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ADAPTIVE DISCRETE-TIME IDENTIFICATION OF CONTINUOUS-TIME SYSTEMS USING ADJUSTED INTEGRATION

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A new approach to the design of on-line discrete-time systems of identification of continuous-time systems is presented. The problem concerns digital signal processing suitable for generation of the regression vector in discrete-time recursive estimation of continuous-time process parameters based on a mixed, discrete-continuous regression model. The approach is related to discrete approximation of continuous-time systems, where a kind of matching to the input and output signals is taken into account. In particular, the normal integrating operator technique is utilised. Two methods of "tempering" the characteristics of discrete-time integrators are proposed. One method consists in the transformation of an original system of matched IIR filters into an equivalent system of FIR filters. The purpose of the other method is simplification of calculations by applying stabilised closed IIR forms of the integrating operators. Such matched-and-tempered normal integrating operators are referred to as the adjusted integrators. Experimental results, obtained in an analogue and digital simulation environment, of the application of the two methods to discrete-time parameter estimation of nonstationary continuous-time systems corrupted by different noise processes illustrate the usefulness of the approach.

1. Introduction

Adaptive mechanisms introduced into control systems generally rely on identified data. Therefore, in various applications of adaptive schemes, routines used for estimating the process dynamics take a cardinal role. Such routines are most essential in continuously adaptive control systems. Many ideas proposed for the purpose of identification of discrete-time models have been proposed—see the bibliography in (Ljung and Söderström, 1983; Kowalczuk, 1992a; Kowalczuk, 1992c).

There are, however, essential drawbacks connected with the discrete-time approach to the design. The loss of information on the value of the relative order of the system transfer function, the residual delay, the problem of choosing the sampling time, the non-minimum phase property, the effect of roots clustering and the resulting system parameter sensitivity, are some of them. These issues justify the

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recently observed return to the continuous-time approach and to the design of digital control systems (Sagara and Zhao, 1990; Unbehauen and Rao, 1990; Pintelon and Kollár, 1991; Kowalczuk, 1993a; 1993b; 1994a; 1994b; 1994c; 1995; Kowalczuk and Marcińczyk, 1995a; 1995b), where the fundamental design (Kowalczuk, 1992c) is carried out in the continuous-time domain prior to digital mechanisation.[1]

This paper presents a new continuous-time approach to the design of on-line systems appropriate for the identification of continuous-time nonstationary systems and processes that can be used in adaptive control applications. Our attention is focused on linear dynamic operations suitable for generating the elements of the regression vector in discrete-time recursive estimation of continuous-time system parameters based on a mixed, discrete-continuous regression model.

The proposed methodology, being representative of the continuous-time design, is related to discrete approximation of continuous-time systems, where a kind of matching to the input and output signals is taken into account (Kowalczuk, 1983a; 1983b; 1991; 1993a; 1993b). In particular, the normal integrating operator technique is applied. Two methods of "tempering" the characteristics of discrete-time integrators for on-line applications are proposed. One method consists in the transformation of an original system of matched IIR filters (with an infinite impulse response) into an equivalent system of FIR filters (of a finite impulse response). Another method allows for a simplification of calculations by applying stabilised closed IIR forms of the integrating operators.

Such "matched-and-tempered" normal integrating operators are referred to as the adjusted integrators. Experimental results of the application of the two methods to discrete-time parameter estimation of nonstationary continuous-time systems corrupted by different noise processes illustrate the usefulness of the approach. The numerical results have been obtained in a hybrid, analogue and digital simulation environment.

2. Identification Problem

Consider a lumped linear continuous-time system described by the n-th order strictly proper rational transfer function

$$\mathcal{G}(s) = \frac{B(s)}{A(s)} \tag{1}$$

where $A(s) = s^n + a_1 s^{n-1} + \cdots + a_n$ and $B(s) = b_0 s^m + b_1 s^{m-1} + \cdots + b_m$ for $0 \leq m < n$.

The input $u(t)$ and output $y(t)$ of the system are related through the following linear integral equation:

$$\left(J^0 + a_1 J^1 + \cdots + a_n J^n\right) y(t) = \left(b_0 J^{n-m} + b_1 J^{n-m+1} + \cdots + b_m J^n\right) u(t) \tag{2}$$

[1] As opposed to the discrete-time approach, where the fundamental design follows a model discretisation procedure (Brogan, 1991; Isermann, 1989; Kowalczuk, 1992a; Kowalczuk and Suchomski, 1995a; 1995b).

where J^i, or $J^i(s) = 1/s^i$, is the operator of the i-th order integration within the limits $(0, t)$.

Note that the above realisable eqn. (2) can be obtained by introducing the n-th order integration $J^n(s)$ that can be interpreted as a type of low pass filtering $H(s) = J^n(s)$ of the system input and output signals $X \in \{U, Y\}$:

$$s^{n-i}H(s)X(s) = s^{n-i}J^n(s)X(s) = J^i(s)X(s)$$

On the other hand, for the purpose of identification and control it is usual to centre the parameter estimation process around a lower-frequencies band by applying a low-pass filter $F(s)$ to both the system's input and output. In the continuous-time domain the system can therefore be represented by the following model:

$$J^0 y_f + a_1 J^1 y_f + \cdots + a_n J^n y_f = b_0 J^{n-m} u_f + b_1 J^{n-m+1} u_f + \cdots + b_m J^n u_f \quad (3)$$

where $u_f = u_f(t)$ and $y_f = y_f(t)$ are the filtered-by-$F(s)$, low-pass versions of the input and output signals, respectively. Note that the effect of filtering by a digitally implemented $H(s) = J^n(s)$ may not be sufficient and a continuous-time filter $F(s)$ is also necessary to avoid the frequency aliasing effects due to sampling.

Consequently, it is clear that the observed dynamical continuous-time system (3) is modelled by the linear-in-the parameters equation, which can be shown in the linear regression form as

$$J^0 y_f(t) = \Theta^T \varphi_f(t) + \eta(t) \quad (4)$$

where $\Theta^T = \left[a_1\ a_2\ \cdots\ a_n \vdots b_0\ b_1\ \cdots\ b_m\right]$ and $\varphi_f^T(t) = [-J^1 y_f(t)\ \cdots\ -J^n y_f(t) \vdots J^{n-m} u_f(t)\ J^{n-m+1} u_f(t)\ \cdots\ J^n u_f(t)]$, $\eta(t)$ being an equation error function.

In the identification process that is to be performed by digital means (in the discrete-time domain), solely samples of the low-pass-filtered input and output signals, i.e. $u_f(kT)$, $y_f(kT)$ and $\varphi_f(kT)$, can be employed:

$$J^0 y_f(kT) = \Theta^T \varphi_f(kT) + \eta(kT) \quad (5)$$

resulting in a mixed discrete-continuous formulation of the regression model that in spite of its seemingly discrete form maintains the original parameterisation of the difference equation of the continuous-time system (1).

The remaining problem is to approximate the continuous-time pre-processing $J^i(s)$ necessary for calculating the elements of the (continuous-time) regression data vector $\varphi_f(kT)$ by a corresponding discrete-time pre-processing of the sampled-data signals $u_f(kT)$ and $y_f(kT)$.

3. Discrete Approximation

The methodology known as the discrete-time approximation of continuous-time systems seems to be most appropriate to this end. A survey of respective methods using closed-form design can be found in (Kowalczuk, 1993a; 1993b).

This type of design methods has several advantages. It is simple and gives closed-form solutions without resorting to iterative optimisation procedures. It enables us to give a natural interpretation of parameters and variables or other system properties of interest. What is more, by maintaining the sampling time as a tunable parameter, it permits implementations at different sampling rates.

The model formulation given in (1)–(5) is suitable to the so-called input-output matching, and, in particular, to the expanded operator method, where a series of discrete operators $\Xi_r^i(z)$ approximating the corresponding set of analogue multiple integrating elements $J^i(s)$ is applied (Kowalczuk, 1993a; 1993b).

The idea of using those operators differs from the approach presented in (Sagara and Zhao, 1990; Pintelon and Kollár, 1991), which can be classified as a simple operator method (Kowalczuk, 1993b).

3.1. Normal Integrating Operators

According to the discretising methodology (Kowalczuk, 1993a; 1993b), the operators $\Xi_r^i(z)$ can be designed so as to ensure their individual matching to the signals sampled at the input and output of the identified system.

There are numerous manifestations of the integrating operators $\Xi_r^i(z)$. The most general class used in the expanded operator scheme comprises the normal integrating operators (Kowalczuk, 1983b; 1991; 1993b)

$$\Xi_r^i(z) = T^i \frac{r!}{(r+i)!} \frac{N_{r+i}(z)}{(z-1)^i N_r(z)} \tag{6}$$

for $r = 0, 1, 2, \ldots$, $i = 1, 2, \ldots$, where $N_r(z)$ are certain normal polynomials. The parameter r is the order of the signal interpolation applied in numerical integration. This order should therefore match the form of the signal integrated: it can, for instance, be fixed as the order of a polynomial spline function advised for approximation of the integrated signal between the sampling points (Kowalczuk, 1983b; 1993b).

Using the integrating operators for the calculation of the discrete-time approximation of (4) results in (5) with $J^0 = 1$ and with the samples of the regression vector

$$\begin{aligned}\varphi_f^T(kT) = \Big[-\Xi_r^1 y_f(kT) \; -\Xi_r^2 y_f(kT) \; \cdots \; -\Xi_r^n y_f(kT) \; \vdots \\ \Xi_q^{n-m} u_f(kT) \; \Xi_q^{n-m+1} u_f(kT) \; \cdots \; \Xi_q^n u_f(kT) \Big]\end{aligned} \tag{7}$$

where r and q are the respective orders of interpolation of the filtered output and input signals of the identified system.

3.2. Tempering of Integrating Operators

A principal problem in using the integrating operators is that they are intrinsically unstable (in the BIBO sense) in the open-loop operation that takes place in (7). The numerical overflow connected with integration can be circumvented by a periodic reset of the estimation algorithm (Unbehauen and Rao, 1990), but for higher-order systems the time interval allowable for integration is usually too short to perform effective identification. A slight displacement of the pole of $\Xi_r^i(z)$ from $z = 1$ inside the unit circle on the z-plane (Pintelon and Kollár, 1991) may be insufficient since for $r \geq 2$ the operators have also other unstable poles. At the same time, a direct way of stabilisation of these operators introduces a phase distortion (Kowalczuk, 1991) and therefore does not seem to be suitable for identification purposes.

Two general ways of tempering characteristics of the operators for on-line identification schemes have recently been proposed in (Kowalczuk, 1994a; 1994b; 1994c; 1995). One method is based on the transformation of the operator transfer functions (6) into a set of equivalent transfer functions, and next into a corresponding set of FIR operations, that leads to the mixed regression model (5), which entails FIR digital filters. The other method is set up on a simple stabilisation procedure of the model (5)–(7) that results in an IIR-type processing of the elements of the vector $\varphi_f(kT)$.

4. FIR Processing of the Regression Vector

The methodology explained below consists in replacing the IIR type of filtering described in (6) by the FIR type of a limited-horizon integration. It is clear that in such a case the problem of unstable poles will be circumvented.

The procedure is based on observation that the elementary single-delay discrete-time rectangular integration scheme can be transformed into its equivalent l-delay form (note that in the resulting algorithm the parameter l will next be interpreted as the length of a FIR-integration horizon):

$$I(z) = \frac{1}{1 - z^{-1}} = \frac{P_l(z^{-1})}{1 - z^{-l}} \tag{8}$$

where $P_l(z^{-1}) = 1 + z^{-1} + z^{-2} + \cdots + z^{-(l-1)}$.

4.1. Using the Expanded Operators

The first technique corresponds to the expanded operator scheme of discrete approximation (Kowalczuk, 1983a; 1983b; 1993a; 1993b). The l-delay operators corresponding to (6) can be shown as

$$\Xi_{r,l}^i(z) = T^i \frac{r!}{(r+i)!} \frac{N_{r+i}(z^{-1})\, P_l^i(z^{-1})}{N_r(z^{-1})\, (1 - z^{-l})^i} \tag{9}$$

For a procedure developed here, all the operators should acquire a form with a common denominator. Therefore, taking into account the order (n) of the continuous-time system (1), the final form of the l-delay integrators having a common denominator is

$$\Xi_{r,l}^{i,n}(z) = \frac{M_{r,l}^{i,T}(z^{-1})\, N_q(z^{-1})\, (1 - z^{-l})^{n-i}}{N_r(z^{-1})\, N_q(z^{-1})\, (1 - z^{-l})^n} \tag{10}$$

where $M_{r,l}^{i,T}(z^{-1}) = \dfrac{r!T^i}{(r+i)!} N_{r+i}(z^{-1})\, P_l^i(z^{-1})$.

Using the operators (10) for the construction of the regression vector (7) and making a simple recalculation of eqn. (5), we obtain

$$J_{r,q}^{0,n} y_f(kT) = \Theta^T \varphi_f(kT) + \eta(kT) \tag{11}$$

with

$$\varphi_f^T(kT) = \Big[-J_{r,q}^{1,n} y_f(kT) \;\; -J_{r,q}^{2,n} y_f(kT) \;\; \dots \;\; -J_{r,q}^{n,n} y_f(kT) \; \vdots \\ J_{q,r}^{n-m,n} u_f(kT) \;\; J_{q,r}^{n-m+1,n} u_f(kT) \;\; \dots \;\; J_{q,r}^{n,n} u_f(kT) \Big] \tag{12}$$

where the discrete-time J-operators, different both in form and interpretation from the continuous-time ones,

$$\begin{aligned} J_{r,q}^{i,n} &= J_{r,q}^{i,n}(z) = J_{r,q,l}^{i,n,T}(z) = M_{r,l}^{i,T}(z^{-1}) N_q(z^{-1})(1 - z^{-l})^{n-i} \\ &= \frac{r! T^i}{(r+i)!} N_{r+i}(z^{-1}) N_q(z^{-1}) P_l^i(z^{-1})(1 - z^{-l})^{n-i} \end{aligned} \tag{13}$$

stand for the limited-horizon integration of the FIR type. Note that for boundary values of the index i

$$J_{r,q}^{0,n}(z) = J_{q,r}^{0,n}(z) = N_r(z^{-1}) \, N_q(z^{-1})(1 - z^{-l})^n \tag{14}$$

yet

$$J_{r,q}^{n,n}(z) = \frac{r! T^n}{(r+n)!} N_{r+n}(z^{-1}) N_q(z^{-1}) P_l^n(z^{-1}) \tag{15}$$

is different from $J_{q,r}^{n,n}(z)$ for $r \neq q$.

4.2. Using the Simple Operators

The simple operator scheme is based on the single normal integrating operator $\Xi_r^1(z)$ from (6)

$$\Xi_{r,l}^i(z) = \left(\frac{T}{r+1} \right)^i \frac{N_{r+1}^i(z^{-1}) P_l^i(z^{-1})}{N_r^i(z^{-1})(1 - z^{-l})^i} \tag{16}$$

that is designed for one "shape" of signals under observation. Note that in such a case, the same interpolation order $q = r$ is applied both at the input and output of the system.

The l-delay operators adjusted for identification of the n-th order system are

$$\Xi_{r,l}^{i,n}(z) = \frac{M_{r,l}^{i,T}(z^{-1}) N_r^{n-i}(z^{-1})(1 - z^{-l})^{n-i}}{N_r^n(z^{-1})(1 - z^{-l})^n} \tag{17}$$

where $M_{r,l}^{i,T}(z^{-1}) = \left(\frac{T}{r+1} \right)^i N_{r+1}^i(z^{-1}) P_l^i(z^{-1})$.

The corresponding regression model takes then the form:

$$J_r^{0,n} y_f(kT) = \Theta^T \varphi_f(kT) + \eta(kT) \tag{18}$$

where $\varphi_f(kT)$ is given by

$$\begin{aligned}\varphi_f^T(kT) = \Big[-J_r^{1,n} y_f(kT) \ \ -J_r^{2,n} y_f(kT) \ \dots \ -J_r^{n,n} y_f(kT) \ \vdots \\ J_r^{n-m,n} u_f(kT) \ \ J_r^{n-m+1,n} u_f(kT) \ \dots \ J_r^{n,n} u_f(kT) \Big]\end{aligned} \tag{19}$$

and

$$\begin{aligned} J_r^{i,n} = J_r^{i,n}(z) = J_{r,l}^{i,n,T}(z) &= M_{r,l}^{i,T}(z^{-1}) N_r^{n-i}(z^{-1})(1 - z^{-l})^{n-i} \\ &= \frac{T^i}{(r+i)^i} N_{r+1}^i(z^{-1}) N_r^{n-i}(z^{-1}) P_l^i(z^{-1})(1 - z^{-l})^{n-i} \end{aligned} \tag{20}$$

The boundary values are

$$J_r^{0,n}(z) = N_r^n(z^{-1})(1 - z^{-l})^n, \quad J_r^{n,n}(z) = \frac{T^n}{(r+1)^n} N_{r+1}^n(z^{-1}) \ P_l^n(z^{-1}) \tag{21}$$

Note that in both techniques (10) and (17) are respectively of the form

$$\Xi_{r,l}^{i,n}(z) = \frac{J_{r,q}^{i,n}(z)}{J_{r,q}^{0,n}(z)} \quad \text{and} \quad \Xi_{r,l}^{i,n}(z) = \frac{J_r^{i,n}(z)}{J_r^{0,n}(z)} \tag{22}$$

and that the integration approach considered in (Sagara and Zhao, 1990) is included in the simple scheme (17)–(20) because the trapezoidal rule is realised here by putting $r = 1$.

5. IIR Processing of Regression Vectors

In the case of low-order continuous-time systems a simple method of stabilisation of the original IIR-type integration, necessary for processing of the regression vector $\varphi(kT)$, is possible.

Consider the system model described by eqns. (5)–(7) and bring it to the form of (11)

$$J_{r,q}^{0,n} y_f(kT) = \Theta^T \varphi_f(kT) + \eta(kT) \tag{23}$$

The regression vector is then given by (12), where

$$J_{r,q}^{i,n}(z) = \frac{r! \, T^i}{(r+i)!} N_{r+i}(z^{-1}) N_q(z^{-1})(1 - z^{-1})^{n-i} \tag{24}$$

with boundary values

$$J_{r,q}^{0,n}(z) = N_r(z^{-1})N_q(z^{-1})(1-z^{-1})^n = J_{q,r}^{0,n}(z) \tag{25}$$

and

$$J_{r,q}^{n,n}(z) = \frac{r!\,T^n}{(r+n)!}N_{r+n}(z^{-1})N_q(z^{-1}) \tag{26}$$

which is different from the filter $J_{q,r}^{n,n}(z)$ for $r \neq q$.

Let us define a polynomial related to $J_{r,q}^{0,n}(z)$

$$D(z) = \gamma + J_{r,q}^{0,n}(z) \tag{27}$$

which has all its zeros inside the unit circle on the z-plane. The parameter γ is a non-negative coefficient used in the accommodation ensuring stability of $D(z)$.

By dividing both sides of (23) by (27) one obtains the regression model that can be expressed as

$$J_{r,q}^{0,n}y_F(kT) = \Theta^T\varphi_F(kT) + \eta_F(kT) \tag{28}$$

where

$$y_F(kT) = \frac{y_f(kT)}{D(z)} = D_F(z)y_f(kT)$$

and

$$\varphi_F(kT) = \frac{\varphi_f(kT)}{D(z)} = D_F(z)\varphi_f(kT)$$

if $D_F(z) \equiv 1/D(z)$, and accordingly

$$\varphi_F^T(kT) = \Big[-J_{r,q}^{1,n}y_F(kT)\ \ -J_{r,q}^{2,n}y_F(kT)\ \dots\ -J_{r,q}^{n,n}y_F(kT)\ \vdots$$

$$J_{q,r}^{n-m,n}u_F(kT)\ \ J_{q,r}^{n-m+1,n}u_F(kT)\ \dots\ J_{q,r}^{n,n}u_F(kT)\Big] \tag{29}$$

Apparently, the above manipulation implies additional filtering of the input and output signals. Note that as before, different orders of interpolation can be used to match individual shapes of the input and output signals of the observed system. It is also worth noticing that the same model equation will be obtained if one adds a quantity of $\gamma y_f(nT)$ to both sides of the basic eqn. (23). Thus it is clear that for $\gamma = 0$ one obtains the ideal integration formula (and exactly, its limited-horizon version).

Both for higher orders (n, i) of the system and higher interpolation orders (r, q), however, the accommodating coefficient γ necessary to stabilise (27) must be of a considerably high value. Therefore in order to avoid undesirable damping of signals, an auxiliary gain coefficient κ has been introduced into the D-filter

$$D_F(z) = \frac{\kappa}{\gamma + J_{r,q}^{0,n}(z)} = \frac{d_0}{1 + d_1 z^{-1} + \cdots} \tag{30}$$

where $d_0 = \kappa/(1+\gamma)$.

6. Simulation Arrangement

Simulations have been performed in a hybrid, analogue and digital simulation environment (Kowalczuk, 1992b) for different types of systems and excitations. In the following sections some results of application of the above algorithms to on-line identification systems are shown. In the simulation study an initial form of the continuous-time process (1) has been assumed to be the following:

$$\mathcal{G}(s) = \frac{B(s)}{A(s)} = \frac{2}{(s+1)^n} \tag{31}$$

The simulation results presented here have been acquired based on sampling and processing at the sampling rate of $f_T = 1/T = 20\,[\text{Hz}]$, unless otherwise stated.

6.1. Estimation Algorithm

For identification of a nonstationary process modelled by (1), the exponentially weighted least-squares (EWLS) algorithm, adopted for the considered versions (I, II and III)[2] of the discrete-continuous regression model (5), has been applied

$$\hat{\Theta}(kT) = \hat{\Theta}(kT - T) + L(kT)\varepsilon(kT) \tag{32}$$

where

$$L(kT) = \frac{P(kT-T)\varphi_f(kT)}{\lambda + \varphi_f^T(kT)P(kT-T)\varphi_f(kT)}$$

$$P(kT) = \frac{1}{\lambda}\Big(P(kT-T) - L(kT)\,\varphi_f^T(kT)\,P(kT-T)\Big)$$

$$\varepsilon(kT) = J_{r,q}^{0,n} y_f(kT) - \hat{\Theta}^T(kT-T)\varphi_f(kT)$$

and

$$\hat{\Theta}(kT) = \Big[\hat{a}_1(kT)\,\hat{a}_2(kT)\cdots\hat{a}_n(kT) \,\vdots\, \hat{b}_0(kT)\,\hat{b}_1(kT)\cdots\hat{b}_m(kT)\Big]^T$$

[2] That is, version I—described by eqns. (11) and (12), version II—by eqns. (18) and (19), and version III—by eqns. (28)–(29). See Appendix for an explicit form of the algorithms of the regression vector processing.

is the vector of the estimated parameters of the continuous-time object given in (1) and λ is the forgetting factor (the sampling period T in (32) might as well be neglected; then, for all the above time functions, $x(k) \equiv x(kT)$ could be practised). In the simulation runs the covariance matrix $P(kT)$ has been assigned an initial value of $P(0) = 10^5 I$, where I is the identity matrix of a proper dimension.

For the considered range of experimental parameters (sampling time, rate of nonstationarity, etc.) the length l of the *core* integrating operator introduced in (8) has been fixed at a value of 20 or 30 [samples]. The effect of a larger value of l on the computation effort can easily be predicted from eqns. (13)–(15) and (20)–(21).

6.2. Performance Indices

Losses σ_j in each of the estimated parameters θ_j have been derived from the mean square of the estimation error

$$\sigma_j^2 = E\left[\left(\theta_j(kT) - \hat{\theta}_j(kT)\right)^2\right] \tag{33}$$

The average estimation loss has been calculated as

$$\sigma = \vartheta^{-1} \sum_{j=1}^{\vartheta} \sigma_j \tag{34}$$

where $\vartheta = n + m + 1$.

The effective settling time has been obtained from

$$\tau = \max_j \tau_j^{90\%} \tag{35}$$

where $\tau_j^{90\%}$ is the time necessary for reaching 90% of the true value of the j-th estimated parameter θ_j, taking into account maximum absolute error. It makes a useful complement to average estimation errors, and allows for the calculation of the average errors at "steady-state".

The average estimation loss σ computed after the effective settling time τ has been denoted by σ_τ. This measure of the estimation performance gives a legible measure of accuracy of the identification process that is not perturbed by transient processes in the estimates' trajectories.

7. Stationary Processes

Characteristics of the $J_{r,q}^{i,n}(z)$-operators and their corresponding performance indices are given in (Kowalczuk, 1994b; 1994c) for stationary processes identified using $\lambda = 1$. A sample of results are recalled here for illustrative purposes.

7.1. FIR Approach

The effect of the zero-order J^0-operation (see eqns. (1)–(4)) performed on the step response of a fifth-order system is shown in Fig. 1. Transients of all the elements of the regression vector $\varphi_f(kT)$, calculated based on the J-operators, during estimation of the continuous-time parameters of the same system are given in Fig. 2.

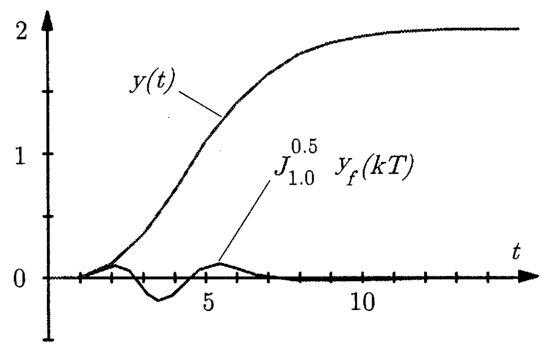


Fig. 1. The generated response of the $J^0 = J^{0,5}_{1,0}$-operator (FIR).

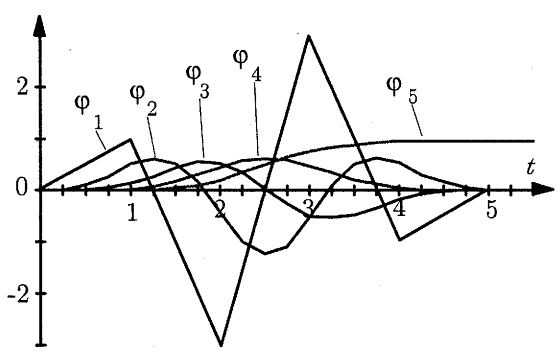


Fig. 2. Transient of the FIR-processed $\varphi_f(kT)$ for a fifth-order system $(n = 5)$.

The output of each J-operator is different from the corresponding integrator's output. Note, however, that the $J^{0,n}_{1,0}$-operator is used to compensate for this effect, and as it will be shown in the following, the identification procedure based on the proposed FIR processing has the desired parameter tracking properties.

The performance indices τ and σ (taken from the whole simulation run), defined in (35) and (34), respectively, are shown in Fig. 3. They have been calculated for an experiment with a step input function and for the "FIR-processed" identification algorithms with different selections of the interpolation orders (r, q).

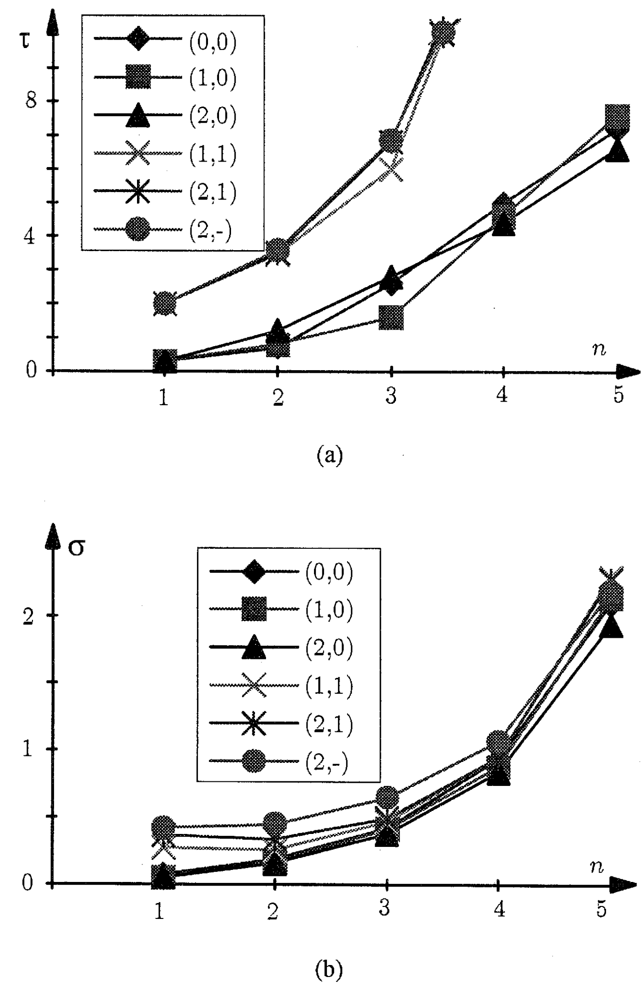


Fig. 3. Settling time τ and estimation loss σ for algorithms (r, q) versus the order of the process n.

The estimation accuracy σ_τ calculated after the settling time τ in experiments with step and ramp input functions are given in Fig. 4. In the step experiment (Figs. 3 and 4) the algorithm with the interpolation pair (r, q) set to $(1, 1)$ yielded the same result as the simple scheme with $r = 1$ and therefore it is not depicted in the figures.

The results corresponding to a sine input are given in Fig. 5. It can be seen from Figs. 4 and 5 that the performance of the algorithms obtained for ramp and sine input functions are virtually the same. Moreover, with both the input functions, tuning the interpolation orders to $(2, 0)$ and $(1, 0)$ led to practically identical results.

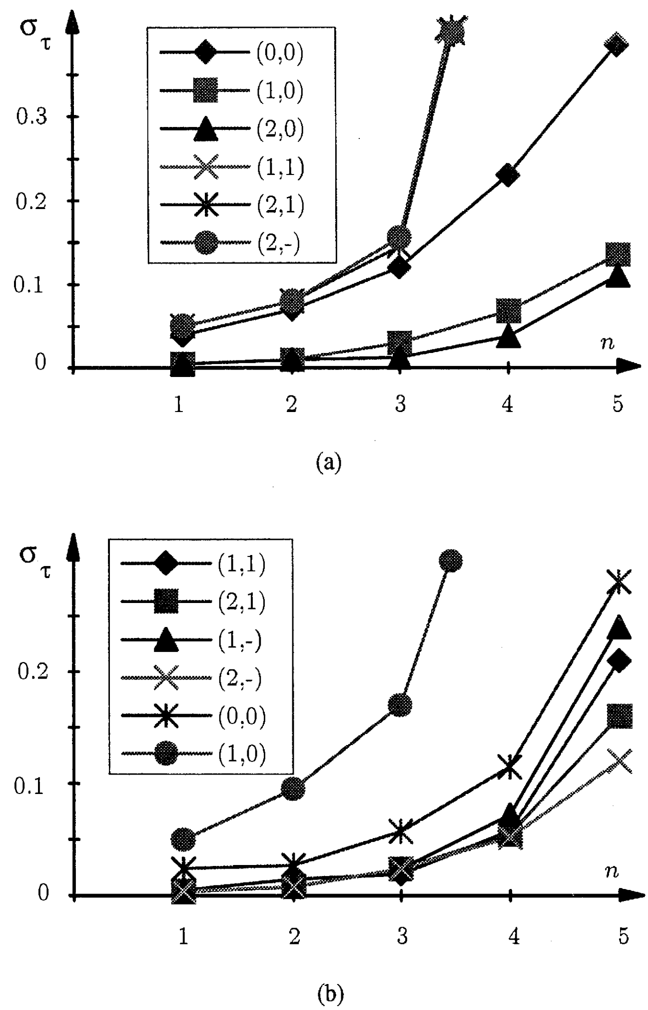


Fig. 4. The estimation accuracy σ_τ versus the order of the process n obtained for step (a) and ramp (b) tests.

As could be expected, in the step experiment, the algorithms with the matched choice $q = 0$ show clear precedence over the unfit case $q = 1$, while with the ramp test the matching choice $q = 1$ results in a superior estimation performance as compared to the inapt case $q = 0$. Note also that a non-matching choice of the interpolation pair (r, q) can have a serious impact on the quality of estimation of the parameters of higher-order systems (see $n = 4,\ 5$).

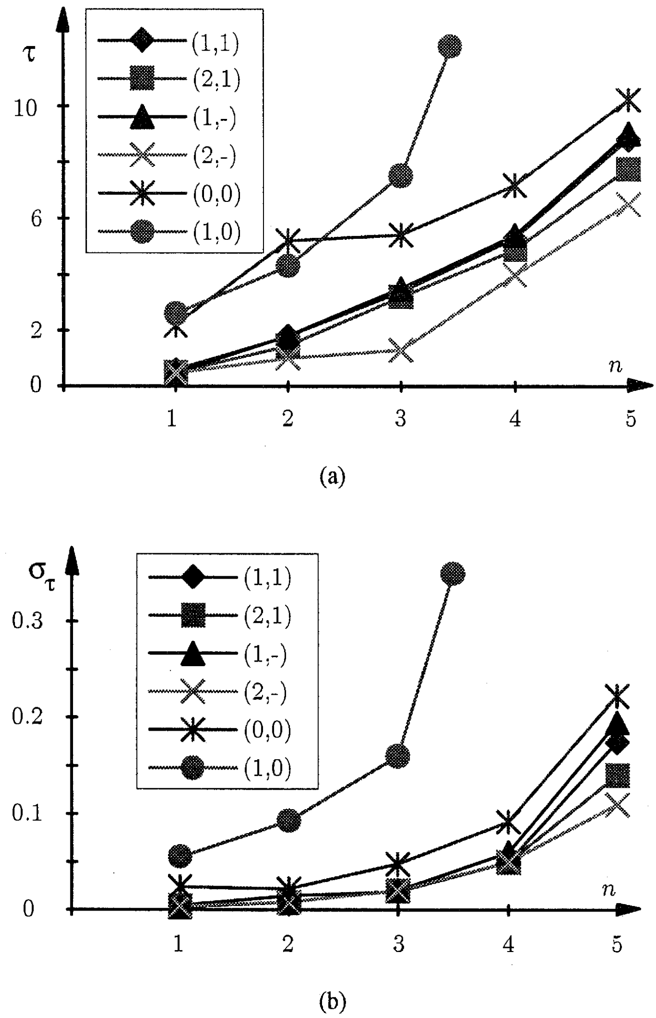


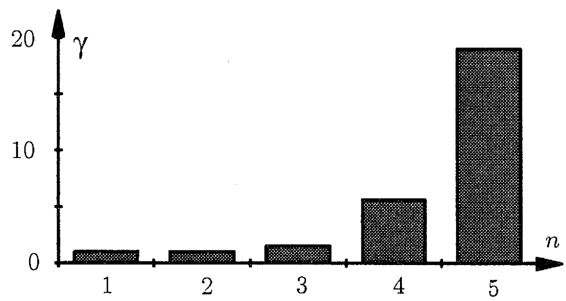
Fig. 5. Effective settling time τ and estimation accuracy σ_τ gained with sine input.

7.2. IIR Approach

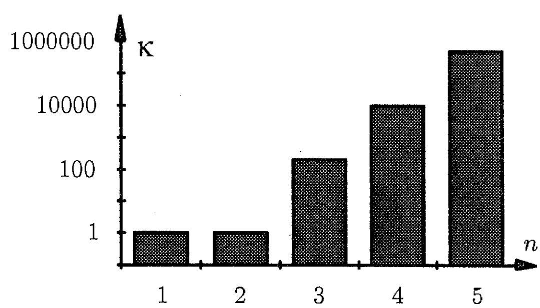
The values of the design parameters: the accommodating coefficient γ and the auxiliary gain coefficient κ of the pre-filter $D_F(z)$ from (30) are depicted in Fig. 6 for different orders n of the identified process.

Transients of the elements of the regression vector $\varphi_F(kT)$, formed via the stabilised IIR-filter processing, during identification of a fourth-order continuous-time system are given in Fig. 7.

Since, in general, the proposed IIR approach gives analogous results to those of the FIR method, we shall only give an illustrative sample of comparison of the two methods.



(a)



(b)

Fig. 6. Pre-filter $D_F(z)$ design: accommodating coefficient γ and auxiliary gain coefficient κ.

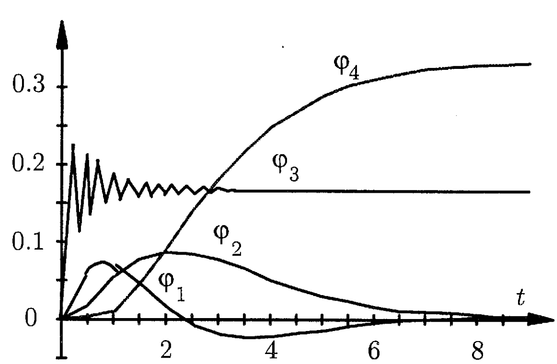


Fig. 7. Transients of regressors for a 4th-order system.

The performance indices τ and σ, given in (35) and (34), respectively, computed for an experiment with a step-wise input function and the identification algorithm with a matching choice of the interpolation orders of the adjusted integration ($r = 1$ for the output signal and $q = 0$ for the input signal), are shown in Figs. 8 and 9.

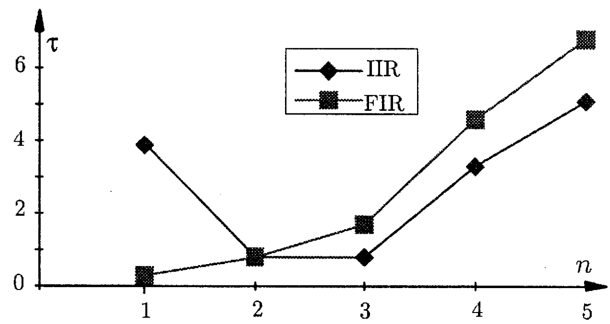
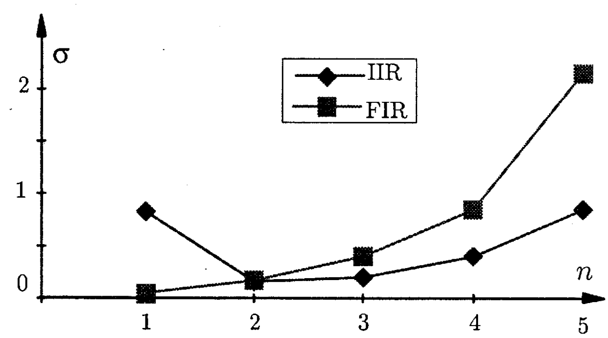
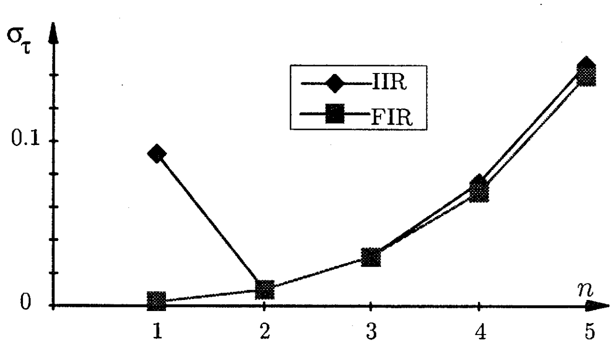


Fig. 8. Comparing FIR and IIR: settling time τ versus the order of the process n.



(a)



(b)

Fig. 9. Comparing FIR and IIR: average estimation errors σ and σ_τ versus the order of the process.

The increased values of the indices obtained for a first order system arise from the resulting Euler backward integration scheme $\Xi_0^1(z) = Tz^{-1}/(1-z^{-1})$ applied to the input signal. Note also that the IIR algorithm, which takes into account matching to the measurement signals on a related basis as the FIR algorithm does, is considerably simpler than the FIR algorithm. The orders of the IIR-filters used for processing the vector φ, for instance, are in practice at least twenty times smaller.

8. Nonstationary Processes

In this section continuous-time processes with variable parameters are considered. Assuming that the observed non-linear plant can have different linear models depending on the value of the input signal applied and that a sufficiently small value of the sampling time can be used, we use a staircase test excitation, and consequently, the simplest matching case of the operators interpolation orders: that of $r = 1$ and $q = 0$. The response of a nonstationary third-order system with the following parameter's change at $t = 10\,[\mathrm{s}]$:

$$\Theta(kT) = \Big[a_1(kT)\; a_2(kT)\; a_3(kT) \,\vdots\, b_0(kT)\Big]^T : \Big[3\;\;3\;\;1 \,\vdots\, 2\Big]^T \to \Big[2\;\;2\;\;1 \,\vdots\, 2\Big]^T$$

obtained for a two-step input function is shown in Fig. 10.

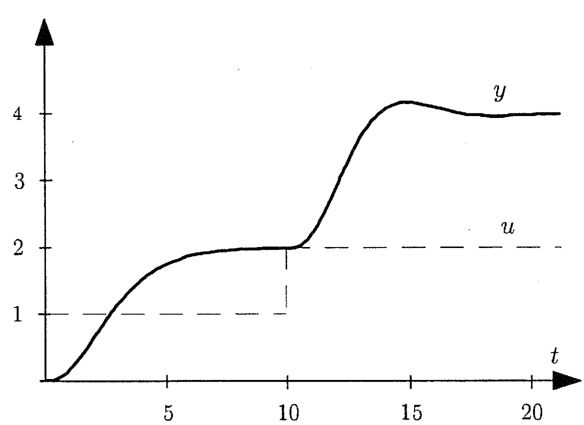
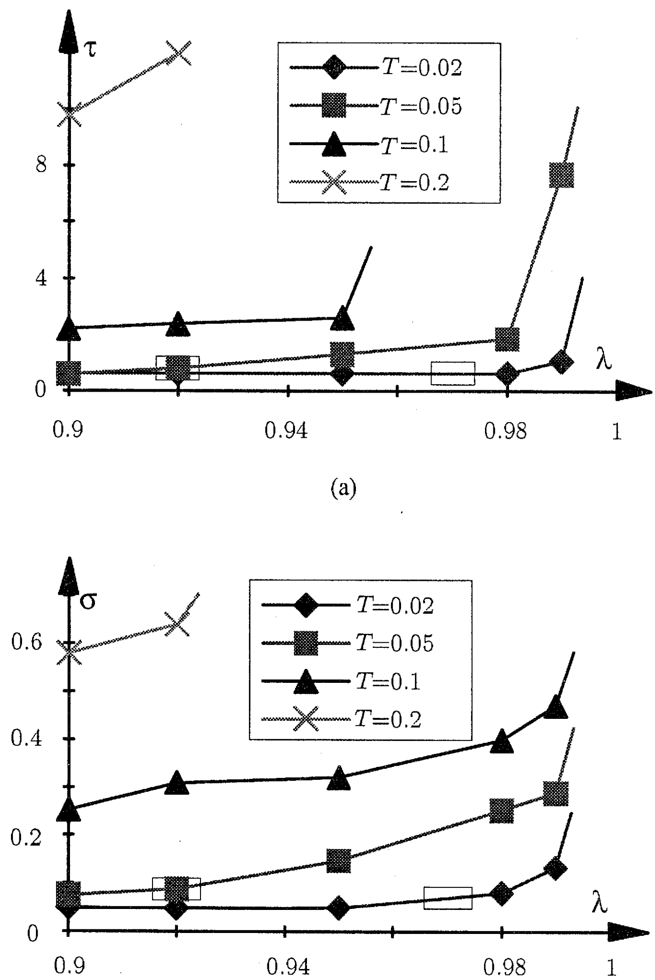


Fig. 10. The input and output trajectories of a nonstationary system.

8.1. Sampling Time T and Forgetting Factor λ

Let us first illustrate the influence of the sampling period T (see eqns. (5), (10), (11)) and the forgetting factor λ (32) on the performance criteria (33)–(35).

The settling time τ and the total estimation error σ for the continuous-time systems of different orders estimated at different sampling periods T using different forgetting factors λ (representing different speeds of adaptation) in two chosen algorithms, denoted by FIRl and IIRγ, are given in Figs. 11–15.



(b)

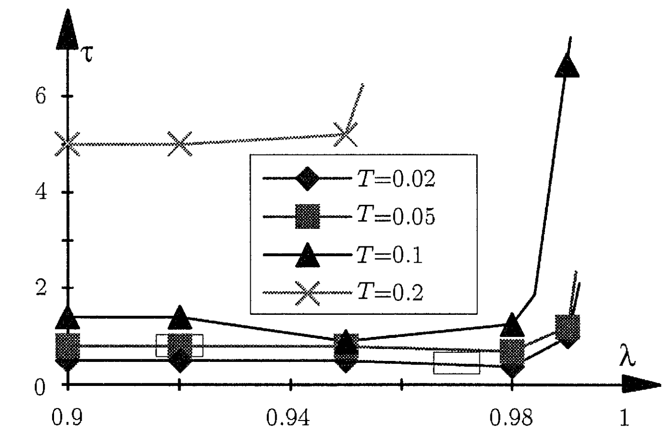
Fig. 11. Algorithm FIR20: Settling time τ and estimation error σ for a 2nd-order system.

Note that by defining an equivalent continuous-time memory length of the considered estimation algorithm

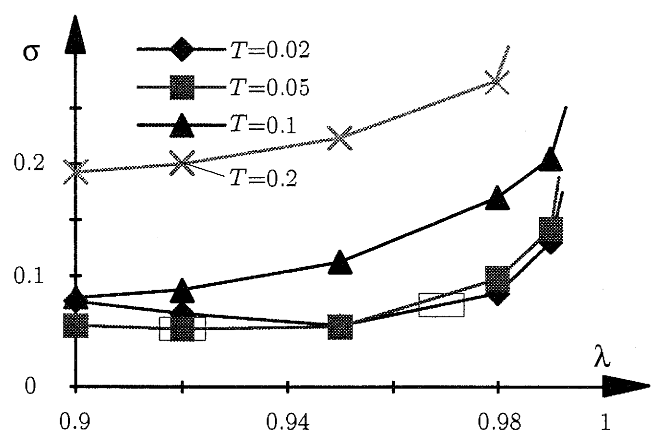
$$\tau_q = N_q T \tag{36}$$

where $N_q = 2/(1-\lambda)$ is the equivalent discrete-time memory length of the EWLS estimator (32), it can readily be observed that a continuous-time memory length should be approximately equal to τ_r,

$$\tau_q \approx \tau_r \tag{37}$$



(a)



(b)

Fig. 12. Algorithm IIR0.11: Settling time τ and estimation error σ for a 2nd-order system.

where τ_r is a rise-time constant of the identified plant, defined in an open-loop operation as the time after which the plant output will be driven to the value of the reference input signal. Based on this reasoning, one obtains the following heuristic rule for tuning the forgetting factor of (32):

$$\lambda \cong 1 - \frac{2T}{\tau_r} \tag{38}$$

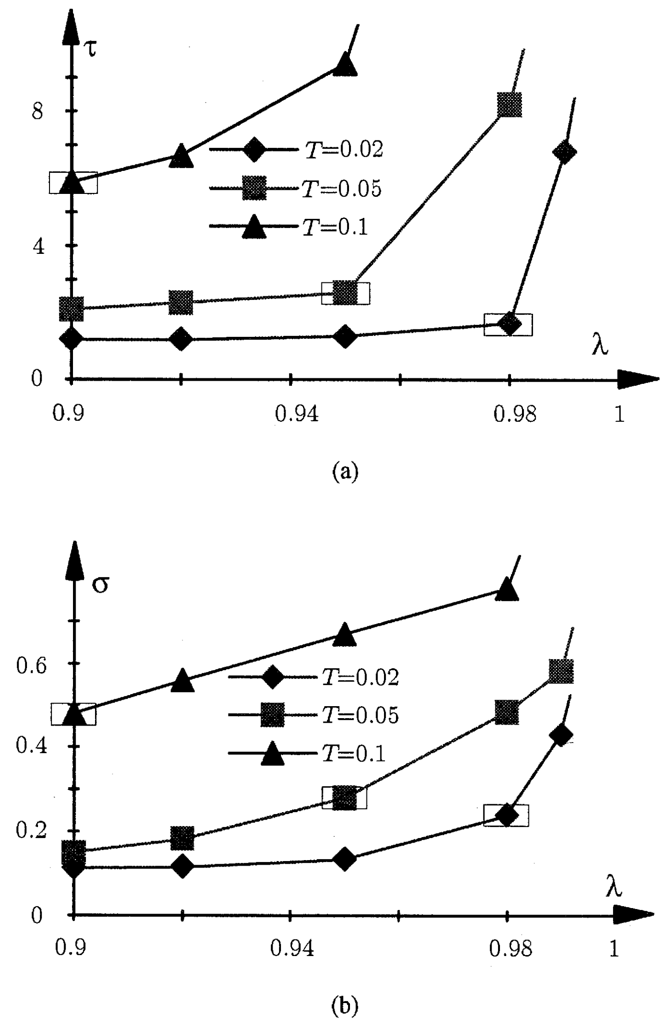


Fig. 13. Algorithm FIR20: Settling time τ and estimation error σ for a 3rd-order system.

Since for the identified continuous-time systems $\tau_r \cong 1.5 \ldots 3.7$ [s], the sampling time from the following well-known guide

$$\frac{\tau_r}{150} < T < \frac{\tau_r}{10} \tag{39}$$

should be $0.01 < T < 0.15$ (for the worst case). The "optimal" values of λ resulting from (38) for the considered sampling times are marked with empty boxes against their respective curves in Figs. 11–15.

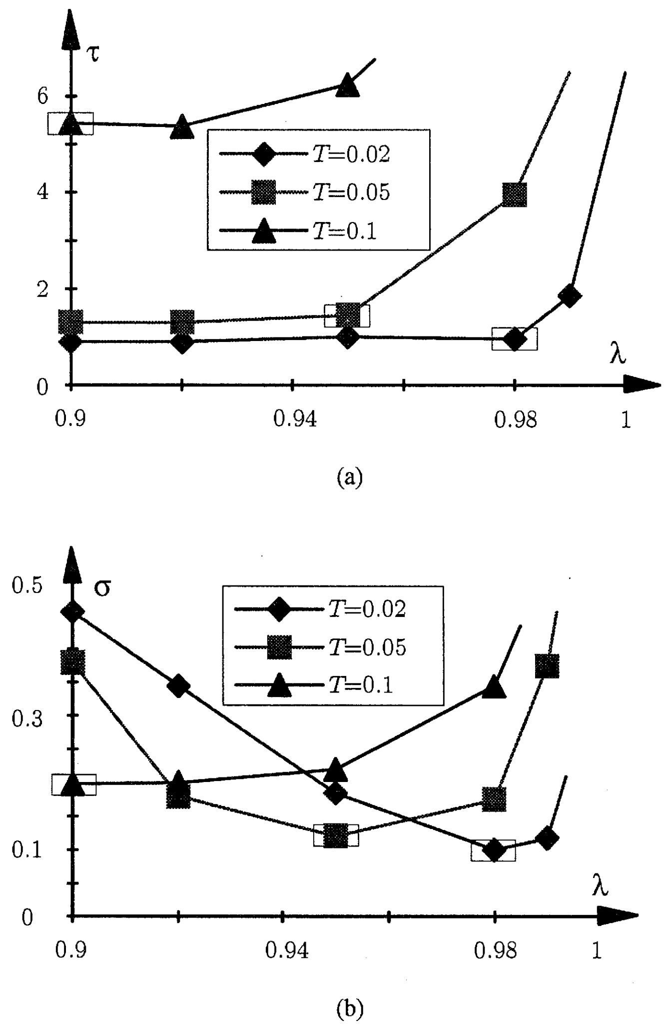
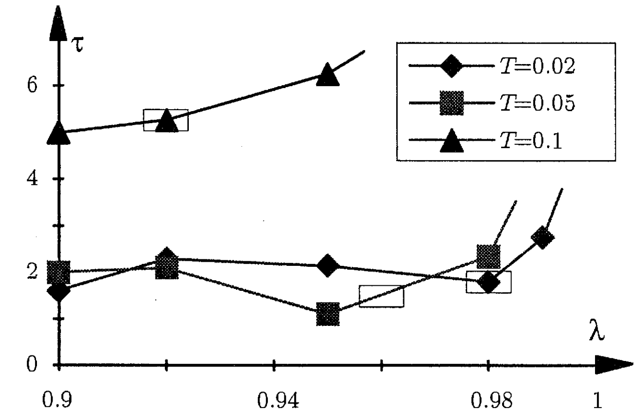


Fig. 14. Algorithm IIR1.5: Settling time τ and estimation error σ for a 3rd-order system.

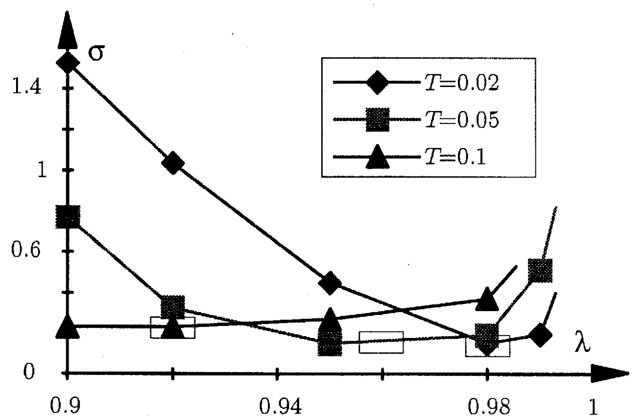
8.2. Comparison of Algorithms for Fixed Sampling Time

The results of a comparative study of the three chosen algorithms FIR20, FIR30, and IIR1.5, performing at the sampling frequency of 20 [Hz], are illustrated in Figs. 16–18.

It can be seen from the above figures that using larger values for the FIR integration horizon l results in a superior accuracy of the parameter estimates after the settling time (τ), but this is obtained at a cost of deterioration and elongation of the



(a)



(b)

Fig. 15. Algorithm IIR5.6: Settling time τ and estimation error σ for a 4th-order system.

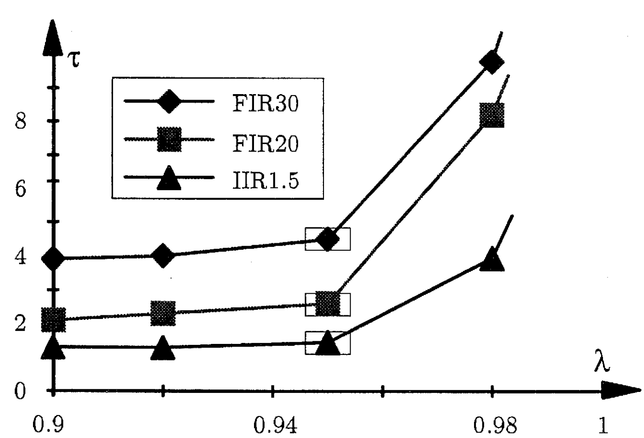


Fig. 16. Settling time τ for estimation of a 3rd-order system.

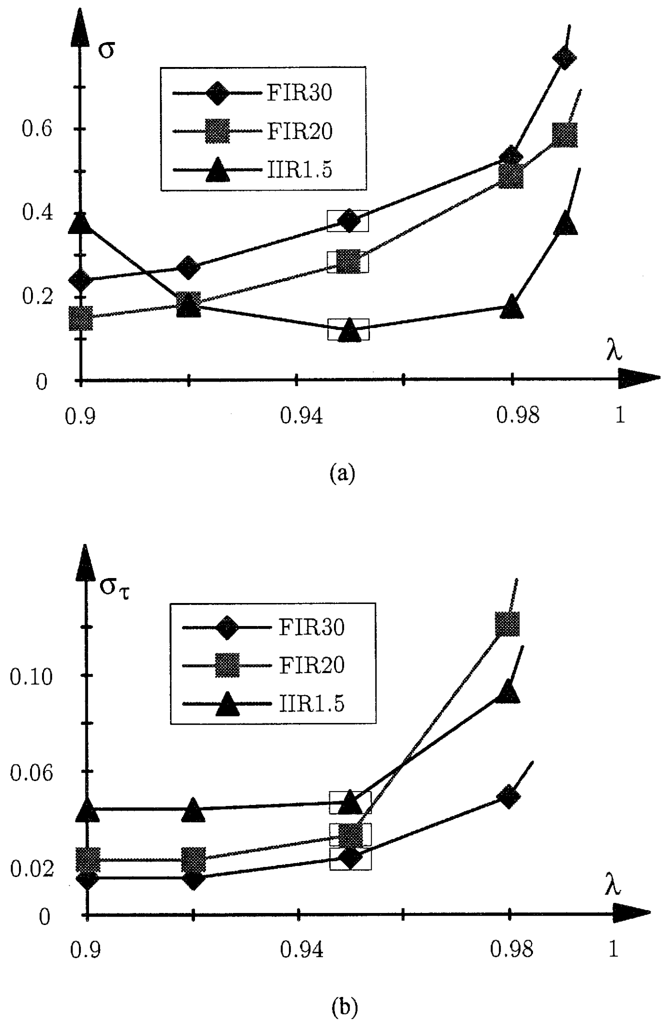


Fig. 17. Estimation errors σ and σ_τ for a 3rd-order system.

transient phase in the estimates' course. In other words, with a longer memory of the limited integration applied one obtains a larger delay in estimation (τ) and a larger global average estimation error (σ), but, at the same time, a smaller estimation error (σ_τ) measured after the settling time (τ). The delay in estimation, as compared to the IIR approach, is clearly seen in Fig. 18. On the other hand, as can be inferred from Fig. 17, the IIR approach leads to a higher sensitivity of the parameter estimation algorithm to small values of λ. In such a case, practical difficulties connected with numerical blow ups should be taken into account (Ljung and Söderström, 1983; Kowalczuk, 1992a; 1992c).

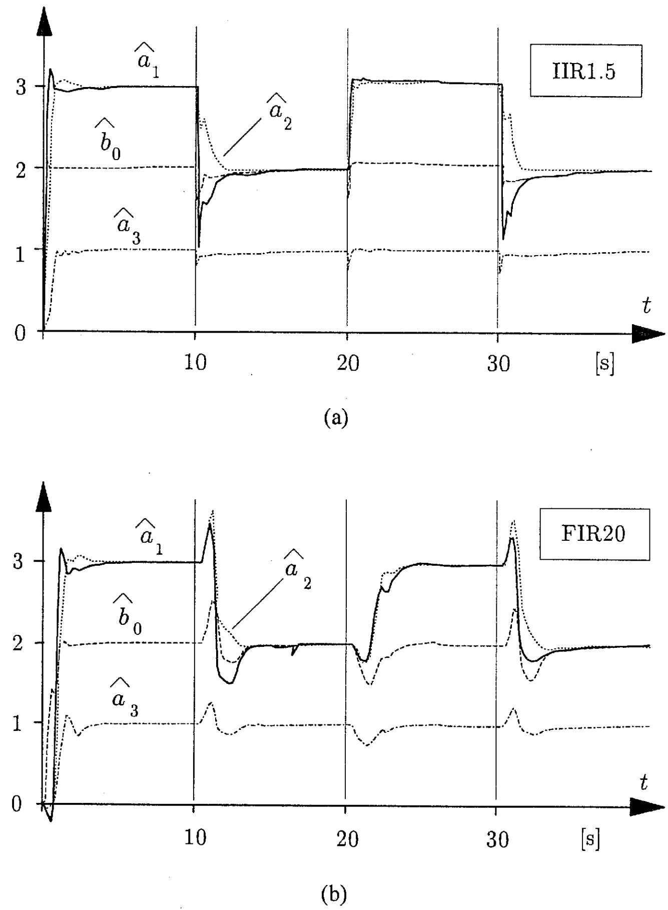


Fig. 18. Tracking the coefficients of a 3rd-order system using $\lambda = 0.95$.

9. Nonstationary Stochastic Systems

In this section, the influence of noise contamination on the identification process is examined. The previously selected algorithms, distinguished by $(r, q) = (1, 0)$, $T = 0.05$ and $\lambda = 0.95$, have been used to estimate a third-order process.

In order to evaluate the effect of a possible contaminating noise process, a discrete-time stochastic generator characterised by a uniformly distributed density function has been used as a source of the noise sequence. The generator has been supplying a stochastic sequence at a multiplicity of the sampling frequency T, at which the EWLS estimator has been executed. The range of the noise distribution can be described by

$$\Delta_u = 10^{-\varpi} \tag{40}$$

where ϖ, given in [dB], will be referred to as the noise range order.

9.1. Discrete-Time Measurement Noise

Let us first consider a case of entirely discrete-time noise generation without any analogue pre-filtering of the sampled-data measurements as shown in Fig. 19, where the noise sequence has been simply decimated during sampling of the process output. A representation of results is given in Fig. 20.

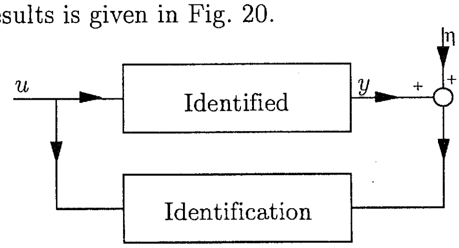


Fig. 19. System with discrete-time output measurement noise.

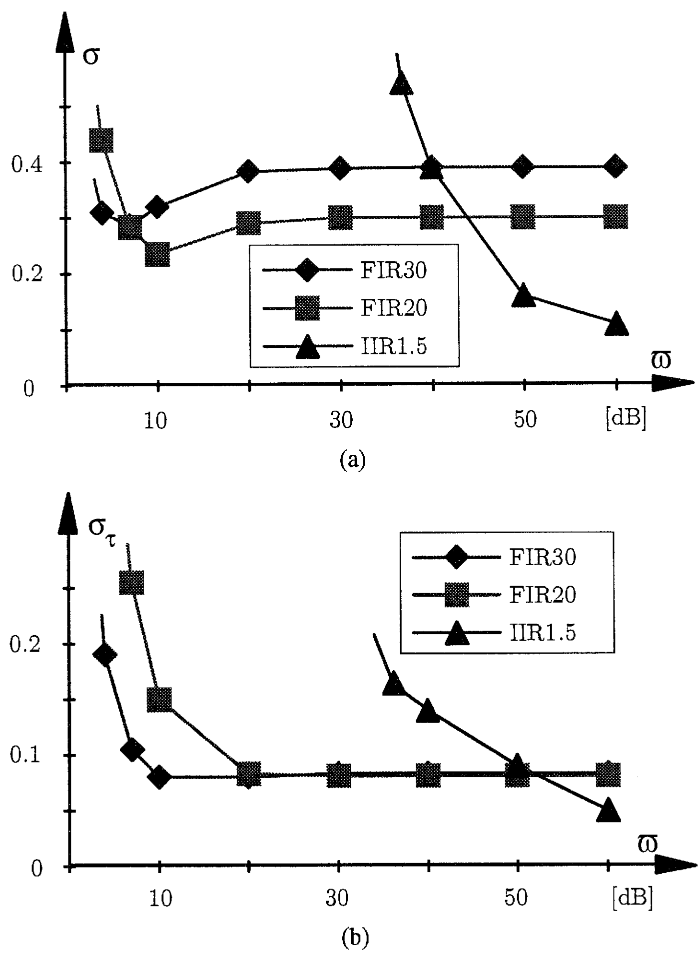


Fig. 20. Estimation errors σ and σ_τ of the algorithms versus the noise range order ϖ.

9.2. Guard Pre-filtering in the Presence of Quasi-Analogue Measurement Noise

In order to simulate a quasi-analogue noise disturbance, the output of the noise generator has been filtered by means of a noise-shaping low-pass continuous-time filter, as depicted in Fig. 21,

$$V(s) = \frac{1}{1 + T_v s} \tag{41}$$

where $T_v = 0.1$ [s]. In order to guarantee the prerequisite conditions for the sampling process, two guard pre-filters of the same lowpass type are applied at the input and output of the observed plant (see Fig. 21):

$$F_f(s) = \frac{1}{1 + T_f s} \tag{42}$$

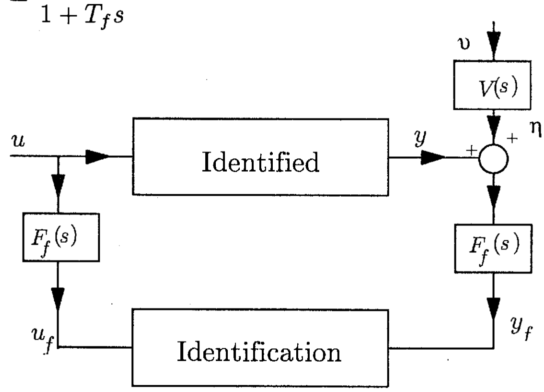


Fig. 21. System with quasi-analogue noise.

It is clear from Figs. 20, 22 and 23 that the algorithms equipped with the guard analogue pre-filters, characterised by time constant T_f, can considerably improve their robustness to a contaminating quasi-analogue noise. An additional time constant $T_f' = 0.1$ [s] introduced to the guard pre-filter with $T_f = 1$ [s], which processes the data for the IIR algorithm marked with "1+" in Fig. 22, brings about a further improvement in this respect.

Note that the pre-filtering can impair the effect induced by the matching choice of the interpolation orders (r, q) for large values of ϖ, which represent noise-free systems. This problem, however, can easily be solved by using a new suitable matching pair (r, q) (see also the results given in Fig. 27).

A comparison of the three chosen algorithms is shown in Fig. 24 and 25. In the latter, which summarises the study of the forgetting factor λ, the suggested values of λ chosen based on (38) are also marked with empty boxes.

The above considered algorithms, used with various settings of the interpolation orders (r, q), have been tested in identification of the continuous-time systems of various orders n (with the design and simulation parameters: $\lambda = 0.95$ and $\lambda = 30$ [dB]). The corresponding results are given in Figs. 26 and 27.

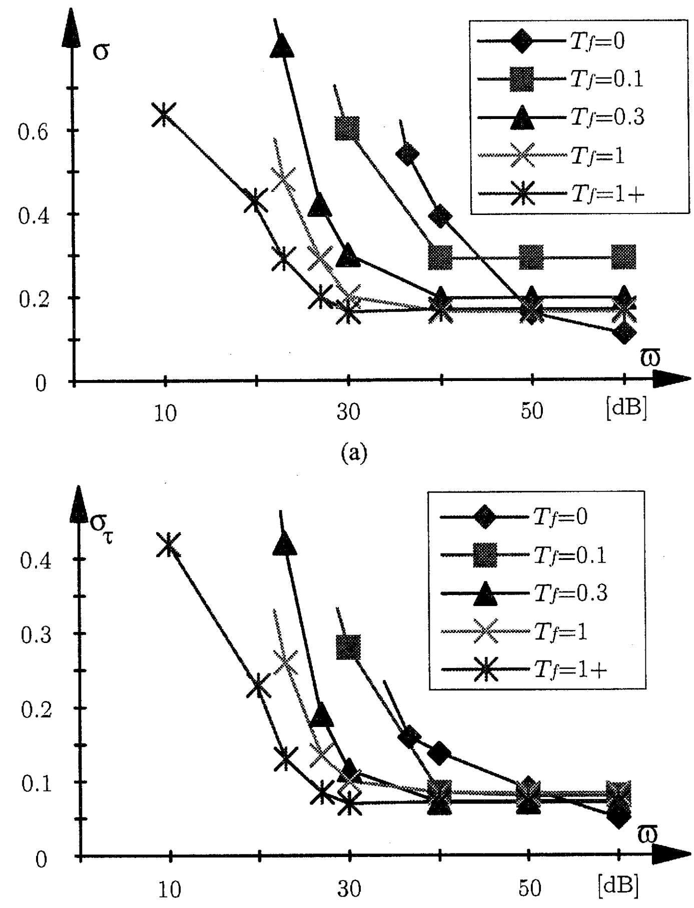


Fig. 22. Estimation errors σ and σ_τ for algorithm IIR1.5 versus noise range order ϖ.

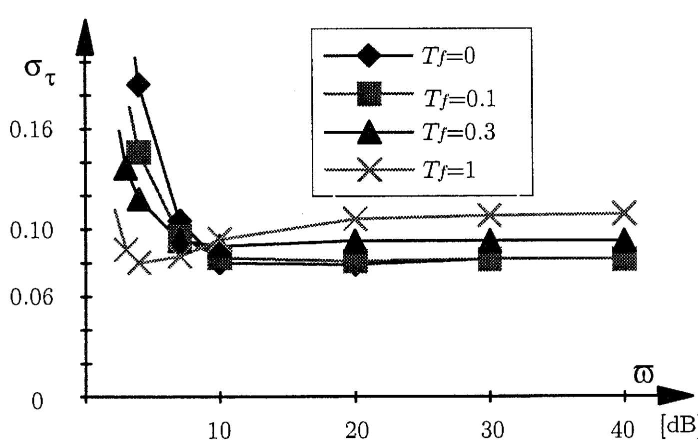
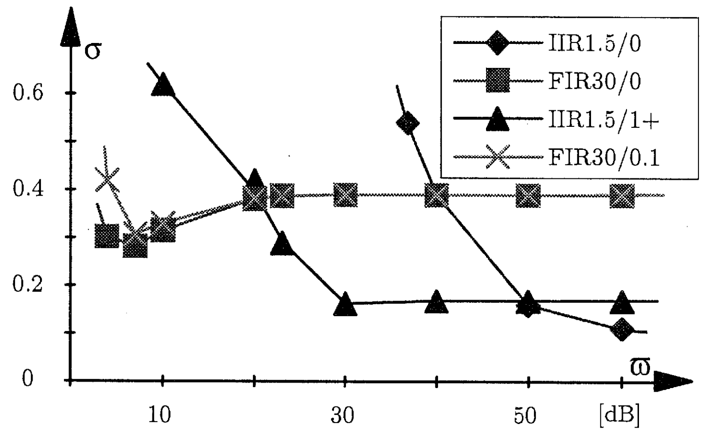
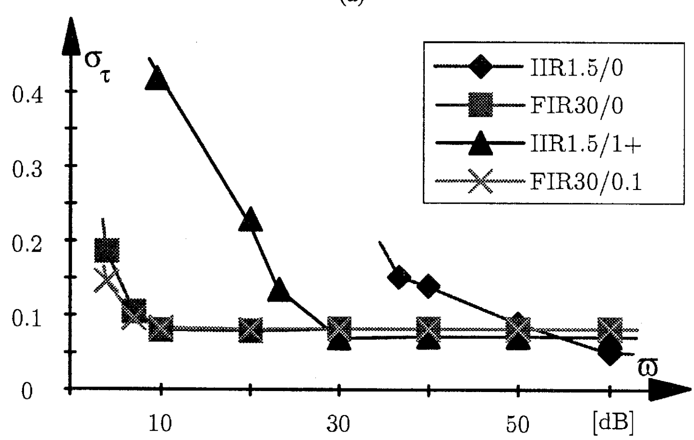


Fig. 23. Estimation error σ_τ for algorithm FIR30 versus the noise range order ϖ.



(a)



(b)

Fig. 24. Comparison of estimation errors σ and σ_τ of the algorithms versus the order ϖ.

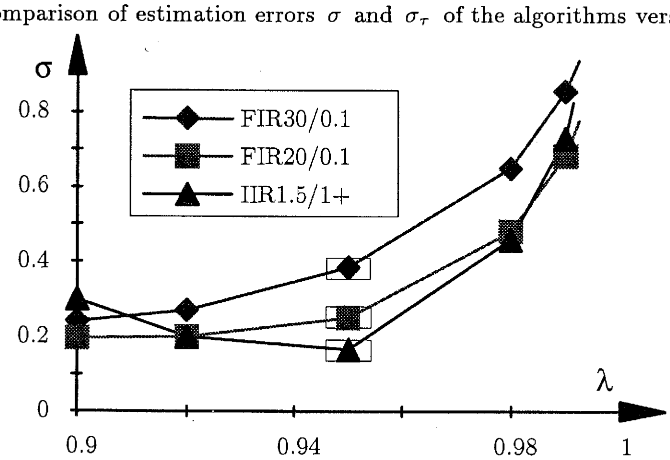


Fig. 25. Estimation error σ of the algorithms vs. the forgetting factor λ (noise order: $\varpi = 30\,[\mathrm{dB}]$).

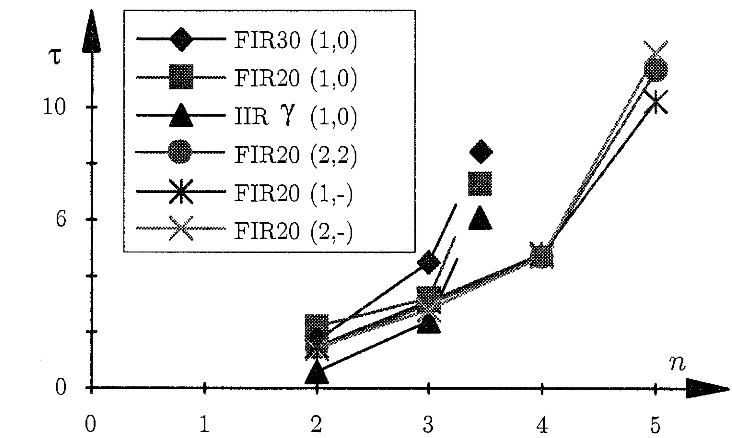
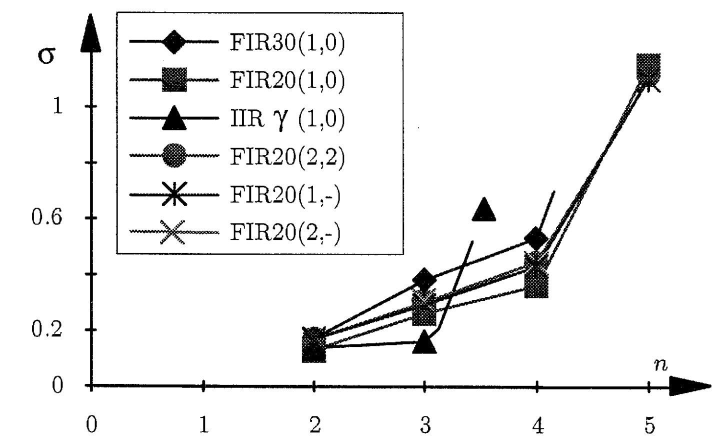
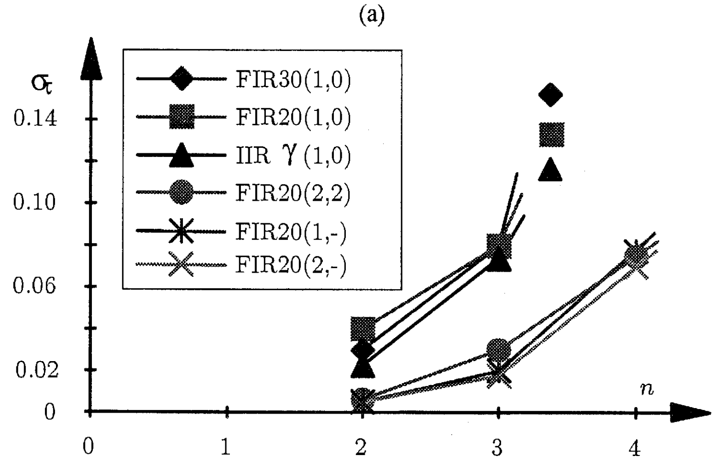


Fig. 26. Settling time with different algorithms (r, q) for various system orders n.



(a)



(b)

Fig. 27. Estimation errors σ and σ_τ with the algorithms (r, q) for various process orders n.

Note that with the stepwise test and pre-filtering included, the necessity of adjusting the interpolation orders is apparent. It also needs to be explained that in the case of fifth-order systems the settling time τ exceeds the period of 10 seconds and therefore the corresponding results are missing in the plot of σ_τ in Fig. 27.

9.3. Plants with Internal Noise Injection

Finally, various schemes of internal plant noise corruption have been considered for a third-order continuous-time system mechanised in a cascaded form shown in Fig. 28. The same type of analogue filtering as in Section 9.2 has been used and the sampling time has been fixed at $T = 0.05$ [s] for parameter estimation of stationary and nonstationary plants.

The stationarity case considered is summarised in Fig. 29, whereas the results concerning nonstationary processes are shown in Fig. 30. Note that in both cases the internal-plant noise injection has an advantageous effect on the performance of the considered identification algorithms using the regression-vector's processing of both FIR and IIR types.

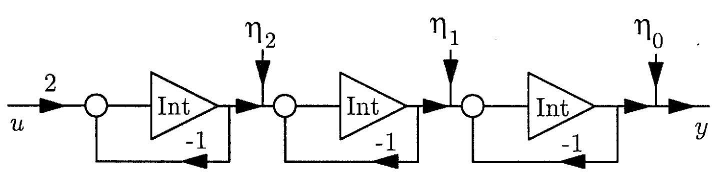


Fig. 28. Cascaded plant with different noise injections.

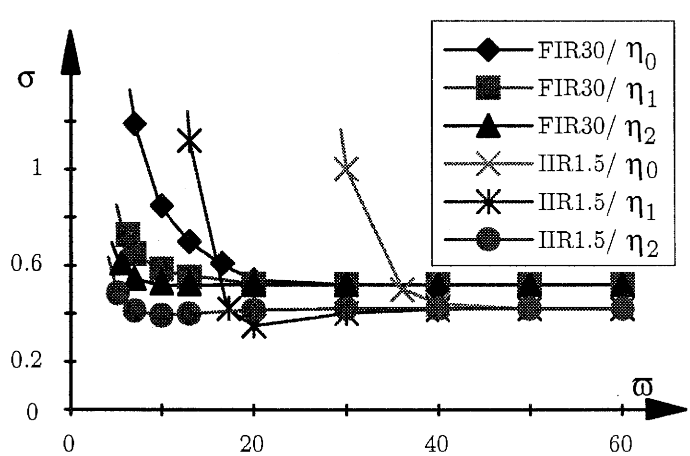


Fig. 29. LS-estimation error σ for stationary plants with different noise injection vs. the noise range order ϖ [dB].

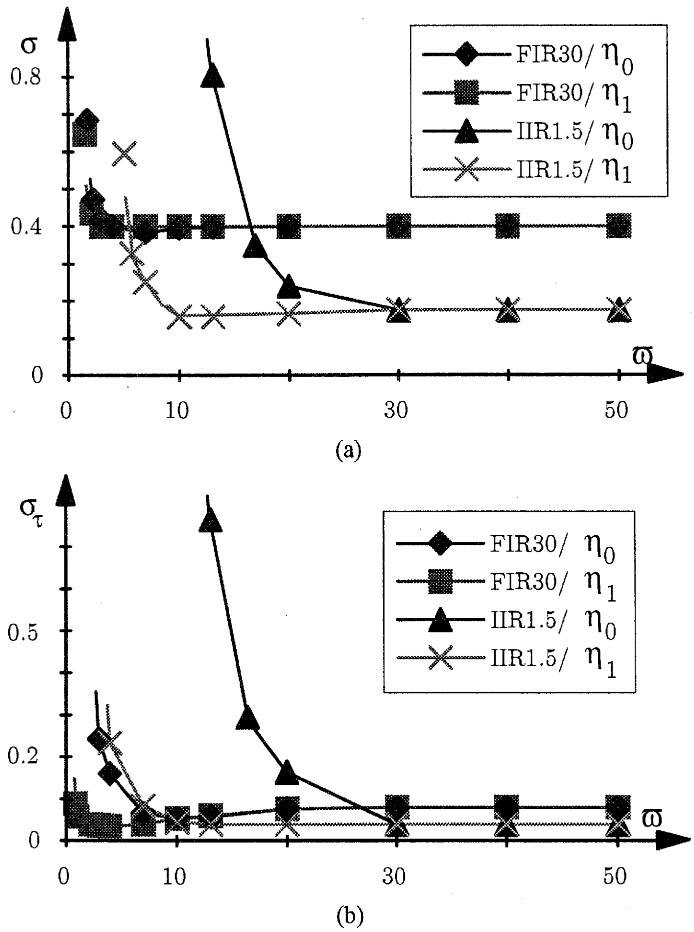


Fig. 30. EWLS-estimation errors σ and σ_τ (at $\lambda = 0.98$) for stationary plants with different noise injection vs. the noise range order ϖ [dB].

10. Concluding Remarks

As a result of the transformation applied (adjusting: matching plus tempering) the pure integration in formation of the regression vector disappears and is replaced either by finite impulse response filters (the FIR approach) with a limited horizon of integration or by approximate stable IIR filters (without any integration pole at $z = 1$).

In terms of the settling time and estimation errors, the simulation results indicate clear advantages of the proposed FIR method over the classical approach, when the type of the observed process input and output signals can be approximately estimated and, based on this, an appropriate choice of the interpolation parameters r and q can be made.

For instance, in the step experiment, the algorithms with the matched choice $q = 0$ have apparent precedence over the case of $q = 1$, while for the ramp test the matching choice $q = 1$ results in a superior estimation performance compared with the case of $q = 0$. Moreover, it is clear that a non-matching choice of the interpolation pair (r, q) can have an extremely negative impact on the quality of estimation of the parameters of higher-order systems.

It is also worth noticing that using a longer memory (l) of the limited (FIR) integration applied one obtains a larger delay in estimation (τ) and a larger global average estimation error (σ) resulting from the deterioration and elongation of the transient phase in the estimates' course, but, at the same time, a smaller estimation error (σ_τ) measured after the settling time (τ) and indicating a better accuracy of the parameter estimates.

Certainly, the simplified approach IIR offers a lower computational load and good transients of estimate for low-order systems, but some precautions have to be taken against possible signal contamination by a system noise, especially against a measurement noise.

Appendix

A. Regression Vector Formation Algorithms

I. FIR (expanded)

$$\varphi_f^T(k) = \Big[-J_{r,q}^{1,n} y_f(k) \;\; -J_{r,q}^{2,n} y_f(k) \;\; \cdots \;\; -J_{r,q}^{n,n} y_f(k) \;\vdots$$

$$J_{q,r}^{n-m,n} u_f(k) \;\; J_{q,r}^{n-m+1,n} u_f(k) \;\; \cdots \;\; J_{q,r}^{n,n} u_f(k)\Big]$$

$$J_{r,q}^{i,n}(z) = \frac{r!T^i}{(r+i)!} N_{r+i}(z^{-1}) N_q(z^{-1}) P_l^i(z^{-1})(1 - z^{-l})^{n-i}$$

for $i = 1, \ldots, n$ and

$$J_{r,q}^{0,n}(z) = N_r(z^{-1}) N_q(z^{-1})(1 - z^{-l})^n$$

II. FIR (simple)

$$\varphi_f^T(k) = \Big[-J_r^{1,n} y_f(k) \;\; -J_r^{2,n} y_f(k) \;\; \cdots \;\; -J_r^{n,n} y_f(k) \;\vdots$$

$$J_r^{n-m,n} u_f(k) \;\; J_r^{n-m+1,n} u_f(k) \;\; \cdots \;\; J_r^{n,n} u_f(k)\Big]$$

$$J_r^{i,n}(z) = \frac{T^i}{(r+1)^i} N_{r+1}^i(z^{-1}) N_r^{n-i}(z^{-1}) P_l^i(z^{-1})(1-z^{-l})^{n-i}$$

for $i = 1, \ldots, n$ and

$$J_r^{0,n}(z) = N_r^n(z^{-1})(1-z^{-l})^n$$

III. IIR (expanded)

$$\varphi_f^T(k) = \Big[-J_{r,q}^{1,n} y_F(k) \;\; -J_{r,q}^{2,n} y_F(k) \;\; \cdots \;\; -J_{r,q}^{n,n} y_F(k) \;\vdots\; J_{q,r}^{n-m,n} u_F(k) \;\; J_{q,r}^{n-m+1,n} u_F(k) \;\; \cdots \;\; J_{q,r}^{n,n} u_F(k) \Big]$$

$$J_{r,q}^{i,n}(z) = \frac{r!T^i}{(r+i)!} N_{r+i}(z^{-1}) N_q(z^{-1})(1-z^{-1})^{n-i}$$

for $i = 1, \ldots, n$ and

$$J_{r,q}^{0,n}(z) = N_r(z^{-1}) N_q(z^{-1})(1-z^{-1})^n$$

where $x_F(k) = D_F x_f(k)$ and $D_F(z) = \kappa/(\gamma + J_{r,q}^{0,n}(z))$.

B. Constituent Polynomials

(a) The l-delay compensator polynomials

$$P_l(z^{-1}) = \frac{1-z^{-l}}{1-z^{-1}} = 1 + z^{-1} + z^{-2} + \cdots + z^{-(l-1)}$$

(b) The normal polynomials (Kowalczuk, 1983a; 1983b; 1993b)

$$N_r(z^{-1}) = N_r(z) = \sum_{j=0}^{r-1} a_{r-j}^r z^j \quad \text{for} \quad r = 1, 2, \ldots, \quad \Big(\text{and} \quad N_0(z) = N_1(z) \equiv 1\Big)$$

where

$$a_j^r = (r+1-j)a_{j-1}^{r-1} + j a_j^{r-1} \qquad \text{for } r = 2, 3, \ldots, \;\; j = 1, 2, \ldots, r$$

Note that the sum of the weighting coefficients in the calculation of a_j^r is constant $(r+1)$ for a given normal polynomial of order $(r-1)$.

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