A NUMERICAL STUDY OF DISSIPATIVE CHEMICALLY REACTIVE RADIATIVE MHD FLOW PAST A VERTICAL CONE WITH NON-UNIFORM MASS FLUX

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A computational model is presented to explore the properties of heat source, chemically reacting radiative, viscous dissipative MHD flow of an incompressible viscous fluid past an upright cone under inhomogeneous mass flux. A numerical study has been carried out to explore the mass flux features with the help of Crank-Nicolson finite difference scheme. This investigation reveals the influence of distinct significant parameters and the obtained outputs for the transient momentum, temperature and concentration distribution near the boundary layer is discussed and portrayed graphically for the active parameters such as the Schmidt number Sc, thermal radiation *Rd*, viscous dissipation parameter ε, chemical reaction parameter λ, MHD parameter *M* and heat generation parameter Δ. The significant effect of parameters on shear stress, heat and mass transfer rates are also illustrated.

Key words: free convection, finite difference, vertical cone, MHD, thermal radiation, viscous dissipation.

1. Introduction

The occurrence of mass transfer commonly exists in numerous chemical processing industries for instance food processing and polymer process.Magneto hydrodynamics is the science, which deals with the motion of an electrically conducting fluid in the presence of magnetic fields. The study of magneto hydrodynamics with heat and mass transfer in the presence of radiation has attracted many researchers and engineers due to diverse applications in different areas of science and technology. MHD plays an important role in geophysics, agriculture, petroleum industries, in exploration and thermal recovery of oil, geothermal reservoirs and underground nuclear waste storage sites. In the field of power generation, MHD is receiving

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considerable attention due to the possibilities of much higher thermal efficiencies in power plants. It is worth mentioning that the basic solution of thermal convection from a heated or cooled surface with suction or injection has numerous practical applications ranging from cooling of manufactured products to local weather forecasting.

The consequences of viscous dissipation takes a vital part in free convection for the range of applications that are subject matter to huge amount of deceleration or which functions at high rotational speeds. Analyzing the temperature and heat transfer have enormous significance to engineers since its universal incidence in several branches of engineering and science.

Rao and Shivaiah [1] presented a finite element method solutions for the governing equations under the influence of chemical reaction on a transient (MHD) flow from a semi infinite vertical plate with viscous dissipation. Siddiqa *et al*. [2] studied the natural convection flow of a viscous fluid above a semi-infinite horizontal plate with internal heat generation and variable viscosity. Srinivasa and Easwara [3] emphasised the transient mixed convection flow over a vertical plate moving in a parallel free flow with heat generation and absorption. Ramli *et al*. [4] examined the MHD convection flow due to external forces and transfer of heat by ferrofluids above a moving plate with consistent heat flux. Lakshmi Narayana and Sibanda [5] showed the Soret and Dufour effects on a natural convection over a vertical wavy surface in a Darcy porous medium. Kumari and Nath [6] investigated the natural convection effects over a flat cone under porous medium with VWT/VWC or VHF/VMF and heat source/sink. Pal and Talukdar [7] used perturbation technique for transient magneto hydro dynamic mixed convection flow in a chemically reactive, radiative micro polar fluid and Ullaha *et al*. [8] studied the above said effects with Maxwell fluid by homotopy analysis method.

A great number of engineering applications takes place at elevated temperatures and hence the knowledge of radiation and heat transfer is necessary for framing various models of industrial apparatus. Large amount of heat is generated when chemical reaction takes place in a process and because of this the temperature of the body increases. Several findings have been accounted on the mass transfer movement of a radiative fluids in the occurrence of MHD was studied by Sambath and Bapuji [9-12]. Muhaimin *et al*. [13] studied variable viscosity with thermally stratified impacts on magneto hydrodynamic free convective mass transfer over a porous wedge in the existence of chemical reaction. Zueco [14] presented the mixed convection mass transfer flow along a porous plate. Devi and Kumarai [15] discussed numerically the slip flow impacts on transient hydro magnetic flow over a enlarged exterior with thermal energy. Alam *et al*. [16] explored the analysis of transient magneto hydro dynamic natural convective flow of the heat transfer over a vertical porous flat plate with internal heat source. The consequence of double diffusion from a vertical truncated cone in a non-Newtonian fluid saturated porous medium with variable heat and mass fluxes wasaddressed by Awad *et al*. [17]. Cheng [18] executed the impact of MHD free convection flow with heat source in transient state. Khan *et al*. [19] examined the consequences of the transfer of heat energy flow of ferrofluids over a horizontal plate with consistent heat flux. Das *et al*. [20] attempted MHD free convective flow of nanofluids past a permeable plate in a rotating system with thermal energy. Also researchers [21-23] portrayed the impact of chemical reaction, MHD with stagnation point flow in porous medium. Chamka *et al*. [24] studied non-Darcy natural convective flow of nanofluid over a cone in a saturated permeable medium with homogeneous mass fraction flux. Ishak *et al*. [25] focused attention with similarity solutions for a vertical plate.

Recently the following investigators [27-33] presented electrically conducting, magneto hydrodynamic transfer of heat in a specific medium like Maxwell fluid, micro polar, nanofluid, Maxwell nanofluid with or without viscous dissipation, heat generation, absorption and mass flux. Magneto hydrodynamic convection flow due to the impacts of external forces and heat transfer over ferrofluids along a moving plate with uniform heat flux and second-order slip effects, chemical reaction with MHD stagnation point flow in porous medium were presented by the researchers [36-39].

In this study, the predominant idea is to simulate the numerical solution to radiative, chemically reactive, viscous dissipative MHD flow past a non- isothermal vertical cone in the occurrence of mass flux and heat generation by Crank-Nicholson scheme [26]. Such a thought is noteworthy in the research fields of biology and chemical engineering. The impacts of dissimilar relevant parameters are analyzed and the concluding results are demonstrated through graphs.

2. Mathematical formulation

In this article, an axi-symmetric, transient boundary layer flow of a viscous incompressible electrically conducting fluid past a vertical cone with inconsistent heat mass fluxes under the influence of chemical reaction, heat generation under steady magnetic field B_0, dissipation effects. The flow is chosen along the direction of the x-coordinate and is chosen along the exterior surface of the cone in the vertical direction and the y axis considered normally outward as shown in Fig.1. Since the outer surface is vast so the fluid flow extended towards infinity. The fluid taken for consideration being gray, absorbing-emitting but non-scattering medium because of this fact Rosseland approximation is employed to illustrate the radiative heat flux. Initially, when time $t' < 0$ the fluid as well as the outer surface of the cone is at rest with temperature T_w' and concentration C_w' at the wall and which is greater than the ambient temperature T_∞' and ambient concentration C_∞'. When time $t' > 0$ the heat energy distributed from the exterior surface of the cone to the fluid medium at the rate of $q_w(x) = ax^m$, $q_w^*(x) = ax^n$ and is retained at this level. With these preceding assumptions, the equations which governs the flow, continuity, momentum, energy and concentration with the help of Boussinesq approximation can be written in a Cartesian form as mentioned below

$$\frac{\partial}{\partial x}(ur) + \frac{\partial}{\partial y}(vr) = 0 , \tag{2.1}$$

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - g\beta(T' - T_\infty')\cos\phi - \upsilon\frac{\partial^2 u}{\partial y^2} - g\beta_c(C' - C_\infty')\cos\phi + \frac{\sigma B_0^2 u}{\rho} = 0 , \tag{2.2}$$

$$\rho C_p \left[\frac{\partial T'}{\partial t'} + u\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial y} - \alpha\frac{\partial^2 T'}{\partial y^2} \right] = Q_0(T' - T_\infty') - \frac{\partial q_r}{\partial y} - \mu\left(\frac{\partial u}{\partial y}\right)^2 , \tag{2.3}$$

$$\frac{\partial C'}{\partial t'} + u\frac{\partial C'}{\partial x} + v\frac{\partial C'}{\partial y} - D\frac{\partial^2 C'}{\partial y^2} + k_1\left(C' - C_\infty'\right) = 0 . \tag{2.4}$$

The significant initial and boundary conditions are given by

$$t' \leq 0 : u = 0, \quad v = 0, \quad T' = T_\infty', \quad C' = C_\infty' \qquad \text{for all} \quad x \quad \text{and} \quad y,$$

$$t' > 0 : u = 0, \quad v = 0, \quad \frac{\partial T'}{\partial y} = \frac{-q_w(x)}{k}, \quad \frac{\partial C'}{\partial y} = \frac{-q_w^*(x)}{D}, \quad \text{at} \quad y = 0,$$

$$u = 0, \quad T' = T_\infty', \quad C' = C_\infty' \quad \text{at} \quad x = 0, \tag{2.5}$$

$$u \to 0, \quad T' \to T_\infty', \quad C' \to C_\infty' \quad \text{as} \quad y \to \infty .$$

The radiative heat flux expression $\frac{\partial q_r}{\partial y}$ appearing in the energy equation in the y- direction is made simpler by utilizing the Rosseland approximation given by Brewster [40]

$$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y} . \tag{2.6}$$

Since the temperature variation within the flow is sufficiently small, we ignore higher order terms more than the first degree in $T' - T'_\infty$. The linear structure of Eq.(2.6) could be attained by escalating T'^4 into Taylor series about T'_∞.

We obtain $\qquad T'^4 \approx T'^3_\infty T' - 3T'^4_\infty$. $\hfill (2.7)$

Applying Eqs (2.5) and (2.6) in Eq.(2.3), we have

$$\frac{\partial T'}{\partial t'} + u\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial y} = \alpha\frac{\partial^2 T'}{\partial y^2} + \frac{Q_0}{\rho c_p}(T' - T'_\infty) - \frac{1}{\rho c_p}\frac{16\sigma * T'^3_\infty}{3k*}\frac{\partial^2 T'}{\partial y^2}. \qquad (2.8)$$

The local skin friction, local Nusselt number and local Sherwood number are

$$\tau_x = \left(\mu\frac{\partial u}{\partial y}\right)_{y=0}, \qquad (2.9)$$

$$Nu_x = \frac{-\left(x\frac{\partial T'}{\partial y}\right)_{y=0}}{T'_w - T'_\infty}, \qquad (2.10)$$

$$Sh_x = \frac{-\left(x\frac{\partial C'}{\partial y}\right)_{y=0}}{C'_w - C'_\infty}. \qquad (2.11)$$

The time dependence assessments of skin friction, heat, mass transfer rate are given by

$$\overline{\tau_L} = \frac{2\mu}{L^2}\int_0^L x\left(\frac{\partial u}{\partial y}\right)_{y=0} dx. \qquad (2.12)$$

Heat transfer coefficient is given by

$$\overline{h} = \frac{-2k}{L^2}\int_0^L \frac{x\left(\frac{\partial T'}{\partial y}\right)_{y=0}}{T'_w - T'_\infty} dx, \qquad (2.13)$$

$$\overline{N_{u_L}} = \frac{L\overline{h}}{k} = -\frac{2}{L}\int_0^L \frac{x\left(\frac{\partial T'}{\partial y}\right)_{y=0}}{T'_w - T'_\infty} dx, \qquad (2.14)$$

$$\overline{S_{h_L}} = \frac{Lk_l}{D} = -\frac{2}{L}\int_0^L \frac{x\left(\frac{\partial C'}{\partial y}\right)_{y=0}}{C'_w - C'_\infty} dx. \qquad (2.15)$$

Establishing the following non dimensional variables for the parametric study

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}(\mathrm{Gr}_L)^{\frac{1}{4}}, \quad R = \frac{r}{L}, \quad \text{where} \quad r = x\sin\phi,$$

$$V = \frac{vL}{\upsilon}(\mathrm{Gr}_L)^{\frac{-1}{5}}, \quad U = \frac{uL}{\upsilon}(\mathrm{Gr}_L)^{\frac{-2}{5}}, \quad t = \frac{\upsilon t'}{L^2}(\mathrm{Gr}_L)^{\frac{2}{5}},$$

$$T = \frac{T' - T'_\infty}{(q_w(L)L/k)}, \quad \mathrm{Gr}_L = \frac{g\beta(T'_w - T'_\infty)L^3\cos\phi}{\upsilon^2}, \quad \Pr = \frac{\upsilon}{\alpha}, \tag{2.16}$$

$$C = \frac{(C' - C'_\infty)}{(q_w(L)L/D)}, \quad \mathrm{Gr}^* = \frac{g\beta_c(C'_w - C'_\infty)L^3\cos\phi}{\upsilon^2}, \quad \mathrm{Sc} = \frac{\upsilon}{D},$$

$$N = \frac{\mathrm{Gr}^*}{\mathrm{Gr}_L}, \quad \Delta = \frac{Q_o L^2}{C_p\mu}(\mathrm{Gr}_L)^{\frac{-1}{5}}, \quad \lambda = \frac{k_1 L^2}{\upsilon}(\mathrm{Gr}_L)^{\frac{-1}{5}}, \quad M = \frac{\sigma B_0^2 L^2}{\mu}\mathrm{Gr}_L^{-1/5},$$

$$R_d = \frac{K_1 * K}{4\sigma * T'^3_\infty}, \quad \varepsilon = \frac{g\beta L}{C_P},$$

where ε is the Eckert number as explained by Gebhert [34] and Jordan [35].

The above Eqs (2.1)-(2.5) be able to be written in dimensionless form

$$\frac{\partial}{\partial X}(UR) + \frac{\partial}{\partial Y}(VR) = 0, \tag{2.17}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} - \frac{\partial^2 U}{\partial Y^2} - (T + NC) + MU = 0, \tag{2.18}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} - \frac{1}{\Pr}\left(1 + \frac{4}{3R_d}\right)\frac{\partial^2 T}{\partial Y^2} - \Delta T - \varepsilon\left(\frac{\partial U}{\partial Y}\right)^2 = 0, \tag{2.19}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} - \frac{1}{\mathrm{Sc}}\frac{\partial^2 C}{\partial Y^2} + \lambda C = 0. \tag{2.20}$$

The dimensionless appropriate initial, boundary conditions are

$$t \le 0 : U = 0, \quad V = 0, \quad T = 0, C = 0 \text{ for all } X \text{ and } Y,$$

$$t > 0 : \quad U = 0, \quad V = 0, \quad \frac{\partial T}{\partial Y} = -X^n, \quad \frac{\partial C}{\partial Y} = -X^m \quad \text{at } Y = 0,$$

$$\tag{2.21}$$

$$U = 0, \quad T = 0, \quad C = 0 \quad \text{at } X = 0,$$

$$U \to 0, \quad T \to 0, \quad C \to 0 \quad \text{as } Y \to \infty.$$

Local shear stress, heat and mass transfer rate in dimensionless form are

$$\tau_X = \mathrm{Gr}_L^{\frac{3}{5}}\left(\frac{\partial U}{\partial Y}\right)_{Y=0}, \tag{2.22}$$

$$\mathrm{Nu}_X = \frac{X}{T_{Y=0}}\left(\frac{-\partial T}{\partial Y}\right)_{Y=0} \mathrm{Gr}_L^{\frac{1}{5}}, \tag{2.23}$$

$$\mathrm{Sh}_X = \frac{X}{C_{Y=0}}\left(\frac{-\partial C}{\partial Y}\right)_{Y=0} \mathrm{Gr}_L^{\frac{3}{5}}. \tag{2.24}$$

Dimensionless average skin friction heat and mass transfer rate are

$$\overline{\tau} = 2\mathrm{Gr}_L^{\frac{3}{4}}\int_0^1 X\left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX, \tag{2.25}$$

$$\overline{\mathrm{Nu}} = 2\mathrm{Gr}_L^{\frac{1}{4}}\int_0^1 \frac{X}{T_Y=0}\left(\frac{-\partial T}{\partial Y}\right)_{Y=0} dX, \tag{2.26}$$

$$\overline{\mathrm{Sh}} = 2\mathrm{Gr}_L^{\frac{1}{4}}\int_0^1 \frac{X}{C_Y=0}\left(\frac{-\partial C}{\partial Y}\right)_{Y=0} dX. \tag{2.27}$$
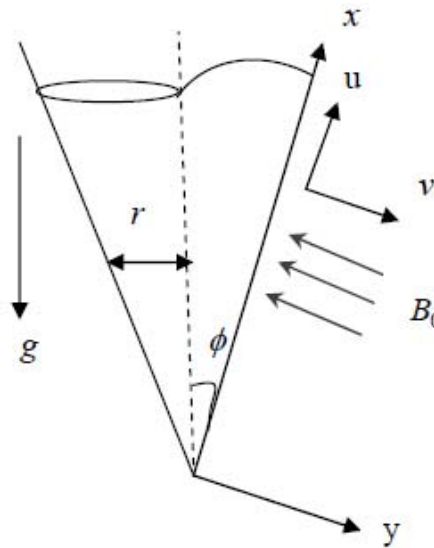

Fig.1. Physical model and coordinate system.

3. Method of solution

The finite difference scheme is one of the dependable procedure for solving partial differential equations. Crank–Nicolson method has been adopted to solve Eqs (2.17)-(2.20) with the help of Eq.(2.15). The integral region has been assumed to be a rectangular mesh with, $X \max = 1$ and $Y \max = 18$, where $Y_{\max} = \infty$ which is farthest from all periphery layers. The assessment of Y max was selected after a few initial examinations, so that the ultimate and penultimate periphery conditions of Eq.(2.15) are contented. The mesh sizes have been predetermined as $\Delta X = 0.05$, $\Delta Y = 0.05$ and $\Delta t = 0.01$. In this assumption, size of the spatial mesh is decreased by *50%* in *x*- direction, followed by in *x* and *y* directions and the outcomes are analyzed. It is noticed that it is contented with correctness up to the lenience limit of 10^{-5}. Hence, the mesh sizes adopted here are believed to be suitable for computations. Computations are conceded out until stability is attained. Central difference procedure is adopted for the computing spatial coordinates. The terms in the

PDE's are transformed to difference equations and the ensuing algebraic problem is deciphered with the help of a well known Thomas algorithm. The differential coefficients associated in Eqs (2.22)-(2.24) are achieved by five point approximation formula then the integrals are assessed using Newton–Cotes formula.

The Crank Nicholson Finite Difference equation equivalent to the equations are specified by

Equation of Momentum

$$
\begin{aligned}
&U_{i,j-1}^{k+1}\left[-V_{i,j}^{k}\frac{\Delta t}{4\Delta Y}-\frac{\Delta t}{2(\Delta Y)^2}\right]+U_{i,j}^{k+1}\left[1+U_{i,j}^{k}\frac{\Delta t}{2\Delta X}+\frac{\Delta t}{(\Delta Y)^2}+M\frac{\Delta t}{2}\right]+U_{i,j+1}^{k+1}\left[V_{i,j}^{k}\frac{\Delta t}{4\Delta Y}-\frac{\Delta t}{2(\Delta Y)^2}\right]+\\
&-U_{i,j}^{k}+U_{i,j}^{k}\left[\frac{\left(U_{i-1,j}^{k+1}-U_{i,j}^{k}+U_{i-1,j}^{k}\right)\Delta t}{2(\Delta X)}\right]-V_{i,j}^{k}\left[\frac{\left(U_{i,j-1}^{k}-U_{i,j+1}^{k}\right)\Delta t}{4\Delta Y}\right]+\\
&-\left[\left\{\left(\frac{T_{i,j}^{k+1}+T_{i,j}^{k}}{2}\right)+N\left(\frac{C_{i,j}^{k+1}+C_{i,j}^{k}}{2}\right)\right\}\Delta t\right]\cos\phi+\left[\left(\frac{U_{i,j-1}^{k}-2U_{i,j}^{k}+U_{i,j+1}^{k}}{2(\Delta Y)^2}\right)\Delta t\right]+M\,U_{i,j}^{k}\frac{\Delta t}{2}.
\end{aligned}
\tag{3.1}
$$

Equation of energy

$$
\begin{aligned}
&T_{i,j-1}^{k+1}\left[-V_{i,j}^{k}\frac{\Delta t}{4\Delta Y}-\frac{1}{\mathrm{Pr}}\left(\frac{3N+4}{3N}\right)\frac{\Delta t}{2(\Delta Y)^2}\right]+T_{i,j}^{k+1}\left[1+U_{i,j}^{k}\frac{\Delta t}{2\Delta X}+\frac{1}{\mathrm{Pr}}\left(\frac{3N+4}{3N}\right)\frac{\Delta t}{(\Delta Y)^2}-\Delta\left(\frac{\Delta t}{2}\right)\right]+\\
&+T_{i,j+1}^{k+1}\left[V_{i,j}^{k}\frac{\Delta t}{4\Delta Y}-\frac{1}{\mathrm{Pr}}\left(\frac{3N+4}{3N}\right)\frac{\Delta t}{2(\Delta Y)^2}\right]-T_{i,j}^{k}+U_{i,j}^{k}\left[\frac{\left(T_{i-1,j}^{k+1}+T_{i-1,j}^{k}-T_{i,j}^{k}\right)\Delta t}{2(\Delta X)}\right]+\\
&-V_{i,j}^{k}\left[\frac{\left(T_{i,j-1}^{k}-T_{i,j+1}^{k}\right)\Delta t}{4\Delta Y}\right]-\frac{1}{\mathrm{Pr}}\left(\frac{3N+4}{3N}\right)\left[\left(T_{i,j-1}^{k}-2T_{i,j}^{k}+T_{i,j+1}^{k}\right)\frac{\Delta t}{2(\Delta Y)^2}\right]+\\
&-\Delta T_{i,j}^{k}\left(\frac{\Delta t}{2}\right)-\varepsilon\left[\frac{U_{i,j+1}^{k}-U_{i,j}^{k}}{\Delta Y}\right]^2.
\end{aligned}
\tag{3.2}
$$

Concentration equation

$$
\begin{aligned}
&C_{i,j-1}^{k+1}\left[V_{i,j}^{k}\frac{\Delta t}{4\Delta Y}-\frac{1}{\mathrm{Sc}}\frac{\Delta t}{2(\Delta Y)^2}\right]+C_{i,j}^{k+1}\left[1+U_{i,j}^{k}\frac{\Delta t}{2\Delta X}+\frac{1}{\mathrm{Sc}}\frac{\Delta t}{(\Delta Y)^2}+\lambda\left(\frac{\Delta t}{2}\right)\right]+\\
&+C_{i,j+1}^{k+1}\left[V_{i,j}^{k}\frac{\Delta t}{4\Delta Y}-\frac{1}{\mathrm{Sc}}\frac{\Delta t}{2(\Delta Y)^2}\right]-\left(C_{i,j}^{k}-U_{i,j}^{k}\right)\left[\frac{\left(C_{i-1,j}^{k+1}-C_{i-1,j}^{k}+C_{i,j}^{k}\right)\Delta t}{2(\Delta X)}\right]+\\
&-V_{i,j}^{k}\left[\frac{\left(C_{i,j+1}^{k}-C_{i,j-1}^{k}\right)\Delta t}{4\Delta Y}\right]-\frac{1}{\mathrm{Sc}}\left[\left(C_{i,j-1}^{k}-2C_{i,j}^{k}+C_{i,j+1}^{k}\right)\frac{\Delta t}{2(\Delta Y)^2}\right]+\lambda\,C_{i,j}^{k}\left(\frac{\Delta t}{2}\right).
\end{aligned}
\tag{3.3}
$$

4. Exchange of views on results

Representative numerical upshots for the transient heat and mass diffusion in the occurrence of MHD, radiation, heat generation, viscous dissipation and chemical reaction will be discussed and which is the prime idea in this section.

A. Momentum profiles

Figures 2a, 2b and 2cdepicts the transient velocity profiles for different values of **Sc**, radiation parameter **Rd** viscous dissipation parameter ε heat generation parameter Δ , chemical reaction parameter λ magnetic field parameter **M**. The tendency confirms the role of mass dispersion is highly dominant in momentum field. The effect of distinct values of Sc, *Rd*, and ε are exhibited in Fig.2a. It is noticed that the momentum rises with reducing *Rd*. This demonstrates that, due to the elevated thermal radiation the momentum reduces enormously. The viscous dissipative heat shoots up the momentum near the exterior of the cone. It is also inferred that the momentum diminishes with rising **Sc**.

The effect of Δ over momentum is viewed from the Fig.2b, it reveals that the velocity rises for the generative response and reduces for a destructive response. The effect of velocity for distinct chemical reaction parameter and MHD values are exposed in Fig.2c. It is studied that the momentum boosts with diminishing values of λ and **M**. This illustrates that the rise in **M** directs to descend in the momentum. This is because the magnetic field put forth a retarding force on natural convection flow.

B. Temperature profiles

The temperature summary for dissimilar quantities of *Rd*, ε and Schmidt number are portrayed in Fig.3a. It is viewed that the temperature amplifies with declining *Rd* and dissipation parameter ε. This demonstrates that the buoyancy upshot on the temperature distribution is most important in air.

It is acknowledged that *Rd* and Pr plays an imperative role in flow phenomenon as, it is a quantity of relative scale of thickness of the boundary layers of viscous to thermal. Lesser temperatures are detected for inferior quantities of Sc. The temperature augments with Schmidt number and MHD is monitored from Fig.3c. Furthermore, we can wind up from Fig.3b that the temperature rises with λ .
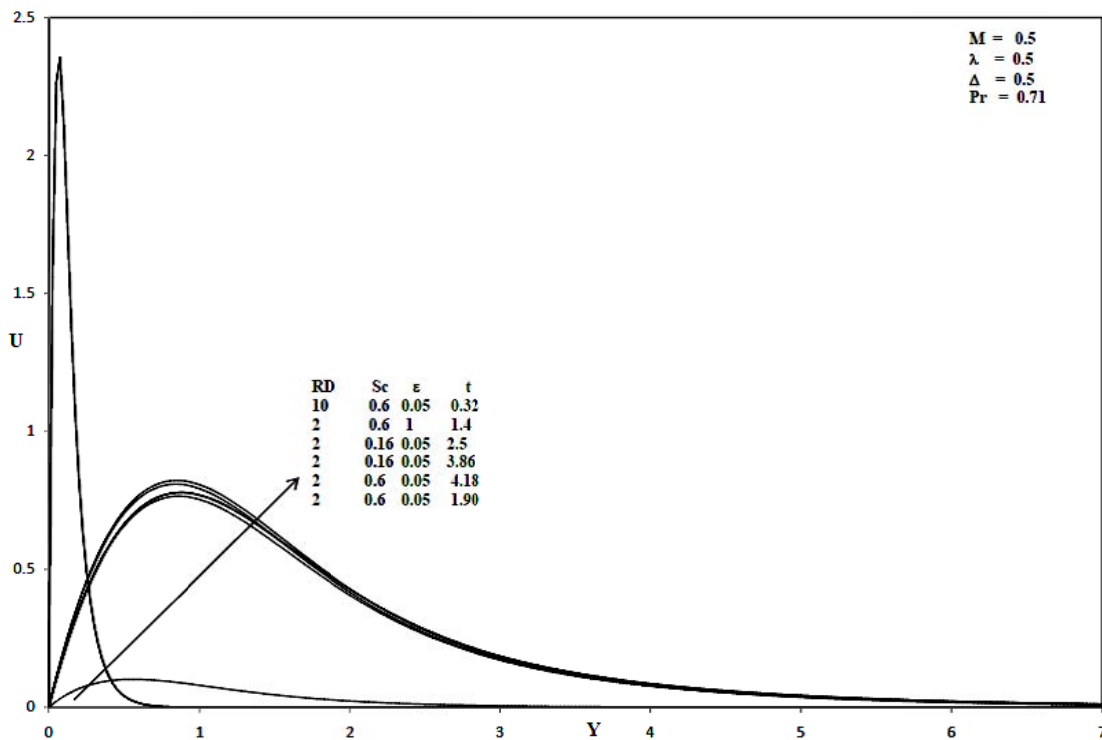

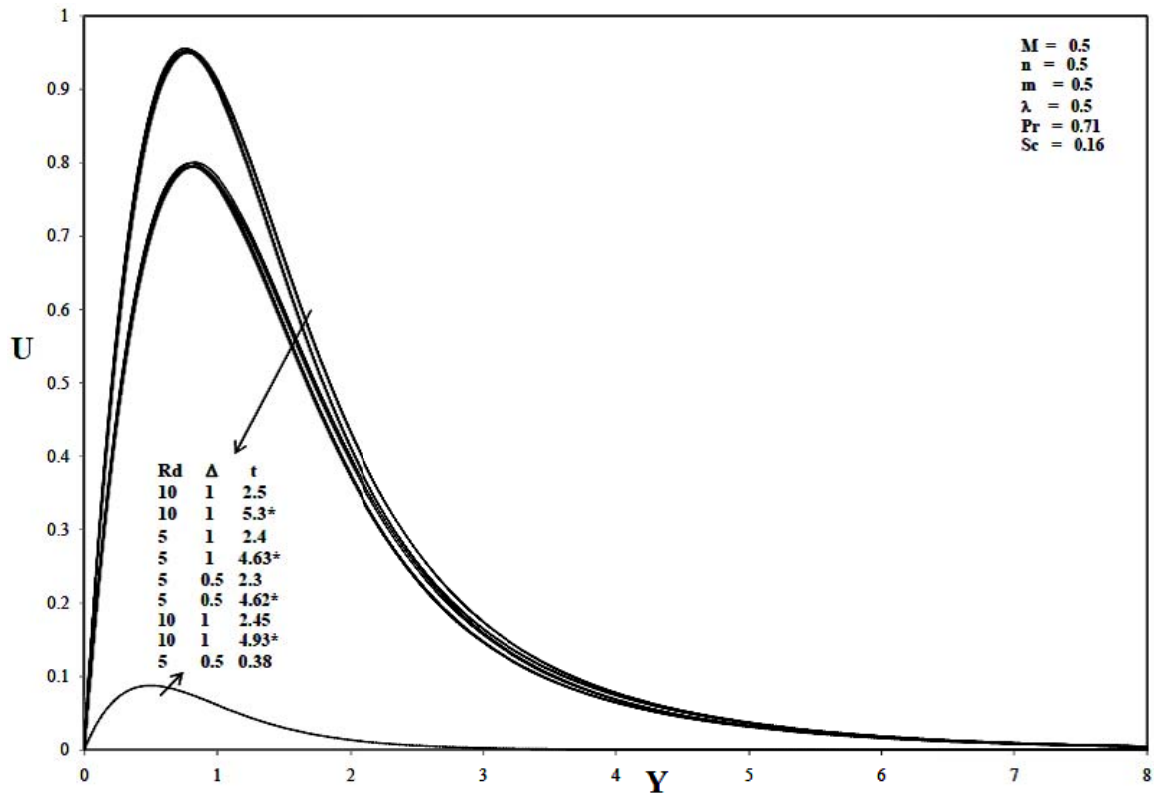Fig.2a. Transient velocity profiles for various values of *Rd*, Sc, ε .

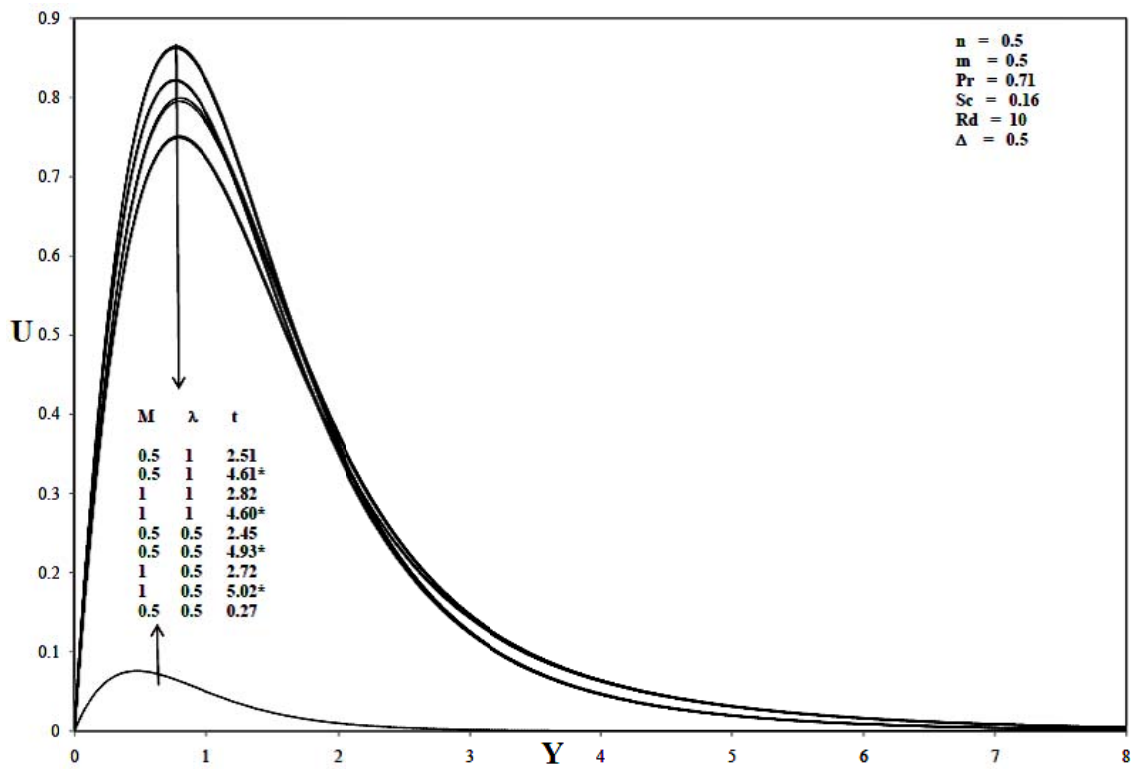Fig.2b. Unsteady velocity profiles for different values of *Rd*, Δ .

Fig.2c. Unsteady velocity profiles for different values of λ , *M.*
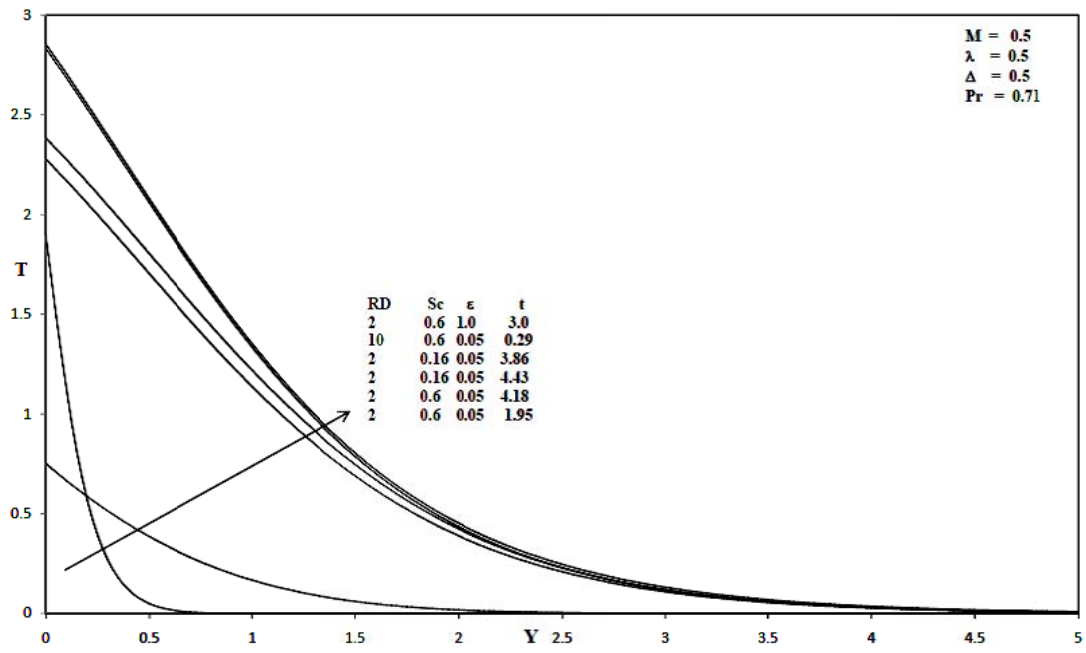
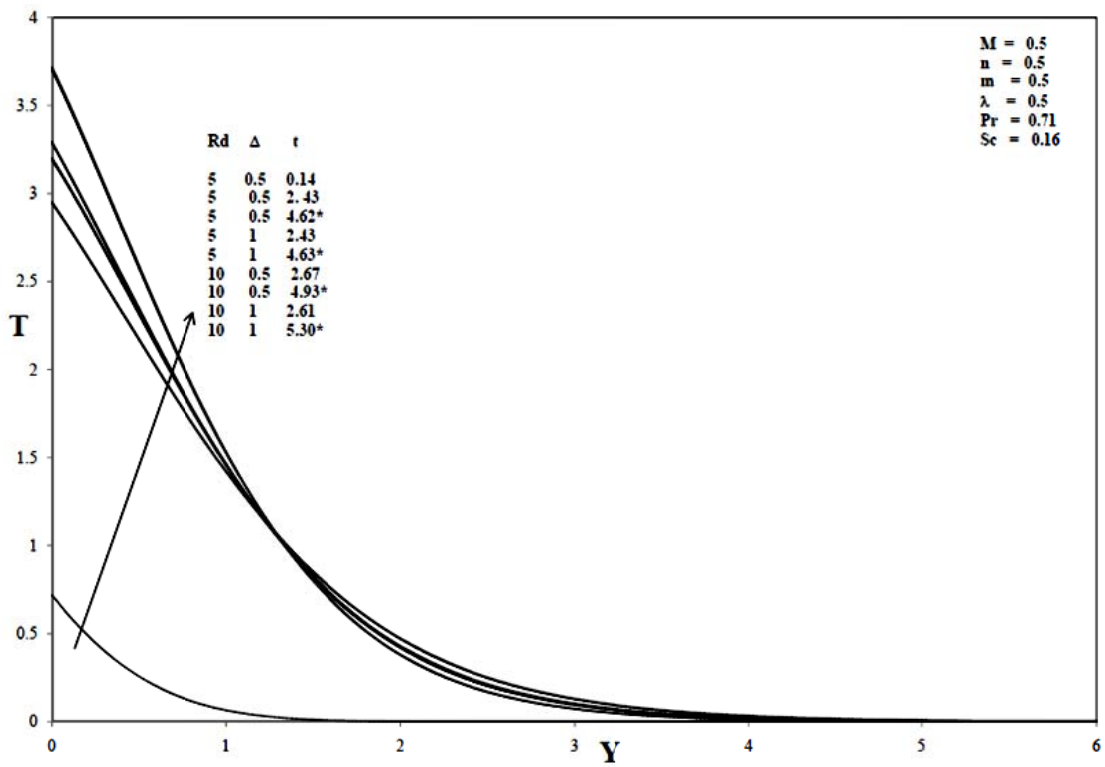Fig.3a. Unsteady temperature profiles for different values of *Rd*, Sc, ε .

Fig.3b. Unsteady temperature profiles for different values of *Rd*, Δ .
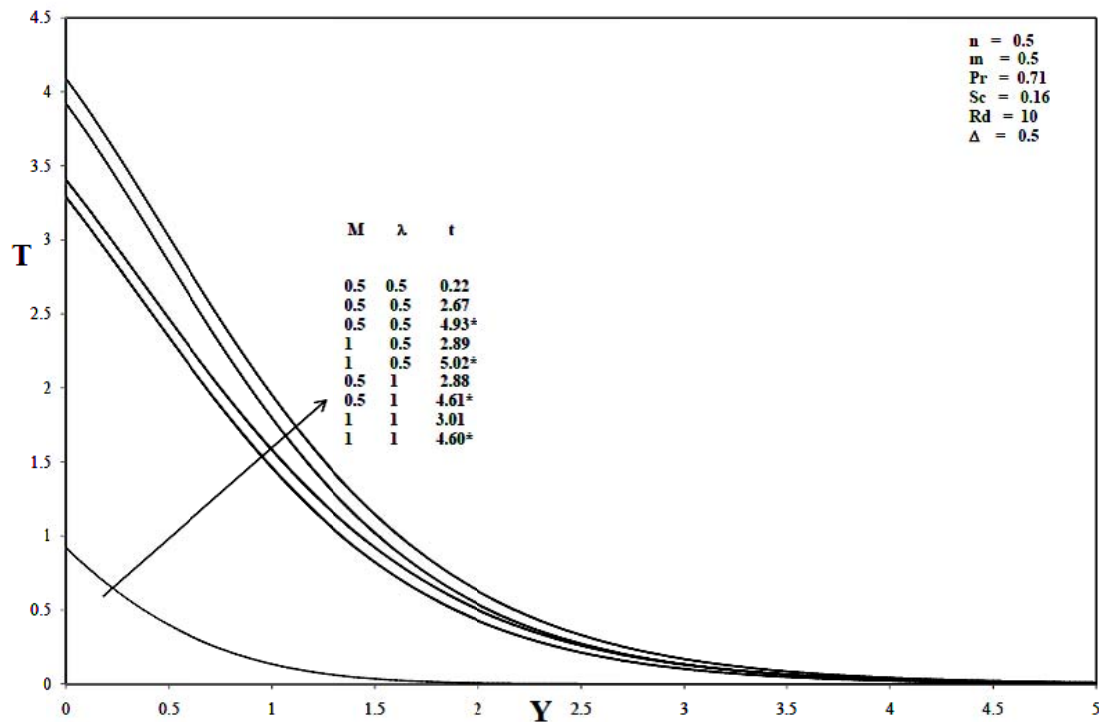
Fig.3c. Unsteady velocity profiles for different values of λ, *M*.

C. Concentration profiles

Unsteady concentration impacts for distinct quantities of the physical parameters are expressed graphically through Figs 4a, 4b and 4c. It is noticed that the surface concentration augments with diminishing *Rd*, Sc and ε. It is perceived that concentration rises with MHD. As anticipated, the species concentration is lesser at elevated quantities of the Sc.

There is a reduction in concentration is viewed by the rising quantities of *Rd* and Δ is inferred in Fig.4b. Figure 4c illustrates the effects of λ and *M* over the concentration profiles, it reveals that the concentration rises for higher numerical values of λ and a smaller amount values of *M*.

D. Profiles of Shear stress, Nusselt number and Sherwood number

The local skin friction, Nusselt number and mass transfer rate consequences are plotted in Figs 5, 6 and 7 respectively. The shear stress decelerates with diminishing radiation parameter. The local heat transfer rate increases with rising radiation parameter. The local Sherwood number increases with λ. For a constructive response, the skin friction increases with escalating numerical values of λ. The heat transfer rate for distinct numerical values of *Rd* is portrayed in Fig.6 as a function of the axial co-ordinate. It is observed, heat transfer rate rises with escalating quantities of *Rd*. The mass transfer rate for distinct quantities of Sc is illustrated in Fig.7. As anticipated, the Sherwood number rises with mounting quantities of Sc. This tendency is just turn round as contrast to the concentration.
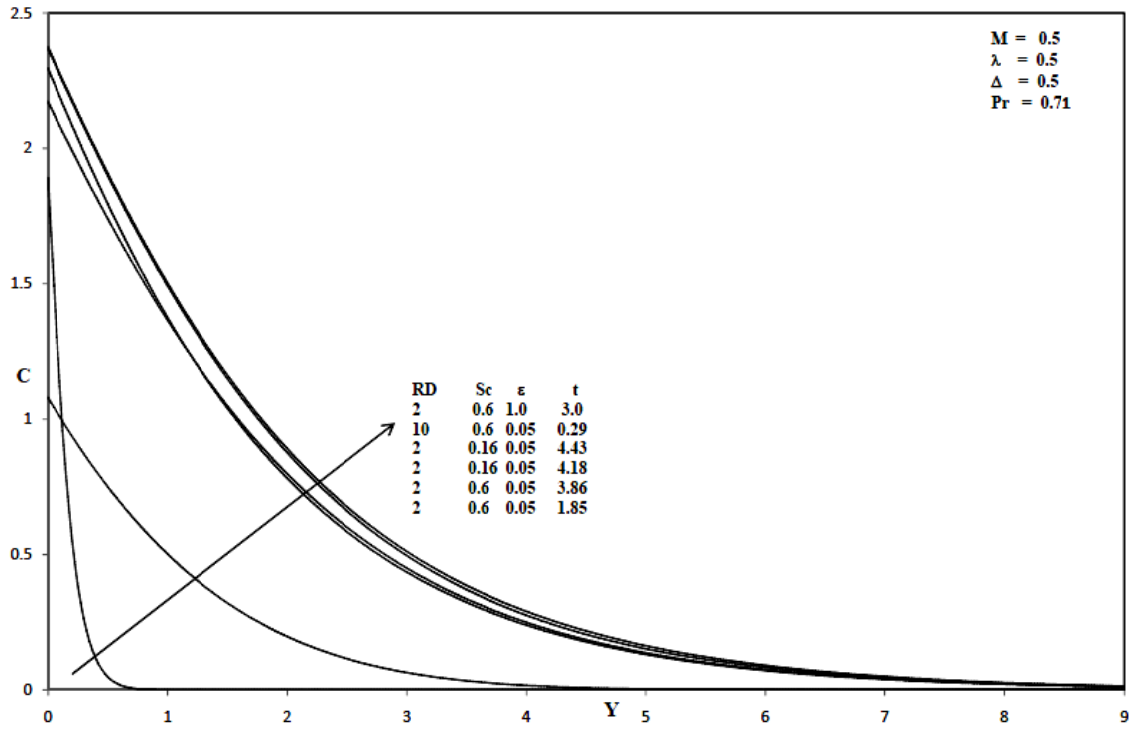
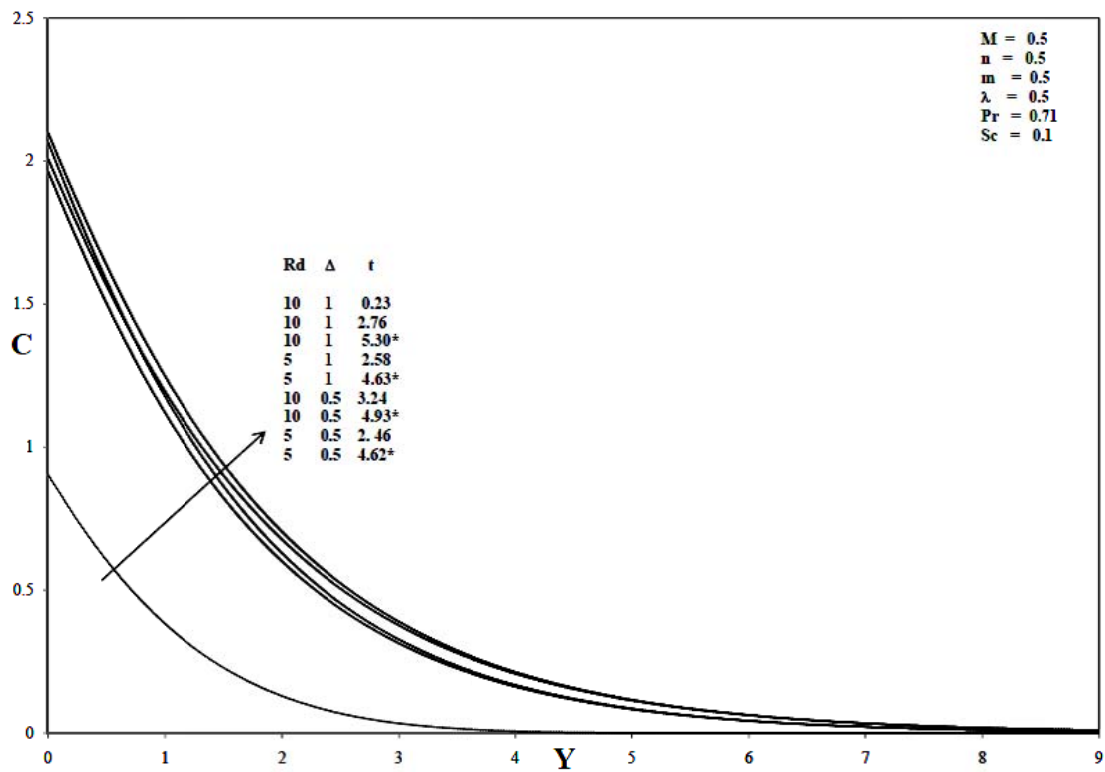Fig.4a. Unsteady concentration profiles for different values of Rd, Sc, ε.

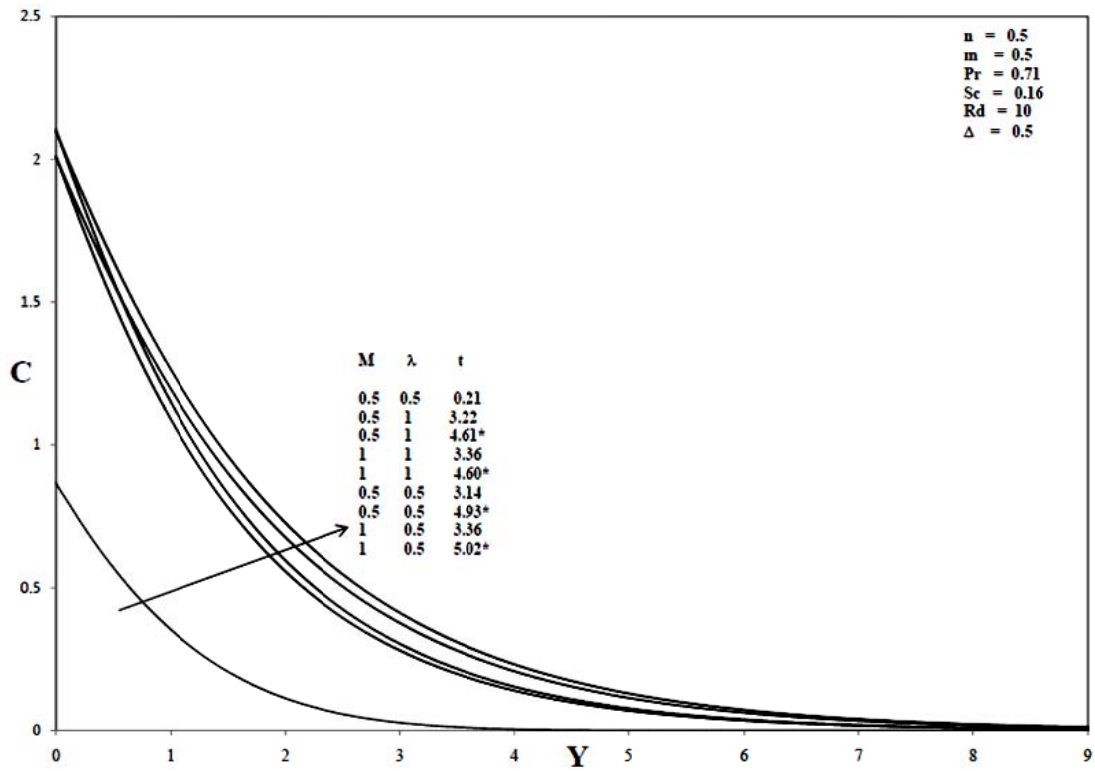Fig.4b. Unsteady concentration profiles for different values of Rd, Δ.

Fig.4c.Unsteady concentration profiles for different values of λ, *M.*
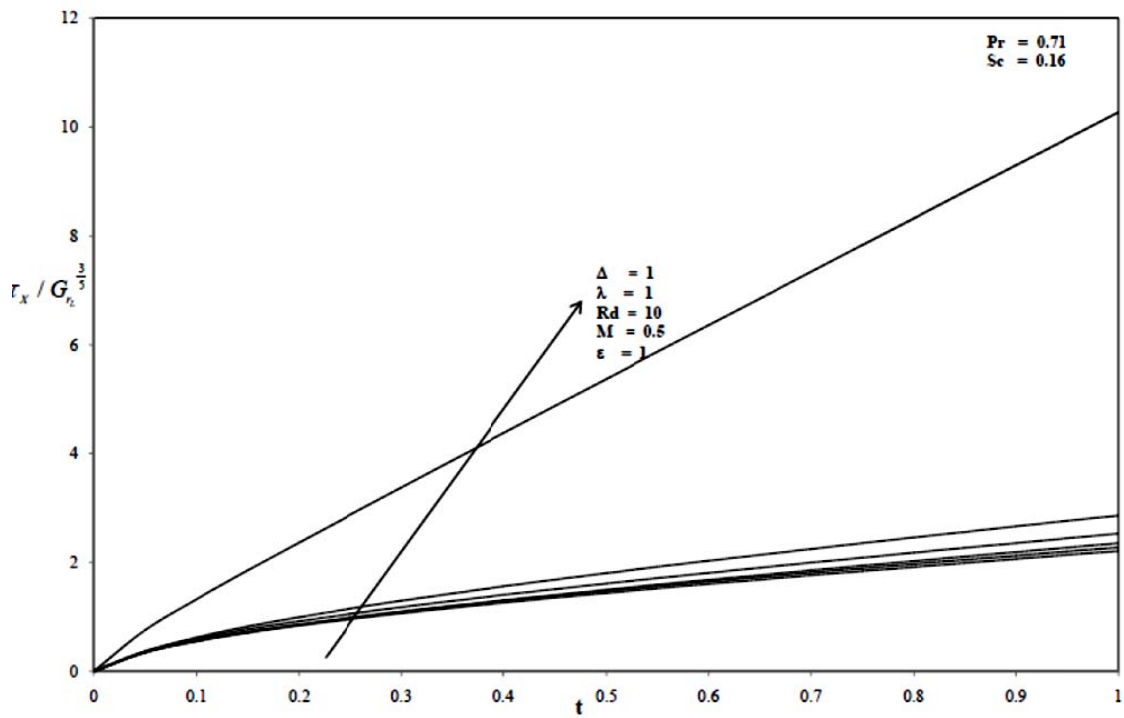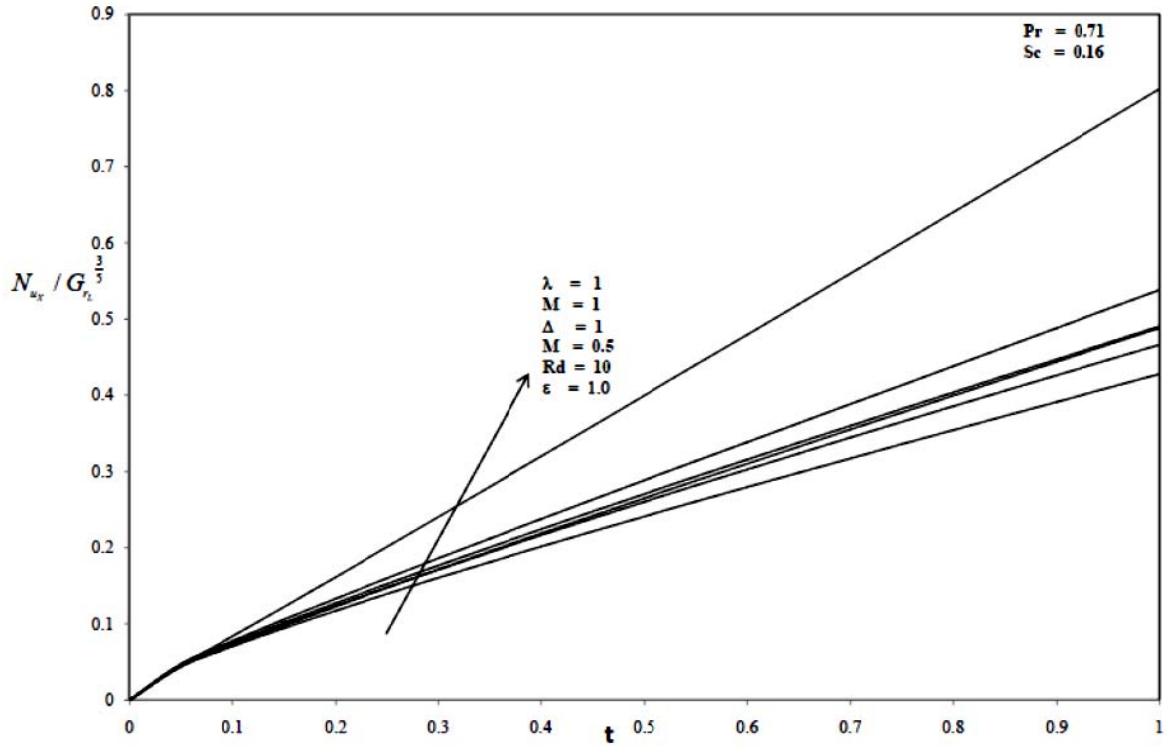

Fig.5. Local skin friction.
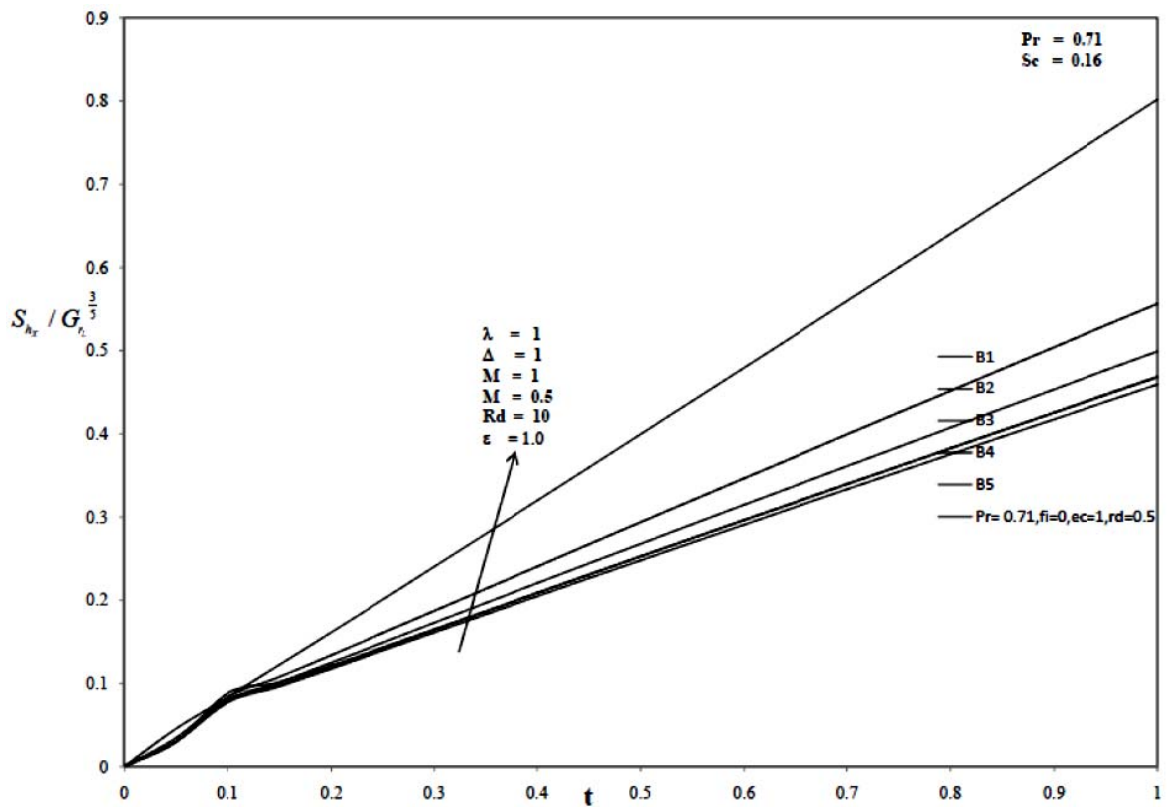
Fig.6. Local Nusselt number.

Fig.7. Local Sherwood number.

5. Conclusion

In this work, we have studied the governing equations for the transient radiative MHD heat and mass transfer flow past an upright cone in the existence of a chemical reaction has been formulated and numerical solutions have been derived using Crank -Nicholson method.

- It is noticed momentum of the fluid flow raises with a rise in *Rd*, Pr, ε.
- Temperature of the fluid decreases with a rise in λ and Schmidt number.
- Temperature of the fluid raises with a rise in ε.
- Concentration of the fluid rises with a decrease in the Schmidt number and λ.

Nomenclature

a, b — constants

B_0 — magnetic field induction

C' — concentration in the fluid

C_P — specific heat

C'_∞ — concentration far-away from cone surface

D — mass diffusivity $\left[m^2 s^{-1} \right]$

Gr_L — Grashof number (thermal)

Gr_c — Grashof number (mass)

g — gravitational force ms^{-2}

k — thermal conductivity $Wm^{-1}K^{-1}$

k_1 — chemical reaction parameter (dimensional)

k^* — mean absorption coefficient

L — referal length m

m — variational power law exponent in surface concentration

N — ratio due to buoyancy force (non dimensional)

Nu_x — heat transfer rate (local)

$\overline{\text{Nu}_L}$ — heat transfer rate (average)

Nu_X — local Nusselt number (non- dimensional)

$\overline{\text{Nu}}$ — average heat transfer rate (non-dimensional)

n — variational power law exponent in surface temperature

Pr — Prandtl number

Q_0 — heat generation and absorption coefficient (dimensional)

q_r — thermal radiation (dimensional)

R — local radius (dimensionless)

R_d — radiation (dimensionless)

r — local radius of the cone m

Sc — Schmidt number

T — temperature (non-dimensional)

T' — temperature K^0

t — time (non-dimensional)

t' — time s

U — momentum along X direction (non-dimensional)

u — momentum over x direction (dimensional) ms^{-1}

V — momentum along Y direction (non-dimensional)

v — momentum over y direction (dimensional) ms^{-1}

X — spatial co-ordinate alongside the cone generator (non-dimensional)

x − spatial co-ordinate alongside the cone generator (dimensional) *m*

Y − spatial co-ordinate perpendicular to cone generator (non-dimensional)

y − spatial co-ordinate perpendicular to cone generator (dimensional) *m*

α − thermal diffusivity $m^2 s^{-1}$

β − volumetric thermal expansion $^0 k^{-1}$

β_c − coefficient of volumetric expansion due to concentration

Δ − heat generation/absorption parameter (non-dimensional)

Δt − time step (non-dimensional)

ΔX − grid size (X direction) (non-dimensional)

ΔY − grid size (Y direction) (non-dimensional)

λ − parameter of chemical reaction (non-dimensional)

ρ − density $kg\ m^{-3}$

ϕ − cone apex half-angle

μ − dynamic viscosity $kg\,m^{-1} s^{-1}$

ν − kinematic viscosity $m^2 s^{-1}$

σ − electrical conductivity

σ^* − Stefan-Boltzmann constant

τ_x − local shear stress

τ_X − local shear stress (non-dimensional)

Subscripts

w − wall condition

∞ − free-stream condition

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