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Brief note

HYDRODYNAMIC FORCES ON A SUBMERGED HORIZONTAL CIRCULAR CYLINDER IN UNIFORM FINITE DEPTH ICE COVERED WATER

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Hydrodynamic forces on a submerged cylinder in uniform finite depth ice-covered water is formulated by using the method of multipoles, the ice-cover being modelled as an elastic plate of very small thickness. The forces (vertical and horizontal) are obtained analytically as well as numerically and depicted graphically for various values of flexural rigidity of the ice-cover to show the effect of its presence. When the flexural rigidity and surface density of the ice-cover are taken to be zero, then the curves for the forces almost coincide with the curves for the case of uniform finite depth water with free surface.

Key words: wave scattering, hydrodynamic forces, submerged cylinder, ice-covered water.

1. Introduction

Havelock [1] considered the problem of radiation and scattering of water waves by spherical object and solved the heave radiation problem for a half immersed sphere in deep water. Ursell [2] solved the problem of surface waves on deep water in the presence of a submerged circular cylinder by using the method of multipoles. This method has been used in various fields of theoretical physics (cf. Jackson [3], Morse and Feshbach [4]). The water wave scattering problem was investigated by Garrett [5] by determining the vertical force, horizontal force and torque for a circular dock in water of finite depth. Problems of radiation and scattering of water waves by a submerged horizontal circular cylinder in finite depth water by using the method of multipoles were solved by Evans and Linton [6]. The evaluation of hydrodynamic forces on a submerged circular cylinder in infinite depth water is investigated (cf. Linton and McIver [7] and Eatock Taylor and Hu [8]).

Wave interaction with an ice-cover (very large floating structure) is a front line area of research due to the practical utility in constructing floating offshore oil platforms, floating airports, floating pleasure cities, etc. The floating structure has elastic properties, and if it is modelled as a thin elastic plate, then the boundary condition at the floating structure, when linearized, involves fifth order partial derivative of the potential function describing the irrotational motion in water in contrast to the first order partial derivative in the free surface condition. This is also the case when the floating ice in an ice-covered ocean is modelled as a thin elastic floating plate. Wave propagation problems in the presence of a floating elastic plate or floating sheet of ice have been investigated recently by mathematicians as well as ocean engineers due to a surge or scientific and ocean- related industrial activities in the polar region. Recently, Sturova [9] also considered the problem of hydrodynamic loads acting on an oscillating cylinder submerged in a stratified fluid with an ice-cover. Thakur and Das [10] investigated the wave scattering by a submerged horizontal circular cylinder in

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water with an ice-cover to obtain the vertical and horizontal forces for infinite depth. Li *et al.* [11] considered the wave radiation and diffraction by a circular cylinder submerged below an ice-sheet with a crack. They used the multipole expansion method and the solution was obtained for a fluid of both finite and infinite depth. Thus we extend the problem of Thakur and Das [10] to examine the scattering by a submerged circular cylinder in uniform finite depth water with an ice-cover to obtain the hydrodynamic forces. When the flexural rigidity and surface density of the ice-cover are taken to be zero, so that the ice-cover tends to a free-surface. Then the curves for forces almost coincide with the curves for the case of water with free surface.

2. Mathematical formulation

A rectangular Cartesian co-ordinate system is chosen such that $y=0$ is the undisturbed position of the ice-cover, y being measured vertically downwards. The central axis of the cylinder with radius a is taken to be $x=0, y=f\,(f>a)$. Assuming linear theory, the velocity potential function describing the resulting motion can be represented by $\mathrm{Re}\left\{\varphi(x,y)e^{-i\sigma t}\right\}$, where the time-independent complex valued potential function $\varphi(x,y)$ satisfies

$$\nabla^2\varphi=0 \text{ in the fluid region,} \tag{2.1}$$

the linearized ice-cover condition (cf. Fox and Squire [12])

$$\left(D\frac{\partial^4}{\partial x^4}+1-\epsilon K\right)\varphi_y+K\varphi=0 \qquad \text{on} \qquad y=0. \tag{2.2}$$

The body boundary condition is

$$\frac{\partial\varphi}{\partial r}=0, \qquad \text{as} \qquad r=a. \tag{2.3}$$

The condition on the uniform finite depth h is given by

$$\frac{\partial\varphi}{\partial y}=0 \qquad \text{as} \qquad y=h. \tag{2.4}$$

3. Method of solution

Here r and θ are polar coordinates defined by

$$x=r\sin\theta, \qquad y=f+r\cos\theta\left(-\pi\le\theta\le\pi\right).$$

The multipoles are (cf. Thorne [13])

$$\varphi_n^s=\frac{\cos n\theta}{r^n}+\int_0^\infty\left[A_1(k)\sinh\sinh ky+B_1(k)\cosh\cosh k(h-y)\right]k^{n-1}\cos kx\,dk, \tag{3.1}$$

$$\varphi_n^a = \frac{\sin n\theta}{r^n} + \int_0^\infty \left[A_2(k) \sinh \sinh ky + B_2(k) \cosh \cosh k(h-y) \right] k^{n-1} \sin kx dk, \tag{3.2}$$

$$B_1(k) = \frac{(-1)^n}{(n-1)!} e^{-kf} \frac{k\left(Dk^4 + 1 - \epsilon K\right) + K}{k\left(Dk^4 + 1 - \epsilon K\right)\sinh kh - K\cosh kh} +$$

$$+ \frac{1}{(n-1)!} e^{-k(h-f)} \frac{k(Dk^4 + 1 - \epsilon K)}{\cosh kh\left[k\left(Dk^4 + 1 - \epsilon K\right)\sinh kh - K\cosh kh\right]},$$

$$B_2(k) = \frac{(-1)^{n+1}}{(n-1)!} e^{-kf} \frac{k\left(Dk^4 + 1 - \epsilon K\right) + K}{k\left(Dk^4 + 1 - \epsilon K\right)\sinh kh - K\cosh kh} +$$

$$+ \frac{1}{(n-1)!} e^{-k(h-f)} \frac{k(Dk^4 + 1 - \epsilon K)}{\cosh kh\left[k\left(Dk^4 + 1 - \epsilon K\right)\sinh kh - K\cosh kh\right]},$$

$$A_1(k) = A_2(k) = \frac{1}{(n-1)!} \frac{e^{-k(h-f)}}{\cosh kh}.$$

We can expand (3.1) and (3.2) in the form of power series which are

$$\varphi_n^s = \frac{\cos n\theta}{r^n} + \sum_{m=0}^{\infty} A_{mn} r^m \cos m\theta \ , \tag{3.3}$$

and

$$\varphi_n^a = \frac{\sin n\theta}{r^n} + \sum_{m=0}^{\infty} B_{mn} r^m \sin m\theta \ , \tag{3.4}$$

where

$$A_{mn} = \frac{1}{2} \frac{1}{m!} \int_0^\infty k^{m+n-1} \left[A_1(k) C_1(k) + B_1(k) C_2(k) \right] dk \ ,$$

$$B_{mn} = \frac{1}{2} \frac{1}{m!} \int_0^\infty k^{m+n-1} \left[A_2(k) C_3(k) + B_2(k) C_4(k) \right] dk \ ,$$

where

$$C_{1,3}(k) = e^{kf} \mp (-1)^m e^{-kf},$$

$$C_{2,4}(k) = e^{-k(h-f)} \pm (-1)^m e^{k(h-f)}.$$

Also, the incident wave potential $\varphi_0(x,y) = \exp(-\lambda y + i\lambda x)$ has the expansion

$$\varphi_0(x,y) = e^{-\lambda f} \sum_{n=0}^{\infty} \frac{(-\lambda r)^n}{n!} (\cos n\theta - i \sin n\theta) \tag{3.5}$$

where λ is the unique positive real root of the dispersion equation

$$k\left(Dk^4 + 1 - \epsilon K\right)\sinh kh = K\cosh kh.$$

Using the multipoles (3.1) and (3.2) , we may express the potential functions φ^+ (symmetric) and φ^- (anti-symmetric) as follows

$$\varphi^+ = e^{-\lambda y}\cos\lambda x + \sum_{n=1}^{\infty} a^n \alpha_n \varphi_n^s, \tag{3.6}$$

$$\varphi^- = i\,e^{-\lambda y}\sin\lambda x + \sum_{n=1}^{\infty} a^n \beta_n \varphi_n^a \tag{3.7}$$

where, the functions φ_n^s are symmetric multipoles and φ_n^a are anti-symmetric multipoles respectively, α_n and β_n are unknown complex constants. Now, using Eq.(2.3) in Eqs (3.6) and (3.7) we get

$$\alpha_m - \sum_{n=1}^{\infty} a^{m+n} A_{mn}\alpha_n = e^{-\lambda f}\frac{(-\lambda a)^m}{m!}, \qquad m = 1,2,3,\ldots\ldots\ldots, \tag{3.8}$$

$$\beta_m - \sum_{n=1}^{\infty} a^{m+n} B_{mn}\beta_n = ie^{-\lambda f}\frac{(-\lambda a)^m}{m!}, \qquad m = 1,2,3,\ldots\ldots\ldots, \tag{3.9}$$

Here Eqs(3.8) and (3.9) are truncated upto five terms. The vertical exciting force is given by

$$X_V = \rho g A\int_{-\pi}^{\pi} a\varphi_n^s(a,\theta)\cos\theta\, d\theta$$

where A is the amplitude. Using the orthogonality condition of the trigonometric function and from Eqs(3.6) and (3.8), we have

$$X_V = 2\pi\rho g A a\alpha_1. \tag{3.10}$$

Similarly, the horizontal force is

$$X_h = \rho g A\int_{-\pi}^{\pi} a\varphi_n^a(a,\theta)\sin\theta\, d\theta,$$

and by using Eqs (3.7) and (3.9), we get

$$X_h = 2\pi\rho g A a\beta_1 . \tag{3.11}$$

4. Numerical results

Curves for the vertical forces X_V are shown in Figs 1 to 3.

Figures 1-3 show X_V (symmetric mode) and Figs 4-6 for X_h (anti-symmetric mode) plotted against Ka for different values of f / a, Viz. f / a = *1.5*, *1.2*, *1.05* and different values of D / a^4. Figure 1 was made for vertical forces and the following values of ratios $D / a^4 = 0, \epsilon / a = 0$. Here we observed that the curves almost coincide with the curves for the case of water with free surface.

Figure 2 was plotted for $D / a^4 = 1, \epsilon / a = 0.01, h / a = 3,$ Fig.3 for $f / a = 1.5, \epsilon / a = 0.01, h / a = 4$ and different values of D / a^4. It is observed that vertical forces first increase as Ka increases; each attains a maximum value and then decrease as Ka farther increases for all cases. Here vertical forces gradually decrease as increases of Ka. Also we noticed that due to the presence of ice-cover, the non-dimensional vertical forces get reduced.

Curves for the horizontal forces $|X_h|$ are shown in Figs 4-6 for uniform finite depth water. All the curves are somewhat similar to those for the vertical forces on the cylinder and display the same characteristics (Figs 4, 5, 6 are similar to Figs 1, 2, 3 respectively). Also, it is observed that the horizontal forces are smaller in comparison to the vertical forces on the cylinder.

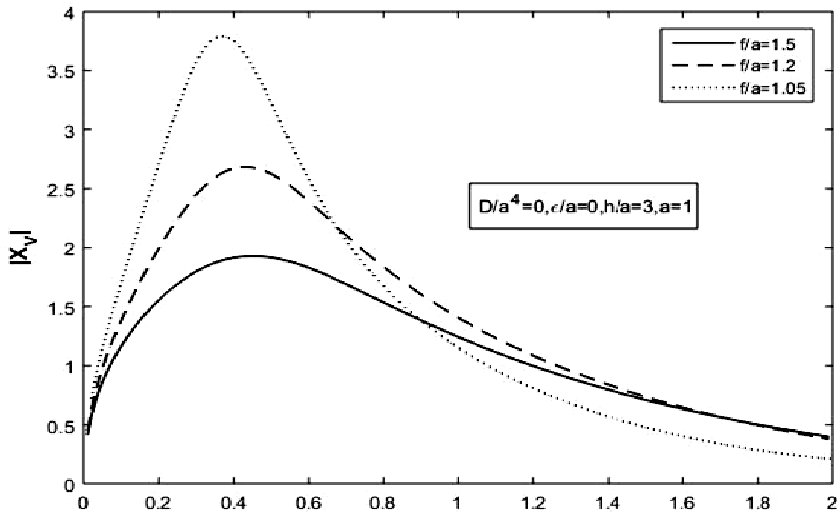

Fig.1. Vertical forces against the wave number *ka*.

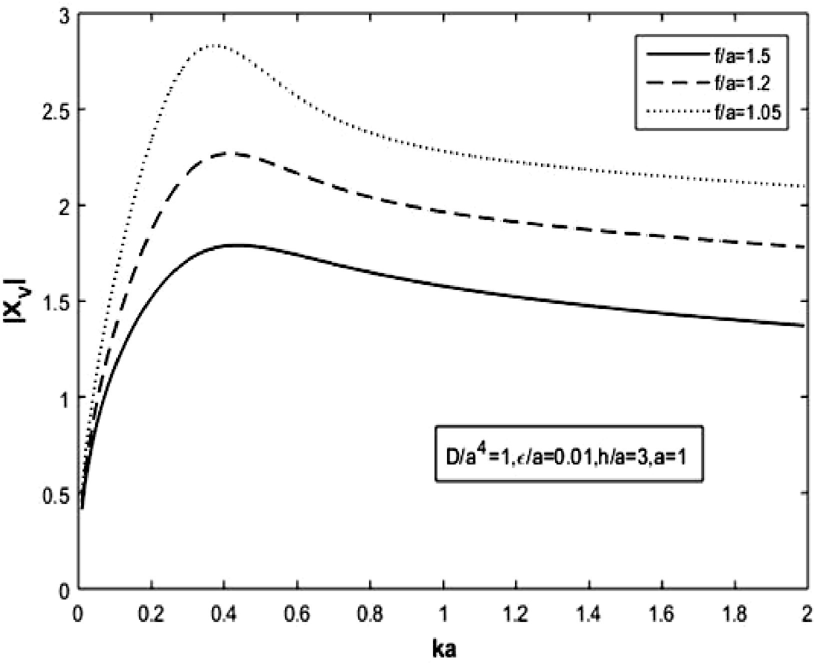

Fig.2. Vertical forces against the wave number *ka*.

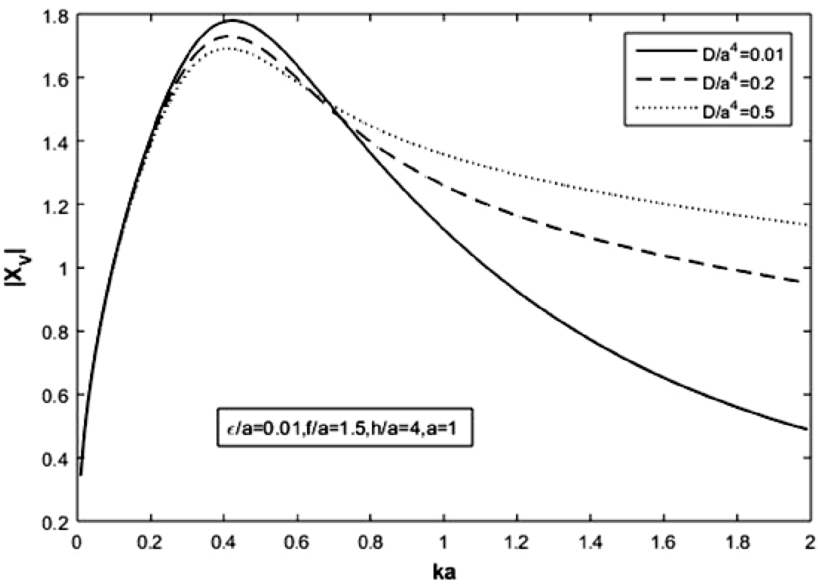

Fig.3. Vertical forces against the wave number *ka*.

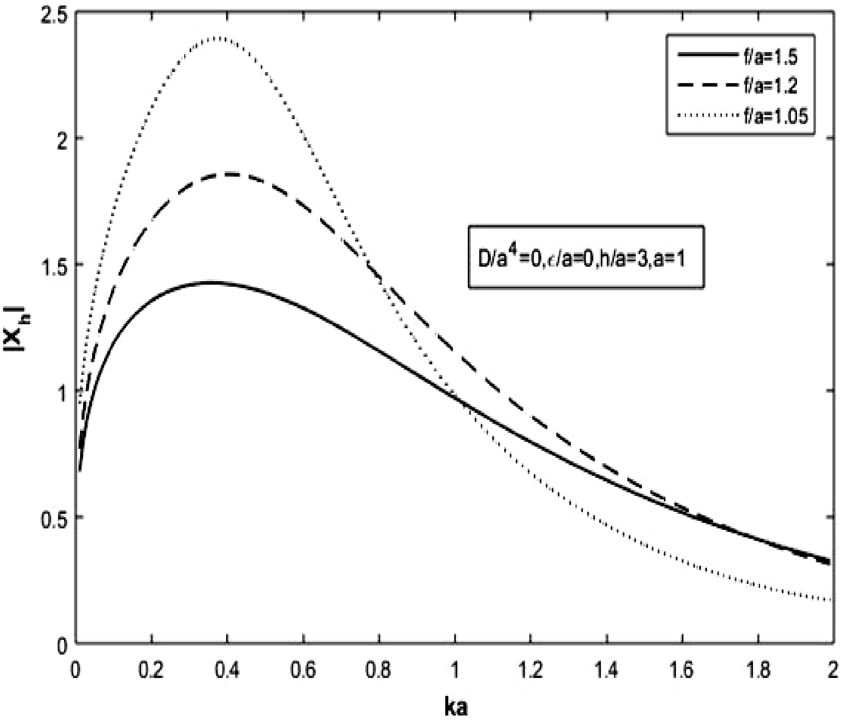

Fig.4. Horizontal forces against the wave number *ka*.

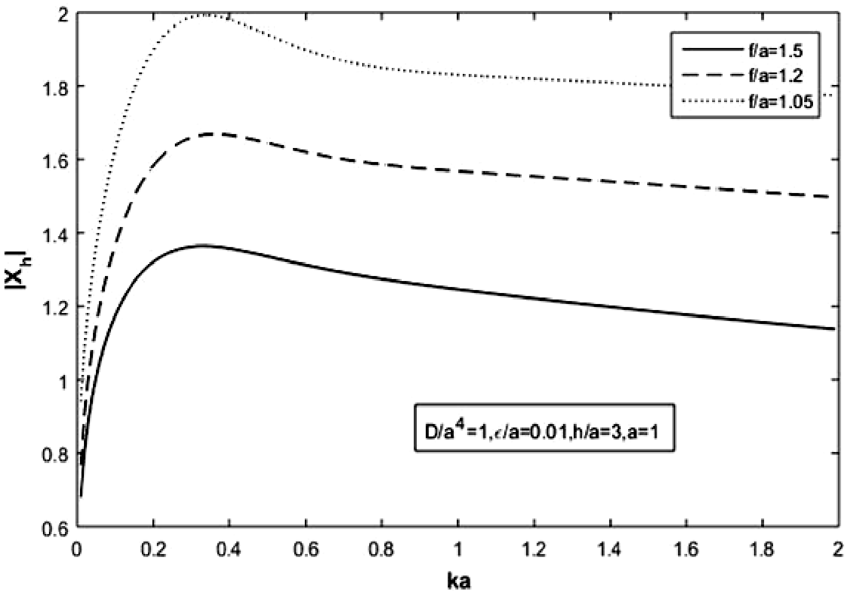

Fig.5. Horizontal forces against the wave number *ka*.

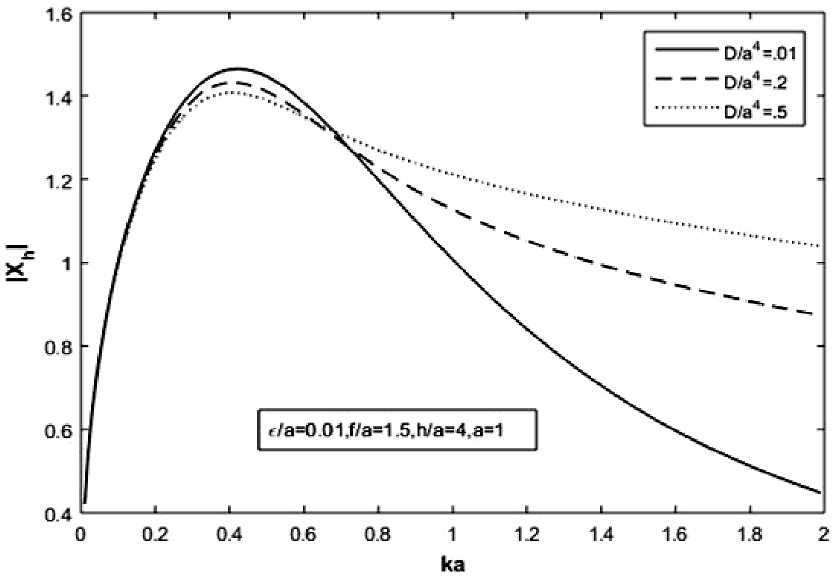

Fig.6. Horizontal forces against the wave number *ka*.

Due to the presence of the ice-cover, the vertical and horizontal forces are oscillatory in nature and tend ultimately to zero for large Ka .

5. Conclusion

The problem of hydrodynamic forces on the submerged circular cylinder in uniform finite depth water beneath the free surface is extended here when the free surface is replaced by a thin ice cover modelled as a thin elastic plate. Numerical results for the vertical and horizontal forces on the cylinder are obtained. It has been shown that the method of multipoles is an extremely powerful method for solving this problem involving submerged circular cylinder. The vertical and horizontal forces on the cylinder are depicted graphically against the wave number in a number of figures. When $D / a^4 = 0$ and $\epsilon / a = 0$, the forces coincide with the curves for the case of water with free surface.

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Nomenclature

- A – amplitude
- a – radius of the circular cross-section of the cylinder
- E – Young's modulus
- f – depth of the centre of cylinder from the ice-cover surface
- g – gravity
- h_0 – small thickness of the ice-cover
- φ – complex valued velocity potential function
- σ – angular frequency
- φ_0 – incident wave potential
- λ – wave number

φ^{+} – symmetric potential function
φ^{-} – anti-symmetric potential function
ν – Poisson's ratio
ρ – density of water
ρ_0 – density of ice

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