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PARAMETER ESTIMATION BASED FAULT DETECTION AND ISOLATION IN WIENER AND HAMMERSTEIN SYSTEMS

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Fault detection and isolation in Wiener and Hammerstein systems via generation and processing of residual sequences is considered. We assume that some models of the unfaulty Wiener and Hammerstein systems under consideration are known. For Wiener systems, we also assume that their static nonlinear subsystems are invertible. Then, based on a serial-parallel definition of the residual error, new fault detection and isolation methods are proposed. To detect and identify all the changes in both the Wiener and Hammerstein system parameters, the sequences of residuals are processed by using linear regression methods or a neural network approach.

Keywords: fault detection, fault isolation, parameter estimation, neural networks, nonlinear systems.

1. Introduction

Model-based fault detection and isolation (FDI) has been investigated intensively for the last two decades. Many different model-based FDI methods have been elaborated using the following two-step procedure. First, by observing the actual system and assuming that its nominal model is known, sequences of residuals are generated and the fault detection step is performed. Then, if any fault occurs, the residuals are processed in the fault isolation step. Note that to isolate a fault, we also need a model of the faulty system. The model-based approach is one of the most common approaches to the FDI problem. There are various solutions available including the parity space, dedicated observer, fault detection filter, and parameter estimation based approaches (Frank, 1990).

The Wiener and Hammerstein models are two examples of simple nonlinear structures composed of a linear dynamic system in cascade with a static nonlinear element. While in the Wiener model the linear dynamic system precedes the static nonlinear element, in the Hammerstein model the same blocks are connected in reverse order (Fig. 1). The nonlinear systems that can be modelled as Wiener and Hammerstein models are widely encountered in different areas including e.g. industry, biology, sociology or psychology. The pH neutralization process is a well-known example of the

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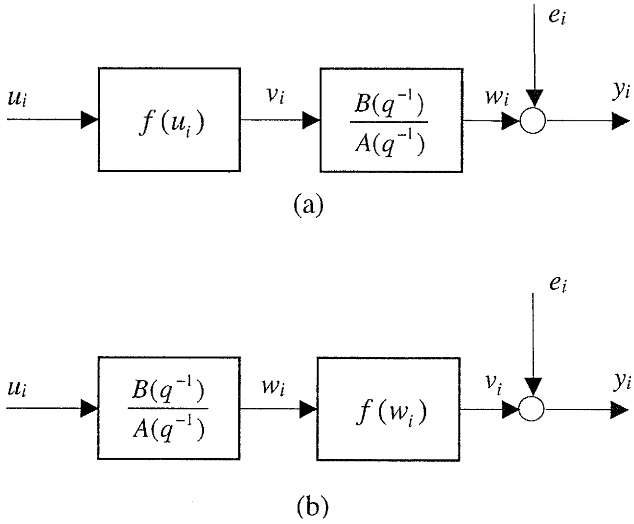

Fig. 1. The SISO Hammerstein (a) and Wiener (b) system.

Wiener system (Kalafatis *et al.*, 1997; Nie and Lee, 1998). Other examples include systems with nonlinear sensors (Wigren, 1994), optimal control systems, fluid flow control (Wigren, 1994), or electrical resistance furnaces (Skoczowski, 1998). Some biological processes can also be considered to be Wiener systems, e.g. a muscle relaxation process (Drewelow *et al.*, 1997), see (Hunter and Korenberg, 1986) for more biological examples. The Hammerstein model (Zi-Qiang, 1994) can describe systems with nonlinear actuators. Two industrial examples of Hammerstein systems are a distillation column and a heat exchanger (Eskinat *et al.*, 1991). Due to their simple block-oriented structure, the models of Wiener and Hammerstein systems are very useful in engineering practice, particularly in the controller design problem.

Identification of Wiener and Hammerstein systems has been investigated intensively for the past few decades. Several methods that have been developed can be subdivided into the following four classes:

A. *Correlation methods.* These methods are based on the theory of separable processes (Billings and Fakhouri, 1978; 1982). The correlation approach makes it possible to separate identification of the linear dynamic system from that of the nonlinear element. For both Wiener and Hammerstein systems, the first-order correlation function is proportional to the linear system impulse response. Then, the linear system impulse response can be parameterized using well-known linear regression methods. Testing the second-order correlation function provides valuable information about the system structure. If the second-order correlation function is equal to the first-order one, up to a constant of proportionality, the system is of Hammerstein type. For Wiener systems, the second-order correlation function is the square of the first-order one, except for a constant of proportionality. Having the linear dynamic system identified, the static nonlinear element can be identified easily by assuming that it can be expressed as a polynomial of a finite and known order and using linear regression techniques.

B. *Linear regression methods.* The linear regression solutions are based on the restrictive assumption that the static nonlinear element can be represented by a series expansion, commonly a polynomial, of a finite and known order. For Hammerstein models, Narendra and Gallman (1966) and Chang and Luus (1971) developed least-squares identification algorithms. Similar algorithms can be used to identify an inverse Wiener model, but this requires the static nonlinear element to be invertible. Another approach presented here is based on a modified definition of the identification error and the assumption of the invertibility of the static nonlinear element.

C. *Nonparametric regression methods.* The class of Wiener and Hammerstein systems which are identified using the correlation or linear regression methods is restricted by the assumption that the nonlinear function $f(\cdot)$ of the static nonlinear element is both continuous and can be expressed as a polynomial of a finite and known order r. This assumption is no longer necessary if we use nonparametric identification methods. For Hammerstein models, the kernel regression estimate

$$\hat{f}(u) = \frac{\sum_{j=0}^{N-1} y_{j+1} K\left(\frac{u-u_j}{l(N)}\right)}{\sum_{j=0}^{N-1} K\left(\frac{u-u_j}{l(N)}\right)} \tag{1}$$

where u_j and y_j are respectively the system input and output, K is a kernel function, and $l(N)$ is a sequence of positive numbers, converges to $f(\cdot)$ as the number of observations increases to infinity (Greblicki and Pawlak, 1986). Greblicki (1994; 1997) applied nonparametric regression methods also to the Wiener system identification.

D. *Neural network approach.* Neural network models of Wiener and Hammerstein systems do not require the power series expansion of the function $f(\cdot)$ to be of a finite order. They consist of a nonlinear multilayer perceptron used as a model of the static nonlinear element and a single linear node used as a model of the linear dynamic system (Janczak, 1995; Korbicz and Janczak, 1996). The serial-parallel models of a feedforward structure can be trained with the computationally effective static backpropagation algorithm. It can be shown that if a system output is corrupted by additive white noise, identification of a serial-parallel model results in correlated residuals. To overcome this, parallel models of recurrent type can be used. As in this case the static backpropagation fails in computing the true gradient, other learning algorithms such as the sensitivity method or backpropagation through time, see (Janczak, 1997a) for Wiener and (Janczak, 1997b) for Hammerstein models, should be used. Both Wiener and Hammerstein models contain a linear dynamic system. Thus, it is also possible to combine the computationally effective least-squares method with gradient descent algorithms, see (Janczak, 1998b) for Wiener and (Al-Duwaish *et al.*, 1997; Janczak, 1998a), for Hammerstein models.

The main idea of this paper is to use known modelling and parameter estimation techniques of Wiener and Hammerstein systems for the model-based FDI. The fault is understood here as an unacceptable change in system parameters. The paper is mainly devoted to parameter estimation based methods. We assume that a perfect model of the system at its nominal (unfaulty) state is known. This model can be defined by parametric, non-parametric or neural network representations. It is clear that faults change the system behaviour. A change of the nonlinear function $f(\cdot)$ and changes in the linear sub-system parameters can express this change in behaviour. The purpose of the paper is to develop fast and reliable estimation methods of system parameter changes caused by system faults.

The paper is organized as follows. First, in Section 2, the problem formulation is given. In Section 3, we discuss different identification error definitions and introduce a modified serial-parallel error definition for Wiener systems. Generation of residual sequences with parallel and serial-parallel models of Wiener and Hammerstein systems is discussed in Section 4. In Section 5, a neural network Residual Generator Model (RGM) is proposed to estimate changes in system parameters. In Section 6, we present linear regression-based estimation methods of the RGM parameters. Section 7 contains five simulation examples for both Wiener and Hammerstein systems. Finally, a few concluding remarks are given in Section 8.

2. Problem Formulation

The problem considered here can be stated as follows: Given system input and output sequences, generate a sequence of residuals using the known model of the unfaulty Wiener or Hammerstein system, and process this sequence to detect and isolate all the changes in system parameters caused by any system fault. Both abrupt (step-like) and incipient (slowly developing) faults are to be considered. Assume that the models of the Wiener or Hammerstein system defined by a nonlinear function $f(\cdot)$ and polynomials $A(q^{-1})$, $B(q^{-1})$ are known,

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \tag{2}$$

$$B(q^{-1}) = b_1 q^{-1} + \cdots + b_m q^{-m} \tag{3}$$

where q^{-1} denotes the backward shift operator. These models describe the systems in their normal operating conditions with no malfunctions (faults). Moreover, assume that at the time moment i a step-like fault occurred, which caused a change of mathematical model of the system. This change can be expressed in terms of additive components of the pulse transfer function polynomials as follows:

$$\Delta A(q^{-1}) = \alpha_1 q^{-1} + \cdots + \alpha_n q^{-n} \tag{4}$$

$$\Delta B(q^{-1}) = \beta_1 q^{-1} + \cdots + \beta_m q^{-m} \tag{5}$$

For Hammerstein systems, we assume that the characteristic of the static nonlinear element $g(\cdot)$ can be expressed as

$$g(u_i) = f(u_i) + \Delta f(u_i) \tag{6}$$

where $\Delta f(\cdot)$ can be parameterized as follows:

$$\Delta f(u_i) = \eta_0 + \eta_2 u_i^2 + \eta_3 u_i^3 + \cdots \tag{7}$$

Note that $g(\cdot)$ is assumed to be properly scaled in such a way that the polynomial $\Delta f(\cdot)$ has no first-order term, i.e. $\eta_1 = 0$.

For Wiener systems, it is assumed that both $f(\cdot)$ and $g(\cdot)$ are invertible. The inverse function $g^{-1}(\cdot)$ can be written as a sum of the inverse function $f^{-1}(\cdot)$ and its change (deviation) $\Delta f^{-1}(\cdot)$:

$$g^{-1}(y_i) = f^{-1}(y_i) + \Delta f^{-1}(y_i) \tag{8}$$

where

$$\Delta f^{-1}(y_i) = \lambda_0 + \lambda_2 y_i^2 + \lambda_3 y_i^3 + \cdots \tag{9}$$

Note that $\Delta f^{-1}(\cdot)$ here does not denote the inverse of $\Delta f(\cdot)$ but only a change of $f^{-1}(\cdot)$. It is also assumed that $\lambda_1 = 0$ unless the expression (8) can be properly scaled.

3. Definition of the Identification Error

3.1. Hammerstein Systems

The residual e_i is defined as a difference between the output of the system y_i and the output of its nominal model $\hat{y}_i$,

$$e_i = y_i - \hat{y}_i \tag{10}$$

The output of the nominal serial-parallel model is given by the following expression:

$$\hat{y}_i = \left[1 - A(q^{-1})\right] y_i + B(q^{-1}) f(u_i) \tag{11}$$

3.2. Wiener Systems

For Wiener systems with an invertible static nonlinear subsystem, the following modified definition of the residual error is introduced:

$$e_i = f^{-1}(y_i) - f^{-1}(\hat{y}_i) \tag{12}$$

where $f^{-1}(\cdot)$ is a known inverse of the static nonlinear function $f(\cdot)$ of the known nominal Wiener model, $\hat{y}_i$ denotes its output, and y_i is the output of the unknown Wiener system. The inverse of the static nonlinear function $f(\cdot)$ is given by the following serial-parallel model:

$$f^{-1}(\hat{y}_i) = \left[1 - A(q^{-1})\right] f^{-1}(y_i) + B(q^{-1})(u_i) \tag{13}$$

We assume that the function $f^{-1}(\cdot)$ can be expressed as a polynomial:

$$f^{-1}(y_i) = y_i + \gamma_2 y_i^2 + \cdots \tag{14}$$

Moreover, the actual Wiener system can be described by the expression

$$y_i = g\left(\frac{B(q^{-1}) + \Delta B(q^{-1})}{A(q^{-1}) + \Delta A(q^{-1})} u_i\right) \tag{15}$$

and the nonlinear function $g(\cdot)$ is assumed to be invertible,

$$g^{-1}(y_i) = \frac{B(q^{-1}) + \Delta B(q^{-1})}{A(q^{-1}) + \Delta A(q^{-1})} u_i \tag{16}$$

From (8), (9) and (14) it follows that the inverse function $g^{-1}(\cdot)$ can be expressed as a polynomial

$$g^{-1}(y_i) = \lambda_0 + y_i + (\gamma_2 + \lambda_2) y_i^2 + \cdots \tag{17}$$

4. Generation of Residuals

4.1. Hammerstein Systems

The nominal model of the Hammerstein system can be used for generation of residual sequences. Two basic configurations are shown in Figs. 2 and 3. The first of them uses a parallel (recurrent) model and the other is based on a serial-parallel (feedforward) model. For the scheme with the parallel model, the residual equation

$$e_i = \frac{A(q^{-1})\Delta B(q^{-1}) - B(q^{-1})\Delta A(q^{-1})}{A(q^{-1})[A(q^{-1}) + \Delta A(q^{-1})]} f(u_i) + \frac{B(q^{-1}) + \Delta B(q^{-1})}{A(q^{-1}) + \Delta A(q^{-1})} \Delta f(u_i) \tag{18}$$

has a quite complex recurrent and nonlinear in parameters form. A much simpler expression can be obtained if we use the feedforward model instead, i.e.

$$e_i = -\Delta A(q^{-1}) y_i + \Delta B(q^{-1}) f(u_i) + \left[B(q^{-1}) + \Delta B(q^{-1})\right] \Delta f(u_i) \tag{19}$$

Neural network models of Wiener and Hammerstein systems are very useful if the static nonlinear function $f(\cdot)$ cannot be expressed as a polynomial of a finite order. In this case, the well-known and computationally effective parametric identification methods that are based on correlation and linear regression cannot be used.

4.2. Wiener Systems

Starting with the definitions of the residual error (12) and model (13), and using (8) and (16) we obtain the following residual equation:

$$e_i = -\Delta A(q^{-1}) f^{-1}(y_i) + \Delta B(q^{-1}) u_i - \left[A(q^{-1}) + \Delta A(q^{-1})\right] \Delta f^{-1}(y_i) \tag{20}$$

The residual sequence can be generated, as illustrated in Fig. 4, and processed to estimate deviations of the polynomials $A(q^{-1})$, $B(q^{-1})$ and $f^{-1}(\cdot)$.

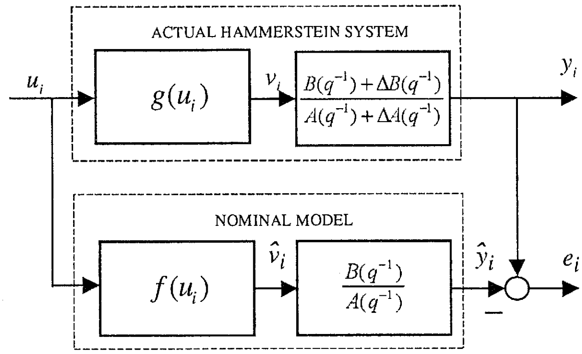

Fig. 2. Residual generation using the parallel model of the Hammerstein system.

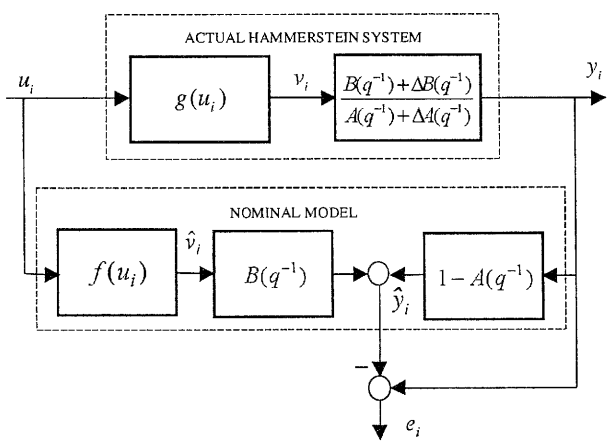

Fig. 3. Residual generation using the serial-parallel model of the Hammerstein system.

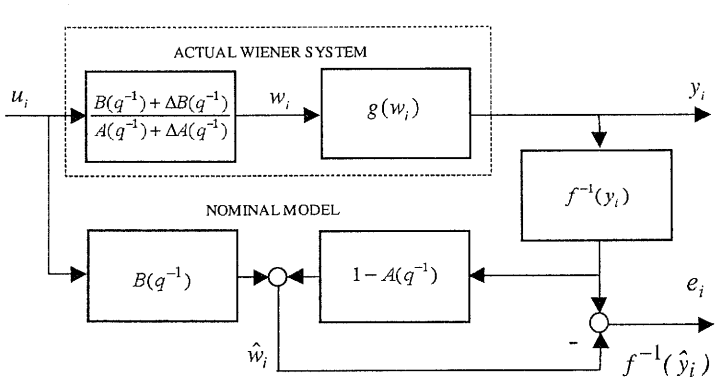

Fig. 4. Residual generation with the modified serial-parallel model of the Wiener system.

5. Models of the Neural Network Residual Generator

5.1. Hammerstein Systems

One way to identify changes in the Hammerstein system parameters is to use another Hammerstein-like neural network shown in Fig. 5. This neural network RGM has three inputs $f(u_i)$, u_i, y_i, and is composed of a multilayer perceptron model of the polynomial $\Delta f(\cdot)$, three tapped delay lines, and a single linear output node. The training of the neural network RGM can be performed by employing the residual sequence (19) as a reference signal to the following model:

$$\hat{e}_i = -\Delta\hat{A}(q^{-1})y_i + \hat{\Delta}B(q^{-1})f(u_i) + \big[B(q^{-1}) + \Delta B(q^{-1})\big]\Delta\hat{f}(u_i) \quad (21)$$

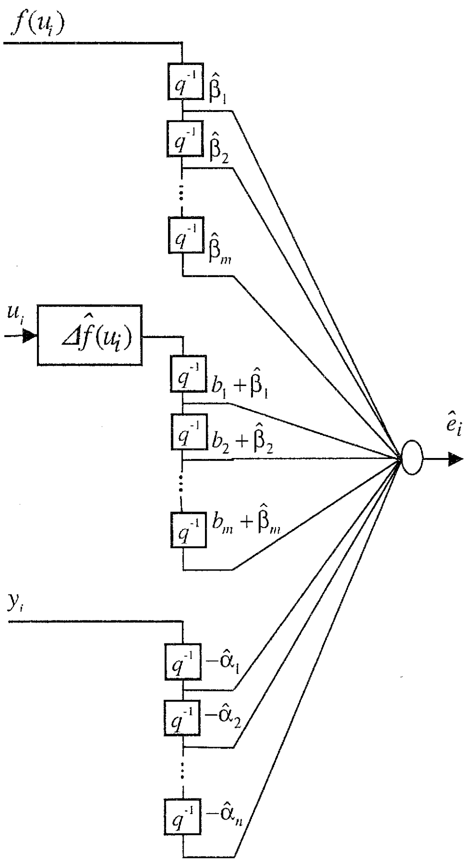

Fig. 5. A neural network model of the residual generator for the Hammerstein system.

5.2. Wiener Systems

For Wiener systems with nonlinear subsystems where the changes in their inverse models cannot be modelled by polynomials of a finite and known order, a neural network model of the residual generator is proposed to estimate the RGM parameters. As neural network learning usually suffers from a slow convergence rate, methods based on neural models of the residual generator can be effectively used only in the case of abrupt faults.

6. Estimation of Residual Generator Parameters

6.1. Hammerstein Systems

Assume that the polynomial $\Delta f(\cdot)$ is of a finite and known order r:

$$\Delta f(u_i) = \eta_0 + \eta_2 u_i^2 + \eta_3 u_i^3 + \cdots + \eta_r u_i^r \tag{22}$$

Then the parameters of the residual eqn. (19) can be computed by solving a set of linear equations or estimated with the use of linear regression methods.

6.1.1. Solving a Set of Linear Equations

The residual eqn. (19) can be transformed into a linear-in-parameters form by introducing parameters d_0 and $d_{j,k}$ as follows:

$$e_i = -\sum_{j=1}^{n} \alpha_j y_{i-j} + \sum_{j-1}^{m} \beta_j f(u_{i-j}) + d_0 + \sum_{k=2}^{r}\sum_{j=1}^{m} d_{j,k} u_{i-j}^k \tag{23}$$

where

$$d_0 = \eta_0 \sum_{j=1}^{m} (b_j + \beta_j) \tag{24}$$

$$d_{j,k} = \eta_k (b_j + \beta_j) \tag{25}$$

For disturbance-free Hammerstein systems, it is possible to compute parameters α_j, β_j, d_0 and $d_{j,k}$ by solving the following set of M linear equations:

$$e_i = -\sum_{j=1}^{n} \alpha_j y_{i-j} + \sum_{j-1}^{m} \beta_j f(u_{i-j}) + d_0 + \sum_{k=2}^{r}\sum_{j=1}^{m} d_{j,k} u_{i-j}^k, \quad i = 0, 1, \ldots, M-1 \tag{26}$$

where $M = m + n + m(r-1) + 1$ is the order of the linear-in-parameter model (23).

Based on b_j and β_j, the parameters η_0 and η_k can be calculated directly from (24) and (25):

$$\eta_0 = \frac{d_0}{\sum_{j=1}^{m}(b_j + \beta_j)} \tag{27}$$

$$\eta_k = \frac{d_{j,k}}{b_j + \beta_j} \tag{28}$$

In this case, no extensive computations are necessary. This results in a fast identification procedure but the solution obtained is sensitive to system disturbances.

6.1.2. RLS Parameter Estimation

It is more realistic to assume that the system is noise corrupted. Then, more measurements can be taken into account, i.e. $N \gg M$, and the RLS algorithm can be used to compute parameters of the RGM. The vectors for the parameter estimates and regression data at time i are respectively defined as follows:

$$\boldsymbol{\theta}_i = \left[\hat{\alpha}_1 \dots \hat{\alpha}_n \ \hat{\beta}_1 \dots \hat{\beta}_m \ \hat{d}_0 \ \hat{d}_{1,2} \dots \hat{d}_{m,2} \dots \hat{d}_{1,r} \dots \hat{d}_{m,r}\right]^T \tag{29}$$

$$\boldsymbol{x}_i = \left[-y_{i-1} \dots -y_{i-n} \ f(u_{i-1}) \dots f(u_{i-m}) \ 1 \ u_{i-1}^2 \dots u_{i-m}^2 \dots u_{i-1}^r \dots u_{i-m}^r\right]^T$$

Parameter estimates of the linear-in-parameters model can be computed on-line using the well-known RLS algorithm (Eykhoff, 1974):

$$\boldsymbol{\theta}_i = \boldsymbol{\theta}_{i-1} + \boldsymbol{P}_i \boldsymbol{x}_i (e_i - \boldsymbol{x}_i^T \boldsymbol{\theta}_{i-1}) \tag{30}$$

$$\boldsymbol{P}_i = \boldsymbol{P}_{i-1} - \frac{\boldsymbol{P}_{i-1}\boldsymbol{x}_i\boldsymbol{x}_i^T\boldsymbol{P}_{i-1}}{1 + \boldsymbol{x}_i^T\boldsymbol{P}_{i-1}\boldsymbol{x}_i} \tag{31}$$

6.1.3. RELS Parameter Estimation

Application of feedforward models often results in correlated residuals. For example, assume that the output of a Hammerstein system is corrupted by additive zero-mean stationary white noise ε_i. Then it can be shown that for the feedforward model, the residual equation has the following form:

$$\begin{aligned} e_i &= -\Delta A(q^{-1})y_i + \Delta B(q^{-1})f(u_i) + \left[B(q^{-1}) + \Delta B(q^{-1})\right]\Delta f(u_i) \\ &\quad + \left[A(q^{-1}) + \Delta A(q^{-1})\right]\varepsilon_i \end{aligned} \tag{32}$$

Owing to the correlated noise term $[A(q^{-1}) + \Delta A(q^{-1})]\varepsilon_i$ in the residual eqn. (32), the least-squares estimation results in biased estimates and correlated residuals. To overcome this problem, other parameter estimation methods such as the extended least squares, generalized least squares or instrumental variables can be used (Åström and Eykhoff, 1971; Söderström *et al.*, 1978).

For the recursive extended least-squares (RELS) method, the vectors of parameter estimates and regression data are respectively defined in the following way:

$$\boldsymbol{\theta}_i = \left[\hat{\alpha}_1 \dots \hat{\alpha}_n \;\; \hat{\beta}_1 \dots \hat{\beta}_m \;\; \hat{d}_0 \;\; \hat{d}_{1,2} \dots \hat{d}_{m,2} \dots \hat{d}_{1,r} \dots \hat{d}_{m,r} \;\; \hat{c}_1 \dots \hat{c}_n\right]^T \quad (33)$$

$$\begin{aligned} \boldsymbol{x}_i = \big[& -y_{i-1} \dots -y_{i-n} \;\; f(u_{i-1}) \dots f(u_{i-m}) \;\; 1 \\ & u_{i-1}^2 \dots u_{i-m}^2 \dots u_{i-1}^r \dots u_{i-m}^r \;\; \widetilde{e}_{i-1} \dots \widetilde{e}_{i-n}\big]^T \end{aligned} \quad (34)$$

$$\widetilde{e}_i = e_i - \boldsymbol{x}_i^T \boldsymbol{\theta}_{i-1} \quad (35)$$

where $\widetilde{e}_i$ denotes the one-step prediction error. The parameter estimates can be computed using the recursive algorithm given by (30) and (31). For systems with slowly varying parameters or slowly developing faults, the RLS with exponential forgetting can be used (Parkum *et al.*, 1992).

6.2. Wiener Systems

Assume that the polynomial $\Delta f^{-1}(\cdot)$ is of a finite and known order r, i.e.

$$\Delta f^{-1}(y_i) = \lambda_0 + \lambda_2 y_i^2 + \dots + \lambda_r y_i^r \quad (36)$$

Then the parameters of the residual generator model can be computed in a similar way as for Hammerstein systems, i.e. by solving a set of linear equations or using the RLS or RELS parameter estimation methods.

6.2.1. RLS Parameter Estimation

To isolate all changes of system parameters, parameters of the ARX (AutoRegressive with eXogenous input) residual generator model (20) can be estimated using the least-squares (LS) or recursive least-squares (RLS) identification methods. Then the regression vector is defined as

$$\begin{aligned} \boldsymbol{x}_i = \big[& -f^{-1}(y_{i-1}) \dots -f^{-1}(y_{i-n}) \;\; u_{i-1} \dots u_{i-m} \;\; 1 \\ & -y_i^2 \dots -y_{i-n}^2 \dots -y_i^r \dots -y_{i-n}^r\big]^T \end{aligned} \quad (37)$$

while the parameter vector is

$$\boldsymbol{\theta}_i = \left[\hat{\alpha}_1 \dots \hat{\alpha}_n \;\; \hat{\beta}_1 \dots \hat{\beta}_m \;\; \hat{d}_0 \;\; \hat{d}_{0,2} \dots \hat{d}_{n,2} \dots \hat{d}_{0,r} \dots \hat{d}_{n,r}\right]^T \quad (38)$$

where

$$\hat{d}_0 = \hat{\lambda}_0 \left[1 + \sum_{j=1}^{n} (a_j + \hat{\alpha}_j)\right] \quad (39)$$

$$\hat{d}_{j,k} = \begin{cases} \hat{\lambda}_k, & j = 0, \quad k = 2, \dots, r \\ (a_j + \hat{\alpha}_j)\hat{\lambda}_k, & j = 1, \dots, n, \quad k = 2, \dots, r \end{cases} \quad (40)$$

It can be shown that for the RLS method, unbiased parameter estimates can be obtained only for disturbance-free systems or systems corrupted by an additive white noise in the following way:

$$y_i = g\left(\frac{B(q^{-1}) + \Delta B(q^{-1})}{A(q^{-1}) + \Delta A(q^{-1})}u_i + \frac{1}{A(q^{-1}) + \Delta A(q^{-1})}\varepsilon_i\right) \tag{41}$$

6.2.2. RELS Parameter Estimation

Assume that a Wiener system is corrupted by an additive white noise ε_i as follows:

$$y_i = g\left(\frac{B(q^{-1}) + \Delta B(q^{-1})}{A(q^{-1}) + \Delta A(q^{-1})}u_i + \varepsilon_i\right) \tag{42}$$

Then the residual equation has the form

$$\begin{aligned} e_i = & -\Delta A(q^{-1})f^{-1}(y_i) + \Delta B(q^{-1})u_i - \big[A(q^{-1}) + \Delta A(q^{-1})\big]\Delta f^{-1}(y_i) \\ & + \big[A(q^{-1}) + \Delta A(q^{-1})\big]\varepsilon_i \end{aligned} \tag{43}$$

Thus, to obtain unbiased parameter estimates, the RELS method can be used. The regression and parameter vectors are respectively defined as

$$\begin{aligned} \boldsymbol{x}_i = & \big[-f^{-1}(y_{i-1}) \dots -f^{-1}(y_{i-n}) \;\; u_{i-1} \dots u_{i-m} \;\; 1 \\ & -y_i^2 \dots -y_{i-n}^2 \dots -y_i^r \dots -y_{i-n}^r \;\; \tilde{e}_{i-1} \dots \tilde{e}_{i-n}\big]^T \end{aligned} \tag{44}$$

$$\boldsymbol{\theta}_i = \big[\hat{\alpha}_1 \dots \hat{\alpha}_n \;\; \hat{\beta}_1 \dots \hat{\beta}_m \;\; \hat{d}_0 \;\; \hat{d}_{0,2} \dots \hat{d}_{n,2} \dots \hat{d}_{0,r} \dots \hat{d}_{n,r} \;\; \hat{c}_1 \dots \hat{c}_n\big]^T \tag{45}$$

7. Simulation Examples

7.1. Hammerstein System

The nominal Hammerstein model with the following second-order pulse transfer function was used in Examples 1 and 2:

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{0.5q^{-1} - 0.3q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} \tag{46}$$

The static nonlinear element of the nominal model was given by the hyperbolic tangent function (Fig. 6)

$$f(u_i) = \tanh(u_i) \tag{47}$$

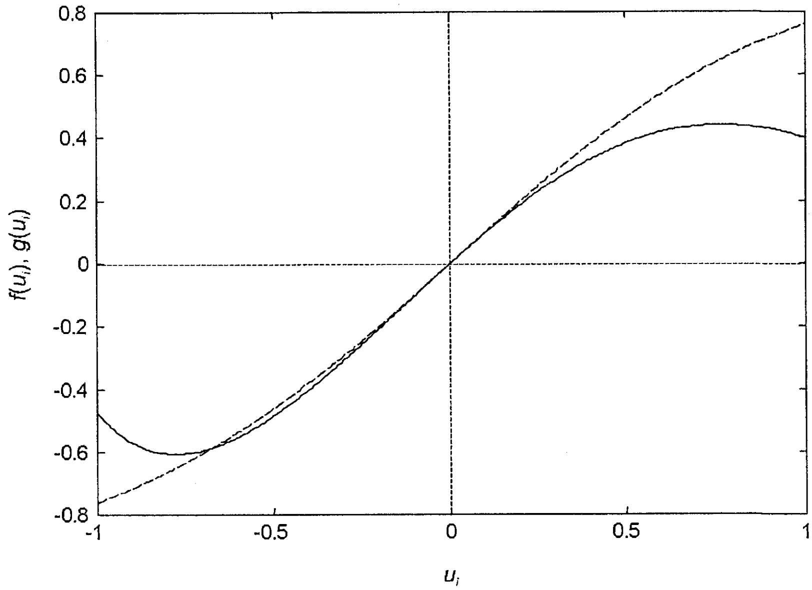

Fig. 6. Nonlinear characteristics of nominal (dashed) and actual (solid) Hammerstein systems.

The system was driven by a pseudorandom sequence uniformly distributed in the interval $(-1, 1)$. The actual Hammerstein system, i.e. a system in its faulty conditions, was given by the following expressions:

$$\frac{B(q^{-1}) + \Delta B(q^{-1})}{A(q^{-1}) + \Delta A(q^{-1})} = \frac{0.3q^{-1} - 0.2q^{-2}}{1 - 1.75q^{-1} + 0.85q^{-2}} \tag{48}$$

$$g(u_i) = \tanh(u_i) - 0.25u_i^2 - 0.2u_i^3 + 0.15u_i^4 - 0.1u_i^5 + 0.05u_i^6 - 0.025u_i^7 + 0.0125u_i^8 \tag{49}$$

Example 1. Noise-corrupted Hammerstein system — the RLS method

The parameters of the RGM were estimated using the RLS method for sequences of 25000 input and output measurements. The system output was disturbed by additive noise uniformly distributed in the interval $(-0.005, 0.005)$. The parameter estimates of the linear system are biased, see the results in Table 1 for comparison with the true values, and Table 2 for the estimation accuracy. The estimation results are also illustrated in Figs. 7–9. ♦

Example 2. Noise-corrupted Hammerstein system — the RELS method

Next, using the same input and output sequences, the parameters of the RGM were estimated with the RELS method. This time the parameter estimates are much more accurate, see Table 1 for the parameter estimates, and Table 2 and Figs. 10 and 11 for comparison with the RLS. ♦

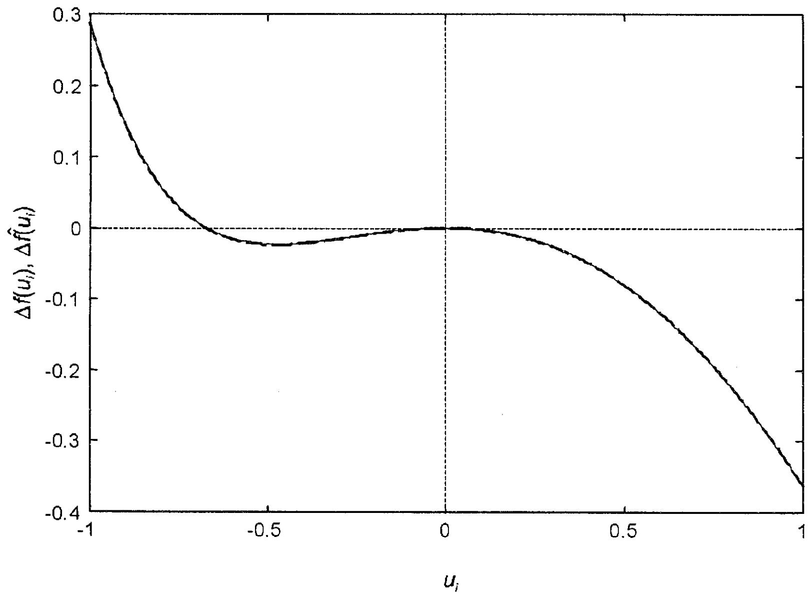

Fig. 7. The change of the nonlinear characteristic of Example 1: true (solid) and estimated (dashed).

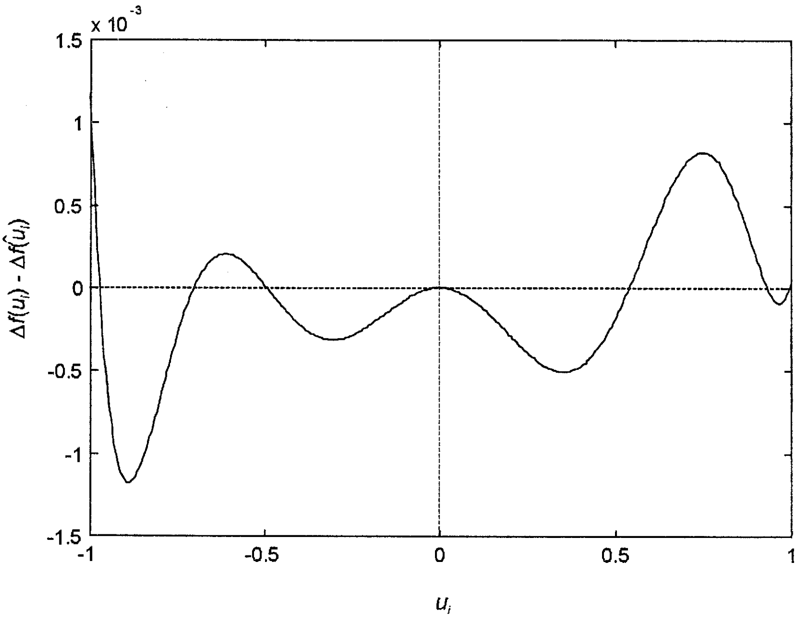

Fig. 8. The estimation error of the nonlinear characteristic change of Example 1.

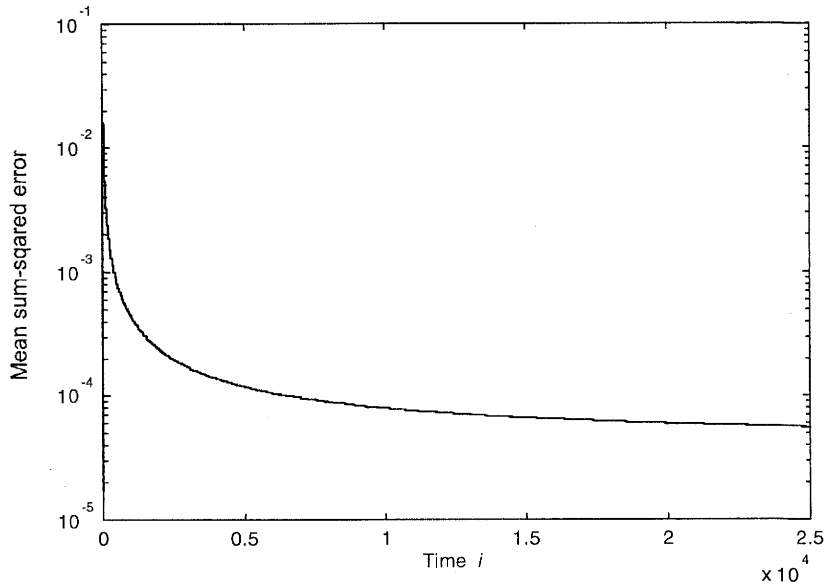

Fig. 9. The mean sum-squared error of Example 1.

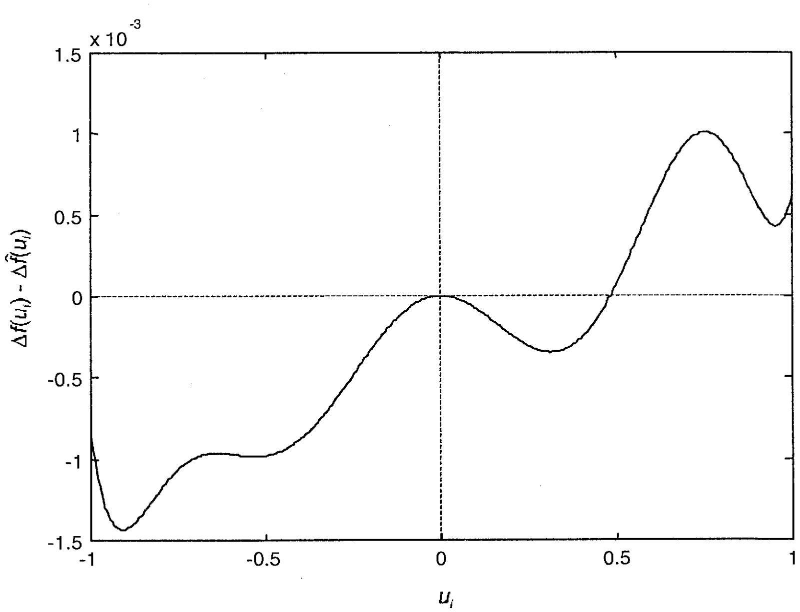

Fig. 10. The estimation error of the nonlinear characteristic change of Example 2.

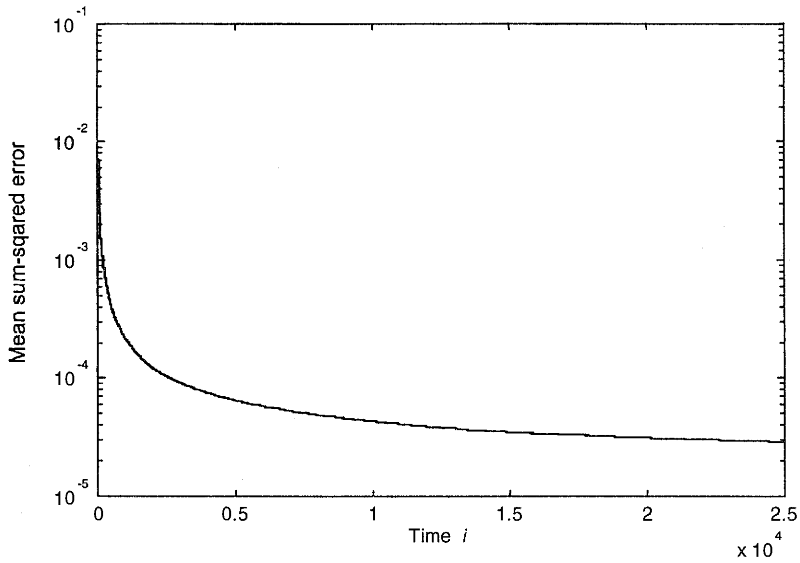

Fig. 11. The mean sum-squared error of Example 2.

Table 1. Parameter estimates for the RLS and RELS methods.

Parameter	True value	RLS	RELS
α_1	−0.2000	−0.1998	−0.1995
α_2	0.1000	0.1011	0.0994
β_1	−0.2500	−0.2456	−0.2499
β_2	0.1500	0.1460	0.1498
η_2	−0.2500	−0.2417	−0.2414
η_3	−0.2000	−0.1955	−0.2059
η_4	0.1500	0.0981	0.1087
η_5	−0.1000	−0.1187	−0.0925
η_6	0.0500	0.1420	0.1136
η_7	−0.0250	−0.0103	−0.0273
η_8	0.0125	−0.0365	−0.0183
c_1	−1.7500		−0.8493
c_2	0.8500		−0.1367

Table 2. Comparison of the parameter estimation accuracy: the RLS and RELS methods.

Performance index	RLS disturbance-free	RLS noise-corrupted	RELS noise-corrupted
$\frac{1}{25000}\sum_{i=1}^{25000}\tilde{e}_i^2$	3.66×10^{-6}	5.56×10^{-5}	2.89×10^{-5}
$\frac{1}{7}\sum_{j=2}^{8}(\eta_j-\hat{\eta}_j)^2$	1.06×10^{-12}	5.08×10^{-4}	2.46×10^{-4}
$\frac{1}{4}\sum_{j=1}^{2}\left[(\alpha_j-\hat{\alpha}_j)^2+(\beta_j-\hat{\beta}_j)^2\right]$	4.85×10^{-16}	9.26×10^{-6}	1.69×10^{-7}

7.2. Wiener System

The nominal Wiener model composed of the second-order linear subsystem (46) and the static nonlinear subsystem of the following inverse characteristic:

$$f^{-1}(y_i) = y_i - \frac{1}{6}y_i^3 \tag{50}$$

see Fig. 12, was used in a simulation study. The pulse transfer function of the actual Wiener system was given by (48) and the nonlinear element was described by the arctangent function

$$g(w_i) = \arctan(w_i) \tag{51}$$

The input to the Wiener system and its nominal model was a pseudo-random sequence of 25000 values uniformly distributed in the interval $(-0.5,\ 0.5)$.

Example 3. Noise-free Wiener system — the RLS method

First, the parameters of the RGM were estimated for a noise-free Wiener system using the RLS method. While the polynomial $\Delta f^{-1}(\cdot)$ was of an infinite order, it was assumed that its estimate was of the order of 11. Thus it can be noticed, see Table 3, that the obtained estimates of the λ_i parameters are biased. ♦

Example 4. Noise-corrupted Wiener system — the RLS method

In Example 4, the parameters of the RGM were estimated for the Wiener system corrupted by an additive noise, as it is defined by (42). The noise was uniformly distributed in the interval $(-0.005,\ 0.005)$. The estimation results shown in Tables 3 and 4 and illustrated in Figs. 13–16, as expected, reveal biased parameter estimates. ♦

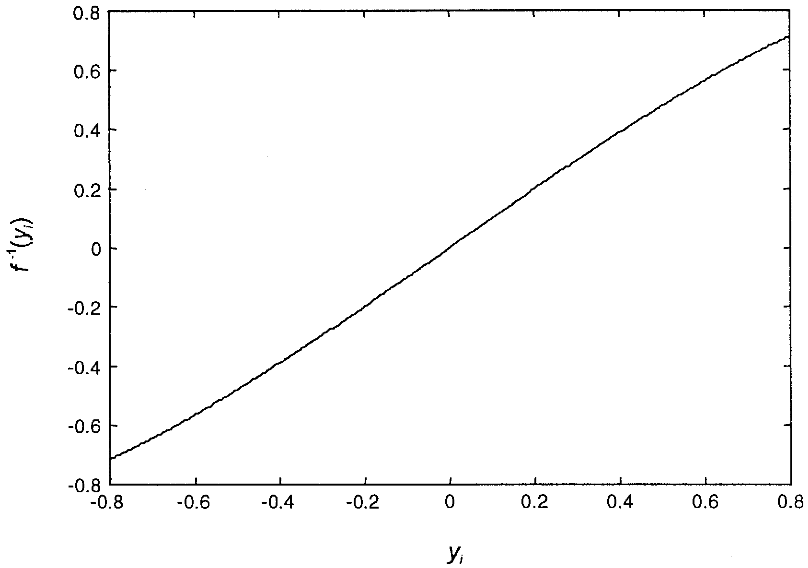

Fig. 12. The inverse function $f^{-1}(\cdot)$ of the nominal Wiener system.

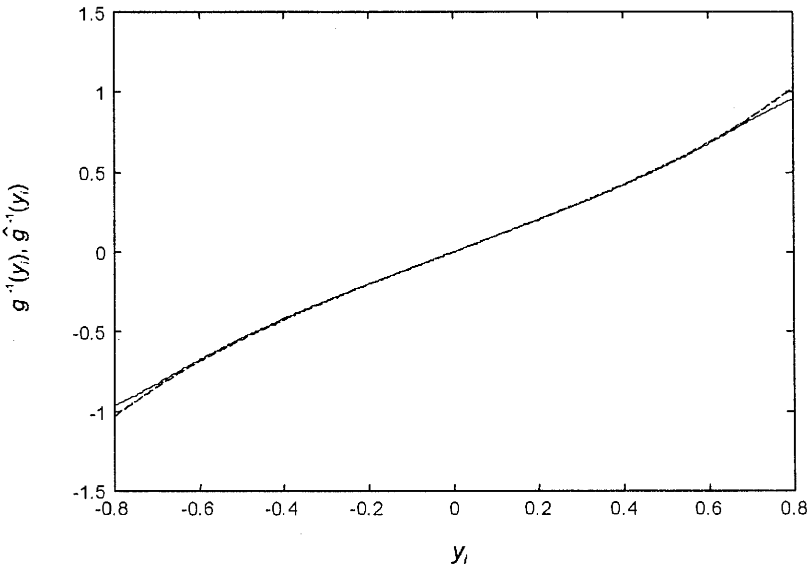

Fig. 13. The true (dashed) and estimated (solid) inverse functions $g(\cdot)$ of Example 4.

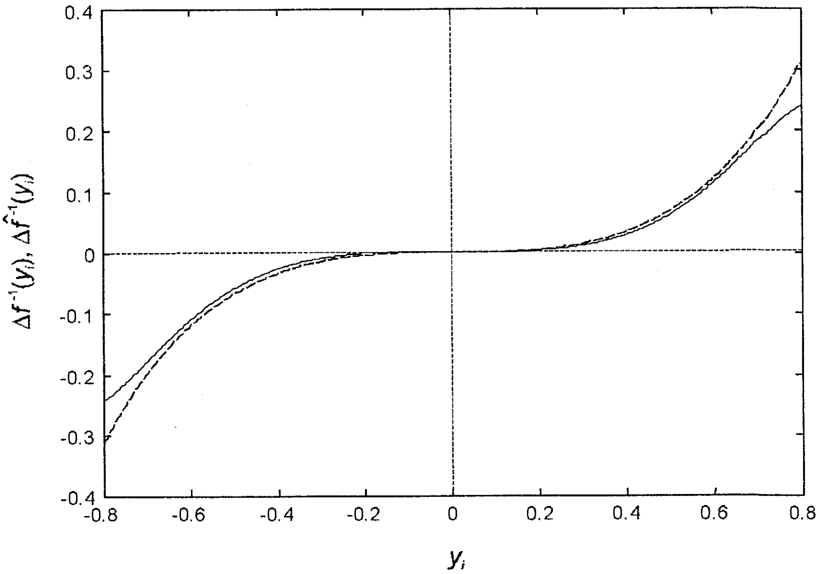

Fig. 14. The true (dashed) and estimated (solid) changes of the function $g(\cdot)$ of Example 4.

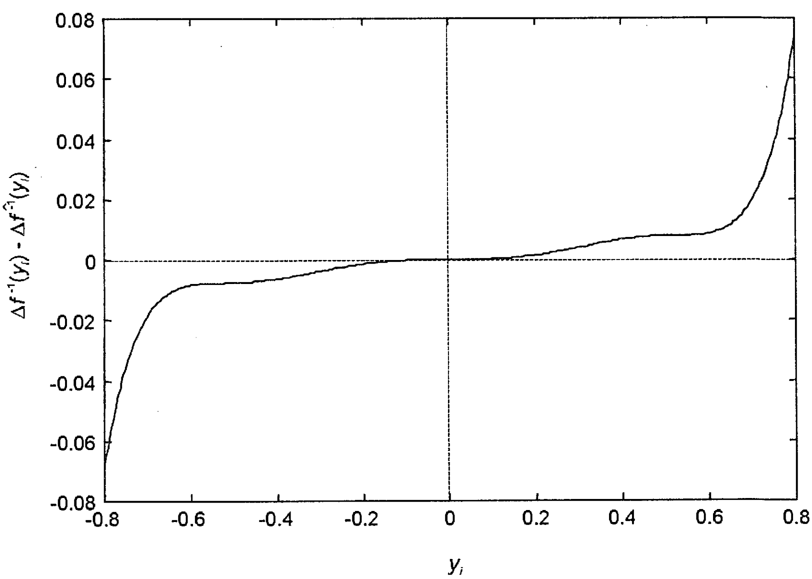

Fig. 15. The estimation error of the inverse nonlinear characteristic change of Example 4.

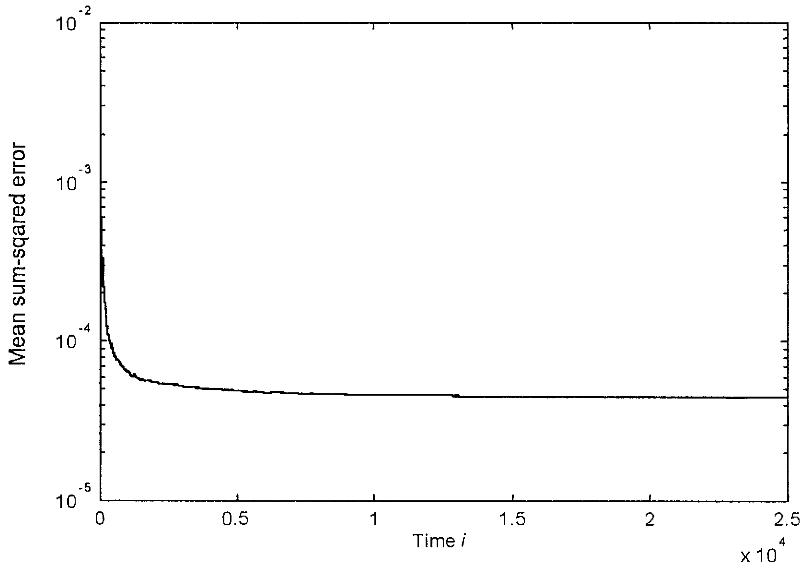

Fig. 16. The mean sum-squared error of Example 4.

Table 3. Parameter estimates for the RLS and RELS methods.

Parameter	True value	RLS disturbance-free	RLS noise-corrupted	RELS noise-corrupted
α_1	−0.2000	−0.2000	−0.2034	−0.2014
α_2	0.1000	0.1000	0.1051	0.1009
β_1	−0.2500	−0.2499	−0.2270	−0.2493
β_2	0.1500	0.1500	0.1338	0.1490
λ_2	0.0000	0.0000	−0.0002	−0.0008
λ_3	0.5000	0.4985	0.2886	0.4126
λ_4	0.0000	0.0000	−0.0115	−0.0014
λ_5	0.1333	0.1394	0.9668	0.4674
λ_6	0.0000	0.0003	0.0507	0.0175
λ_7	0.0540	0.0463	−0.7752	−0.2736
λ_8	0.0000	0.0010	−0.0308	−0.0006
λ_9	0.0219	0.0188	−0.3777	−0.1032
λ_{10}	0.0000	0.0013	−0.0592	−0.0100
λ_{11}	0.0089	0.0085	−0.0978	−0.0012
c_1	−1.7500			−1.0949
c_2	0.8500			0.1145

Table 4. Comparison of the parameter estimation accuracy: the RLS and RELS methods.

Performance index	RLS disturbance-free	RLS noise-corrupted	RELS noise-corrupted
$\frac{1}{25000}\sum_{i=1}^{25000} \tilde{e}_i^2$	4.45×10^{-7}	4.45×10^{-5}	1.97×10^{-5}
$\frac{1}{10}\sum_{j=2}^{11}(\lambda_j - \hat{\lambda}_j)^2$	1.11×10^{-5}	1.61×10^{-1}	2.43×10^{-2}
$\frac{1}{4}\sum_{j=1}^{2}\left[(\alpha_j - \hat{\alpha}_j)^2 + (\beta_j - \hat{\beta}_j)^2\right]$	1.12×10^{-9}	2.07×10^{-4}	1.05×10^{-6}

Example 5. Noise-corrupted Wiener system the RELS method

Finally, the parameters of the RGM were estimated using the RELS method. It can be seen from Tables 3 and 4, and Figs. 17–20 that, in comparison with the previous example, a higher estimation accuracy of both the linear dynamic and nonlinear static sub-models was achieved. ◆

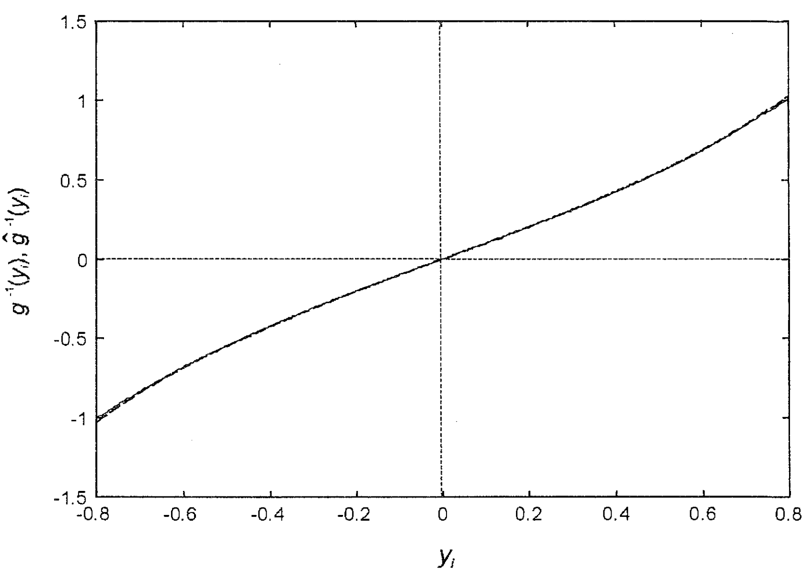

Fig. 17. The true (dashed) and estimated (solid) inverse functions $g(\cdot)$ of Example 5.

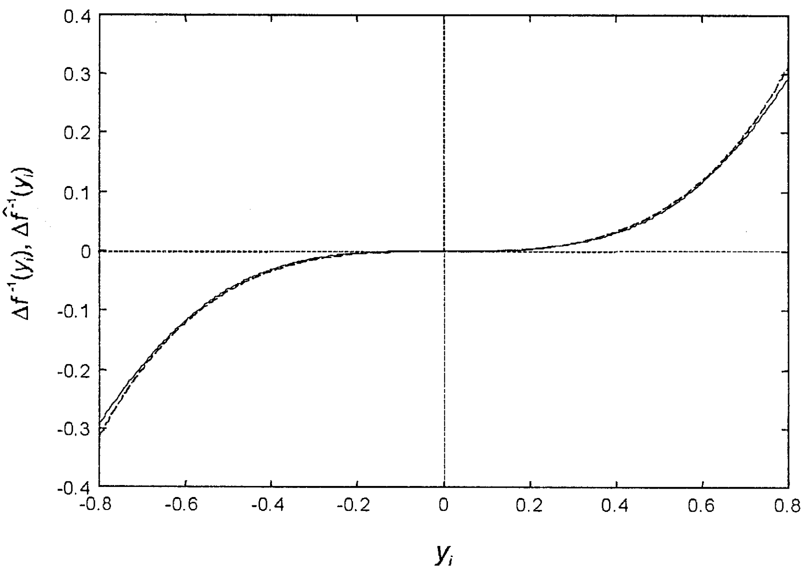

Fig. 18. The true (dashed) and estimated (solid) changes of the function $g(\cdot)$ of Example 5.

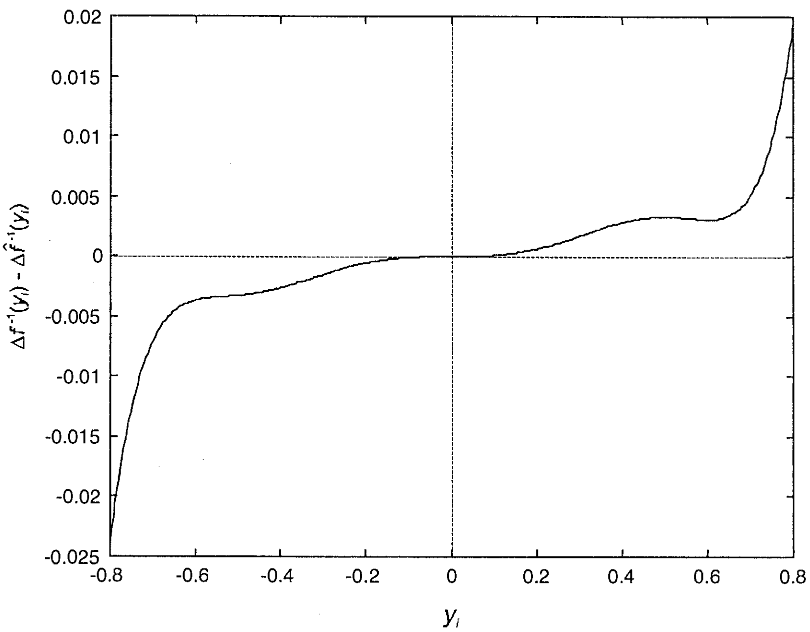

Fig. 19. The estimation error of the inverse nonlinear characteristic change of Example 5.

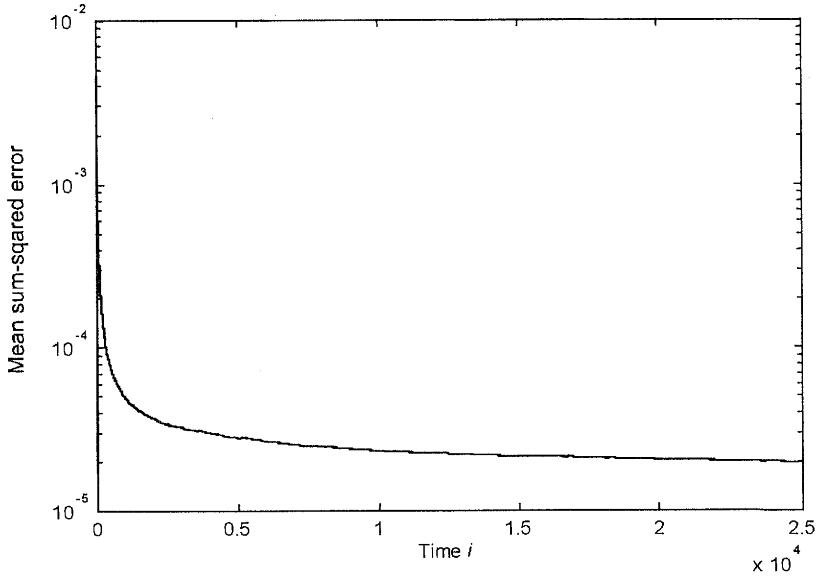

Fig. 20. The mean sum-squared error of Example 5.

8. Conclusions

The model-based FDI methods presented here use the RGM to estimate parameter deviations of Wiener and Hammerstein systems. These methods are suitable for both disturbance-free and noise-corrupted systems. The methods make it possible to detect and isolate all the changes in the system parameters quickly and reliably. For the linear regression based approach, no extensive computations are necessary. It is also possible to use these methods in the case of slowly varying parameters or slowly developing faults.

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