sciendo

Int. J. of Applied Mechanics and Engineering, 2018, vol.23, No.4, pp.1025-1033
DOI: 10.2478/ijame-2018-0059

Technical note

DECISION OF THE DIRECT POSITION PROBLEM OF THE JOINT RELATIVE MANIPULATION MECHANISM WITH FIVE DEGREES OF FREEDOM

V.N. PASHCHENKO* and D.I. MELESHCHENKO
KF MSTU. N.E. Bauman
Kaluga, RUSSIA

G.V. RASHOYAN
Institute of Engineering Science. Blagonravov RAS
Moscow, RUSSIA

D.I. MALYSHEV and V.S. KUZMINA
Belgorod State Technological University
Belgorod, RUSSIA
E-mail: rl_bgtu@intbel.ru

One of the mechanisms of joint relative manipulation with five degrees of freedom is considered. An approach to solving the direct positional problem is described. A simplified 3D model was created to verify the correctness of the solution. The results of a comparison of calculated and experimental data are presented.

1. Introduction

The creation of high-performance technological equipment used in the field of machining and measurement is one of the directions for the development of machine building. The approach is based on the principle of using mechanisms of parallel structure, which have a number of advantages over traditional manipulation mechanisms [1-3]. These mechanisms have found their application in robotic machines. They allow handling details of complex shape with higher accuracy and speed than traditional equipment [4-7]. However, limited working space is one of disadvantages. Minimizing this disadvantage will allow the organization of joint relative manipulation of several mechanisms of a parallel structure [8]. The total number of degrees of freedom of the system will be the sum of the degrees of freedom of each mechanism for the joint manipulation of several mechanisms [9]. The system of such mechanisms is also a system of relative manipulation — "a mechanism that reproduces a given trajectory of a point and/or the orientation of a body in a moving coordinate system and the motion of the coordinate system itself. The general structural feature of the mechanisms of relative manipulation is the presence of two output mobile links " [10].

The direct problem of position is one of the main tasks of the synthesis and analysis of manipulation mechanisms [11-13].

There are various approaches to solving the problem of the position of mechanisms of parallel structure. Most of these approaches can be divided into two groups according to a formal characteristic. The first uses the apparatus of vector algebra: the position and orientation of the platform are expressed in terms

* To whom correspondence should be addressed

of the units of the mobile coordinate system, which are determined using vectors connecting the base and the moving platform, or through the components of the rotation matrix [14-16]. The other approach is based on the use of geometric relationships, while the mechanism is considered as a complex spatial construction [17].

Despite the variety of methods, the solution of the direct problem is reduced to solving a system of nonlinear equations that a priori require knowledge of a large number of parameters related to the structural features of the parallel structure mechanism, the location of the hinges, and so on. Numerical methods are used to solve the system of equations [18, 19].

2. Formulation of the problem

Finding an analytical solution of the direct kinematics problem for the whole joint relative manipulation mechanism as a whole is a rather complicated task, because, firstly, the system includes two parallel structure mechanisms; secondly, the position and orientation of the output link of the entire mechanism is determined in the mobile coordinate system output link of one of the mechanisms, the position and orientation of which is determined in the basic fixed coordinate system.

The position and orientation of the output link of the mechanism can be represented as a matrix of a homogeneous transformation, the general form of which is

$$T_{jrmm} = \begin{pmatrix} R(\alpha,\beta,\gamma) & p(x,y,z) \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix}$$

where $R(\alpha,\beta,\gamma)$ is the rotation matrix that uniquely determines the orientation of the output (the mobile coordinate system of the moving platform) of the mechanism (the rotation matrix is functionally dependent on the Euler angles), $p(x,y,z)$ is a transfer vector that specifies the Cartesian coordinates (coordinates of the mobile coordinate system origin) of the mechanism output link, 0 is a zero vector of dimension 1x3.
Consider one of the less well-known representatives of a wide class of joint relative manipulation mechanisms (Fig.1).

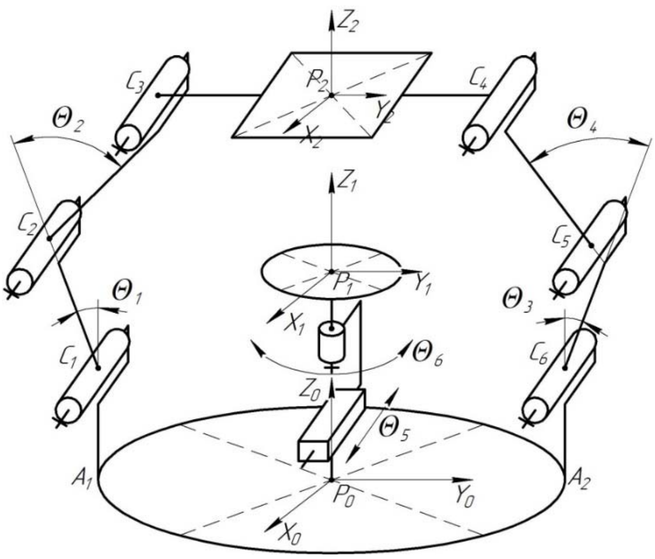


Fig.1. The structure of the joint relative manipulation mechanism.

The mechanism consists of two modules (mechanisms of a parallel structure). Since the mechanism is made in the form of a single spatial structure, and the kinematic pairs are arranged so that the output link of the upper module (the six-link mechanism) can move along one of the horizontal axes and rotate about the vertical axis, and the output link of the lower module (rotary mechanism) can move along the vertical and one of the horizontal axes, and rotate around the other horizontal axis, the degrees of freedom are thus distributed between the modules in two and four, respectively.

Let us determine the number of degrees of freedom of the joint relative manipulation mechanism. It is known that the number of mechanism freedom degrees is defined as the sum of the number of degrees of each component of its mechanisms, so we determine the number of degrees of freedom for each mechanism separately. We use Chebyshev's structural formula for plane mechanisms to determine the number of freedom degrees of a flat six-link mechanism

$$W = 3n - 1P_4 - 2P_5 = 3 \cdot 5 - 2 \cdot 6 = 3.$$

To determine the number of degrees of freedom of the rotational mechanism, we use the Somov-Malyshev structural formula

$$W = 6 \cdot n - 5 \cdot p_5 - 4 \cdot p_4 - 3 \cdot p_3 - 2 \cdot p_2 - p_1 = 6 \cdot 2 - 5 \cdot 2 = 2.$$

Accordingly, the number of degrees of freedom of the entire joint relative manipulation mechanism is five. Let us connect the basic fixed frame of reference $X_0Y_0Z_0$ with the base of the joint relative manipulation mechanism. The joint relative manipulation mechanism output link is the mobile platform of the upper mechanism (coordinate systems $X_2Y_2Z_2$). We connect the basic reference frame $X_1Y_1Z_1$ of the joint relative manipulation mechanism with the base of the lower mechanism. The upper module is made in the form of a flat six-link mechanism and is intended for tool installation. The output link of the mechanism can move vertically along the axes P_oY_o and P_oZ_o and rotate about the P_oX_o axis.

The lower module is designed as a rotary mechanism and is intended for installation of the processed object (part). The output link of the mechanism can perform a rotation around the P_oZ_o axis and displacement along the P_oX_o axis.

We formulate the direct problem of the position of the relative manipulation mechanism as follows: let the generalized coordinates of the planar six-link mechanism $\theta_1, \theta_2, \theta_3, \theta_4$ and the generalized coordinates of the rotary mechanism θ_5, θ_6 be known; it is necessary to find the position and orientation of the joint relative manipulation mechanism output link, that is, the output link of the flat six-link mechanism $X_2Y_2Z_2$ in the coordinate system X_1, Y_1, Z_1 of the output link rotary mechanism. The solution of the problem allows us to establish the relationships between the generalized coordinates and the coordinates of the output link in the base coordinate system.

The solution of the direct positional problem is described by means of a successive transition

$$P_1X_1Y_1Z_1 \to P_0X_0Y_0Z_0 \to P_2X_2Y_2Z_2.$$

In the matrix form this transition is represented as

$$T_{jrmm} = T_{rot}^{-1} \cdot T_s \tag{2.1}$$

where, is, T_s — the matrix of homogeneous transformations of the six-link mechanism;
T_{rot} — the matrix of homogeneous transformations of the rotary mechanism;
T_{jrmm} — the matrix describing the direct problem of the position of the entire joint relative manipulation mechanism.

3. Research methodology

Proceeding from the kinematics of the rotary mechanism, the matrix of homogeneous transformations of the rotary mechanism is equal to

$$T_{rot} = R_{X,Y} \cdot R_{OZ} = \begin{pmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & \theta_5 \\ \sin(\theta_6) & \cos(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & Z_{P_1} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.1}$$

where, $R_{X,Y} = \begin{pmatrix} 1 & 0 & 0 & \theta_5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & Z_{P_1} \\ 0 & 0 & 0 & 1 \end{pmatrix}$ — the transfer matrix on the X_0 axis taking into account the displacement Z_{P_1},

$R_{OZ} = \begin{pmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & 0 \\ \sin(\theta_6) & \cos(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ — the rotation matrix R_{OZ} around the axis Z_0.

Let us find the matrix of homogeneous transformations of the six-link mechanism T_s (Fig.2).

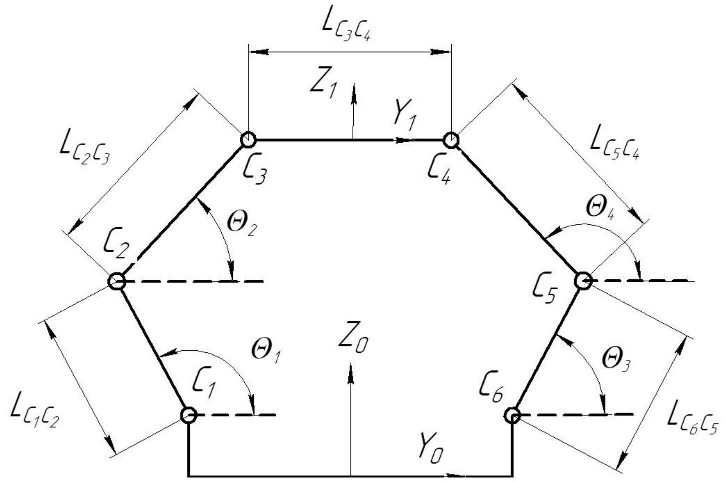


Fig.2. The six-point mechanism.

We use the polar coordinate system. Equate

$$\theta_1 = \theta_1 + 90,$$

$$\theta_2 = (\theta_1 + 90) + \theta_2,$$

$$\theta_3 = \theta_3 + 90,$$

$$\theta_4 = (\theta_3 + 90) + \theta_4.$$

To find the coordinates of points C_i we form a system of coupling equations

$$C_2 = L_{C_1C_2} \cdot e^{i\theta_1} + C_1,$$

$$C_3 = L_{C_2C_3} \cdot e^{i\theta_2} + C_2,$$

$$C_4 = L_{C_5C_4} \cdot e^{i\theta_4} + C_5,$$

$$C_5 = L_{C_6C_5} \cdot e^{i\theta_3} + C_6,$$

$$L_{C_3C_4} = |C_4 - C_3|$$

where
$C_1, C_2, C_3, C_4, C_5, C_6$ - points with coordinates $C_i(0, \mathrm{Re}(C_i), \mathrm{Im}(C_i))$, $L_{C_1C_2}, L_{C_2C_3}, L_{C_5C_4}, L_{C_6C_5}, L_{C_3C_4}$ - lengths of links of a flat mechanism, $\theta_1, \theta_2, \theta_3, \theta_4$ - generalized coordinates.

From the system of equations we express the coordinates of points C_3 and C_4 taking into account the fact that the coordinates of points C_1 and C_6 are known

$$C_3 = L_{C_2C_3} \cdot e^{i\theta_2} + L_{C_1C_2} \cdot e^{i\theta_1} + C_1,$$

$$C_4 = L_{C_5C_4} \cdot e^{i\theta_4} + L_{C_6C_5} \cdot e^{i\theta_3} + C_6.$$

Let us find the coordinates of the point P_2 (Fig.3).

$$Y_{P_2} = \mathrm{Re}\left(\frac{C_3 + C_4}{2}\right) = \mathrm{Re}\left(\frac{L_{C_2C_3} \cdot e^{i\theta_2} + L_{C_1C_2} \cdot e^{i\theta_1} + C_1 + L_{C_5C_4} \cdot e^{i\theta_4} + L_{C_6C_5} \cdot e^{i\theta_3} + C_6}{2}\right),$$

$$Z_{P_2} = \mathrm{Im}\left(\frac{C_3 + C_4}{2}\right) = \mathrm{Im}\left(\frac{L_{C_2C_3} \cdot e^{i\theta_2} + L_{C_1C_2} \cdot e^{i\theta_1} + C_1 + L_{C_5C_4} \cdot e^{i\theta_4} + L_{C_6C_5} \cdot e^{i\theta_3} + C_6}{2}\right),$$

$$\cos(\beta)=\frac{\mathrm{Re}\left(\left(L_{C_5C_4}\cdot e^{i\theta_4}+L_{C_6C_5}\cdot e^{i\theta_3}+C_6\right)-\left(L_{C_2C_3}\cdot e^{i\theta_2}+L_{C_1C_2}\cdot e^{i\theta_1}+C_1\right)\right)}{L_{C_3C_4}},$$

$$\sin(\beta)=\frac{\mathrm{Im}\left(\left(L_{C_5C_4}\cdot e^{i\theta_4}+L_{C_6C_5}\cdot e^{i\theta_3}+C_6\right)-\left(L_{C_2C_3}\cdot e^{i\theta_2}+L_{C_1C_2}\cdot e^{i\theta_1}+C_1\right)\right)}{L_{C_3C_4}}$$

where β is the angle of rotation of the upper platform relative to the working element (Fig.3).

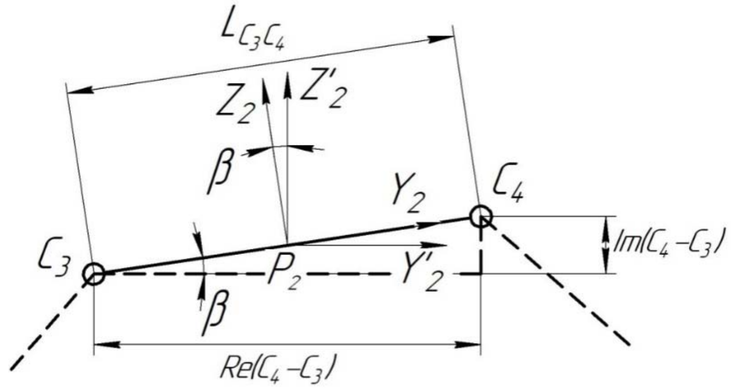


Fig.3. Output link of the six-link mechanism.

We form the transition matrix for the six-link mechanism

$$T_{плоск}=R_{Y,Z}\times R_{OX}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos(\beta) & -\sin(\beta) & Y_{P_2}\\ 0 & \sin(\beta) & \cos(\beta) & Z_{P_2}\\ 0 & 0 & 0 & 1\end{pmatrix}. \tag{3.2}$$

We substitute (3.1) and (3.2) in Eq.(2.1) and obtain a transition matrix that describes the direct problem of the position of the entire joint relative manipulation mechanism

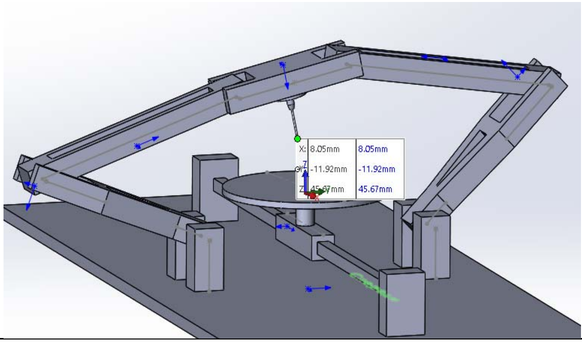
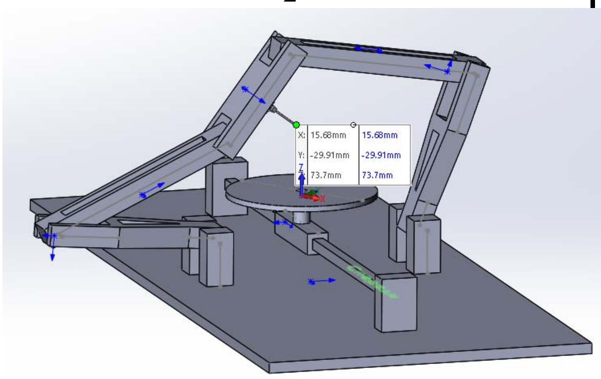
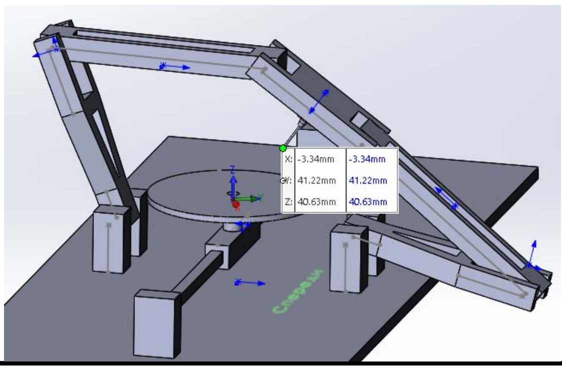
$$T_{rjmm}=\begin{bmatrix}t_{11} & t_{12} & t_{13} & t_{14}\\ t_{21} & t_{22} & t_{23} & t_{24}\\ 0 & t_{32} & t_{33} & t_{34}\\ 0 & 0 & 0 & 1\end{bmatrix}$$

where $t_{11}\,t_{12}\,t_{13}$, $t_{21}\,t_{22}\,t_{23}$, $t_{32}\,t_{33}\,t_{34}$ — expressions describing the orientation of the joint relative manipulation mechanism output link(the output link of a flat six-link mechanism) relative to the base coordinate system $P_1X_1Y_1Z_1$ of the whole mechanism. $t_{14}\ t_{24}t_{34}$ - expressions describing the position of the output link.

4. Computational experiment

A series of experiments was performed using a simplified 3D model of the mechanism to confirm the correctness of the solution obtained. Table 1 provides a comparative analysis of theoretical and experimental data.

Table 1. Analysis of theoretical and experimental data.

№	**Generalized coordinate**	**Initial data**	**Result of the decision of the CAP**		**Experiment result**
1 	θ_1	*69.57*	*X*	*8.0521*	*8.05*
	θ_2	*-140.92*			
	θ_3	*-44.96*	*Y*	*-11.9222*	*-11.92*
	θ_4	*126.99*			
	θ_5	*10*	*Z*	*45.6583*	*45.67*
	θ_6	*10*			
2 	θ_1	*85*	*X*	*15.6874*	*15.68*
	θ_2	*-150*			
	θ_3	*-20*	*Y*	*-29.8994*	*-29.91*
	θ_4	*100.94*			
	θ_5	*30*	*Z*	*73.7018*	*73.7*
	θ_6	*35*			
3 	θ_1	*21.66*	*X*	*-3.3748*	*-3.34*
	θ_2	*-112.71*			
	θ_3	*-103.63*	*Y*	*41.1419*	*41.14*
	θ_4	*154.39*			
	θ_5	*-20*	*Z*	*40.6149*	*40.62*
	θ_6	*24.29*			

The resulting error is not significant.

5. Conclusions

Thus, this paper proposed the solution of the direct problem of position for a variation of the joint relative manipulation mechanism with five degrees of freedom. To verify the correctness of the task, experiments were carried out. The small discrepancies between the calculated and experimental data confirmed the correctness of the proposed solution of the direct positional problem.

Acknowledgments

This work was supported by the Russian Science Foundation, the agreement number 16-19-00148.

References

[1] Glazunov V.A. and Chunikhin A.D. (2014): *Development of research on the mechanisms of parallel structure*. – Problems of Machine Building and Machine Reliability, No.3, pp.37-43.

[2] Glazunov V.A., Lastochkin A.B., Shalyukhin K.A. and Danilin P.O. (2009): *To the analysis and classification of devices of relative manipulation.* – Problems of Mechanical Engineering and Reliability of Machines, No.4. pp.81-85.

[3] Rybak L.A., Chichvarin A.V. and Erzhukov V.V. (2011): *Effective methods for solving the kinematics and dynamics problems of a parallel robot machine.* – M.: Fizmatlit. pp.148.

[4] Hailo S.V., Glazunov V.A., Shirinkin M.A. and Kalendarev A.V. (2013): *Possible applications of mechanisms of parallel structure*. – Problems of Machine Building and Machine Reliability, No.5. pp.19-24.

[5] Bushuev V.V. and Kholshev I.G. (2001): *Mechanisms of parallel structure in machine building*. – Machines and Tools, No.1. pp.3-8.

[6] Krainev A.F., Kovalev L.K., Vasetsky V.G. and Glazunov V.A. (1994): *Development of installations for laser cutting on the basis of mechanisms of a parallel structure*. – Problems of Mechanical Engineering and Reliability of Machines, No.6, pp.84-93.

[7] Rybak L.A., Gaponenko E.V. and Zhukov Y.M. (2012): *Investigation of the accuracy of machining on machines with parallel kinematics taking into account the displacements of the drive mechanisms and the cutting force*. – Technology of Machine Building, No.12. pp.25-28.

[8] Pashchenko V.N., Sharapov I.V., Rashoyan G.V. and Bykov A.I. (2017): *Construction of a working area for the manipulation mechanism of simultaneous relative manipulation*. – Journal of Machinery Manufacture and Reliability, vol.46, No.3, pp.225-231. © Allerton Press, Inc..

[9] Glazunov V.A., Lastochkin A.B., Terekhova A.N. and Wu Ngok Bik (2007): *On the peculiarities of relative manipulation devices*. – Problems of Machine Building and Machine Reliability, No.2. pp.77-85.

[10] Krainev A.F. and Glazunov V.A. (1994): *New mechanisms of relative manipulation*. – Problems of Mechanical Engineering and Reliability of Machines, No.5. pp.106-117.

[11] Lebedev P.A. (1987): *Kinematics of Spatial Mechanisms*. – M.: Mechanical Engineering, 280 p.

[12] Pashchenko V.N., Romanov A.V., Artemiev A.V., Men'shova E.V. and Loginov N.A. (2017): *The solution of the direct problem of the position of the six-stage manipulator of the parallel structure on the basis of the crank-and-rod mechanism*. – Electronic Information Systems, No.4 (15), pp.91-101.

[13] Angeles J. (2004): *The qualitative synthesis of parallel manipulators*. – Journal of Mechanical Design, vol.126, pp.617-624.

[14] Behi F. (1988): *Kinematic analysis for a six-degree-of-freedom 3-PRPS parallel mechanism*. – IEEE J. Robot, and Automat, N 4 / 5, pp.561-565.

[15] Zenkevich S.L. and Yushchenko A.S. (2004): *Fundamentals of manipulation robots*. – Moscow: MSTU Them, N.E. Bauman, 576 p.

[16] Bykov R.E., Glazunov V.A., Glazunova O.V. and Chan Dyk Hai (2005): *Simulation of the working space of the parallel structure mechanism with four kinematic chains*. – Problems of Machine-Building and Machine Reliability, No.5, pp.10-15.

[17] Pashchenko V.N., Glazunov V.A. and Ulyanov D.O. (2017): *Solution of the problem of the velocities of the spatial mechanism of a parallel structure*. – Engineering Journal. Directory, pp.23-30.

[18] Rybak L.A., Erzhukov V.V., Pochekaev S.G. and Chichvarin A.V. (2010): *Construction of neural network algorithms for solving a direct kinematics problem for a parallel robot machine*. – Journal Neurocomputers, No.5, pp.53-60.

[19] Evtushenko Y.G., Posypkin M.A., Rybak L.A. and Turkin A.V. (2018): *Approximating a solution set of nonlinear inequalities*. – Journal of Global Optimization, vol.71, No.1, pp.129-145.

[20] Evtushenko Y.G., Posypkin M.A., Rybak L.A. and Turkin A.V. (2017): *Finding sets of solutions to systems of nonlinear inequalities.* – Computational Mathematics and Mathematical Physics, vol.57, No.8, pp.1248-1254.

Received: July 31, 2018

Revised: August 13, 2018