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HEAT AND MASS TRANSFER IN A VERTICAL CHANNEL FLOW THROUGH A POROUS MEDIUM IN THE PRESENCE OF RADIATION

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An analysis is made of heat and mass transfer in a three dimensional flow between two vertical porous plates through a porous medium. Analytical solutions have been obtained using the perturbation technique. The effect of non-dimensional parameters on velocity, temperature and concentration field are shown graphically. It is seen that the main flow velocity decreases with an increase in both the radiation parameter and Schmidt number but increases with an increase in the thermal Grashoff number, mass Grashoff number as well as the permeability parameter. Variations of the shear stress at the left plate are given in a tabular form. It is seen that the shear stress due to the primary flow at the left plate increases with an increase in the Reynolds number but decrease with an increase in the Schmidt number. With the increase of both the radiation parameter and Reynolds number the temperature decreases. The concentration field also decreases with an increase of the Schmidt number. Variations of mass flux at the left plate are given in tabular form. It is seen that the mass flux at the left plate increases with increase in both Schmidt number or Reynolds number.

Key words: mass transfer, permeability, porous medium, periodic suction, radiation.

1. Introduction

Free convective flow with heat and mass transfer has many application in science and technology. Guria and Jana [1] studied the effect of periodic suction on a three dimensional vertical channel flow.Guria *et al.* [2] extended the problem by adding radiation. The effect of radiation on the flow past a vertical plate was discussed by Takhar *et al.* [3]. Guria *et al.* [4] also studied the effect of radiation on flow past a vertical plate in the presence of a magnetic field.Sing and Thakar [5], Ahmed [6] and Ahmed and Liu [7] studied mixed convection flow and mass transfer.If the fluid temperature is rather high, radiation effects play an important role and this situation does exist in space technology. Nuclear power plants and various propulsion devices for aircrafts, satellites are examples of such engineering areas.

In this cases, one has to take into account the effects of radiation and free convection. Also, when the temperature of the plate is high, the radiation effects are not negligible. Reddy and Reddy [8] studied the effects of radiation and mass transfer flow with viscous dissipation. Free convective flows with periodic permeability through porous media has many applications in filtration and purification processes. Rapits [9], Rapits and Perdikis [10], Varshney [11], Ahmed and Ahmed [12] studied the flows through a porous medium. Singh *et al.* [13] studied a three dimensional flow through a porous medium past a vertical porous plate. Heat and mass transfer flow through a porous medium has attracted many investigators. Sachin [14] studied a three dimensional flow through a porous media with periodic permeability. Dwivedi *et al.* [15] studied a MHD vertical channel flow through a porous medium. Usman *et al.* [16] also studied two dimensional heat and mass transfer in a vertical channel flow through a porous medium with slip condition and radiation. Reddy *et al.* [17] investigated a magnetohydrodynamic heat and mass transfer flow past an acceletated vertical porous plate. Sumathi *et al.* [18] sudied an unsteady heat and mass transfer flow past an infinite vertical porous plate with fluctuating temperature.

Guria [19] investigated the heat and mass transfer flow past a vertical porous plate in the presence of radiation. Guria [20, 21] also studied the heat and mass transfer flow through a vertical channel in the presence of radiation. In this paper, we study the heat and mass transfer flow throgh the vertical channel in the presence of radiation in a porous medium.Our problem is non-trivial extension of the results obtained by Guria [20] by applying porosity of the medium.

2. Basic equations

Consider the flow between two vertical parallel porous plates apart at a distance d . Here the $x^{\star}$-axis is chosen along the direction of the flow, (see Fig.1). T_w and T_0 $(T_w > T_0)$ are the temperature at the left and right plates $y^* = 0$ and $y^* = d$, respectively. There is uniform injection V_0 and variable suction

$$v^{\star} = -V_0\left[1+\epsilon\cos\left(\frac{\pi z^{\star}}{d}\right)\right], \tag{2.1}$$

at the left and right plates, respectively.

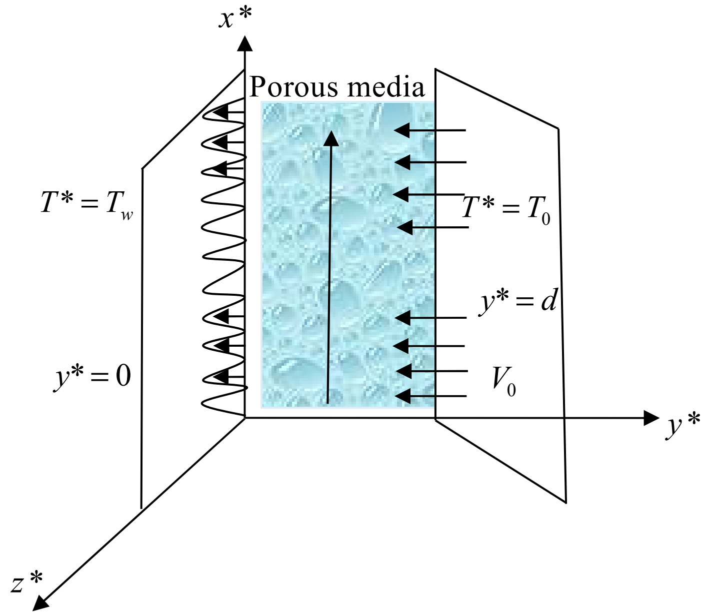

Fig.1. Physical model and co-ordinate system.

The Navier-Stokes equations are

$$\frac{\partial v^{\star}}{\partial y^{\star}} + \frac{\partial w^{\star}}{\partial z^{\star}} = 0, \tag{2.2}$$

$$v^{\star}\frac{\partial u^{\star}}{\partial y^{\star}} + w^{\star}\frac{\partial u^{\star}}{\partial z^{\star}} = \nu\left(\frac{\partial^2 u^{\star}}{\partial y^{\star 2}} + \frac{\partial^2 u^{\star}}{\partial z^{\star 2}}\right) + g\beta\left(T^* - T_0\right) + g\beta\left(C^* - C_0\right) - \frac{\nu u^{\star}}{K}, \tag{2.3}$$

$$v^\star \frac{\partial v^\star}{\partial y^\star} + w^\star \frac{\partial v^\star}{\partial z^\star} = -\frac{1}{\rho}\frac{\partial p^\star}{\partial y^\star} + \nu\left(\frac{\partial^2 v^\star}{\partial y^{\star 2}} + \frac{\partial^2 v^\star}{\partial z^{\star 2}}\right) - \frac{\nu v^\star}{K}, \tag{2.4}$$

$$v^\star \frac{\partial w^\star}{\partial y^\star} + w^\star \frac{\partial w^\star}{\partial z^\star} = -\frac{1}{\rho}\frac{\partial p^\star}{\partial z^\star} + \nu\left(\frac{\partial^2 w^\star}{\partial y^{\star 2}} + \frac{\partial^2 w^\star}{\partial z^{\star 2}}\right) - \frac{\nu w^\star}{K}, \tag{2.5}$$

$$v^\star \frac{\partial T^*}{\partial y^\star} + w^\star \frac{\partial T^*}{\partial z^\star} = \frac{1}{\rho C_p}\left(\frac{\partial^2 T^*}{\partial y^{\star 2}} + \frac{\partial^2 T^*}{\partial z^{\star 2}}\right) - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*}, \tag{2.6}$$

$$v^\star \frac{\partial C^*}{\partial y^\star} + w^\star \frac{\partial C^*}{\partial z^\star} = D\left(\frac{\partial^2 C^*}{\partial y^{\star 2}} + \frac{\partial^2 C^*}{\partial z^{\star 2}}\right) \tag{2.7}$$

where ν is the kinematic coefficient of viscosity, ρ is the density, $p^\star$ is the fluid pressure, g is the acceleration due to gravity, β is the thermal expansion and C_p is the specific heat at constant pressure. K^* is the permeability of the medium.

The boundary conditions of the problem are

$$\begin{aligned} &u^\star = 0, \quad v^\star = -V_0\left[1 + \epsilon\cos\left(\frac{\pi}{d}z^\star\right)\right], \quad w^\star = 0, \quad T^* = T_w, \quad C^* = C_w \quad \text{at} \quad y^\star = 0, \\ &u^\star = 0, \quad v^\star = -V_0, \quad w^\star = 0, \quad T^* = T_0, \quad C^* = C_\infty, \quad p^\star = p_\infty \quad \text{at} \quad y^\star = d. \end{aligned} \tag{2.8}$$

Assuming

$$y = \frac{y^\star}{d}, \quad z = \frac{z^\star}{d}, \quad p = \frac{p^\star}{\rho V_0^2}, \quad u = \frac{u^\star}{V_0}, \quad v = \frac{v^\star}{V_0}, \quad w = \frac{w^\star}{V_0}, \quad \theta = \frac{\left(T^* - T_0\right)}{\left(T_w - T_0\right)}, \tag{2.9}$$

Eqs (2.2)-(2.7) become

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{2.10}$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{Re}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + Gr\theta + GmC - \frac{u}{K}, \tag{2.11}$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{v}{K}, \tag{2.12}$$

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{w}{K}, \tag{2.13}$$

$$v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}=\frac{1}{RePr}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)-F\theta, \tag{2.14}$$

$$v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\frac{1}{SRe}\left(\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right) \tag{2.15}$$

where $Re=V_0 d/\nu$ is the Reynolds number, $Pr=\nu/\rho$ is the Prandtl number and $Gr=dg\beta\left(T_w-T_0\right)/V_0^2$ is the Grashof number, $Gm=dg\beta\left(C_w-C_0\right)/V_0^2$ is the mass Grashof number, $F=4Id/\rho C_p V_0$ is the radiation parameter, $S=\nu/D$ is the Schmidt number. Using (2.9), the boundary conditions (2.8) become

$$\begin{gathered}u=0,\quad v=-\left[1+\epsilon\cos(\pi z)\right],\quad w=0,\quad \theta=1,\quad C=1\quad \text{at}\quad y=0,\\ u=0,\quad v=-1,\quad w=0,\quad \theta=0,\quad C=0,\quad p=\frac{p_\infty}{\rho V^2}\quad \text{at}\quad y=1.\end{gathered} \tag{2.16}$$

3.Solution of the problem

We assume

$$\begin{gathered}u(y,z)=u_0(y)+\epsilon u_1(y,z)+\epsilon^2 u_2(y,z)+\cdots,\\ v(y,z)=v_0(y)+\epsilon v_1(y,z)+\epsilon^2 v_2(y,z)+\cdots,\\ w(y,z)=w_0(y)+\epsilon w_1(y,z)+\epsilon^2 w_2(y,z)+\cdots,\\ p(y,z)=p_0(y)+\epsilon p_1(y,z)+\epsilon^2 p_2(y,z)+\cdots,\\ \theta(y,z)=\theta_0(y)+\epsilon\theta_1(y,z)+\epsilon^2\theta_2(y,z)+\cdots.\\ C(y,z)=C_0(y)+\epsilon C_1(y,z)+\epsilon^2 C_2(y,z)+\cdots.\end{gathered} \tag{3.1}$$

On substituting Eqs (3.1) in Eqs (2.10)-(2.15), we get(terms free from ϵ)

$$v_0'=0, \tag{3.2}$$

$$u_0''-Rev_0u_0'-\frac{Reu_0}{K}=-ReGr\theta_0-ReGmC_0, \tag{3.3}$$

$$\theta_0''-RePrv_0\theta_0'-FRePr\theta_0=0, \tag{3.4}$$

$$C_0''-SRev_0C_0'=0, \tag{3.5}$$

with

$$u_0 = 0, \quad v_0 = -1, \quad \theta_0 = 1, \quad C_0 = 1 \quad \text{at} \quad y = 0,$$
$$u_0 = 0, \quad v_0 = -1, \quad \theta_0 = 0, \quad C_0 = 0 \quad \text{at} \quad y = 1. \tag{3.6}$$

The solutions of Eqs (3.2) to (3.5), subject to the boundary conditions (3.6) are

$$v_0(y) = -1, \tag{3.7}$$

$$\theta_0(y) = \frac{1}{\left(e^{-m_1} - e^{-m_2}\right)}\left[e^{-m_1}e^{-m_2 y} - e^{-m_2}e^{-m_1 y}\right], \tag{3.8}$$

$$C_0(y) = \frac{1}{\left(e^{-SRe} - 1\right)}\left[e^{-SRe} - e^{-SRey}\right], \tag{3.9}$$

$$u_0(y) = \left[\sum_{i=1}^{4} A_i e^{-m_i y} + A_5 + A_6 e^{-SRey}\right] \tag{3.10}$$

where

$$m_{1,2} = \frac{1}{2}\left\{RePr \pm \sqrt{Re^2 Pr^2 + 4FRePr}\right\}, \quad m_{3,4} = \frac{1}{2}\left\{Re \pm \sqrt{Re^2 + 4Re/K}\right\},$$

$$K_1 = \frac{-ReGr}{\left(e^{-m_1} - e^{-m_2}\right)}, \; K_2 = \frac{ReGm}{\left(e^{-SRe} - 1\right)}, \; A_1 = \frac{-K_2 e^{-m_2}}{\left(m_1^2 - m_1 Re - Re/K\right)},$$

$$A_2 = \frac{K_1 e^{-m_1}}{\left(m_2^2 - m_2 Re - Re/K\right)}, \; A_4 = -\left(A_1 + A_2 + A_3 + A_5 + A_6\right), \tag{3.11}$$

$$A_3 = \frac{-1}{(e^{-m_3} - e^{-m_4}}\left[A_1\left(e^{-m_1} - e^{-m_4}\right) + A_2\left(e^{-m_2} - e^{-m_4}\right) + A_5\left(1 - e^{-m_4}\right) + \right.$$
$$\left. + A_6\left(e^{-SRe} - e^{-m_4}\right)\right],$$

$$A_5 = \frac{KK_2 e^{-SRe}}{Re}, \; A_6 = \frac{K_2}{\left(S^2 Re^2 - SRe^2 - Re/K\right)}.$$

On substituting Eqs (3.1) in Eqs (2.10)-(2.15), we get (with the coefficient of ϵ),

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \tag{3.12}$$

$$v_0 \frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_0}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) + Gr\theta_1 + GmC_1 - \frac{u_1}{K}, \tag{3.13}$$

$$v_0 \frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{v_1}{K}, \tag{3.14}$$

$$v_0 \frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - \frac{w_1}{K}, \tag{3.15}$$

$$v_0 \frac{\partial \theta_1}{\partial y} + v_1 \frac{\partial \theta_0}{\partial y} = \frac{1}{RePr}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) - F\theta_1, \tag{3.16}$$

$$v_0 \frac{\partial C_1}{\partial y} + v_1 \frac{\partial C_0}{\partial y} = \frac{1}{SRe}\left(\frac{\partial^2 C_1}{\partial y^2} + \frac{\partial^2 C_1}{\partial z^2}\right), \tag{3.17}$$

with

$$\begin{aligned} &u_1 = 0, \quad v_1 = -\cos(\pi z), \quad w_1 = 0, \quad \theta_1 = 0, \quad C_1 = 0 \quad \text{at} \quad y = 0, \\ &u_1 = 0, \quad v_1 = 0, \quad w_1 = 0, \quad \theta_1 = 0, \quad C_1 = 0 \quad \text{at} \quad y = 1. \end{aligned} \tag{3.18}$$

We assume

$$\begin{aligned} &u_1(y,z) = u_{11}(y)\cos(\pi z), \\ &v_1(y,z) = v_{11}(y)\cos(\pi z), \\ &w_1(y,z) = -\frac{1}{\pi} v'_{11}(y)\sin(\pi z), \\ &p_1(y,z) = p_{11}(y)\cos(\pi z), \\ &\theta_1(y,z) = \theta_{11}(y)\cos(\pi z), \\ &C_1(y,z) = C_{11}(y)\cos(\pi z). \end{aligned} \tag{3.19}$$

Substituting Eqs (3.19) in Eqs (3.12)-(3.17), we obtain

$$v''_{11} + Rev'_{11} - \left(\pi^2 + \frac{Re}{K}\right)v_{11} = Rep'_{11}, \tag{3.20}$$

$$v'''_{11} + Rev''_{11} - \left(\pi^2 + \frac{Re}{K}\right)v'_{11} = Re\pi^2 p_{11}, \tag{3.21}$$

$$\theta_{11}'' + RePr\theta_{11}' - \left(FRePr + \pi^2\right)\theta_{11} = RePrv_{11}\theta_0', \tag{3.22}$$

$$C_{11}'' + SReC_{11}' - \pi^2 C_{11} = SRev_{11}C_0', \tag{3.23}$$

$$u_{11}'' + Reu_{11}' - \left(\pi^2 + \frac{Re}{K}\right)u_{11} = Rev_{11}u_0' - Re\left(Gr\theta_{11} + GmC_{11}\right). \tag{3.24}$$

The corresponding boundary conditions are

$$\begin{aligned} &u_{11} = 0, \quad v_{11} = -1, \quad v_{11}' = 0, \quad \theta_{11} = 0, \quad C_{11} = 0 \quad \text{at} \quad y = 0, \\ &u_{11} = 0, \quad v_{11} = 0, \quad v_{11}' = 0, \quad \theta_{11} = 0, \quad C_{11} = 0 \quad \text{at} \quad y = 1. \end{aligned} \tag{3.25}$$

Solutions of the Eqs (3.20)-(3.24) subject to conditions(3.25) and on using Eqs (3.19) yield

$$v_1(y,z) = \left[B_1 e^{-\lambda_1 y} + B_2 e^{-\lambda_2 y} + B_3 e^{\pi y} + B_4 e^{-\pi y}\right]\cos(\pi z), \tag{3.26}$$

$$w_1(y,z) = \frac{1}{\pi}\left[B_1\lambda e^{-\lambda_1 y} + B_2\lambda_2 e^{-\lambda_2 y} - B_3\pi e^{\pi y} + B_4\pi e^{-\pi y}\right]\sin(\pi z), \tag{3.27}$$

$$p_1(y,z) = \frac{1}{\pi}\left[B_3(\pi - 1/K)e^{\pi y} + B_4(\pi + 1/K)e^{-\pi y}\right]\cos(\pi z), \tag{3.28}$$

$$\begin{aligned} \theta_1(y,z) = \Big[&C_1 e^{-\lambda_3 y} + C_2 e^{-\lambda_4 y} + C_3 e^{-(\lambda_1 + m_2)y} + C_4 e^{-(\lambda_2 + m_2)y} + C_5 e^{(\pi - m_2)y} + \\ &+ C_6 e^{-(\pi + m_2)y} + C_7 e^{-(\lambda_1 + m_1)y} + C_8 e^{-(\lambda_2 + m_1)y} + C_9 e^{(\pi - m_1)y} + C_{10} e^{-(\pi + m_1)y}\Big]\cos(\pi z), \end{aligned} \tag{3.29}$$

$$\begin{aligned} C_1(y,z) = \Big[&G_1 e^{-\alpha_1 y} + G_2 e^{-\alpha_2 y} + G_3 e^{-(\lambda_1 + SRe)y} + G_4 e^{-(\lambda_2 + SRe)y} + \\ &+ G_5 e^{(\pi - SRe)y} + G_6 e^{-(\pi + SRe)y}\Big]\cos(\pi z), \end{aligned} \tag{3.30}$$

$$\begin{aligned} u_1(y,z) = \Big[&D_1 e^{-\lambda_1 y} + D_2 e^{-\lambda_2 y} + D_3 e^{-(\lambda_1 + m_1)y} + D_4 e^{-(\lambda_2 + m_1)y} + D_5 e^{(\pi - m_1)y} + \\ &+ D_6 e^{-(\pi + m_1)y} + D_7 e^{-(\lambda_1 + m_2)y} + D_8 e^{-(\lambda_2 + m_2)y} + D_9 e^{(\pi - m_2)y} + D_{10} e^{-(\pi + m_2)y} + \\ &+ D_{11} e^{-(\lambda_1 + m_3)y} + D_{12} e^{-(\lambda_2 + m_3)y} + D_{13} e^{(\pi - m_3)y} + D_{14} e^{-(\pi + m_3)y} + D_{15} e^{-(\lambda_1 + m_4)y} + \\ &+ D_{16} e^{-(\lambda_2 + m_4)y} + D_{17} e^{(\pi - m_4)y} + D_{18} e^{-(\pi + m_4)y} + D_{19} e^{-(\lambda_1 + SRe)y} + D_{20} e^{-(\lambda_2 + SRe)y} + \\ &+ D_{21} e^{(\pi - SRe)y} + D_{22} e^{-(\pi + SRe)y} + D_{23} e^{-\lambda_3 y} + D_{24} e^{-\lambda_4 y} + D_{25} e^{\alpha_1 y} + D_{26} e^{-\alpha_2 y}\Big]\cos(\pi z) \end{aligned} \tag{3.31}$$

where

$$\lambda_{1,2} = \frac{1}{2}\left\{Re \pm \sqrt{Re^2 + 4\left(\pi^2 + Re/K\right)}\right\},$$

$$\lambda_{3,4} = \frac{1}{2}\left\{RePr + \sqrt{Re^2Pr^2 + 4\left(FRePr + \pi^2\right)}\right\},$$

$$\alpha_{1,2} = \frac{1}{2}\left\{SRe \pm \sqrt{S^2Re^2 + 4\pi^2}\right\},$$

$$B_1 = \left[\pi r_2\left(e^{\pi} - e^{-\pi}\right) + r_4\left(e^{\pi} + e^{-\pi}\right)\right] / 2\left(r_1r_4 - r_2r_3\right),$$

$$B_2 = -\left[\pi r_1\left(e^{\pi} - e^{-\pi}\right) + r_3\left(e^{\pi} + e^{-\pi}\right)\right] / 2\left(r_1r_4 - r_2r_3\right),$$

$$B_3 = -\frac{1}{2\pi}\left[\pi + A_5\left(\pi - m_5\right) + A_6\left(\pi - m_6\right)\right], \tag{3.32}$$

$$B_4 = -\frac{1}{2\pi}\left[\pi + A_5\left(\pi + m_5\right) + A_6\left(\pi + m_6\right)\right],$$

$$r_1 = e^{-m_5} - \frac{1}{2\pi}\left[e^{\pi}\left(\pi - m_5\right) + e^{-\pi}\left(\pi + m_5\right)\right],$$

$$r_2 = e^{-m_6} - \frac{1}{2\pi}\left[e^{\pi}\left(\pi - m_6\right) + e^{-\pi}\left(\pi + m_6\right)\right],$$

$$r_3 = m_5e^{-m_5} + \frac{1}{2}\left[e^{\pi}\left(\pi - m_5\right) - e^{-\pi}\left(\pi + m_5\right)\right],$$

$$r_4 = m_6e^{-m_6} + \frac{1}{2}\left[e^{\pi}\left(\pi - m_6\right) - e^{-\pi}\left(\pi + m_6\right)\right].$$

We omit the other constants to save space.

4. Results and discussion

In order to get a physical insight into the problem, we have studied the velocity field, shear stress, temperature and concentration field, mass flux for several values of non-dimensional parameters. To be realistic, we have chosen $Sc = 0.30,\ 0.6,\ 0.78$ to represent concentration distribution of helium, water vapour and ammonia, respectively. The effects of the Grashoff number, mass Grashoff number, radiation parameter, Schmidt number and permeability parameter on the primary flow velocity are shown in Figs 2-6. Figure 2 represents the primary velocity for different values of $Gr = 2,5,8,10$ It is observed that the primary velocity increases with an increase in the Grashoff number. Figure 3 represents the primary velocity for different values of $Gm = 2,5,8,10$. It is seen that the primary velocity increases with an increase in the mass Grashoff number. Figure 4 illustrates the primary velocity for different values of $F = 2,3,4,5$. The primary velocity decreases with an increase in the radiation parameter. Figure 5 represents the primary velocity for

different values of $S = 0.3, 0.6, 0.66, 0.78$. The primary velocity decreases with an increase in the Schmidt number. Figure 6 represents the primary velocity for different values of $K = 0.1, 0.5, 1, 2$. It is observed that the primary velocity increases with an increase in the permeability parameter.
The shear stress at the plate $y^* = 0$ due to the primary flow is given by

$$\tau_x^* = \mu\left(\frac{\partial u^*}{\partial y^*}\right)_{y^*=0} = \frac{\mu V_0}{d}\left(\frac{\partial u}{\partial y}\right)_{y=0} . \tag{4.1}$$

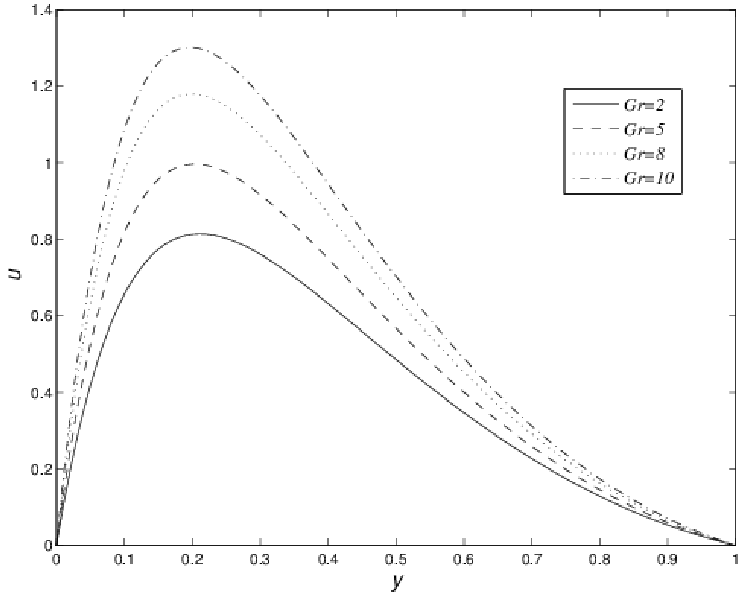

Fig.2. Primary velocity u for $S = 0.3, Gm = 5, F = 2, Pr = 0.71, Re = 5, K = 1$.

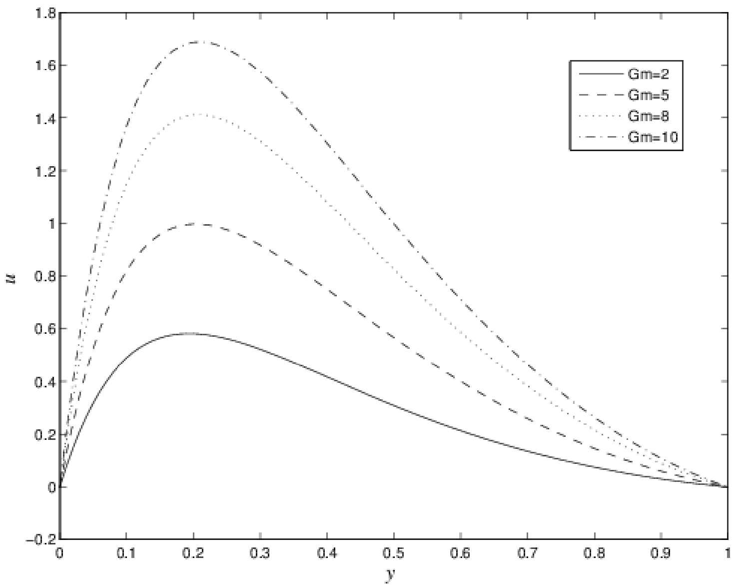

Fig.3. Primary velocity u for $S = 0.3, Gr = 5, F = 2, Pr = 0.71, Re = 5, K = 1$.

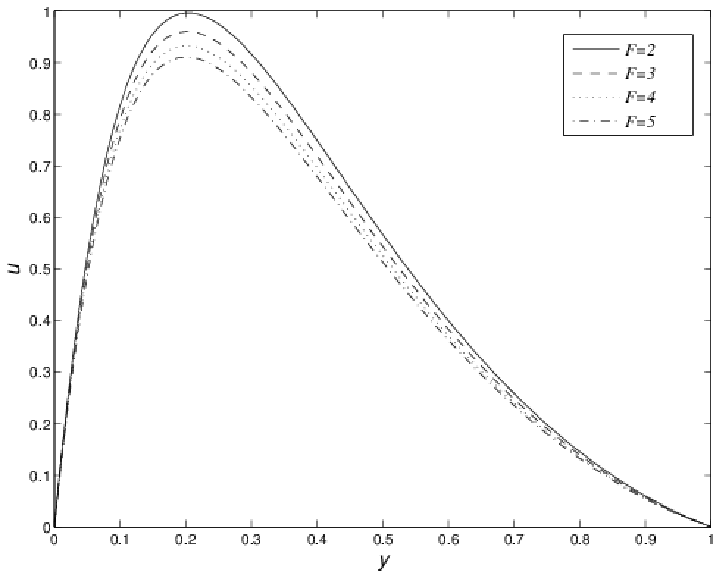

Fig.4. Primary velocity u for $S = 0.3, Gm = 5, Gr = 5, Pr = 0.71, Re = 5, K = 1$.

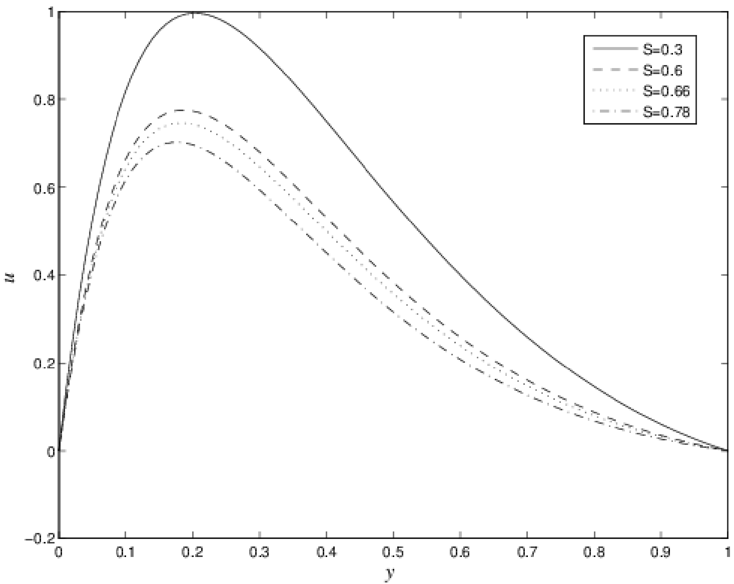

Fig.5. Primary velocity u for $F = 2, Gm = 5, Gr = 5, Pr = 0.71, Re = 5, K = 1$.

In a non-dimensional form, it can be written as

$$\tau_x = \frac{\tau_x^* d}{\mu V_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0},$$

$$\tau_x = u_0^{'}(0) + \epsilon u_1^{'}(0),$$

$$\tau_x = -\sum_{i=1}^{4} A_i m_i - A_6 SRe - \epsilon\big[D_1\lambda_1 + D_2\lambda_2 + D_3(\lambda_1 + m_1) + D_4(\lambda_2 + m_1) - D_5(\pi - m_1) +$$
$$+D_6(\pi + m_1) + D_7(\lambda_1 + m_2) + D_8(\lambda_2 + m_2) + D_9(\pi - m_1) + D_{10}(\pi + m_2) +$$
$$+D_{11}(\lambda_1 + m_3) + D_{12}(\lambda_2 + m_3) - D_{13}(\pi - m_3) + D_{14}(\pi + m_3) + D_{15}(\lambda_1 + m_4) + \quad (4.2)$$
$$+D_{16}(\lambda_2 + m_4) - D_{17}(\pi - m_4) + D_{18}(\pi + m_4) + D_{19}(\lambda_1 + SRe) + D_{20}(\lambda_2 + SRe) +$$
$$+D_{21}(\pi - SRe) + D_{22}(\pi + SRe) + D_{23}\lambda_3 + D_{24}\lambda_4 + D_{25}\alpha_1 + D_{26}\alpha_2\big]\cos(\pi z).$$

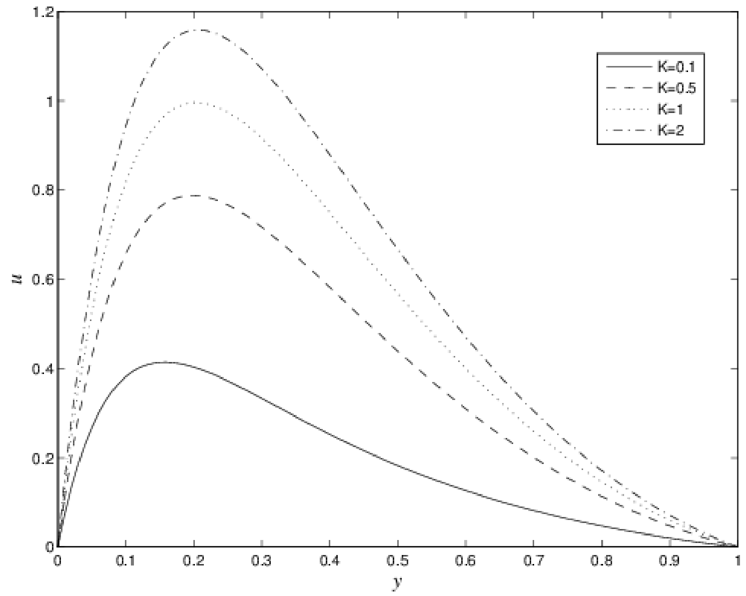

Fig.6. Primary velocity u for $F = 2, Gm = 5, Gr = 5, Pr = 0.71, Re = 5, K = 1$.

Variation of τ_x is shown in Tab.1 for different values of $Re = 2,3,4,5$ and $S = 0.3,0.6,0.66,0.78$. It is seen that τ_x increases with an increase in the Reynolds number but it decreases with an increase in the Schmidt number.

Table.1. Shear stress component due to primary flow for $Gr = 5.0$, $Pr = 0.71$, $\epsilon = 0.25$, $z = 0.0$.

	τ_x			
Re	$S = 0.3$	$S = 0.6$	$S = 0.66$	$S = 0.78$
2	3.08	2.58	2.43	2.00
3	4.34	3.31	3.04	2.27
4	5.49	3.84	3.44	2.35
5	6.50	4.15	3.62	2.27

The temperature profile θ is plotted for different values of F and Re in Figs7 and 8. Figure 7 represents the temperature profile for different values of $F = 2,3,4,5$ It is observed that it decreases with an increase in the radiation parameter. Figure 8 illustrates the temperature profile for different values of $\mathrm{Re} = 2,3,4,5$. It is seen that it decreases with an increase in the Reynolds number. The variations of $C(y)$ for different values of S and Re are shown in Figs 9 and 10. In Figure 9 we have plotted $C(y)$ for several values of $S = 0.3, 0.6, 0.66, 0.78$. It is observed that the concentration field decreases with an increase in the Schmidt number. In Figure 10, we have plotted $C(y)$ for several values of $Re = 2, 3, 4, 5$. It is observed that the concentration field decreases with an increase in the Reynolds number.

The non-dimensional mass flux at the plate $y = 0$ in terms of the Sherwood number Sh is

$$Sh = \left(\frac{\partial C}{\partial y}\right)_{y=0}$$

$$Sh = -C_0'(0) - \epsilon C_1'(0),$$

$$Sh = -C_0'(0) - \epsilon C_{11}'(0)\cos(\pi z). \tag{4.3}$$

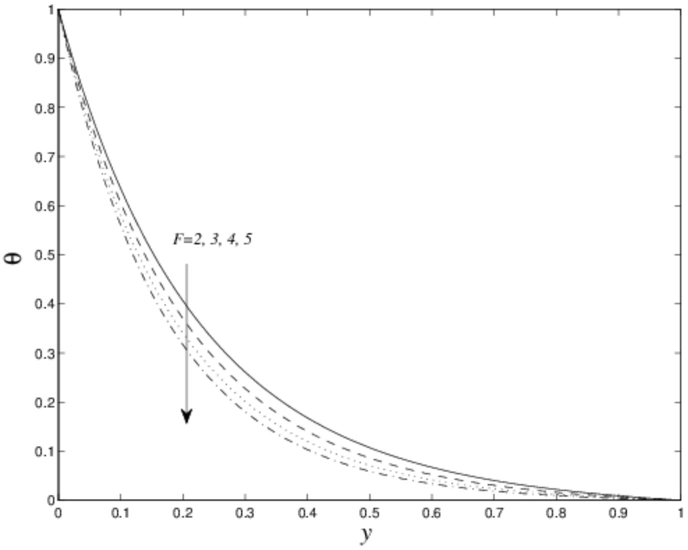

Fig.7. Temperature profile θ for $Gr = 5.0$, $Re = 5.0$, $Pr = 0.71$.

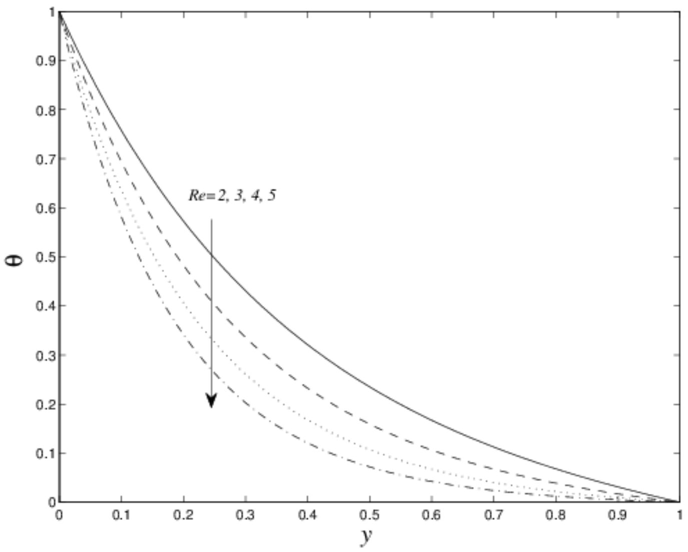

Fig.8. Temperature profile θ for $Gr = 5.0$, $Re = 5.0$, $Pr = 0.71$, $\epsilon = 0.25$.

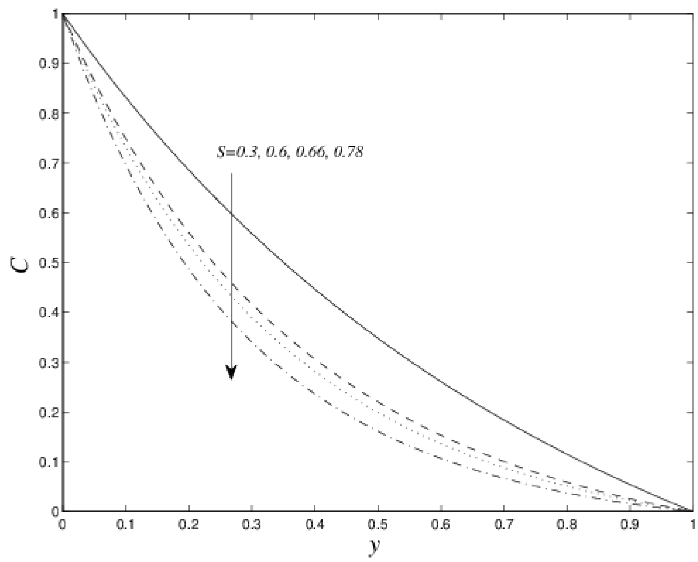

Fig.9. Variations of concentration field for $Re = 5$.

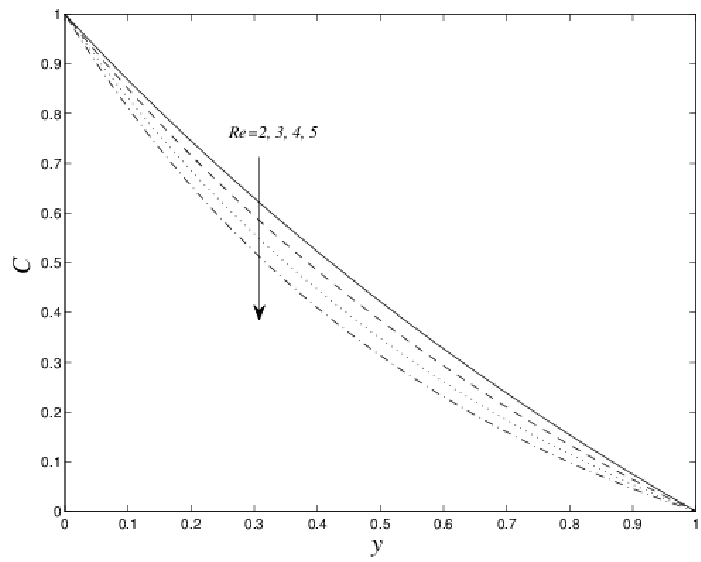

Fig.10. Variations of concentration field for $S = 0.3$.

Table.2. Sherwood number for $Gr = 5.0$, $Pr = 0.71$, $\epsilon = 0.25$, $z = 0.0$.

	Sh			
Re	$S = 0.3$	$S = 0.6$	$S = 0.66$	$S = 0.78$
2	*1.29*	*1.62*	*1.69*	*1.83*
3	*1.45*	*1.98*	*2.10*	*2.33*
4	*1.62*	*2.38*	*2.54*	*2.87*
5	*1.80*	*2.79*	*3.01*	*3.44*

The mass flux increseases with an increase in the both Schmidt number and Reynolds number.

5. Conclusion

Heat and mass transfer flow through a porous medium has been studied in the presence of radiation. It is found that an increase in the thermal Grashoff number or mass Grashoff number or permeability parameter leads to an increase in the primary velocity while an increased radiation parameter or the Schmidt number decrease the primary velocity. Varations of shear stress at the left plate are given in tabuler form. It is seen that the shear stress due to the primary flow at the left plate increases with an increase in the Reynolds number but decreases with an increase in the Schmidt numer. It is observed that the temperature profile decreases with an increase in either the radiation parameter or the Reynolds number. The concentration field also decreases with an increase of the both Schmidt number and Reynolds number. Variations of the mass flux at the left plate are given in a tabuler form. It is seen that the mass flux at the left plate increseases with an increase in the both Schmidt number and Reynolds number.

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Nomenclature

$A_i, i = 1,..6$ – constants
$B_i, i = 1,\cdots 4$ – constants
$C_i, i = 1,\cdots 10$ – constants
$D_i, i = 1,\cdots 26$ – constants
d – channel width
F – radiation parameter
$G_i, i = 1,\cdots 6$ – constants
g – gravitational acceleration
Gr – Grashoff number
Gm – mass Grashoff number
K^* – permeability of the porous medium
K_1, K_2 – constants
$m_i, i = 1,\cdots 6$ – constants
Nu_1, Nu_2 – Nusselt number at the left and right plates
p^* – pressure
p – dimensionless pressure
Pr – Prandtl number
q – local heat transfer at the plate
$r_i, i = 1,\cdots 4$ – constants
Re – Reynolds number
T^* – temperature of the fluid
T_w – plate temperature ($y^* = 0$)
T_0 – plate temperature ($y^* = d$)

u^*, v^*, w^* – velocity components in x, y, z axes

u,v,w – dimensionless velocity components in x, y, z axes respectively

V_0 – constant suction velocity

x^*, y^*, z^* – Cartesian coordinates system

x,y,z – dimensionless Cartesian coordinate system

β – coefficient of thermal expansion

θ – non-dimensional temperature

ν – kinematic viscosity

ε – amplitude of the suction velocity

ρ – density of the fluid

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