theta'_1(t) \div \Theta'_5(t)$.

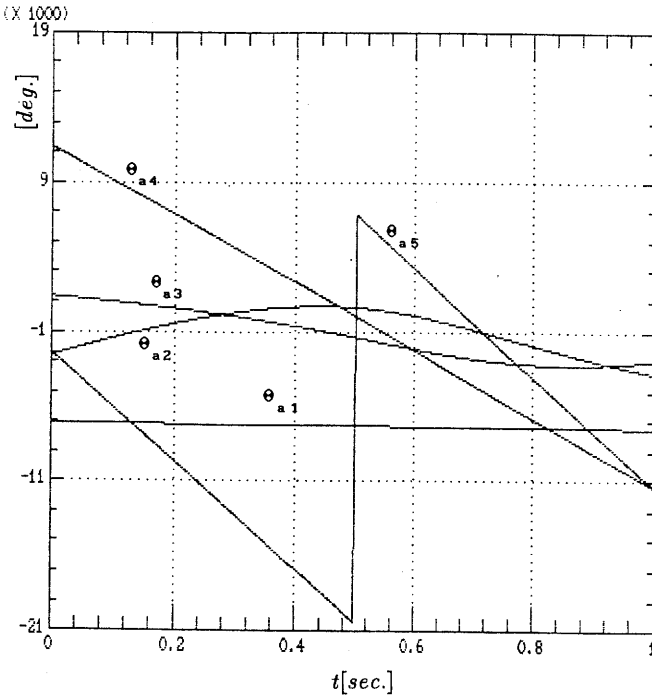


Fig. 10. Actuator natural coordinates $\Theta_{a1}(t) \div \Theta_{a5}(t)$.