Int. J. of Applied Mechanics and Engineering, 2019, vol.24, No.4, pp.106-124
DOI: 10.2478/ijame-2019-0052

ELECTRO-HYDRODYNAMIC CONVECTION IN A ROTATING DIELECTRIC MICROPOLAR FLUID LAYER

H. KAUR*
I.K.G.P.T.U
Deptt. of Applied Sciences, CEC Landran
Jalandhar, Punjab, INDIA
E-mail: simran1905@gmail.com

G.N VERMA
I.K.G.P.T.U, Jalandhar, Punjab, INDIA
E-mail: drvermagn@gmail.com

Thermal convection of a rotating dielectric micropolar fluid layer under the action of an electric field and temperature gradient has been investigated. The dispersion relation has been derived using normal mode analysis. The effects of the electric Rayleigh number, micropolar viscosity, Taylor number and Prandtl number on stability and over stability criteria are discussed. It is found that rotation postpones the instability in the fluid layer, while the Prandtl number and rotation both have a stabilizing effect. It is also observed that the micropolar fluid additives have a stabilizing effect, whereas the electric field has a destabilizing effect on the onset of convection stability.

Key words: convection, rotation, electric, Rayleigh number, Taylor number, normal-mode.

1. Introduction

Electro-hydrodynamics (EHD) is concerned with the mechanics involved in the fluid motion under the effect of an electric field. Electro-hydrodynamics(EHD) has wide range of applications in several areas including engineering, bio-technology, telecommunication, aerospace engineering, etc. For example, EHD enhanced heat transfer may be used in nano-fluids, EHD may be used for the cooling process in mechanical systems, EHD pumps are used to enhance the motion of ions in the fluids.

The theory of electro-dynamics of continuous media and the expression of electric force in fluid dielectric has been presented in Landau and Lifshitz [1]. Later, the convection problem in EHD was studied by Roberts [2]. Here, the fluid layer is generally considered under the action of AC or DC current and the main objective is to find the effect of the electric Rayleigh number on the stability and over stability of fluid system. Of course, the fluid layer considered in the system should be electrically conductive. Melcher and Taylor [3] analyzed the convection in a liquid layer in the presence of an AC or DC electric field and he later proposed the leaky dielectric model. In order to understand possible control of convection in liquid dielectrics and a control of heat and mass transfer in high-voltage devices by electric field, several studies have been carried out to assess the effect of AC or DC electric field on free convection in a horizontal dielectric fluid layer. Some of the related theories are given by Gelmont and Loffe [4], Gross and Porter [5], Takashima and Aldridge [6] among several others. Detailed explanations of thermal convection problems in

* To whom correspondence should be addressed

a thin horizontal layer of a Newtonian fluid heated from below under varying assumptions of hydro-magnetic and hydro-dynamics can be found in the monograph by Chandrasekhar [7].

Eringen [8] introduced the theory of microfluids and defined a subclass of these fluids called micropolar fluids. Micropolar fluids consist of rigid randomly oriented fluid filaments suspended in a viscous medium, where the deformation of filament is ignored. Industrial colloidal fluids, liquid crystals and animal blood are few examples of micropolar fluids.Compared to classical Newtonian fluids, micropolar fluids are characterizedby two variables, i.e., spin vector (micro-rotation vector) responsible for the micro-rotations and micro- inertia tensor (gyration parameter) describing the distributions of atoms or molecules inside the fluid element in addition to the velocity vector. The micropolar theory of fluids is extended by Eringen [9] to include the heat conduction and heat dissipation effects. The instability of a fluid layer heated from below or above produces a fixed temperature difference and is known as Rayleigh-Benard convection problem in the literature. Linear and non-linear instability problems of micropolar fluids have been investigated by Perez-Garcia and her co-workers [10, 11], Ahmadi [12], Datta and Sastry [13], Rama Rao [14, 15], Sharma and Gupta [16], Sharma and Kumar [17, 18], Rana *et al*. [19], Sharma and Gupta [20], Rani and Tomar [21] among several others. The effect of a vertical AC electric field on the onset of convective instability in a dielectric micropolar fluid layer heated from below under a simultaneous action ofrotation of the system and the vertical temperature gradient has been studied by Ezzat and Othman [22]. They used the power series method to obtain the dispersion equation and solved them numerically for stability motion of the system. It should be noted that they could not perform an analysis for over-stability motions. Recently, Rani and Tomar [23] have studied EHD convection in dielectric micropolar fluidlayer.

The effect of rotation on the thermal convection in micropolar fluids is important in certain chemical engineering and biochemical situations. Qin and Kaloni [24] studied a thermal instability problem in a rotating micropolar fluid. The effect of rotation on thermal convection in micropolar fluids in a porous medium has been analysed by Sharma and Kumar [25]. Othman and Zaki [26] studied the effect of a vertical magnetic field on the onset of convective instability in a conducting micropolar fluid (Oldroyd fluid) layer heated from below confined between two horizontal planes under the simultaneous action of rotation of the system and the vertical temperature gradient. Shivkumara *et al*. [27] studied EHD instability of a rotating couple stress dielectric fluid layer.

In the present paper, we have studied electro-hydrodynamic convection in a rotating dielectric micropolar fluid layer using the normal mode analysis method. The normal mode analysis method has superiority over the power series method in the sense that it can give complete information about the instability of a system including the growth rate of any unstable perturbation. The proposed study may help in understanding the stability characteristics of the system and provide theoretical results for the control of unstable convection occurring in many micro fluid devices. It is found that micropolar viscosity and rotation parameter has a stabilizing effect on the onset of convection, while the electric Rayleigh number has a destabilizing effect on the system. The thermal Rayleigh number is found to be independent of the Prandtl number in case of stability motion.

2. Mathematical formulation

Consider a horizontal layer of an incompressible dielectric micropolar fluid of uniform thickness 'd'. Let the fluid layer be rotating about the vertical axis with constant angular velocity Ω under the gravitational field. With reference to the rectangular Cartesian coordinate system OXYZ, we take the z-axis normal to the fluid layer such that $z = 0$ and $z = d$ defines the lower and upper surfaces of the layer, which are maintained at constant temperature T_L and $T_U(< T_L)$ respectively. We shall assume that rotation of the system will not affect the isotropy of the micropolar fluid.

Under the Boussinesq approximation, the mass, momentum, internal angular momentum, internal energy balance equations in the absence of body load and body couple densities (see Eringen [8]) and relevant Maxwell equations (see Landau and Liftshitz [1]) are given by

$$\nabla \cdot \boldsymbol{V} = 0\,, \tag{2.1}$$

$$\rho_0\left(\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\nabla\right)\boldsymbol{V} = -\nabla\left(p-\frac{\rho_0}{2}|\boldsymbol{\Omega}\times r|^2\right)+\rho g + \\ +(\mu+k)\nabla^2\boldsymbol{V}+\kappa\nabla^2\times\boldsymbol{\Pi}+f_e+2\rho_0(\boldsymbol{V}\times\boldsymbol{\Omega}), \tag{2.2}$$

$$\rho_0 j\left(\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\nabla\right)\boldsymbol{\Pi} = (\varepsilon+\beta)\nabla(\nabla\cdot\boldsymbol{\Pi})+\gamma\nabla^2\Pi+\kappa(\nabla\times\boldsymbol{V}-2\Pi)\,, \tag{2.3}$$

$$\rho_0 c_v\left(\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\nabla\right)T = k_t\nabla^2 T+\delta(\nabla\times\Pi)\cdot\nabla T\,, \tag{2.4}$$

$$\nabla\cdot(\epsilon\boldsymbol{E}) = 0\,, \tag{2.5}$$

$$\nabla\times\boldsymbol{E} = 0, \quad \Rightarrow \boldsymbol{E} = -\nabla\varphi \tag{2.6}$$

where $V = (u,v,w)$, $\Pi = (\Pi_x,\Pi_y,\Pi_z)$, $g = (0,0,-g)$, $\boldsymbol{\Omega} = (0,0,\Omega)$ and $\boldsymbol{E} = (0,0,E_z)$ denote respectively the velocity vector, spin, gravitational acceleration, angular velocity and electric field, while the quantities ρ, p, j, c_v, k_t, T, δ, ϵ, φ are respectively the density, pressure, micro-inertia, specific heat at constant volume, thermal conductivity, temperature, coupling coefficient between heat flux and spin flux, dielectric constant and electric potential. The symbol ρ_0 denotes fluid density at reference temperature and r is the position vector. In Eq.(2.2), the term $\frac{1}{2}\nabla|\boldsymbol{\Omega}\times r|^2$ represents the '*Centrifugal force*', which arises due to rotation of the system and the term $2(\boldsymbol{\Omega}\times\boldsymbol{V})$ represents the '*Coriolis acceleration*'. The quantities ϵ, β, γ, κ denote the micropolar fluid viscosities and f_e denotes the electric force given by

$$f_e = \rho_e - \frac{1}{2}\boldsymbol{E}^2\nabla\epsilon + \frac{1}{2}\nabla\left(\rho\frac{\partial\epsilon}{\partial\rho}\boldsymbol{E}^2\right). \tag{2.7}$$

In the expression of electric force, the first term corresponds to the '*Coulomb force*', the second term corresponds to the '*electrophoretic force*' and the last term corresponds to the '*electrostrictive force*'. The Coulomb force is very poor in comparison to the electrophoretic force for most dielectric fluids in a *60* Hz AC electric field (see Takashima, [28]), so the first term can be neglected. With these considerations, Eq.(2.2) can be written as

$$\rho_0\left[\frac{\partial\boldsymbol{V}}{\partial t}+(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}\right] = -\nabla P+\rho g+(\mu+\kappa)\nabla^2\boldsymbol{V}+\kappa\nabla\times\boldsymbol{\Pi}-\frac{1}{2}\boldsymbol{E}^2\nabla\epsilon+2\rho_0(\boldsymbol{V}\times\boldsymbol{\Omega}), \tag{2.8}$$

where $P = p - \frac{1}{2}\rho\frac{\partial\epsilon}{\partial\rho}\boldsymbol{E}^2$ is the modified pressure. The mass density and dielectric constant are assumed to be a linear function of temperature in the form

$$\rho = \rho_0\left[1-\alpha(T-T_0)\right], \qquad \epsilon = \epsilon_0\left[1-e(T-T_0)\right] \tag{2.9}$$

where $\alpha > 0$ is the coefficient of thermal expansion and $e > 0$ is the coefficient of relative variations of the dielectric constant with temperature, which is assumed to be small. The basic state of the system is given by

$$V = V_b = 0, \quad \Pi = \Pi_b, \quad P = P_b(z), \quad T = T_b(z) = T_0 - \overline{\beta} z, \quad \epsilon = \epsilon_b(z) = \epsilon_0\left(1 + e\overline{\beta} z\right),$$

$$E = E_b(z) = \frac{E_0 \hat{k}}{1 + e\overline{\beta} z}, \qquad \phi_b(z) = -\frac{E_0}{e\overline{\beta}} \log\left(1 + e\overline{\beta} z\right)$$

where the subscript b denotes the basic state. Also, $\overline{\beta} = \left(T_L - T_U\right)/d$ is the adverse temperature gradient, $E_0 = \dfrac{\phi_1 e\overline{\beta} z}{\log\left(1 + e\overline{\beta} z\right)}$ is the root mean square value of the electric field at $z = 0$, $\hat{k}$ is the unit vector in the direction of the positive z-axis.

To study the stability of the system, we shall perturb the variables from the basic state. Let the small perturbations $V' = \left(u', v', w'\right)$, $\Pi' = \left(\Pi'_x, \Pi'_y, \Pi'_z\right), P', T', E', \rho', \phi'$ and ε' be as follows

$$\boldsymbol{V} = \boldsymbol{V}_b + \boldsymbol{V}', \quad \boldsymbol{\Pi} = \boldsymbol{\Pi}_b b + \boldsymbol{\Pi}', \quad P = P_b + P', \quad T = T_b + T', \quad \varepsilon = \varepsilon_b + \varepsilon',$$

$$\boldsymbol{E} = \boldsymbol{E}_b + \boldsymbol{E}', \quad \phi = \phi_b + \phi', \quad \rho = \rho_b + \rho'.$$

Eliminating the pressure term by operating curl twice to Eq.(2.8) and once to Eqs (2.3) and (2.8), then using the above perturbation and retaining the z-component, we obtain the following equations after suppressing primes

$$\left[\rho_0 \frac{\partial}{\partial t} - (\mu + \kappa)\nabla^2\right]\nabla^2 w = \left(\alpha\rho_0 g + \epsilon e^2 E_0^2 \overline{\beta}\right)\nabla_1^2 T + \\ + \kappa\nabla^2\Omega_z + E_0 e\overline{\beta}\epsilon_0 \nabla_1^2 \frac{\partial\varphi}{\partial z} - 2\rho_0\Omega_z \frac{\partial\zeta_z}{\partial z}, \tag{2.10}$$

$$\rho_0 \frac{\partial\zeta_z}{\partial t} = (\mu + \kappa)\nabla^2\zeta_z + 2\rho_0\Omega_z \frac{\partial w}{\partial z}, \tag{2.11}$$

$$\left(\rho_0 j \frac{\partial}{\partial t} - \gamma\nabla^2 + 2\kappa\right)\Omega_z = -\kappa\nabla^2 w \tag{2.12}$$

where the change in density given by $\rho' = -\alpha\rho_0 T$ has been used. And

$$\zeta_z = \left(\nabla \times \boldsymbol{V}'\right)_z, \qquad \Omega_z = \left(\nabla \times \Pi'\right)_z, \qquad \nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \qquad \nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}.$$

Next, using the perturbation defined above in Eqs (2.5) to (2.6), one obtain after suppressing the primes

$$\rho_0 c_v\left(\frac{\partial T}{\partial t}-\bar{\beta}w\right)=k_t\nabla^2 T-\bar{\beta}\delta\Omega_z , \tag{2.13}$$

$$\nabla^2\phi+eE_0\frac{\partial T}{\partial z}=0. \tag{2.14}$$

Now, introducing the non-dimensional quantities given by

$$z^*=\frac{z}{d},\quad t^*=\frac{t\mu}{\rho_0 d^2},\quad T^*=\frac{T}{\bar{\beta}d},\quad V^*=\frac{Vd}{k_t},\quad \phi^*=\frac{\phi}{eE_0\bar{\beta}d^4},\quad \Omega^*=\frac{\Omega_z\rho_0}{\mu}.$$

Equations (2.10) to (2.14) reduce to a non-dimensional form (after suppressing the asterisk for convenience) as

$$\left[\frac{\partial}{\partial t}-(1+K)\nabla^2\right]\nabla^2 w=\left(R_T+R_{ea}\right)\nabla_1^2 T+R_{ea}\nabla_1^2\frac{\partial\varphi}{\partial z}+K\nabla^2\Omega_z-\sqrt{T_a}\frac{\partial\zeta_z}{\partial z}, \tag{2.15}$$

$$\frac{\partial\zeta_z}{\partial t}=(1+K)\nabla^2\zeta_z+\sqrt{T_a}\frac{\partial w}{\partial z}, \tag{2.16}$$

$$\left(\bar{j}\frac{\partial}{\partial t}-c_0\nabla^2+2K\right)\zeta_z=-K\nabla^2 w , \tag{2.17}$$

$$\left(p_1\frac{\partial}{\partial t}-\nabla^2\right)T=w-\bar{\delta}\zeta_z , \tag{2.18}$$

$$\nabla^2\varphi+\frac{\partial T}{\partial z}=0 \tag{2.19}$$

where

$$K=\frac{\kappa}{\mu},\quad \bar{j}=\frac{j}{d^2},\quad c_0=\frac{\gamma}{\mu d^2},\quad \bar{\delta}=\frac{\delta}{d^2},\quad \nu=\frac{\mu}{\rho_0},\quad p_1=\frac{\nu}{k_t},$$

$$R_T=\frac{g\alpha\bar{\beta}d^4}{\nu k_t},\quad R_{ea}=\frac{\epsilon_0 e^2E_0^2\bar{\beta}d^4}{\mu k_t},\quad T_a=\frac{4\Omega_z^2 d^4}{\nu^2}.$$

Here ν is the kinematic viscosity, p_1 is the Prandtl number, R_T is the thermal Rayleigh number, R_{ea} is the electric Rayleigh number (also known as the Robert number) and T_a is the Taylor number.

3. Normal mode analysis

Our aim is to find the solution of system of Eqs (2.15) – (2.19). For this purpose, we assume the form of various variables given by

$$\{w,T,\varphi,\zeta_z,\Omega_z\}(x,y,z)=\{W(z),\Theta(z),\Phi(z),Z(z),\Omega(z)\}\times\exp\left[\sigma t+i\left(k_x x+k_y y\right)\right] \tag{3.1}$$

where σ is the stability parameter, which is a complex constant in general, k_x and k_y are respectively the *x* and *y* components of the wave number $k=\sqrt{k_x^2+k_y^2}$, t is the time variable and $i=\sqrt{-1}$. The quantities W,Θ,Φ,Z and Ω are the arbitrary functions of spatial coordinate *z*. It should be noted that $\sigma = 0$ corresponds to the case of stability motion. In other words, the principle of exchange of stabilities is valid. The onset of over-stability corresponds to real $(\sigma) = 0(\ \sigma \neq 0)$, Inserting Eq.(3.1) into Eqs (2.15) – (2.19), we obtain

$$\begin{aligned}&\left[\sigma-(1+K)\left(D^2-k^2\right)\right]\left(D^2-k^2\right)W=\\&=K\left(D^2-k^2\right)\Omega-k^2\left(R_{ea}+R_T\right)\Theta-R_{ea}k^2D\Phi-\sqrt{T_a}DZ\ ,\end{aligned} \tag{3.2}$$

$$\left[\sigma-(1+K)\left(D^2-k^2\right)\right]Z=\sqrt{T_a}DW, \tag{3.3}$$

$$\left[\left(D^2-k^2\right)-l\sigma-2A\right]\Omega=A\left(D^2-k^2\right)W, \tag{3.4}$$

$$\left[\left(D^2-k^2\right)-p_1\sigma\right]\Theta=\bar{\delta}\Omega-W, \tag{3.5}$$

$$\left(D^2-k^2\right)\Phi=-D\Theta \tag{3.6}$$

where $l=\dfrac{\bar{j}A}{K}$, $A=\dfrac{K}{c_0}$ and $D\equiv\dfrac{d}{dz}$. Here, A is the ratio between the micropolar viscous effects and micropolar diffusion effects. Eliminating the quantities Z, Ω, Θ and Φ from Eqs (3.2) – (3.6), we obtain the following relation

$$\begin{aligned}&\left[\left(D^2-k^2\right)-l\sigma-2A\right]\left[\left(D^2-k^2\right)-p_1\sigma\right]\times\left[\sigma-(1+K)\left(D^2-k^2\right)\right]\left(D^2-k^2\right)^2W=\\&=R_Tk^2\left(D^2-k^2\right)\left[\left(D^2-k^2\right)-l\sigma-2A-\bar{\delta}A\left(D^2-k^2\right)\right]W+\\&+KA\left[\left(D^2-k^2\right)-p_1\sigma\right]\left(D^2-k^2\right)^3W-R_{ea}k^4\left[\left(D^2-k^2\right)-l\sigma-2A-\bar{\delta}A\left(D^2-k^2\right)\right]W+\\&-T_a\left[\left(D^2-k^2\right)-l\sigma-2A\right]\times\left[\left(D^2-k^2\right)-p_1\sigma\right]\left(D^2-k^2\right)D^2W.\end{aligned} \tag{3.7}$$

The boundary surface of the considered fluid layer (i.e., $z = 0$ and $z = 1$) are considered to be stress-free. Therefore, the boundary conditions can be expressed as (Chandrasekhar [7])

$$W=D^2W=\Theta=D\Phi=DZ=\Omega=0, \quad \text{at} \quad z=0 \quad \text{and} \quad z=1. \tag{3.8}$$

Using Eq.(3.8) in Eqs (3.2) – (3.6), we must have

$$D^4W=0,\quad D^2\Theta=D^2\Omega=0,\quad D^3Z=D^3\Phi=0,\quad \text{at} \quad z=0 \quad \text{and} \quad z=1. \tag{3.9}$$

In view of boundary conditions given in Eq.(3.9), one can see that all the even derivatives of *W* vanish, e.g., $D^6W=D^8W=..........=0$. We also note that Eq.(3.7) is in *W* only. Therefore, the boundary conditions given in Eq.(3.8) suggest that the proper solution of Eq.(3.7) characterizing the lowest mode should be (Chandrasekhar [7])

$$W = W_0 \sin \pi z \tag{3.10}$$

where w_0 is a non-zero constant quantity. Inserting Eq.(3.10) into Eq.(3.7), we obtain the following relation

$$\begin{gathered}R_T b k^2 \left[\sigma + (1+K)b\right]\left[l\sigma + 2A + b(1-\overline{\delta}\,A)\right] = \\ = -R_{ea} k^4 \left[\sigma + (1+K)b\right]\left[l\sigma + 2A + b(1-\overline{\delta}\,A)\right] + \\ -KAb^3 \left[\sigma + (1+K)b\right](b + p_1\sigma) + b\pi^2 T_a (b + p_1\sigma)(l\sigma + 2A + b) + \\ +\left[\sigma + (1+K)b\right]^2 (l\sigma + 2A + b)(b + p_1\sigma) b^2\end{gathered} \tag{3.11}$$

where $b = \pi^2 + k^2$. This is the dispersion relation giving us the information about various non-dimensional numbers. For example, the expression for the thermal Rayleigh number can be written as

$$\begin{gathered}R_T = -\frac{R_{ea} k^2}{b} + \frac{\left[T_a \pi^2 (l\sigma + 2A + b) - KAb^2 \{\sigma + (1+K)b\}\right]}{k^2 \{\sigma + (1+K)b\}\left[l\sigma + 2A + b(1-\overline{\delta}\,A)\right]} \times \\ \times (b + p_1\sigma) + \frac{\{\sigma + (1+K)b\}[l\sigma + 2A + b](b + p_1\sigma) b}{k^2 \left[l\sigma + 2A + b(1-\overline{\delta}\,A)\right]}.\end{gathered} \tag{3.12}$$

It is easy to find from Eq.(3.12) that

$$\frac{dR_T}{dR_{ea}} = -\frac{k^2}{b}, \tag{3.13}$$

which is negative, showing that the electric field has a destabilizing effect on the system. The expression for the electric Rayleigh number can be written from Eq.(3.11) as

$$\begin{gathered}R_{ea} = -\frac{R_T b}{k^2} + \frac{\left[T_a \pi^2 (l\sigma + 2A + b) - KAb^2 \{\sigma + (1+K)b\}\right]}{k^4 \{\sigma + (1+K)b\}\left[l\sigma + 2A + b\left(1-\overline{\delta}\,A\right)\right]} \times b(b + p_1\sigma) + \\ + \frac{\{\sigma + (1+K)b\}(l\sigma + 2A + b)(b + p_1\sigma) b^2}{k^4 \left[l\sigma + 2A + b\left(1-\overline{\delta}\,A\right)\right]}.\end{gathered} \tag{3.14}$$

We note that the thermal Rayleigh number and electric Rayleigh number are linear functions of the Prandtl number and Taylor number.

4. Stability convection

The state of stability of the system is obtained by putting $\sigma = 0$ into Eq.(3.11). In this case, the thermal and electric Rayleigh numbers are obtained from Eq.(3.12) as

$$R_T = -\frac{R_{ea}k^2}{b} + \frac{\left[(2A+b)(1+K)-KA\right]b^3}{k^2\left[2A+b\left(1-\overline{\delta}A\right)\right]} + \frac{T_a\pi^2(2A+b)}{k^2(1+K)\left[2A+b\left(1-\overline{\delta}A\right)\right]} \tag{4.1}$$

and

$$R_{ea} = -\frac{R_T b}{k^2} + \frac{\left[T_a\pi^2(2A+b)-KAb^3(1+K)\right]b}{k^4(1+K)\left[2A+b\left(1-\overline{\delta}A\right)\right]} + \frac{(1+K)(2A+b)b^4}{k^4\left[2A+b\left(1-\overline{\delta}A\right)\right]} . \tag{4.2}$$

We note that the expression of the thermal and electric Rayleigh numbers are independent of the Prandtl number (p_1). Hence the stability of the system will not depend on the kinematic viscosity, however, it depends on micropolar viscosities. Further, we note that in the absence of rotation, the problem reduces to the problem already investigated by Rani and Tomar [23]. It is easy to verify that when $T_a = 0$, the expression of R_T and R_{ea} given in Eqs (4.1) and (4.2) exactly reduce to Eqs (3.11) and (3.13) of Rani and Tomar [23] for their corresponding problem.

4.1. Special cases

(i): In the absence of the electric field, i.e., when $R_{ea} = 0$, relation (4.1) becomes

$$R_T = \frac{\left[(2A+b)(1+K)-KA\right]b^3}{k^2\left[2A+b\left(1-\overline{\delta}A\right)\right]} + \frac{T_a\pi^2(2A+b)}{k^2(1+K)\left[2A+b\left(1-\overline{\delta}A\right)\right]} . \tag{4.3}$$

In the absence of the coupling between heat and spin fluxes, i.e., $\overline{\delta} = 0$ or in the case when $\overline{\delta} < 0$, we see that the second term on the right side of Eq.(4.3) is always positive. Thus, in this case, we can conclude that the effect of rotation on the system is likely to be stabilizing.

(ii): In the absence of the electric field and rotation, i.e., when $R_{ea} = T_a = 0$, relation (4.1) becomes

$$R_T = \frac{\left[(2A+b)(1+K)-KA\right]b^3}{k^2\left[2A+b\left(1-\overline{\delta}A\right)\right]} . \tag{4.4}$$

This is the result obtained by Datta and Sastry [13] for the corresponding problem.

(iii): Further, in the absence of coupling between heat and spin fluxes, i.e., $\overline{\delta} = 0$ and in the absence of micropolar viscosity, i.e., $K = 0$, relation (4.3) becomes

$$R_T = \frac{b^3}{k^2} = \frac{\left(\pi^2+k^2\right)^3}{k^2} . \tag{4.5}$$

When the Rayleigh number is less than given by Eq.(4.5), the disturbance with wave number k will be stable. The disturbance will be marginally stable when the Rayleigh number equals the value given by Eq.(4.5). When the Rayleigh number exceeds the value given by Eq.(4.5), the same disturbance will be unstable. The critical Rayleigh number for the onset of instability is given by

$$\frac{dR_T}{dk^2}=\frac{3k^2\left(\pi^2+k^2\right)^2-\left(\pi^2+k^2\right)^3}{\left(k^2\right)^2}=0, \qquad \Rightarrow k\equiv k_c=\frac{\pi}{\sqrt{2}}=2.2214\ . \tag{4.6}$$

The corresponding value of $R_T=\dfrac{27\pi^4}{4}=657.5$, which is the classical result of Chandrasekhar [7] in the corresponding problem of Newtonian fluid.

(iv): In the absence of coupling between heat and spin fluxes and micropolar viscosity, i.e., $\overline{\delta}=K=0$, one obtains from Eq.(4.1)

$$R_T=-\frac{R_{ea}k^2}{b}+\frac{b^3}{k^2}+\frac{T_a\pi^2}{k^2}. \tag{4.7}$$

To find the critical value of R_T Eq.(4.7) is differentiated with respect to k^2 and equated to zero to get a polynomial in (k_c^2) in the form

$$\begin{aligned}&2\left(k_c^2\right)^5+7\pi^2\left(k_c^2\right)^4+8\pi^2\left(k_c^2\right)^3+\pi^2\left(2\pi^4-R_{ea}-T_a\right)\left(k_c^2\right)^2+\\&-2\pi^4\left(\pi^4+T_a\right)\left(k_c^2\right)^1-\pi^6\left(\pi^4+T_a\right)\left(k_c^2\right)^0=0.\end{aligned} \tag{4.8}$$

It is observed that the critical wave number varies with R_{ea} and T_a. The result is identical with the equation obtained by Shivakumara [27] in the absence of the couple stress parameter.

(v): In the absence of rotation, coupling between heat and spin fluxes and micropolar viscosity, but in the presence of the electric field, i.e., $T_a=K=\overline{\delta}=A=0$ and $R_{ea}\neq 0$, one obtains from Eq.(4.1) that

$$R_T=-\frac{R_{ea}k^2}{b}+\frac{b^3}{k^2}\ , \tag{4.9}$$

which coincides with the results of Roberts [6] for the corresponding problem.

(vi): In the absence of the electric field and micropolar viscosity, i.e., when $R_{ea}=K=0$, together with $\overline{\delta}=0$, one obtains from Eq.(4.1) that

$$R_T=\frac{T_a\pi^2}{b}+\frac{b^3}{k^2}\ , \tag{4.10}$$

which coincides with the result of Chandrasekhar [7] for the relevant problem. From Eq.(4.10), we obtain

$$\frac{dR_T}{dT_a}=\frac{\pi^2}{k^2}\ , \tag{4.11}$$

which is positive; therefore, rotation has a stabilizing effect on the system which is an agreement with results derived by Takashima [28] and Rana *et al*. [19].

(vii): In the absence of micropolar viscosity, i.e., when $K=0$, together with $\overline{\delta}=0$, one obtains from Eq.(4.2) that

$$R_{ea} = -\frac{R_T b}{k^2} + \frac{b\left(T_a \pi^2 + b^3\right)}{k^4}. \tag{4.12}$$

Further, in the absence of rotation, i.e., when $T_a = 0$, we get from the above equation

$$R_{ea} = -\frac{R_T b}{k^2} + \frac{b^4}{k^4}. \tag{4.13}$$

From this we can obtain that

$$\frac{dR_{ea}}{dR_T} = -\frac{b}{k^2}, \tag{4.14}$$

which is the same result as obtained in Eq.(3.13) showing that the electric Rayleigh number has a destabilizing effect on the system.

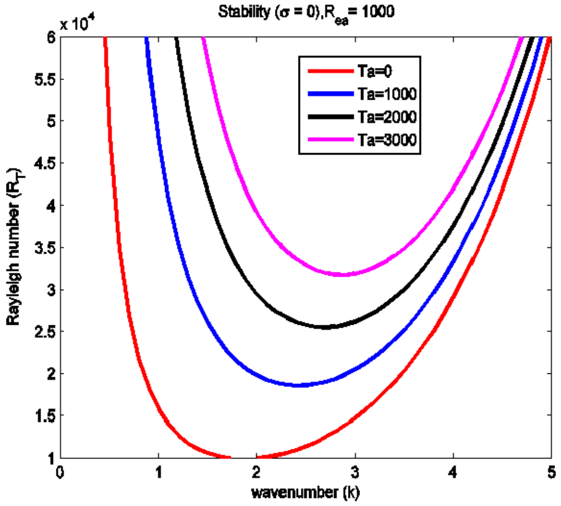

Fig.1. Effect of T_a on R_T in stability motions.

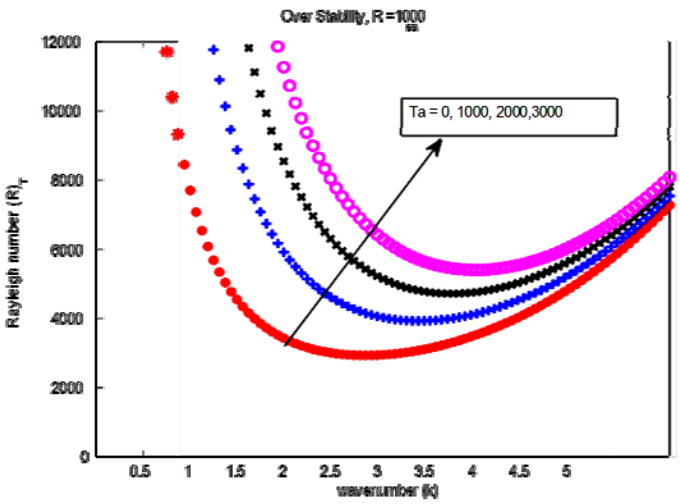

Fig.2. Effect of T_a on R_T in over stability motions.

5. Over stability motions

Since σ is a complex in general, so we write $\sigma = \sigma_R + i\sigma_I$, $\sigma_R, \sigma_I \in R$. For over stability convection, $\sigma \neq 0$ and $\sigma_R = 0$, which gives $\sigma_I \neq 0$. Hence $\sigma = i\sigma_I$ and in this case, the dispersion relation (3.12) yields

$$R_T = X + iY \tag{5.1}$$

where X and Y are real valued functions of $p_1, b, l, k, A, K, \overline{\delta}$ and σ_I. The explicit expression of X and Y are given by

$$X = \frac{C_1 C_3 + C_2 C_4}{bk^2\left(C_1^2 + C_2^2\right)}, \qquad Y = \frac{C_1 C_4 - C_2 C_3}{bk^2\left(C_1^2 + C_2^2\right)} \tag{5.2}$$

where

$$C_1 = \left[2A + b(1-\bar{\delta}A)\right](1+K)b - l\sigma_I^2, \quad C_2 = \left[2A + b(1-\bar{\delta}A) + (1+K)bl\right]\sigma_I \ ,$$

$$\begin{aligned} C_3 &= b^5(2A+b)(1+K)^2 - R_{ea}k^4 b(1+K)\left\{2A + b(1-\bar{\delta}A)\right\} - KAb^5(1+K) + \\ &+ T_a\pi^2 b^2(2A+b) - \left[b^3(2A+b) + p_1 b^4 l(1+K)^2 + 2b^4 l(1+K) + \right. \\ &\left. + 2p_1 b^3(2A+b)(1+K) - R_{ea}k^4 l - KAp_1 b^3 + T_a b\pi^2 p_1 l\right]\sigma_I^2 + p_1 b^2 l\sigma_I^4, \end{aligned} \tag{5.3}$$

$$\begin{aligned} C_4 &= \left[2b^4(2A+b)(1+K) + (1+K)^2 b^5 l + p_1 b^4(2A+b)(1+K)^2 + \right. \\ &- R_{ea}k^4\left\{bl(1+K) + 2A + b(1-\bar{\delta}A)\right\} - KAb^4\left\{(1+K)p_1 + 1\right\} + \\ &\left. + T_a\pi^2\left\{b^2 l + p_1 b(2A+b)\right\}\right]\sigma_I - \left[2p_1 b^3 l(1+K) + b^3 l + p_1 b^2(2A+b)\right]\sigma_I^3. \end{aligned}$$

For over stability motion, since $\sigma_I \neq 0$, therefore

$$R_T = \frac{C_1 C_3 + C_2 C_4}{bk^2\left(C_1^2 + C_2^2\right)}, \qquad C_1 C_4 - C_2 C_3 = 0. \tag{5.4}$$

Similarly, from Eq.(3.14), we can write

$$R_{ea} = X' + iY' \tag{5.5}$$

where

$$X' = \frac{C'_1 C'_3 + C'_2 C'_4}{bk^2\left(C'^2_1 + C'^2_2\right)}, \qquad Y = \frac{C'_1 C'_4 - C'_2 C'_3}{bk^2\left(C'^2_1 + C'^2_2\right)}, \tag{5.6}$$

$$C'_1 = \left[2A + b(1-\bar{\delta}A)\right](1+K)b - l\sigma_I^2, \; C'_2 = \left[2A + b(1-\bar{\delta}A) + (1+K)bl\right]\sigma_I,$$

$$\begin{aligned} C'_3 &= b^5(2A+b)(1+K)^2 - R_{ea}k^4 b(1+K)\left\{2A + b(1-\bar{\delta}A)\right\} - KAb^5(1+K) + \\ &+ T_a\pi^2 b^2(2A+b) - \left[b^3(2A+b) + p_1 b^4 l(1+K)^2 + 2b^4 l(1+K) + \right. \\ &\left. + 2p_1 b^3(2A+b)(1+K) - R_{ea}k^4 l - KAp_1 b^3 + T_a b\pi^2 p_1 l\right]\sigma_I^2 + p_1 b^2 l\sigma_I^4, \end{aligned} \tag{5.7}$$

$$\begin{aligned} C'_4 &= \left[2b^4(2A+b)(1+K) + (1+K)^2 b^5 l + p_1 b^4(2A+b)(1+K)^2 + \right. \\ &- R_{ea}k^4\left\{bl(1+K) + 2A + b(1-\bar{\delta}A)\right\} - KAb^4\left\{(1+K)p_1 + 1\right\} + \\ &\left. + T_a\pi^2\left\{b^2 l + p_1 b(2A+b)\right\}\right]\sigma_I. \end{aligned}$$

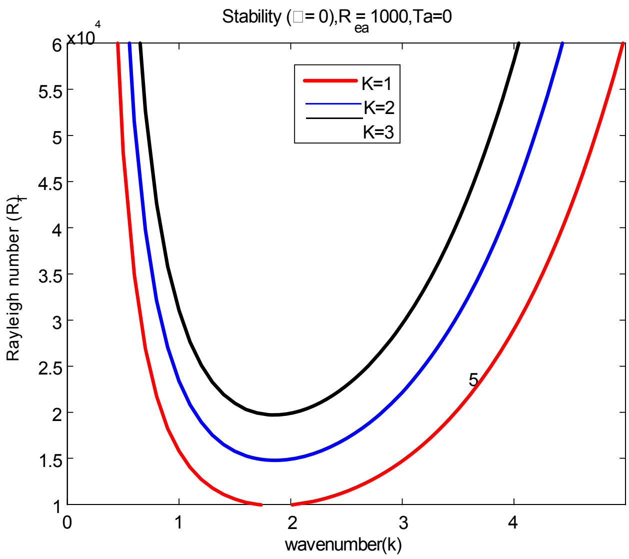

Fig.3. Effect of K and T_a in stability motions.

For oscillatory convection that is over stability motion, we shall determine the critical thermal and electric Rayleigh numbers from formulae (5.4) and (5.6). It is clear from these formulae that these numbers depend on the Prandtl number contrary to the case of stability motion. In the absence of rotation, the results given in Eqs (5.4) and (5.6) must reduce to those given by Rani and Tomar [23] for the corresponding problem. It is found that the expression of C_1, C_2, C_3 and C_4 given by Rani and Tomar [23] are erranous. The correct expressions of these quantities are given by

$$C_1 = \left[2A + b(1-\bar{\delta}A)\right](1+K)b - l\sigma_I^2, \qquad C_2 = \left[2A + b\left(1-\bar{\delta}A\right) + (1+K)bl\right]\sigma_I \ ,$$

$$\begin{aligned} C_3 = {} & b^5(2A+b)(1+K)^2 - R_{ea}k^4 b(1+K)\left\{2A + b\left(1-\bar{\delta}A\right)\right\} - KAb^5(1+K) + \\ & + T_a\pi^2 b^2(2A+b) - \left[b^3(2A+b) + p_1 b^4 l(1+K)^2 + 2b^4 l(1+K) + \right. \\ & \left. + 2p_1 b^3(2A+b)(1+K) - R_{ea}k^4 l - KAp_1 b^3 + T_a b\pi^2 p_1 l\right]\sigma_I^2 + p_1 b^2 l\sigma_I^4, \end{aligned} \tag{5.8}$$

$$\begin{aligned} C_4 = {} & \left[2b^4(2A+b)(1+K) + (1+K)^2 b^5 l + p_1 b^4(2A+b)(1+K)^2 + \right. \\ & - R_{ea}k^4\left\{bl(1+K) + 2A + b(1-\bar{\delta}A)\right\} - KAb^4\left\{(1+K)p_1 + 1\right\} + \\ & \left. + T_a\pi^2\left\{b^2 l + p_1 b(2A+b\right\}\right]\sigma_I - \left[2p_1 b^3 l(1+K) + b^3 l + p_1 b^2(2A+b)\right]\sigma_I^3. \end{aligned}$$

The correct expression of C_1, C_2, C_3 and C_4 can be obtained from Eq.(5.7) likewise.

6. Results and discussion

Numerical computations have been carried out to study the detailed effect of various parameters present in the system on the onset of convection in the fluid layer. The numerical values of relevant non-dimensional parameters are taken as $p_1 = \bar{\delta} = l = 1$ and $K = 1$, the value of $\sigma_I = 0$ and $\sigma_I \neq 0$ correspond to the case of stability and over stability motions respectively. The thermal Rayleigh (R_T) and electric Rayleigh number (R_{ea}) are computed for different values of the wave number (k) for stability convection from formulae (3.12) and (3.14). For over stability motion, the thermal Rayleigh number (R_T) is computed from formulae (5.2). This computation is a little bit tricky. For a given k, the values of σ_I are obtained from Eq.$(5.4)_2$ and then this value of σ_I is used in Eq.$(5.4)_1$ to compute R_T. Figures 1 and 2 show the behaviour of the thermal Rayleigh number (R_T) against thewave number (k) for the case of stability and over stability motions at a fixed value of the electric Rayleigh number (R_{ea}), namely $R_{ea} = 1000$ and for different values of the Taylor number (T_a), namely $T_a = 0, 1000, 2000, 3000$. From Fig.1, we note that the thermal Rayleigh number is significantly affected by the Taylor number. The value of the critical thermal number is found to be minimum in a non-rotating micropolar fluid layer. As the Taylor number increases, the critical value of the thermal Rayleigh number also increases. This shows that rotation has a stabilizing effect on the system. Further, we also observe that with an increase in the Taylor number, the critical thermal Rayleigh number shifts towards the right side, showing that the critical Rayleigh number is not only a function of T_a but also of k. From Fig.2, we notice almost a similar behaviour as in the case of stability motion. Here, we also note that the critical thermal Rayleigh number increases with an increase in the Taylor number. The effect of the Taylor number is seen to be more prominent before the critical wave number (k^*)(the wave number at which the critical value of the thermal Rayleigh number occurs) than that after k^*. It is observed that rotation has a stabilizing effect on the system in the case of over stability motion also. In Figs 3-5, we have shown the graphs of the thermal Rayleigh number (R_T) versus the wave number (k) for different values of the Taylor number ($T_a = 0, 1000, 5000$). The value of the critical thermal Rayleigh number increases with an increase of micropolar viscosity, showing that the micropolar viscosity has a stabilizing effect on the system. We also note that as the rotation parameter increases, the critical thermal Rayleigh number also increases. Thus the combined effect of micropolar viscosity and rotation is to stabilize the system.

In Figs 6-9, we have shown the graphs of the thermal Rayleigh number (R_T) versus the wave number (k) for different values of the micropolar viscosity parameter $K = 1, 2, 3$ in the case of over stability motion at fixed Rayleigh number ($R_{ea} = 1000$) and for different values of the Taylor number ($T_a = 0, 1000, 2000, 3000$).

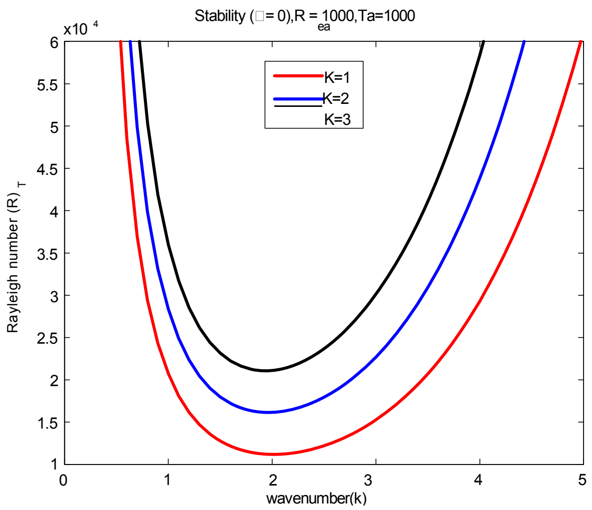

Fig.4. Effects of K and T_a in stability motions.

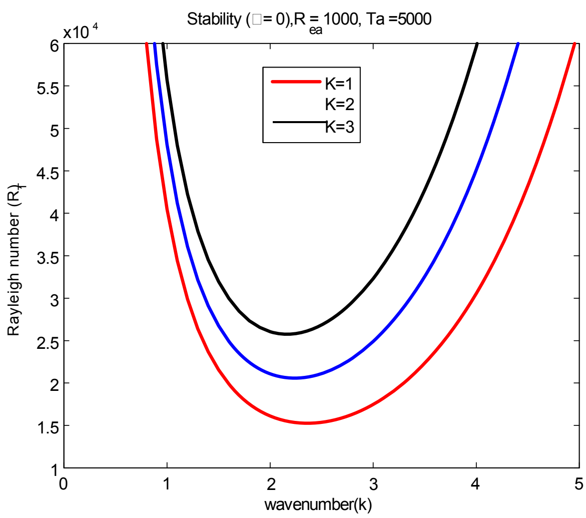

Fig.5. Effect of K and T_a in stability motions.

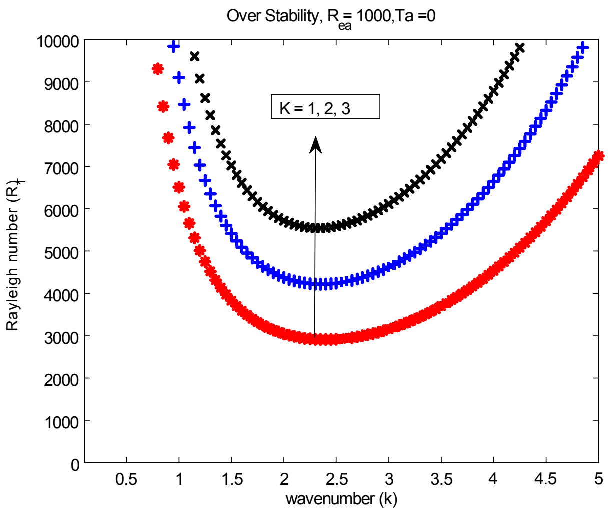

Fig.6. Effect of K and T_a in over stability motions.

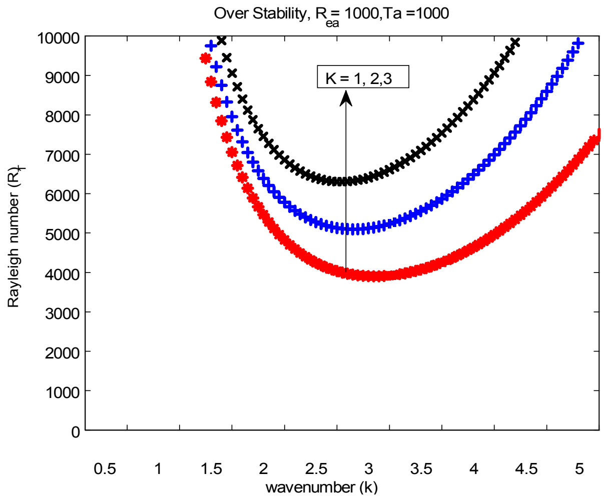

Fig.7. Effect of K and T_a in over stability motions.

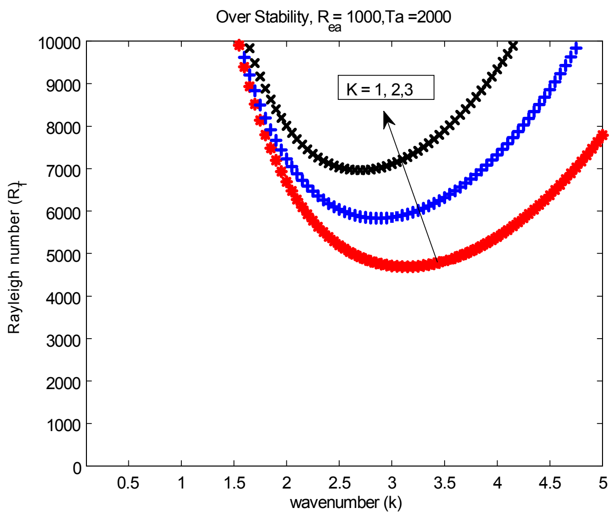

Fig.8. Effect of K and T_a in over stability motions.

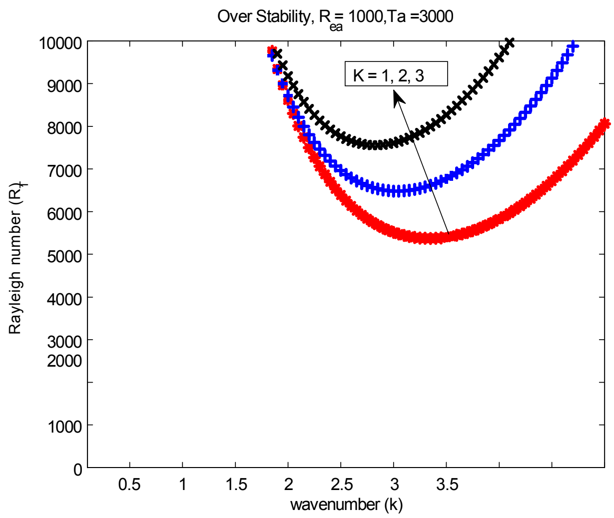

Fig.9. Effect of K and T_a in over stability motions.

We note from these figures that the thermal Rayleigh number increases as the value of the micropolar viscosity parameter increases. It also increases with an increase of the Taylor number, indicating that rotation and micropolar viscosity parameters both have a stabilizing effect on the system in the case of over stability motion. The effect of micropolar viscosity (K) on the thermal Rayleigh number curve is much less in the range $0 < k < k^*$.

Figures 10-12 depict the effect of rotation (T_a) on the electric Rayleigh number (R_{ea}) for fixed values of the thermal Rayleigh number (R_T), namely ($R_T = 1000, 3000, 5000$) at different values of the Taylor number ($T_a = 0, 1000, 2000, 3000$) in the case of stability motion. It is noticed that the electric Rayleigh number increases with an increase of T_a, but decreases with an increase of R_T. This is also clear from formula (4.14). Next, we infer that the Taylor number has a prominent effect on the electric Rayleigh number curve in the range $0< k < k^*$, while beyond k^*, the said effect is very poor and goes on diminishing as k takes larger and larger values. This means that the electric Rayleigh number does not depend on the rotation parameter at very large wave number. This fact can be made clear from formula (4.2) as k takes larger and larger values, the last two terms (including the term containing Ta) diminish much faster than the term containing R_T.

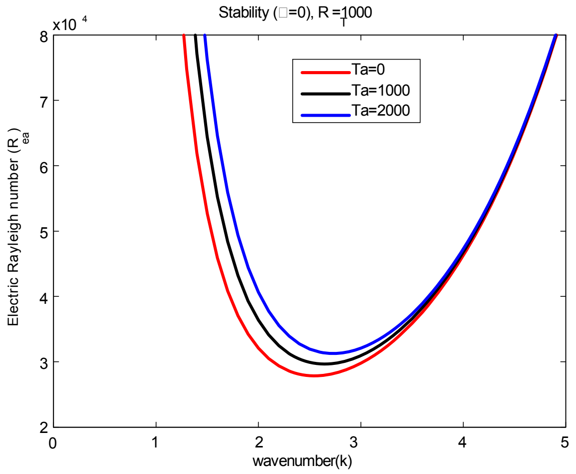

Fig.10. Effect of R_T and T_a on R_{ea} in stability motions.

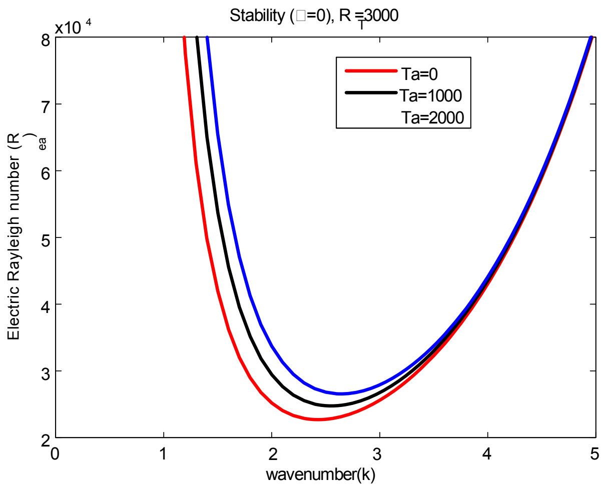

Fig.11. Effects of R_T and T_a on R_{ea} in stability motions.

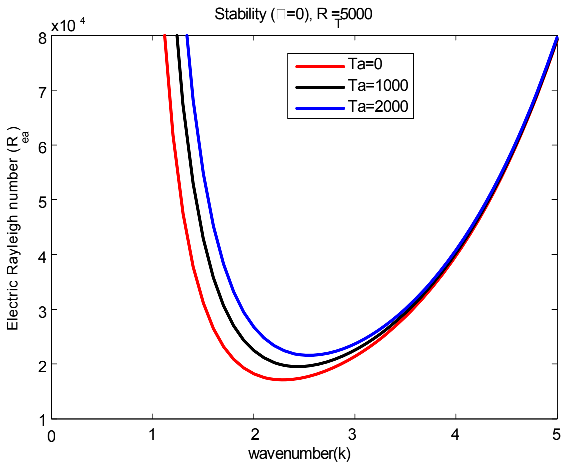

Fig.12. Effect of R_T and T_a on R_{ea} in stability motions.

Figures 13-15 show that the effect of rotation (T_a) on the electric Rayleigh number (R_{ea}) for fixed values of the thermal Rayleigh number (R_T), namely (R_T = *100*, *200*, *300*) and at different values of the Taylor number (T_a = *0*, *1000*, *2000*) in cases of over-stability motion. As the thermal Rayleigh number increases, the value of the critical electric Rayleigh number decreases. However, the critical electric Rayleigh number increases with an increase of the rotation parameter.

Figures 16-18 depict the effect of the Prandtl number (p_1) for given values of the electric Rayleigh number (R_{ea} = *1000*) and for a fixed value of the Taylor number (T_a = *10*, *500*, *1000*). We observe that as the Prandtl number increases, the thermal Rayleigh number also increases. Careful observation indicates that the critical thermal Rayleigh number also increases with an increase of the Taylor number. Thus we can conclude that the combined role of the Prandtl and Taylor number is to stabilize the system.

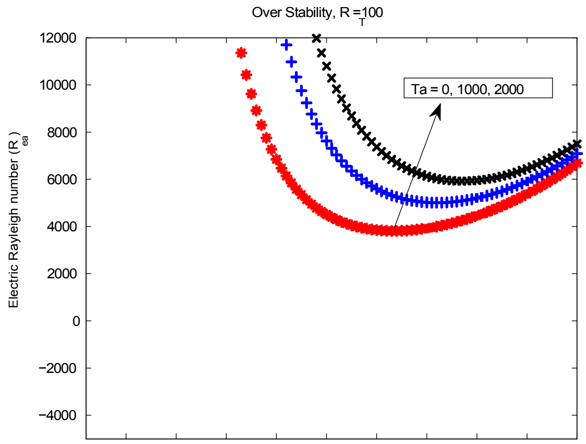

Fig.13. Effect of R_T and T_a on R_{ea} in over stability motions.

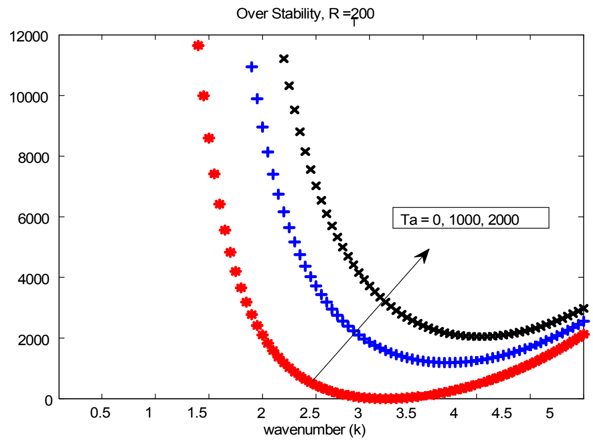

Fig.14. Effect of R_T and T_a on R_{ea} in over-stability motions.

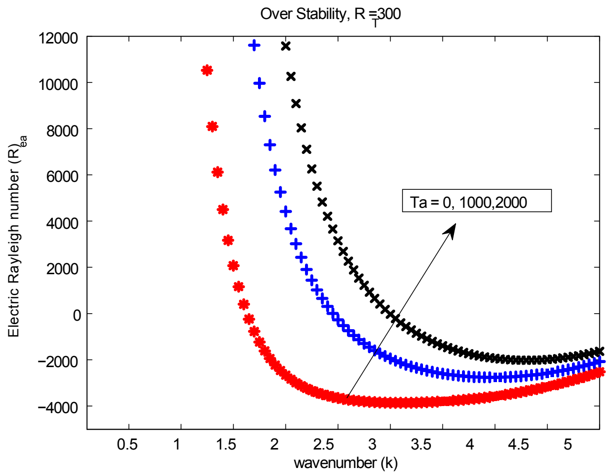

Fig.15. Effect of R_T and T_a on R_{ea} in over stability motions.

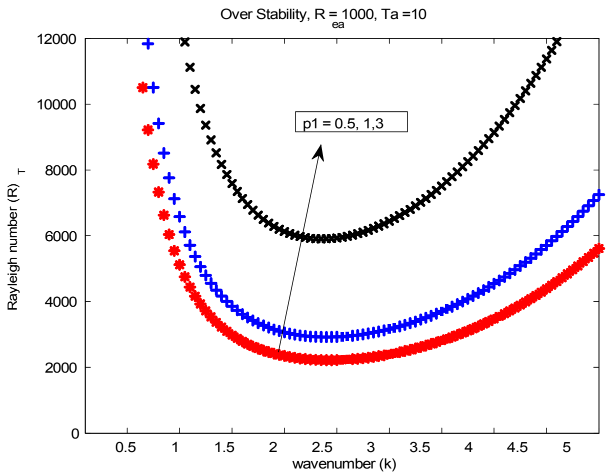

Fig.16. Effect of T_a and p_1 on R_T in over stability motions.

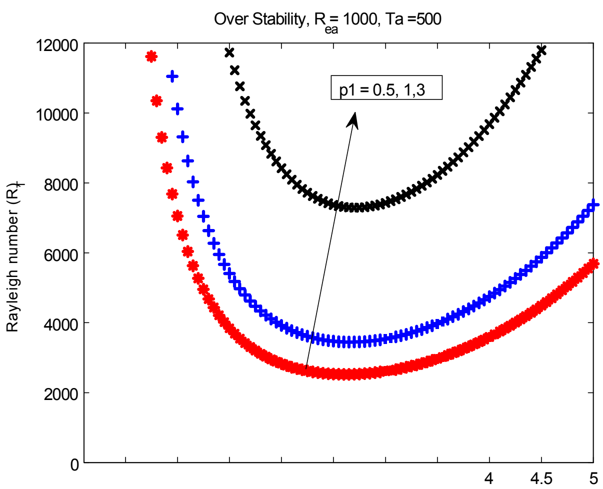

Fig.17. Effect of T_a and p_1 on R_T in over stability motions.

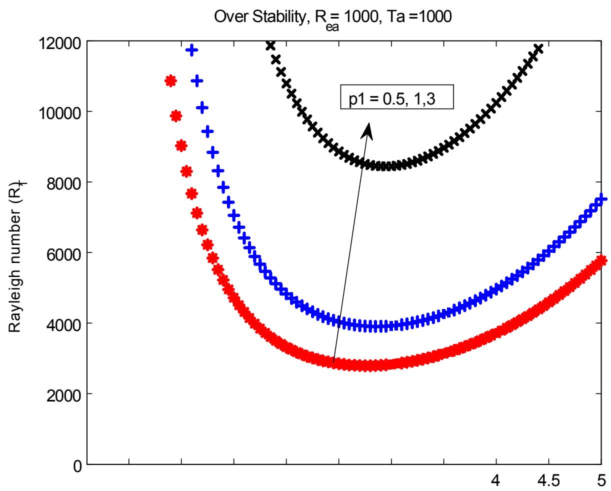

Fig.18. Effect of T_a and p_1 on R_T in over stability motions.

Figures 19-21 depict the effect of the Prandtl number (p_1) on the electric Rayleigh number curve for given values of the thermal Rayleigh number (R_T =*100*) and for different values of the Taylor number (T_a = *0*, *1000*, *3000*) in over stability motion. It is observed that as the Taylor number increases, the critical electric Rayleigh number shifts to the right. The role of the Prandtl number is to stabilize the system.

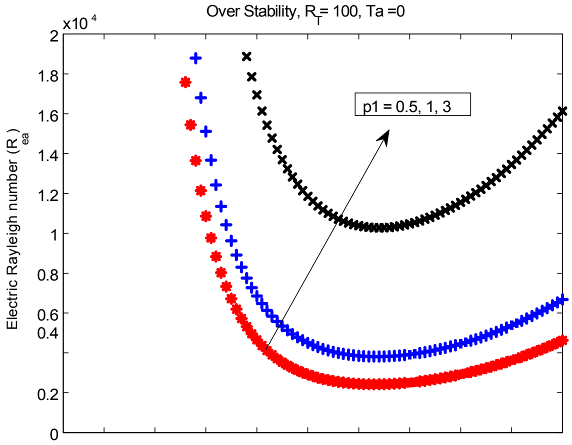

Fig.19. Effect of T_a and p_1 on R_{ea} in over stability motions.

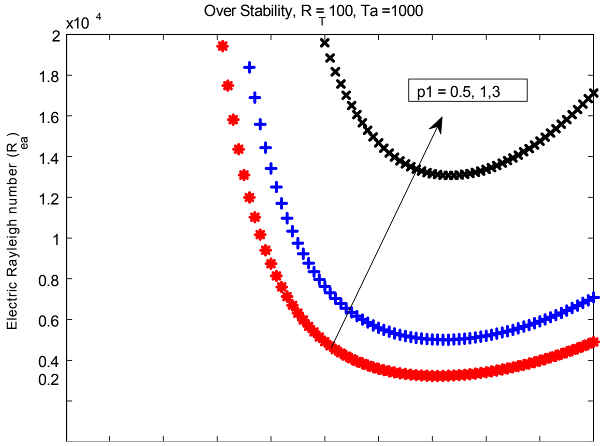

Fig.20. Effect of T_a and p_1 on R_{ea} in over stability motions.

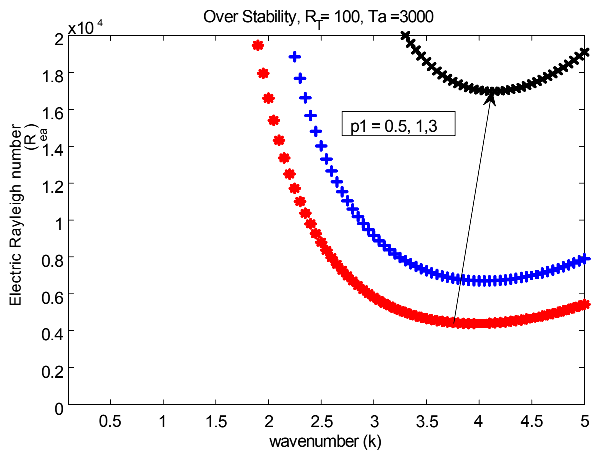

Fig.21. Effect of T_a and p_1 on R_{ea} in over stability motions.

7. Conclusion

Natural convection of a rotating micropolar fluid layer heated from below and in the presence of an electric field has been investigated. The case of free-free boundaries has been considered. Mainly, the study is focused on studying the effect of rotation, micropolar viscosity and electric field on the convection phenomenon. From this study, we conclude that the micropolar additives and rotation of the fluid layer have a stabilizing effect on the system. The role of the electric field is to stabilize the fluid layer in the case of over stability motion. The expression of the thermal Rayleigh number is found to be independent of the Prandtl number in the case of stability motion.

Acknowledgement

The support from I.K. Gujral Punjab Technical University, Jalandhar (Punjab, India) is gratefully acknowledged. The authoris also grateful to Professor S.K. Tomar, Department of Mathematics, Panjab University, Chandigarh, INDIA for his valuable discussion, checking the manuscript and providing facilities during the completion of this study.

Nomenclature

d – thickness of the dielectric fluid layer
E – root-mean square value of the electric field
E_0 – root-mean square value of the electric field at $z = 0$
g – acceleration due to gravity
k – wave number
R_T – thermal Rayleigh number
R_{ea} – electric Rayleigh number
T_a – Taylor number
$\boldsymbol{V}$ – velocity vector
α – thermal expansion coefficient
ϵ – dielectric constant
ν – kinematic viscosity
μ – dynamic viscosity
σ – growth rate
ρ – density
Ω – angular velocity
$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ – horizontal Laplacian operator
$\nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}$ – Laplacian operator

References

[1] Landau L.D. and Lifshitz E.M. (1960): *Electro-Hydrodynamics of Continuous Media*. – Pergamon Press Oxford.

[2] Roberts P.H. (1969): *Electrohydrodynamics convection*. – Quart. J. Mech. Appl. Math., vol.22, pp.211-220.

[3] Melcher J.R. and Taylor G.L. (1969): *Electrohydrodynamics:a review of role of interfacial shear stresses*. – Annu. Rey. Fluid Mech., vol.1, pp.111-146.

[4] Gelmont B.L. and Loffe I.V. (1968): *The electric field influence on the convection in the liquid dielectric*. – Phys. Lett. 26 (A), pp.253-254.

[5] Gross M.J. and Porter J.E. (1966): *Electrically induced convection in dielectric liquids*. – Nature, vol.212, pp.1343-1345.

[6] Takashima M. and Aldrige K.D. (1976): *The stability of horizontal layer of dielectric fluid under the simultaneous action of vertical DC electric field and a vertical temperature gradient*. – Quart. J. Mech. Appl. Math., vol.29, No.1, pp.71-87.

[7] Chandrasekhar S. (1981): *Hydrodynamics and Hydromagnetic Stability.* – Dover Publication Inc.

[8] Eringen A.C. (1966): *Theory of micropolar fluids.* – Int. J. Engng. Sci., vol.16.

[9] Eringen A.C. (1972): *Theory of thermal micropolar fluids*. – J. Math. Anal. Appl., vol.38, No.2, pp.480-496.

[10] Perez-Gracia C. and Rubi J.M. (1982): *On the possibility of overstable motions of micropolar fluids heated from below.* – Int. J. Engng. Sci., vol.20, pp.873-878.

[11] Perez-Garcia C., Rubi J.M. and Casas-Vazquez J. (1981): *On the stability of micropolar fluids.* – J. Non-Equilib., Thermodyn, vol.6, pp.65-78.

[12] Ahmadi G. (1976): *Stability of micropolar layer heated from below.* – Int. J. Engng. Sci., vol.14, pp.81-89.

[13] Datta B. and Sastry V.U.K. (1976): *Thermal instability of horizontal layer of micropolar fluid heated from below.* – Int. J. Engng. Sci., vol.14, No.7, pp.631-637.

[14] Rama Rao K.V. (1979): *Numerical solution of the thermal instability in micropolar fluid layer between rigid boundaries.* – Acta Mech., vol.32, pp.79-88.

[15] Rama Rao K.V. (1980): *Thermal instability in a micropolar fluid layer subjected to a magnetic field*. – Int. J. Engng. Sci., vol.18, pp.741-750.

[16] Sharma R.C. and Gupta U. (1995): *Thermal convection in micropolar fluids in porous medium.* – Int. J. Engng. Sci., vol.33, pp.1887-1892.

[17] Shrama R.C. and Kumar P. (1994): *Effect of rotation on thermal convection in micropolar fluids.* – Int. J. Engng. Sci., vol.32, pp.545-551.

[18] Sharma R.C. and Kumar P. (1998): *Effect of rotation on the thermal convection in micropolar fluids in porous media*. – Ind. J. Pure Appl. Math., vol.29, No.1, pp.95-104.

[19] Rana G.C. Chand R. and Yadav D. (2015): *The onset of electrohydrodynamics instability of an elastico-viscous Walters'(Model B') dielectric fluid layer*. – FME Trans., vol.43, pp.154-160.

[20] Sharma V. and Gupta S. (2008): *Thermal convection of micropolar fluid in the presence of suspended particles in rotation.* – Arch. Mech., vol.60, No.4, pp.403-419.

[21] Rani N. and Tomar S.K. (2010): *Thermal convection problem of micropolar fluid subjected to hall current*. – Appl. Math. Model., vol.34, No.2, pp.508-519.

[22] Ezzat M.A. and Othman M.I.A. (2000): *Thermal instability in a rotating micropolar fluid layer subject to an electric field*. – Int. J. Engng. Sci., vol.38, No.16, pp.1851-1867.

[23] Rani N. and Tomar S.K. (2015): *Electro-hydrodynamics convection in dielectric micropolar fluid layer.* – J. Electrostatics, vol.78, pp.60-67.

[24] Qin Y. and Kaloni P.N. (1992): *A thermal instability problem in rotating micropolar fluid.* – Int. J. Engng. Sci., vol.30, pp.1117-1126.

[25] Sharma R.C. and Kumar P. (1997): *On micropolar fluids heated from below in hydromagnetics in porous medium*. – Czec. J. Phys., vol.47, No.6, pp.637-647.

[26] Othman M.I.A. and Zaki S.A. (2004): *Thermal relaxation effect on magnetohydrodynamics instability in a roataing micropolar fluid layer heated from below*. – Acta Mechanica, vol.170, pp.187-197.

[27] Shivakumra I.S., Akkanagamma M. and Chiu-On-Ng (2013): *Electrohydrodynamics instability of a rotating couple stress dielectric fluid layer*. – Int. J. Heat. Mass Trans., vol.62, pp.761-771.

[28] Takashima M. (1976): *The effect of rotation on electrohydrodynamics instability.* – Canadian J. Physics, vol.54, No.3, pp.342-347.

Received: January 23, 2018
Revised: July 13, 2019