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EFFECT OF MHD ON UNSTEADY OSCILLATORY COUETTE FLOW THROUGH POROUS MEDIA

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This paper describes the effects of a magnetic field on unsteady free convection oscillatory systems. When temperature and species concentration fluctuate with time around a non-zero constant, "Couette flow" across a porous medium occurs. The system of non-linear ODEs that governs the flow is solved analytically using the perturbation approach because the amplitude of fluctuations is very tiny. Mean flow and transient velocity, transient concentration, transient temperature, heat transfer, mean skin friction and phase and amplitude of skin friction. All have approximate solutions. The influence of different parameters on flow characteristics has been specified and discussed.

Key words: MHD, porous media, free convection, Couette flow.

1. Introduction

Modern technology has made fluid flow studies a hot topic in recent years due to the interplay of multiple phenomena. To get the most out of geo-thermal energy, it goes without saying that you need a thorough understanding of the number of perturbations required to produce flow in geo-thermal fluids. Furthermore, knowing the number of perturbations required to start flow in mineral fluids located in the earth's crust enables the most efficient mineral harvesting. Raptis *et al.* [1] conducted a series of research considering geophysical applications of flow through porous media, where the porous medium is surrounded by porous plates that are either horizontal, vertical, or parallel. The free convective flow past a vertical wall was explored extensively by Kulacki [2]. Nield [3] also investigated convection flow in a porous material with an inclined temperature gradient.

Regarding many industrial and aerodynamic flow problems, the reaction of a laminar boundary layer flow to free stream oscillations is critical. For a growing number of scientific and technical applications, Kelleher *et al.* [4] examined how surface temperature oscillations affected the laminar free convection boundary layers' heat transfer response along heated vertical plates. Free convection oscillatory flow via a porous medium with periodic temperature changes was studied by Sharma *et al.* [5]. Couette oscillations in rotating systems were investigated by Muzumder [6]. Oscillatory flow through porous media with convection was investigated by Raptis and Peridikis [7]. Singh and Verma [8] and Sharma *et al.* [9] explored a three-dimensional oscillatory flow in porous media. Periodic solutions for oscillatory channel flow in a rotating porous medium were investigated by Singh *et al.* [10].

For more than three decades, many studies have focused on buoyancy-induced flows with the combined effect of heat and mass diffusion. As a result of its importance various scientific and industrial applications, fluid mechanics and heat and mass transfer are now viewed as crucial fields. Buoyancy forces act on fluid constituents when temperature changes cause density changes, causing free convection.

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Temperature variations drive air flow in our daily lives. Ostrach [11] and others carried out detailed investigations on the free convective flow across plates oriented vertically. During these experiments, only steady-state flows were studied. Soundalgekar [12] examined the effects of viscous dissipation on the flow across an infinite vertical porous plate in the case of unstable free convective flows. As a result, it was believed that the temperature of the plate fluctuated with a small amplitude. Martynenko *et al.* [13] studied vertical plate laminar free convection in a laboratory setting. Harris *et al.* [14] examined free convection through a porous medium from a vertically oriented plate. Using the combined buoyancy effects of mass and thermal diffusion, Gebhart and Pera [15] studied natural convection flows along horizontal and vertical surfaces. Tripathi and Sharma [16-17] discussed the effects of heat and mass transfer on a two-phase blood flow with Joule heating and variable viscosity in the presence of a magnetic field.

Magnetohydrodynamics (MHD) involves the study of electrically conducting fluids. Liquid metals, plasmas and salt water or electrolytes are examples of such fluids. The problems of flow of electrically conducting fluids under the influence of a magnetic field have gained the attention of many writers because of their applications in geophysics, astronomy, engineering, and boundary layer management in aerodynamics. Many existing viscous hydrodynamic solutions should be enhanced to include the effects of a magnetic field in contexts where the viscous fluid is electrically conducting, given the growing number of technical applications that utilize the magnetohydrodynamics effect. Hydromagnetic effects on flow through a plate were thoroughly investigated by Greenspan and Carrier [18]. According to Attia and Kotab [19], a hydromagnetic channel's flow and temperature fields were analyzed. Using an MHD free convection flow with varying surface heat flux, Hossain *et al.* [20] examined. A porous material was studied by Sharma *et al.* [21] under periodic temperature variations. Sharma and Singh [22] investigated the effects of a magnetic field and thermal diffusion on an oscillatory free convection flow past a plate with viscous heating in great detail. Recently, Sharma *et al.* [23] examined the Soret and Dufour effects in a biomagnetic fluid flow through a tapered porous stenosed artery.

Many engineering applications rely on the impact of simultaneous heat and mass diffusion in an oscillatory flow on low electrical conductivity fluid boundary layer flows. As a result, the primary goal of this paper is to investigate the effects of porosity and a magnetic field on the flow of an incompressible, viscous and electrically conducting, fluid between two parallel, vertically oriented porous plates with fluctuating free stream concentration, temperature, and velocity fluctuating with time around a non-zero constant mean.

2. Formulation of the problem

We investigate an unsteady Couette flow of a incompressible, viscous, electric conducting fluid over a highly porous media limited by two infinite porous flat plates oriented vertically. One of them is startled awake by the free stream velocity, which oscillates in time around a constant mean. The $x^* -$axis runs vertically up the moving vertical plate, while the y^*-axis runs perpendicular to it. At temperature T_b^*, the other fixed plate is at $y^* = b$. The shape's free-stream velocity distribution is defined as:

$$U^*\left(t^*\right) = U_0\left(1 + \varepsilon e^{i\omega^* t^*}\right) \tag{2.1}$$

where U_0 is the average constant free-stream velocity, ω^* is the oscillation frequency, and t^* is the period.

Magnetofluid-dynamics differs from standard fluid dynamics. Due to the electromagnetic field a force term is added. Maxwell's equations must be met on the whole field. The following assumptions are made to construct the basic equations for the problem at hand:

1) The magnetic field is applied perpendicular to the plane of the disc, and the flow is steady and laminar.
2) The fluid is viscous, incompressible, finitely conducting, and has constant physical properties.
3) The magnetic Reynolds number is set to a low enough value to ignore the induced magnetic field.
4) The Hall Effect, as well as electrical and polarization effects, are not considered.

The problem is governed by the following equations:

$$\frac{\partial u^*}{\partial t^*}=\frac{\partial U^*}{\partial t^*}+\nu\frac{\partial^2 u^*}{\partial y^{*2}}+g\beta\left(T^*-T_b^*\right)+g\beta_c\left(C^*-C_b^*\right)-\frac{\left(\vec{J}\times\vec{B}\right)}{\rho}-\frac{\nu}{K^*}\left(u^*-U^*\right). \tag{2.2}$$

The "Lorentz force" due to magnetic field is represented by the fifth term on the RHS of Eq.(2.2)

$$\vec{J}\times\vec{B}=\sigma\left(\vec{v}\times\vec{B}\right)\times\vec{B}\,. \tag{2.3}$$

The equation (2.2) becomes

$$\frac{\partial u^*}{\partial t^*}=\frac{\partial U^*}{\partial t^*}+\nu\frac{\partial^2 u^*}{\partial y^{*2}}+g\beta\left(T^*-T_b^*\right)+g\beta_c\left(C^*-C_b^*\right)-\frac{\left(u^*-U^*\right)\sigma B^2}{\rho}-\frac{\nu}{K^*}\left(u^*-U^*\right). \tag{2.4}$$

The energy equation is as follows:

$$\frac{\partial T^*}{\partial t^*}=\alpha\frac{\partial^2 T^*}{\partial y^{*2}}\,. \tag{2.5}$$

The concentration equation is as follows:

$$\frac{\partial C^*}{\partial t^*}=D\frac{\partial^2 C^*}{\partial y^{*2}}\,. \tag{2.6}$$

The fluid motion's boundary conditions are:

$$y^*=0: u^*=U_0\left(1+\varepsilon e^{i\omega^* t^*}\right), T^*=T_0^*+\varepsilon\left(T_0^*-T_b^*\right)e^{i\omega^* t^*},$$

$$C^*=C_0^*+\varepsilon\left(C_0^*-C_b^*\right)e^{i\omega^* t^*}, \tag{2.7}$$

$$y^*=b: u^*=0,\quad T^*=T_b^*,\quad C^*=C_b^*$$

where U^*, u^*, g, v, β, β_c, α, T^*, T_b^*, T_0^*, C^*, C_b^*, C_0^*, D, K^*, are respectively, the free-stream velocity, velocity, gravity, kinematic viscosity, volumetric coefficient of thermal expansion, volumetric coefficient of thermal expansion with concentration, thermal diffusivity, fluid temperature in the boundary layer, temperature of the moving plate, temperature of the stationary plate, fluid concentration in the boundary layer, concentration of the moving plate, concentration of the stationary plate, species concentration, electrical conductivity and permeability of the porous medium. The $(*)$ denotes dimensional quantities.

The non-dimensional quantities are described as follows

$$y=y^*/b,\quad u=u^*/U_0,\quad U=U^*/U_0,\quad t=\omega^* t^*,\quad \omega=\omega^* b^2/\nu,$$

$$\theta=\left(T^{*}-T_{b}^{*}\right)/\left(T_{0}^{*}-T_{b}^{*}\right),\quad Gr\left(\text{Grassoff number}\right)=\frac{g\beta b^{2}\left(T_{0}^{*}-T_{b}^{*}\right)}{\nu U_{0}},$$

$$Gc\left(\text{modified Grassoff number}\right)=\frac{g\beta_{c}b^{2}\left(C_{0}^{*}-C_{b}^{*}\right)}{\nu U_{0}},\quad K=\frac{K^{*}}{b^{2}},$$

$$C=\left(C^{*}-C_{b}^{*}\right)/\left(C_{0}^{*}-C_{b}^{*}\right),\qquad Sc\left(\text{Schmidt number}\right)=\frac{\nu}{D},$$

$$M\left(\text{Hartmann number}\right)=\sqrt{\frac{\sigma B^{2}b^{2}}{\rho\nu}},\quad \Pr\left(\text{Prandtl number}\right)=\nu/\alpha\,.$$

Equations (2.4), (2.5), and (2.6) are transformed into

$$\omega\frac{\partial u}{\partial t}=\omega\frac{\partial U}{\partial t}+\frac{\partial^{2}u}{\partial y^{2}}+Gr\theta+GcC-M^{2}\left(u-U\right)-\frac{\left(u-U\right)}{K}\,, \tag{2.8}$$

$$\omega\Pr\frac{\partial\theta}{\partial t}=\frac{\partial^{2}\theta}{\partial y^{2}}\,, \tag{2.9}$$

$$\omega Sc\frac{\partial C}{\partial t}=\frac{\partial^{2}C}{\partial y^{2}}\,, \tag{2.10}$$

with corresponding boundary conditions

$$\begin{aligned}&y=0:\quad u=1+\varepsilon e^{it},\quad \theta=1+\varepsilon e^{it},\quad C=1+\varepsilon e^{it},\\&y=1:\quad u=0,\quad \theta=0,\quad C=0.\end{aligned} \tag{2.11}$$

3. Solution of the problem

Because the amplitudes of the concentration, temperature, and free-stream velocity fluctuations are relatively small, we now assume the following solutions:

$$\begin{aligned}&u\left(y,t\right)=u_{0}\left(y\right)+\varepsilon u_{1}\left(y\right)e^{it}\,,\\&\theta\left(y,t\right)=\theta_{0}\left(y\right)+\varepsilon\theta_{1}\left(y\right)e^{it}\,,\\&C\left(y,t\right)=C_{0}\left(y\right)+\varepsilon C_{1}\left(y\right)e^{it}\,,\end{aligned} \tag{3.1}$$

and for the free-stream velocity

$$U = 1 + \varepsilon e^{it} \ . \tag{3.2}$$

We find the following equations by using Eqs (2.8), (2.9) and (2.10) in (3.1) and (3.2) and comparing the coefficients of ε and ignoring those of ε^2:

$$u_0'' - M^2 u_0 - \frac{u_0}{K} = -Gr\theta_0 - GcC_0 - M^2 - \frac{1}{K}, \tag{3.3}$$

$$\theta_0'' = 0, \tag{3.4}$$

$$C_0'' = 0, \tag{3.5}$$

$$u_1'' - \left(i\omega + M^2\right)u_1 - \frac{u_1}{K} = -Gr\theta_1 - GcC_1 - \left(i\omega + M^2\right) - \frac{1}{K}, \tag{3.6}$$

$$\theta_1'' - i\omega \mathrm{Pr}\,\theta_1 = 0, \tag{3.7}$$

$$C_1'' - i\omega ScC_1 = 0, \tag{3.8}$$

with the corresponding boundary conditions:

$$\begin{aligned} &y=0: \ u_0 = 1, \quad u_1 = 1, \quad \theta_0 = 1, \quad \theta_1 = 1, \quad C_0 = 1, \quad C_1 = 1, \\ &y=1: \ u_0 = 0, \quad u_1 = 0, \quad \theta_0 = 0, \quad \theta_1 = 0, \quad C_0 = 0, \quad C_1 = 0 \end{aligned} \tag{3.9}$$

where differentiation with respect to y is denoted by primes. We find the solutions by solving Eqs (3.3) to (3.8) under the corresponding boundary conditions (3.9):

$$\theta_0(y) = (1 - y), \tag{3.10}$$

$$C_0(y) = (1 - y), \tag{3.11}$$

$$u_0(y) = k_1 e^{dy} + k_2 e^{-dy} + \frac{(Gr + Gc)}{\left(M^2 + \frac{1}{K}\right)}(1 - y) + 1, \tag{3.12}$$

$$\theta_1(y) = k_3 e^{n_1 y} + k_4 e^{-n_1 y}, \tag{3.13}$$

$$C_1(y) = k_5 e^{n_2 y} + k_6 e^{-n_2 y}, \tag{3.14}$$

$$u_1(y) = k_{13} e^{n_3 y} + k_{14} e^{-n_3 y} + k_7 e^{n_1 y} + k_8 e^{-n_1 y} + k_9 e^{n_2 y} + k_{10} e^{-n_2 y} + 1 \tag{3.15}$$

where:

$$k_1 = \frac{M^2 + \frac{1}{K} - (Gr + Gc)e^{-d}}{\left(M^2 + \frac{1}{K}\right)\left(e^{-d} - e^d\right)}, \quad k_2 = \frac{M^2 + \frac{1}{K} - (Gr + Gc)e^{d}}{\left(M^2 + \frac{1}{K}\right)\left(e^{d} - e^{-d}\right)}, \quad k_3 = \frac{1}{1 - e^{2n_1}},$$

$$k_4 = \frac{1}{1 - e^{-2n_1}}, \quad d = \sqrt{M^2 + \frac{1}{K}}, \quad n_1 = \sqrt{i\omega Pr}, \quad k_5 = \frac{1}{1 - e^{2n_2}}, \quad k_6 = \frac{1}{1 - e^{-2n_2}},$$

$$n_2 = \sqrt{i\omega Sc}, \quad n_3 = \sqrt{i\omega + M^2 + \frac{1}{K}}, \quad k_7 = -\frac{Grk_3}{n_1^2 - \left(i\omega + M^2 + \frac{1}{K}\right)},$$

$$k_8 = -\frac{Grk_4}{n_1^2 - \left(i\omega + M^2 + \frac{1}{K}\right)}, \quad k_9 = -\frac{Gck_5}{n_2^2 - \left(i\omega + M^2 + \frac{1}{K}\right)}, \quad k_{10} = -\frac{Gck_6}{n_2^2 - \left(i\omega + M^2 + \frac{1}{K}\right)},$$

$$k_{11} = -\left[k_7 + k_8 + k_9 + k_{10}\right], \quad k_{12} = -\left[1 + k_7 e^{n_1} + k_8 e^{-n_1} + k_9 e^{n_2} + k_{10} e^{-n_2}\right],$$

$$k_{13} = \frac{k_{12} - k_{11} e^{-n_3}}{e^{n_3} - e^{-n_3}}, \quad k_{14} = \frac{-k_{12} + k_{11} e^{n_3}}{e^{n_3} - e^{-n_3}}.$$

4. Results and discussion

The influence of the magnetic field and convection on the mean and transient temperature, concentration, and velocity is discussed when the plate is subjected to oscillatory temperature, concentration and velocity. The numerical values of the Prandtl number *Pr*, Grashoff number *Gr*, modified Grashoff number *Gc*, Schmidt number *Sc*, fluctuation frequency ω, permeability parameter *K*, and Hartmann number *M* are all computed. The Prandtl numbers are chosen to be around 0.71 and 7, that represent air and water at $20^\circ C$, respectively. The Schmidt number values have been chosen to represent the most prevalent diffusing chemical species of interest in air and water. *Sc* values are $0.60, 1.002$, and 617 in air and water, respectively, representing the species H_2O, CO_2, and Cl_2 in air and water at $25^\circ C$ and one atmospheric pressure. The species are thought to be in low abundance. The values for *Gr*, *Gc*, *M*, *K*, and are chosen at random.

(a) Mean flow

Equation (3.12) gives the mean flow velocity. Figure 1 depicts this velocity component. The graphic shows that for the same value of *M*, the mean flow velocity drops as the Grashoff number increases. The mean flow velocity decreases when the magnetic field parameter *M* is increased up to the mid half of the channel, then drops towards the plate at $y = b$, but the permeability *K* has the opposite effect. It can also be seen in the graph that it rises with rising *Gc*.

It is critical to understand the impact of the Grashoff numbers and magnetic field on the mean-skin friction at $y=0$ after learning about the mean flow velocity field. It is provided by:

$$\tau^* = \mu\left(\frac{du^*}{dy^*}\right)_{y^*=0}, \tag{4.1}$$

and in non-dimensional form it is given by:

$$\tau = \frac{\tau^* b}{\mu U_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0}{\partial y}\right)_{y=0} + \varepsilon\left(\frac{\partial u_1}{\partial y}\right)_{y=0} e^{it}. \tag{4.2}$$

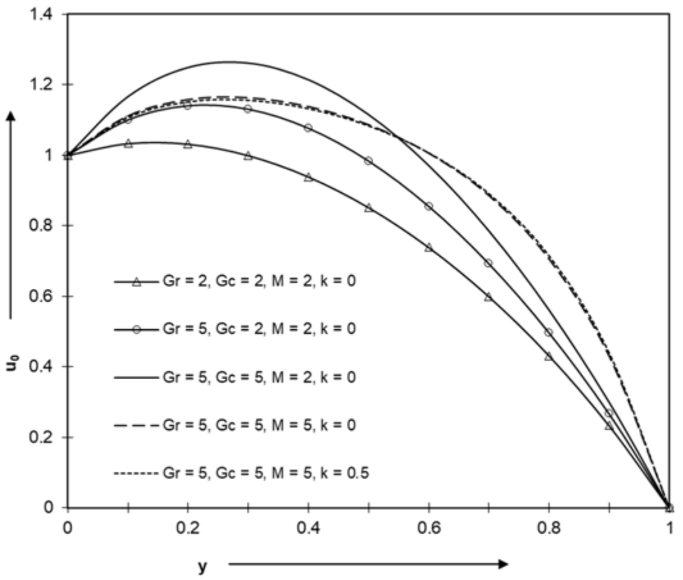

Fig.1. Mean velocity profiles.

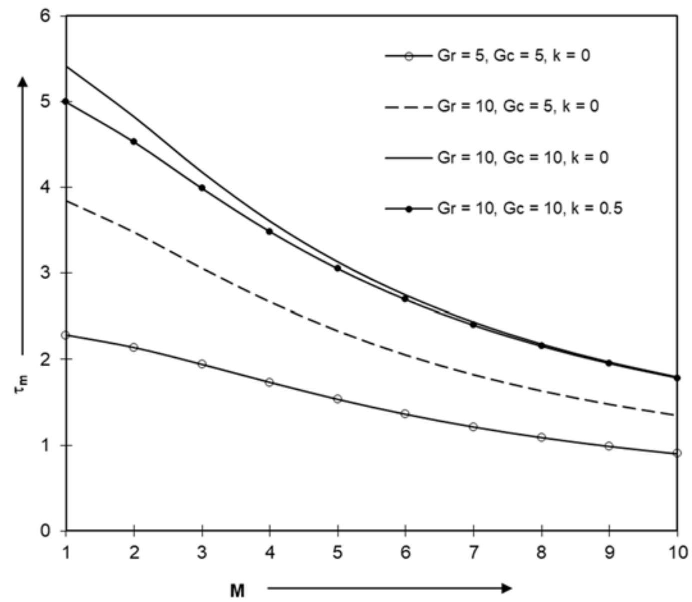

Fig.2. Mean skin friction.

Denoting the mean skin friction by:

$$\tau_m = \left(\frac{du_0}{dy} \right)_{y=0} . \tag{4.3}$$

Substituting Eq.(3.12) in Eq.(4.3), we have:

$$\tau_m = \left(k_1 - k_2 \right) d - \frac{\left(Gr + Gc \right)}{M^2 + \frac{1}{K}} . \tag{4.4}$$

Figure 2 shows the average skin friction profile. There is an increase in the mean skin friction as *Gc* and *Gr* both increase, but *M* has the opposite effect. It is also noted that as *K* increases, the skin friction reduces. Physically, porous media do not allow the fluid to pass through them. It is worth noting that the increase in *Gr* and *Gc* has a greater impact on the mean skin friction.

(b) Unsteady flow

Equations (3.10)-(3.15) produce the temperature and velocity field, which may now be represented in terms of fluctuating parts as follows:

$$u\left(y,t \right) = u_0\left(y \right) + \varepsilon e^{it}\left(N_r + iN_i \right), \tag{4.5}$$

$$\theta\left(y,t \right) = \theta_0\left(y \right) + \varepsilon e^{it}\left(Q_r + iQ_i \right), \tag{4.6}$$

$$C\left(y,t \right) = C_0\left(y \right) + \varepsilon e^{it}\left(C_r + iC_i \right), \tag{4.7}$$

where:

$$N_r + iN_i = u_1\left(y \right), Q_r + iQ_i = \theta_1\left(y \right), C_r + iC_i = C_1\left(y \right). \tag{4.8}$$

For $t = \pi / 2$, we can generate the following formulas for the transient velocity, temperature, and concentration:

$$u\left(y,\pi / 2 \right) = u_0\left(y \right) - \varepsilon N_i , \tag{4.9}$$

$$\theta\left(y,\pi / 2 \right) = \theta_0\left(y \right) - \varepsilon Q_i , \tag{4.10}$$

$$C\left(y,\pi / 2 \right) = C_0\left(y \right) - \varepsilon C_i . \tag{4.11}$$

The transient velocity profiles for $Pr = 0.71$, $Sc = 0.60$ (H_2O), and $Gc = 0.2$ are presented in Fig.3 for a tiny value of $= 0.2$. Because the buoyancy force rises in the upward direction, the graph shows that there is an increase in the transient velocity with an increase in *Gr* and *Gc*. The image also shows that as the magnetic field parameters *M*, *K* grow, the velocity drops until it reaches the middle part of the channel, where it reverses. This is because the flow is resisted by the porous material, resulting in a decrease in velocity.

Figure 4 shows the impact of *Gc* and the magnetic field parameter *M* on the transient velocity profile with fixed values of $Pr = 0.71$, $Gr = 5$, $Sc = 1.002$ (CO_2). As shown in the graph, the transient velocity increases as *Gc* increases. As the magnetic field parameter *M* intensity increases, the transient velocity decreases. Up to the center portion of the channel, the velocity reduces slightly due to a rise in *K*, but then increases near the opposite plate at $y=b$. Figure 5 shows the transient velocity for $Sc=617$ (Cl_2) in water with fixed values of other parameters. It has been discovered that when the frequency of oscillations increases ω, the velocity increases. The image also shows that as *M* and *K* are increased, velocity drops up to half of the channel, then reverses. The transient temperature is depicted in Figure 6. For air and water the transient temperature rises with increasing frequency of fluctuations up to $y=0.4$, then rises with distance *y*. Near the plate, temperature values are higher, and the effect reverses as you go away from the plate. It has also been discovered that as you get further away from the plates, the temperature declines.

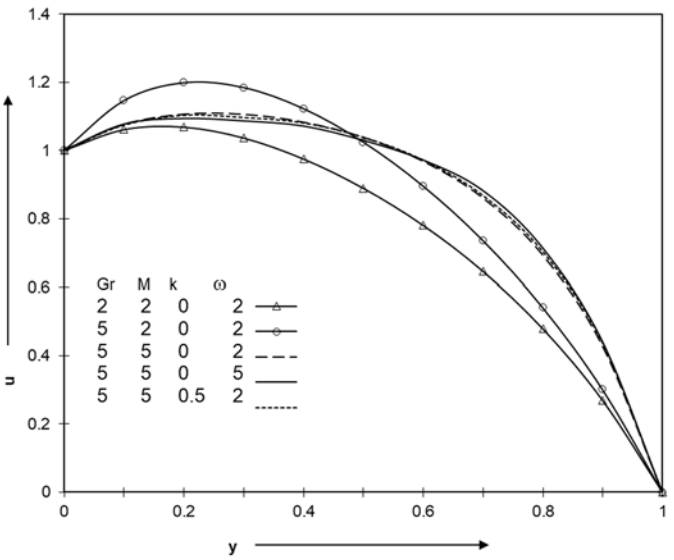

Fig.3. Transient velocity profiles for $\varepsilon=0.2$, $t=\pi/2$, $Pr=0.71$ (air), $Gc=2$, $Sc=0.60$ (H_2O).

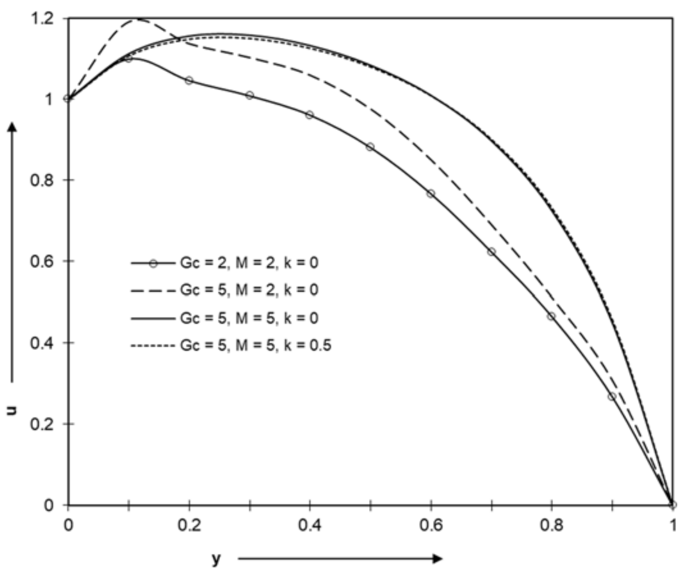

Fig.4. Transient velocity profiles for $\varepsilon=0.2$, $t=\pi/2$, $Pr=0.71$ (air), $Gr=5$, $\omega=5$ $Sc=1.002\left(CO_2\right)$.

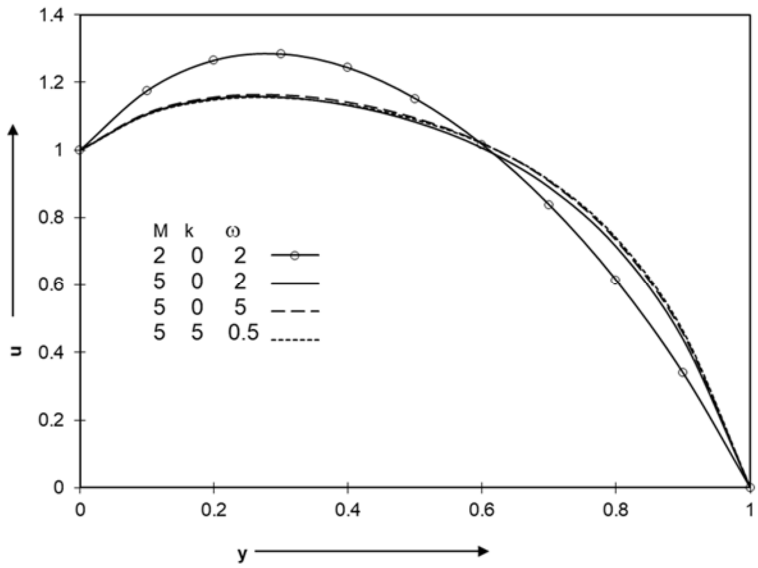

Fig.5. Transient velocity profiles for $\varepsilon = 0.2$, $t = \pi / 2$, $Gr = 5$, $Gc = 5$, $Pr = 7$ and $Sc = 617$ (water).

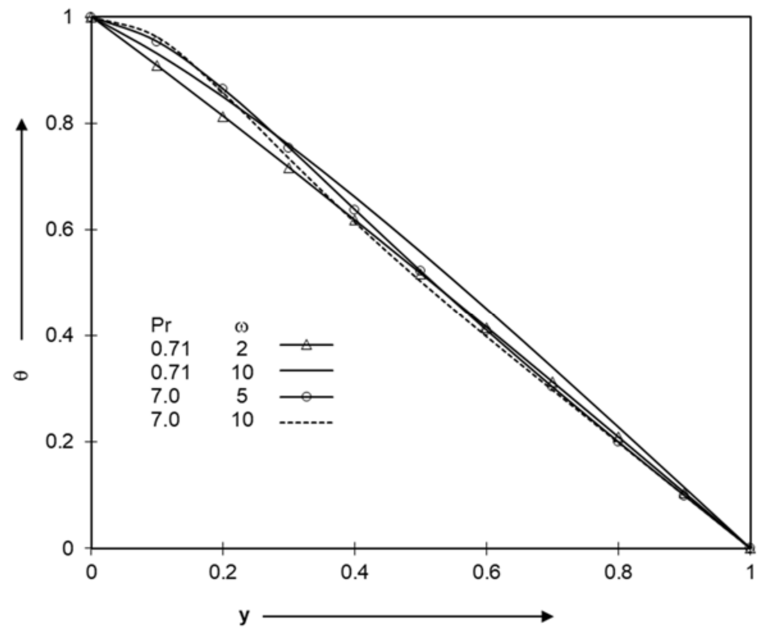

Fig.6. Transient temperature profiles for $\varepsilon = 0.2$, and $t = \pi / 2$.

Figure 7 shows the transient concentration profile. For the same values of ω, it is seen that the transitory concentration rises as *Sc* rises. In the case of CO_2, the magnitude of concentration is greater than in the case of H_2O. The image also shows that concentration falls as you get further away from the plates.

It is now proposed to investigate the phase and amplitude behaviours of the skin friction. We get from Eqs (4.2), (4.4), and (3.15).

$$\tau = \tau_m + \varepsilon e^{it} \left[n_3 k_{13} - n_3 k_{14} + k_7 n_1 - k_8 n_1 + k_9 n_2 - k_{10} n_2 \right]. \tag{4.12}$$

Equation (4.12) can be expressed in terms of the phase and amplitude of the skin friction as:

$$\tau = \tau_m + \varepsilon |T| \cos(t + \varphi) , \tag{4.13}$$

where:

$T = T_r + iT_i$ = coefficients of εe^{it} in Eq.(4.12)

$|T| = \sqrt{T_r^2 + T_i^2}$, and $\tan\varphi = T_i / T_r$.

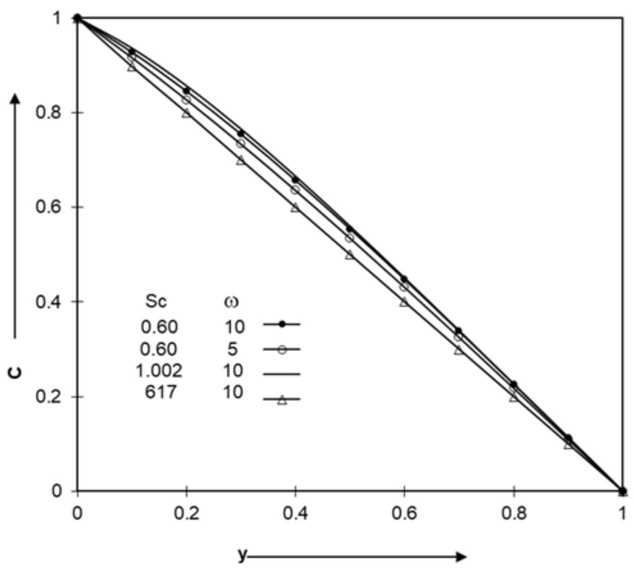

Fig.7. Transient concentration profiles for $\varepsilon = 0.2$, and $t = \pi / 2$.

In Fig.8., the amplitude of skin friction $|T|$ is shown for $Pr = 7$ (water) $\omega = 5$ and $Sc = 617$ (Cl_2). The figure shows that $|T|$ increases as *Gc* and *Gr* increase. It also decreases slightly with an increase in *M*.

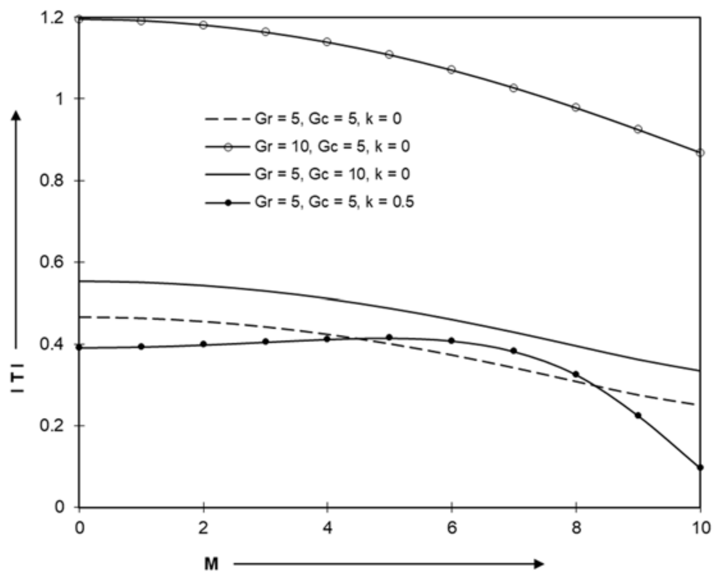

Fig.8. Amplitude of skin friction $Pr = 7$ (water), $Sc = 617$ (Cl_2) and $\omega = 5$.

Figure 9 shows the tangent of phase of the skin friction for $Pr = 7$ (water), $\omega = 5$ and $Sc = 617$ (Cl_2). The phase of the skin friction increases as *Gr* increases, whereas *Gc* has the opposite effect. For the same value of *Gr*, it is almost constant for *M* and *Gc*.

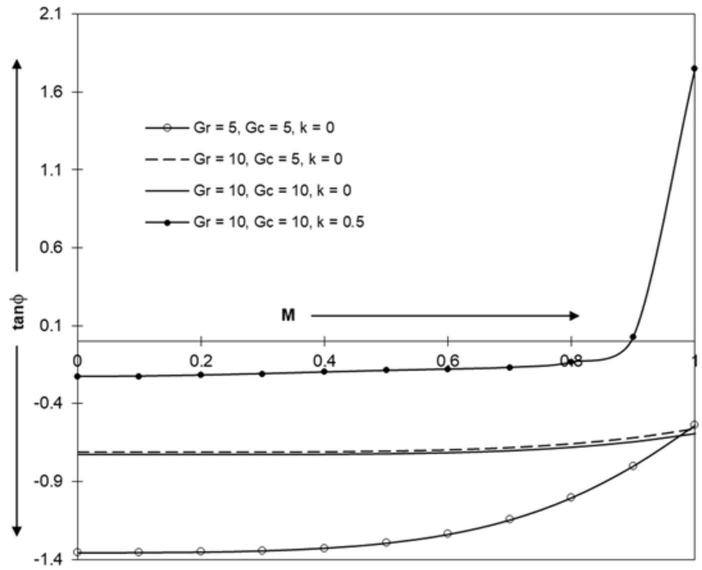

Fig.9. Phase of skin friction for $Pr = 7$, $\omega = 5$ and $Sc = 617$.

Now we will examine how the rate of heat transfer is affected by ω. The Nusselt number can be used to calculate the rate of heat transfer.

$$Nu = -\frac{q_\omega^* b}{k\left(T_0^* - T_b^*\right)} = \left(\frac{\partial\theta}{\partial y}\right)_{y=0} = \left(\frac{\partial\theta_0}{\partial y}\right)_{y=0} + \varepsilon\left(\frac{\partial\theta_1}{\partial y}\right)_{y=0} e^{it} \tag{4.14}$$

$$N_u = -1 + \varepsilon e^{it}\left[k_3 n_1 - k_4 n_1\right]. \tag{4.15}$$

We can express (4.15) in terms of phase and amplitude of heat transfer as:

$$Nu = -1 + \varepsilon\left|H\right|\cos(t + \psi) \tag{4.16}$$

where:

$H = H_r + iH_i$ = coefficients of $\varepsilon\, e^{it}$ in expression (4.16)

$\left|H\right| = \sqrt{H_r^2 + H_i^2}$ and $\tan\psi = H_i / H_r$.

Figure 10 depicts the phase and amplitude of heat transmission for different values of ω, which increase the phase and amplitude of heat transfer, but for $Pr=7$ the phase of heat transfer is not constant. For the same quantities of the frequency of fluctuations ω, the phase and the amplitude of heat transfer in air ($Pr = 0.71$) is smaller than that in water $(Pr = 7)$.

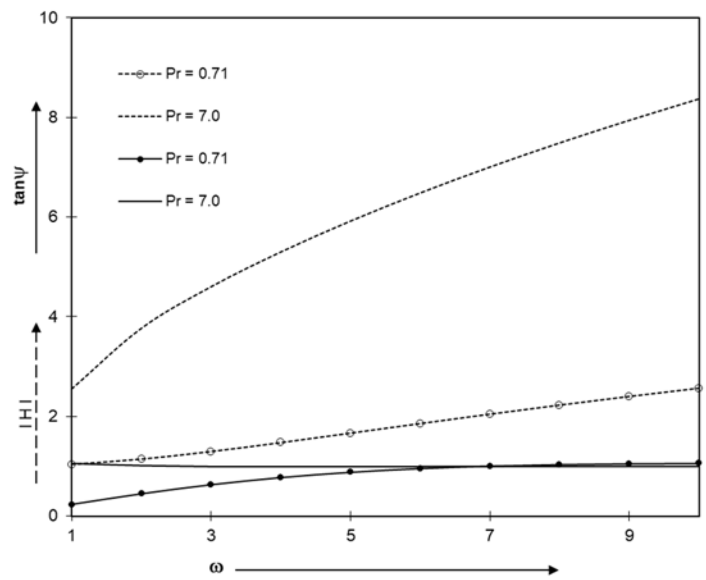

Fig.10. Amplitude and phase of heat transfer.

Nomenclature

B	− applied magnetic field
C^*	− dimensional species concentration
D	− mass diffusivity
g	− acceleration due to gravity
Gc	− Modified Grashoff number based on concentration
Gr	− Grashoff number
k	− permeability of porous medium
M	− Hartmann number
Nu	− Nussult number
Pr	− Prandtl number
q_w	− heat flux at wall
Sc	− Schmidt number
t^*	− time period
T_b	− temperature at the plate
u	− velocity in the clear fluid
U	− velocity in free stream
U_0	− constant amplitude of free stream velocity
α	− the thermal diffusivity
θ	− dimensionless temperature
ε	− amplitude of oscillations
κ	− thermal conductivity
μ	− viscosity

ν – kinematic viscosity
ρ – density
σ – electrical conductivity
τ – skin friction
τ_m – mean skin friction
ω – frequency

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