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POSITIONING ACCURACY ANALYSIS OF THE PARALLEL MECHANISM NEAR SINGULAR POSITIONS

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This paper presents a method of numerical modelling of parallel mechanisms with clearances in their kinematic pairs taken into account. The pairs with clearances are modelled as shape connections based on constraints in the form of contact interactions. Using the created models simulations were run to determine the positioning errors of the links in a parallel mechanism with three degrees of freedom (MR2120). In particular, the accuracy of positioning the links close to the mechanism singular configurations was studied.

Key words: clearances in the joint, parallel mechanism, singular positions.

1. Introduction

Parallel mechanisms are characterized by the multibranch connection of the driven link to the base or to the driving links and the base, which means that they are closed kinematic chain systems. A peculiar variety of parallel mechanisms are systems with three degrees of freedom. They are used mainly as assembling or packing manipulators (e.g., ABB IRB 340, Fanuc M-1iA, M-3iA, KOCH KRH-D), translational positioners or experimental testing systems (e.g., WUT MR2120 (Fig.1) developed at the Wrocław University of Technology) (Merlet, 2000; Tsai, 1999; Bałchanowski and Gronowicz, 2001; Bałchanowski, 2008; 2014b).

Fig.1. Parallel mechanism WUT MR2120 developed and built at the Wrocław University of Technology.

One of the advantages of parallel mechanisms over series mechanisms is the high precision with which the driven member can be positioned. In the real mechanism its positioning accuracy may deteriorate due to different causes, such as the deformation of its members under external load, the poor workmanship of the members, and clearances in its kinematic pairs. The effect of some of the causes leading to positioning inaccuracies can be eliminated in a simple way, e.g,. by increasing the stiffness of the links, decreasing the tolerances and lowering the accuracy grade of the components or by calibrating the finished mechanism. Clearances in the kinematic pairs (unavoidable in real mechanisms) still remain a source of significant positioning and driven link motion repeatability errors. As a result of the wear of the interacting parts the clearances can significantly increase, even in well fitted systems. Therefore it is necessary to develop a method of predicting the accuracy of positioning the links in a mechanism, which will take the actual clearances in its kinematic pairs into account (Bałchanowski, 2014a). It is particularly essential to describe the accuracy with which the links assume their position close to singular configurations.

The aim of this research was to develop a method of numerical modelling of parallel mechanisms, which would take into account the presence of clearances in their kinematic pairs. Such a method has been developed and is presented below. The method can be used to analyze and predict errors in the positioning of the mechanism links, caused by clearances in order to determine the allowable clearances ensuring a desired positioning accuracy, especially in the neighbourhood of singular configurations. The method was used to carry out exemplary numerical analyses of the influence of clearances on the operation of parallel mechanisms. Simulations of the accuracy of positioning the driven member during the performance of typical working motions were run for the WUT MR2120 mechanism (Fig.1). The simulation studies were particularly focused on the influence of clearances on the position of the driven member close to singular configurations.

2. Structure of parallel mechanism WUT MR2120

The subject of analysis in this paper is the WUT MR2120 mechanism (Fig.1), the structure and geometry of which were defined in the course of basic research on the topology of parallel mechanisms (Bałchanowski, 2000; 2008; 2014b; 2014c; Bałchanowski and Gronowicz, 2001) (Fig.2).

In the mechanism its driven member 7 (a platform) is connected to the base by means of three identical branches. Since the mechanism has three degrees of freedom it is necessary to use three kinematic excitations q_1, q_2 and q_3 describing the angular displacements in rotation couples A, B and C (Fig.2).

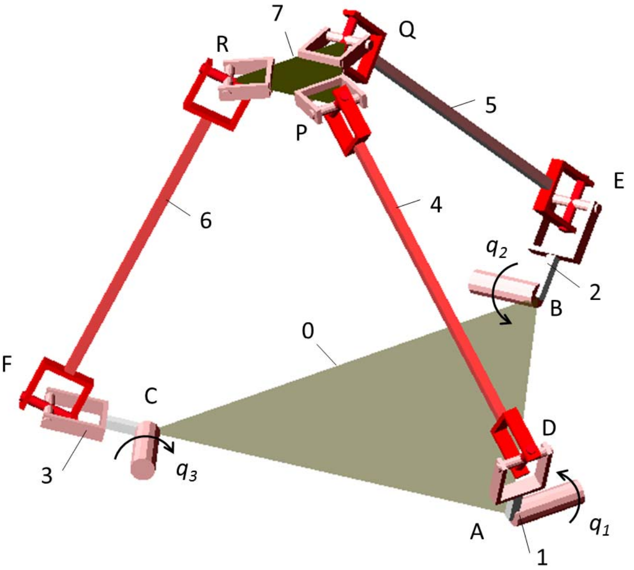

Fig.2. Kinematic scheme of parallel manipulator WUT MR2120.

WUT MR2120 is a translational parallel mechanism, i.e., a system in which the driven mechanism (the platform) can perform only three translational motions relative to the base while maintaining a constant orientation. The conditions for translational motion performance are satisfied only when a particular geometry of the system members and a defined mutual position of the axes relative to which the relative motions are executed in the kinematic pairs are adopted (Bałchanowski, 2000; 2014; Bałchanowski and Gronowicz, 2001; Tsai, 1999) (Fig.2).

One of the disadvantages of the mechanism is the occurrence of singular positions in its work zone. The singular configurations in this mechanism are known (Bałchanowski, 2008; 2014b). They have been analytically determined by, among others, the author through an analysis of the system speed equations (Bałchanowski, 2014b; 2014c). Figure 3 shows the mechanism in its singular positions defined for the forward and inverse kinematics problems (Bałchanowski, 2014b).

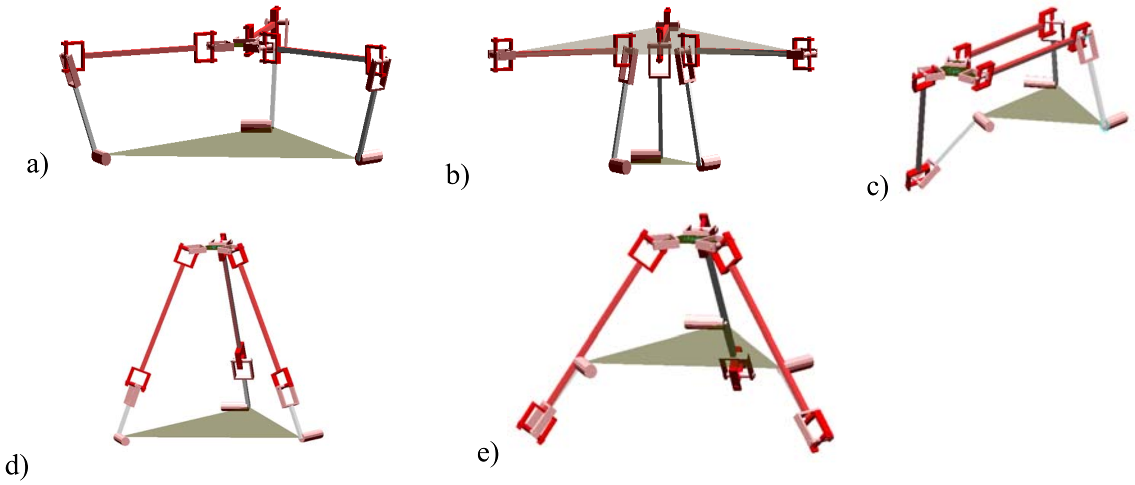

Fig.3. Singular configurations of parallel mechanism WUT MR2120 for the direct kinematics problem (a, b, c) and inverse kinematics problem (c, d, e) (Bałchanowski, 2014b).

The design of such a mechanism should prevent it from entering a singular position. In some cases, the entering of singular positions by the mechanism links may result in system failure or damage. Due to clearances in its kinematic pairs the mechanism being close to a singular position may enter it despite the fact that theoretically it is at a safe distance from the singularity (Bałchanowski 2014b; 2014c).

Clearances in the kinematic pairs are difficult to eliminate in real mechanisms. Therefore there is a need to describe and examine the effect of clearances on the operation of kinematic systems. The problem is complex and difficult to describe explicitly. It has been approached from different angles in many papers. Ting (Ting *et al*., 2000) modelled the clearance in kinematic pairs by introducing additional small members, which he called clearance members. Each such member introduces one additional degree of freedom, increasing thereby the total number of the mechanism's degrees of freedom. This method offers a geometrical model for determining the influence of clearances on the orientation and positions of the links in planar linkages and manipulators.

Kosuge (Kosuge *et al*., 1991) undertook an interesting attempt to analytically describe the radial clearances in the couples of rotation in planar parallel micromechanisms. He introduced the so-called clearance fields in the particular pairs and examined their mutual influences on the positioning of the driven link. Also, stochastic methods are used to describe the clearance phenomenon in bar linkages (Malik and Dhande, 1987) and in manipulators (Zhu and Ting, 2000) in order to predict link positioning errors and determine the clearances permissible with regard to the accuracy of positioning.

Chebi *et al.* (2009) presented an interesting analytical-numerical description of the influence of clearances on the positioning errors of the driven link for a translational parallel manipulator with three degrees of freedom, and maps of the distribution of errors in the system work zone with singular positions taken into account.

In the case of the existing methods of describing clearances one cannot simply write clearance models into the dynamics equations and in this way determine the actual positions of the links during their motion along work trajectories. In the present paper the thus created a numerical method of building models of kinematic pairs with clearances is proposed. The models can be implemented in standard multibody system dynamic analysis programs (e.g., LMS DADS, MD Adams) in order to construct computing models of complete mechanisms and run simulations aimed at predicting the effect of clearances on the motion of the mechanism.

3. Structure of computing model of WUT MR2120 mechanism with clearances in kinematic pairs

In the WUT MR2120 mechanism the links of the system branches are connected with driven link 7 by means of universal (Cardan) joints. There are pairs: D, E, F in the branches and P, Q, R at the platform. With base 0 the branches form rotational pairs A, B and C (Fig.2). Rotational pairs A, B and C on base 0 and pairs P, Q, R on platform 7 are located in the vertexes of equilateral triangles with sides AB=BC=AC= *0.74 m* and PQ=QR=RP= *0.15 m* (Fig.2). The dimensions of the mechanism branches are: AD=BE=CF=*0.3 m* and DP=EQ=FR=*0.68 m*.

In the real mechanism (Fig.1) clearances can occur in all the kinematic pairs. Figures 4 and 5 show the cross sections of the system rotational pairs and universal joints. Rotational pair A (Fig.4) is formed by branch arm 1 screwed to the exit of the strain wave gear (HFUC 25-100) which is connected via a rigid coupling with the servo drive (MKD-025B). The gear case is fixed to mechanism base 0. Because of the character of the meshing and the way of bearing the shafts in the harmonic drive this joint is assumed to be without clearances.

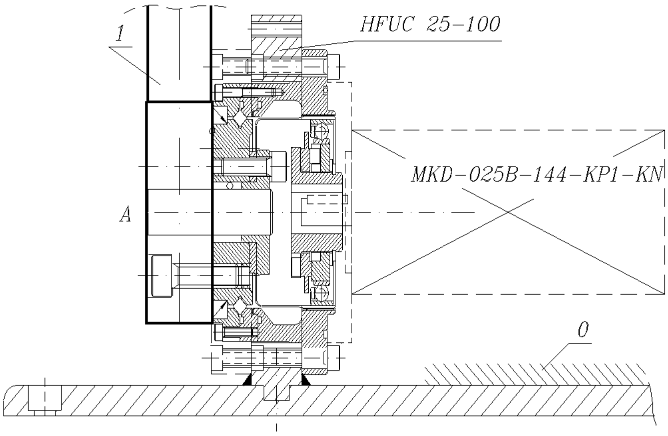

Fig.4. Cross section through rotational pair A.

The universal joint structure includes four rotational pairs and an intermediate member (a cross) (Fig.5). In this design solution, roller bearings are used instead of the rotational pairs (Bałchanowski, 2014a). Details of the bearing arrangement are shown in Fig.5. Because of its design, radial clearances h_{LR} and axial clearances h_{LW} occur in the bearing between the cross elements and the fork end of the branch. The size of clearances h_{LR} and h_{LW} depends on the adopted tolerances, the workmanship of the components and the degree of wear of the interacting parts. In the universal joint similar clearances occur in the three other bearings. Even a slight play in all the six Cardan joints can affect the accuracy of positioning and the repeatability of the motion of the mechanism driven link. Therefore in order to estimate the effect of clearance on the system motion one should build a computing model of the mechanism, which takes into account the radial and axial clearances in its kinematic pairs.

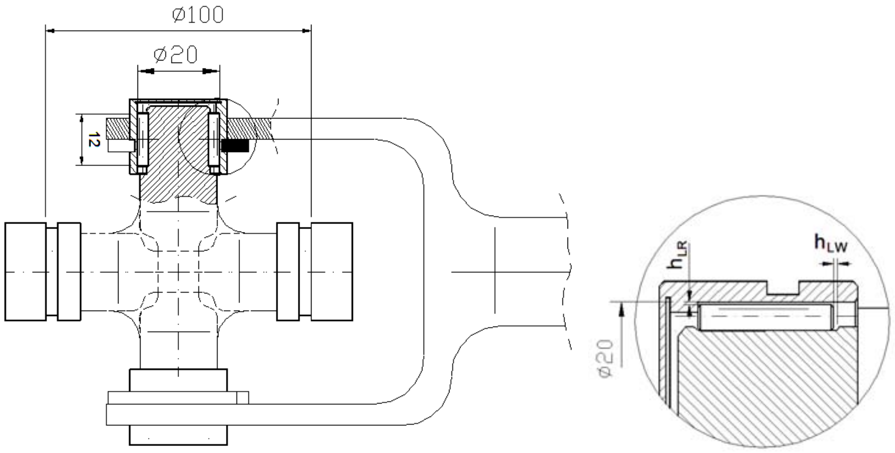

Fig.5. Universal joint (Bałchanowski, 2014a): a) general view, b) cross section through cross bearing, view of radial clearance h_{LR} and axial clearance h_{LW}.

A kinematic pair ensures the relative motion of its two constitutive links in accordance with the defined degree of freedom. For example, a rotational pair has one degree of freedom and ensures rotational motion relative to one axis, a spherical pair has three degrees of freedom and enables rotations along three axes, etc. When clearances appear, a kinematic pair receives additional degrees of freedom, which are dynamically executed.

In the case of a pair with clearances, the relative motion of its two constitutive links is not defined by exact kinematic constraints, but depends on the shape of their contact surface. The mutual definition of the positions of the members in the mechanism is not kinematically determined in this case, but depends on the equilibrium of forces and torques, which means that this problem belongs to dynamic analysis (Bałchanowski, 2014a).

In the present paper the method of building models of kinematic pairs with clearances as shape connections and constraints in the form of interactions between two surfaces is applied (Bałchanowski, 2014a). Using as an example the universal joint one can trace the way in which clearances are modelled in such pairs. Figure 6 shows the created model of the universal joint with clearances. The four rotational bearings connecting the intermediate member with the members forming the joint were modelled by ball ends set in cylindrical box sockets. The difference between the inside radius of the cylinder and that of the ball defines the radial clearance h_{LR} while the difference between the spacing of the sockets and that of the balls defines the axial clearance h_{LW} (Fig.6).

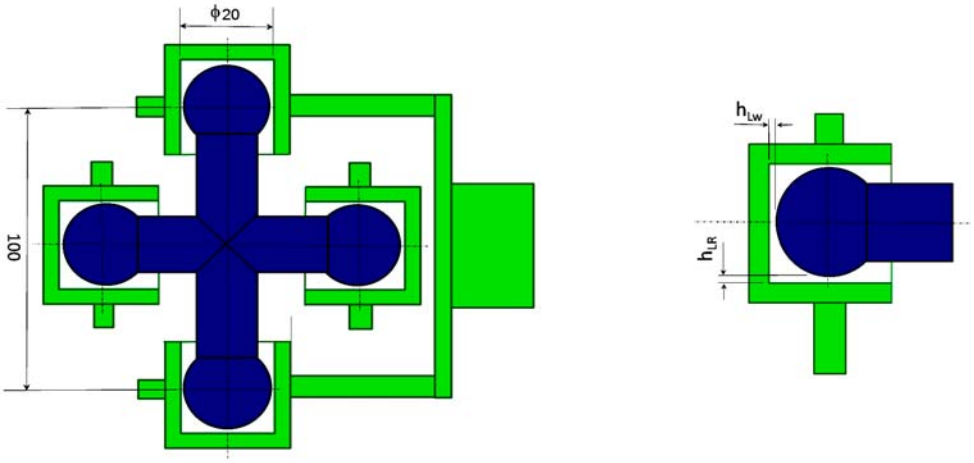

Fig.6. Universal joint model (Bałchanowski, 2014a): a) general view, b) longitudinal section and cross section through bearings, view of the radial clearance h_{LR} and axial clearance h_{LW}.

The contact forces between the inner surfaces of the box sockets and the outer surfaces of the ball ends of the joints form dynamically executed constraints between the links. If as a result of a displacement of the links the latter come in contact, a contact force will arise in the point of contact. The interaction force values resulting from the surface pressures are calculated on the basis of the displacements and deformations of the interacting links. The linkage built in this way ensures the transfer of the required rotations and the motions resulting from the clearances.

Two computing solid body models of the WUT MR2120 mechanism were built in the LMS DADS computer environment for the dynamic analysis of multibody systems (Haug, 1989). One model was built from links connected by ideal theoretical kinematic pairs without clearances. Ready models of pairs, of the "universal joint" and "revolute joint" type, from the system constraints library were used for the connections. This model of the mechanism has three degrees of freedom determined by three kinematic excitations q_1, q_2 and q_3.

The other model of the mechanism was built from links connected by kinematic pairs with clearances (modelled as shape connections) (Fig.6). Three rotational pairs A, B, C (of the revolute type) were left in the system. The other pairs: universal joints D, E, F, P, Q and R were defined as shape connections with clearances (Fig.6). The Force-Contact-SegSeg model (from the LMS DADS library of system loads) (Haug, 1989) was used to define the interaction between the links. It was assumed that interactions between the links are ideally elastic and the joints are made of steel (E=2.1e5 MPa). This model has 63 degrees of freedom and three excitations: drives q_1, q_2 and q_3.

4. Simulation studies of the MR2120 translational parallel mechanism

In the WUT MR2120 mechanism the driven link (platform 7) moves translationally maintaining a constant orientation relative to the base. The occurrence of clearances in the pairs disturbs the character of this motion. The aim of the simulation studies was to predict the influence of the size of the clearances on the accuracy of the positioning of the driven member in different points of the work zone.

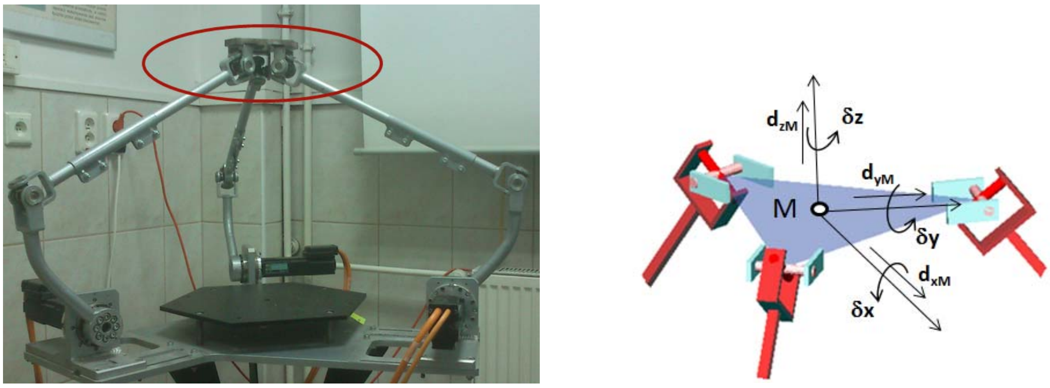

Fig.7. Schematic of possible platform positioning errors: linear errors d_{xM}, d_{yM}, d_{zM} and angular errors δx, δy, δz.

Displacements d_{xM}, d_{yM}, d_{zM} of centre M of driven member 7 relative to the same point for the system without clearances, and changes in the angles of platform orientation δx, δy, δz relative to the orientation of the platform of the system without clearances (Fig.7), can be presented in the form of relations

$$
\begin{aligned}
d_{xM} &= x_M{}^L - x_M, \\
d_{yM} &= y_M{}^L - y_M, \\
d_{zM} &= z_M{}^L - z_M, \\
\delta x &= \alpha_x{}^L - \alpha_x, \\
\delta y &= \alpha_y{}^L - \alpha_y, \\
\delta z &= \alpha_x{}^L - \alpha_z,
\end{aligned}
\tag{4.1}
$$

where

x_M, y_M, z_M – the coordinates of point M for the system without clearances,
$x_M{}^L$, $y_M{}^L$, $z_M{}^L$ – the coordinates of point M for the system with clearances,
α_x, α_y, α_z – the platform orientation angles for the system without clearances,
$\alpha_x{}^L$, $\alpha_y{}^L$, $\alpha_z{}^L$ – the platform orientation angles for the system without clearances,

were adopted as the measure of positioning accuracy.

The simulation results for the motion of the mechanism along one selected trajectory are presented below. They show the positioning accuracy in selected points of the work zone for the different three combinations of the radial clearance h_{LR} and axial clearance h_{LW} given in Tab.1. Set L_1 describes the values of the axial clearance h_{LW} and radial clearance h_{LR} in the adopted universal joints (Fig.5), set L_2 models the joint with radial clearances and practically without axial clearances ($h_{LW} = 0.001\ mm$) and set L_3 describes the joint with axial clearances and without radial clearances ($h_{LR} = 0.001\ mm$). All the kinematic pairs and universal joints in the mechanism were assumed to be identical.

Tab.1. Clearance values used in computations.

Name of the set of clearances	L_1	L_2	L_3
h_{LR} [mm]	*0.020*	*0.020*	*0.001*
h_{LW} [mm]	*0.020*	*0.001*	*0.020*

4.1. Simulation studies of the WUT MR2120 mechanism along assigned trajectory

Simulation studies consisted in moving the driven member (platform 7) from position M along trajectory MM'M'' to position M''. The trajectory was so planned that the mechanism assumed two singular configurations during its motion. Platform position M' is a singular position for the inverse kinematics problem and lies on the work zone boundary (Fig.3d) while end configuration M'' is a singular position for the direct kinematics problem (Fig.3a). During its motion the mechanism is loaded with mass forces and the motion is forced by drives q_1, q_2, q_3. A schematic of the simulation and the graphs of kinematic excitations q_1, q_2, q_3 of the drives which make the execution of the assigned trajectory possible are shown in Fig.8 and Fig.9, respectively.

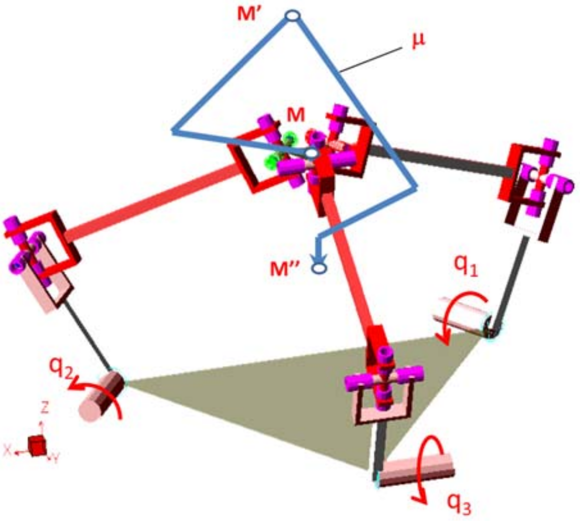

Fig.8. Schematic of simulation of motion along trajectory *MM'M''*.

Four simulations: one for the model of the mechanism without clearances and three for the model with the three combinations of clearance values shown in Tab.1 were carried out. The results of the simulations are presented in successive figures below. Figure 10 shows the graphs of the coordinates of point M in the course of its motion along the assigned trajectory for the system without clearances. Figure 11 shows successively positioning errors d_{xM}, d_{yM}, d_{zM} of point *M* on the driven link while Fig.12 presents the graphs of the changes in platform orientation angles δx, δy, δz for the three sets of clearances in the course of motion along the assigned trajectory. Values of errors were calculated using formula (4.1).

The positioning error value depends on the position of the link in the work zone. As point M on the platform approaches the singular positions ($t = 2.0$ s and $t = 4.0$ s – Figs 11-12) positioning errors sharply increase. As in time $t=2.0$ s the mechanism passes through the singular position of the inverse kinematics problem (point *M'*), errors d_{xM} and d_{yM} in graphs 11a and 11b appreciably increase to *0.5 mm*. Positioning and orientation errors sharply increase as the platform approaches point *M''*, in which the mechanism is in the singular configuration for the direct kinematics problem.

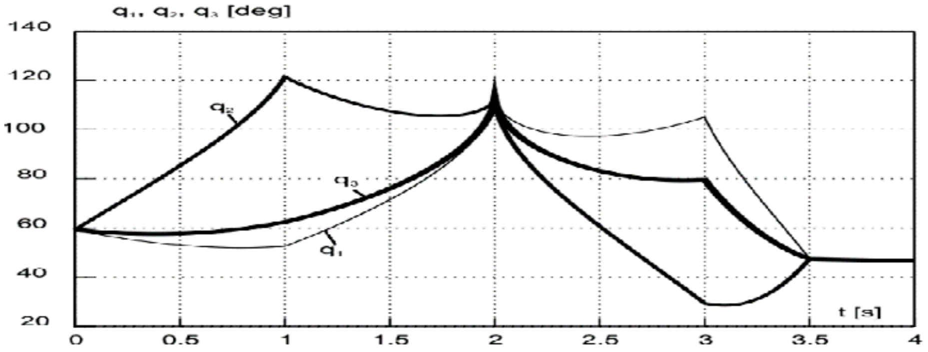

Fig.9. Graphs of kinematic excitations q_1, q_2, q_3 of drives, describing system motion along path *MM'M''*.

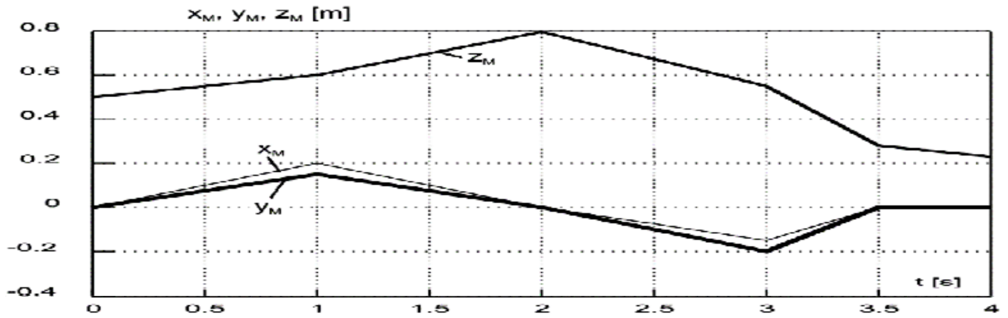

Fig.10. Graphs of coordinates x_M, y_M, z_M of point *M*, describing system motion along path *MM'M''* without clearances.

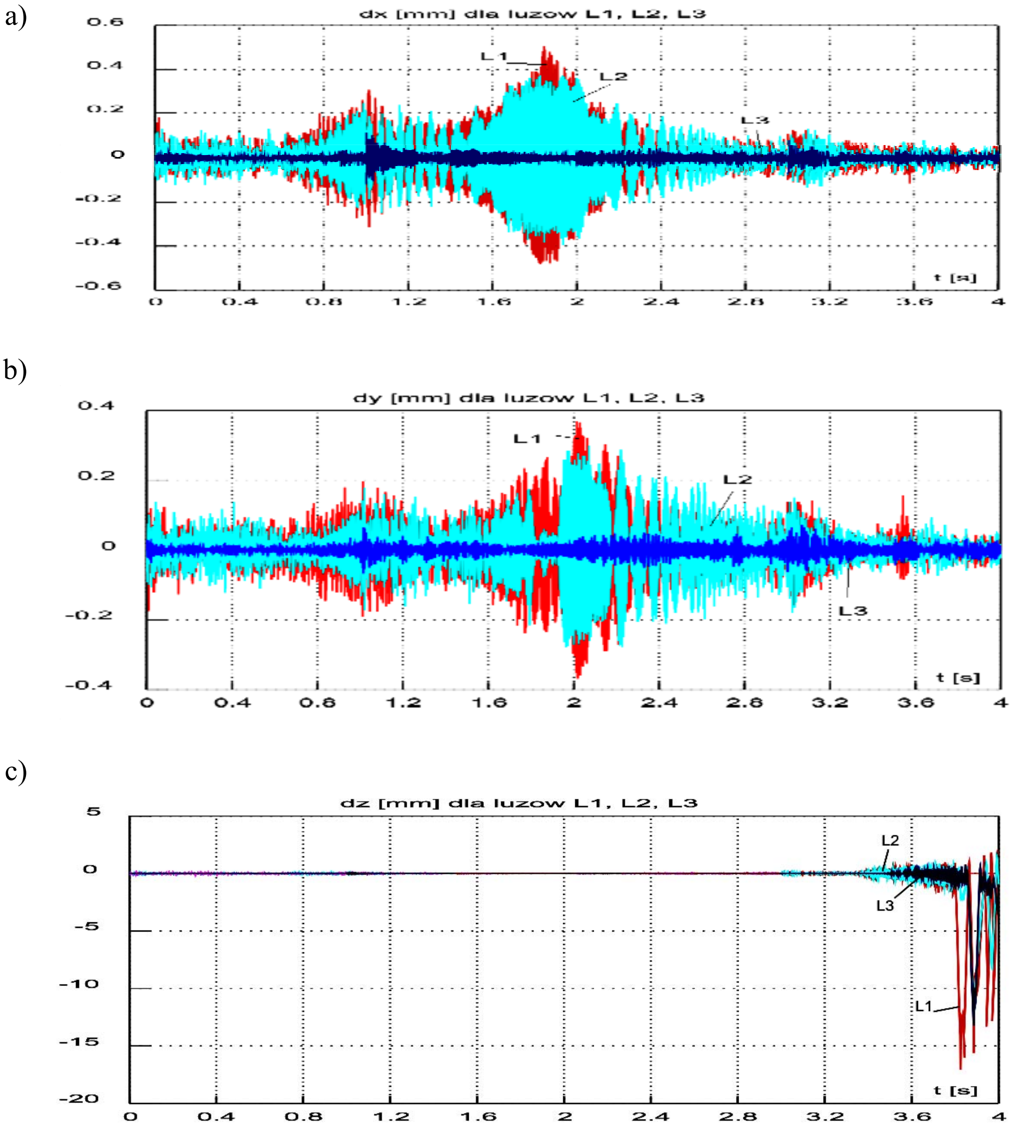

Fig.11. Graphs of a) d_{xM}, a) d_{yM} and c) d_{zM} positioning errors along axes *x*, *y* and *z*, describing system motion along path *MM'M''* with L_1, L_2 and L_3 sets of clearances.

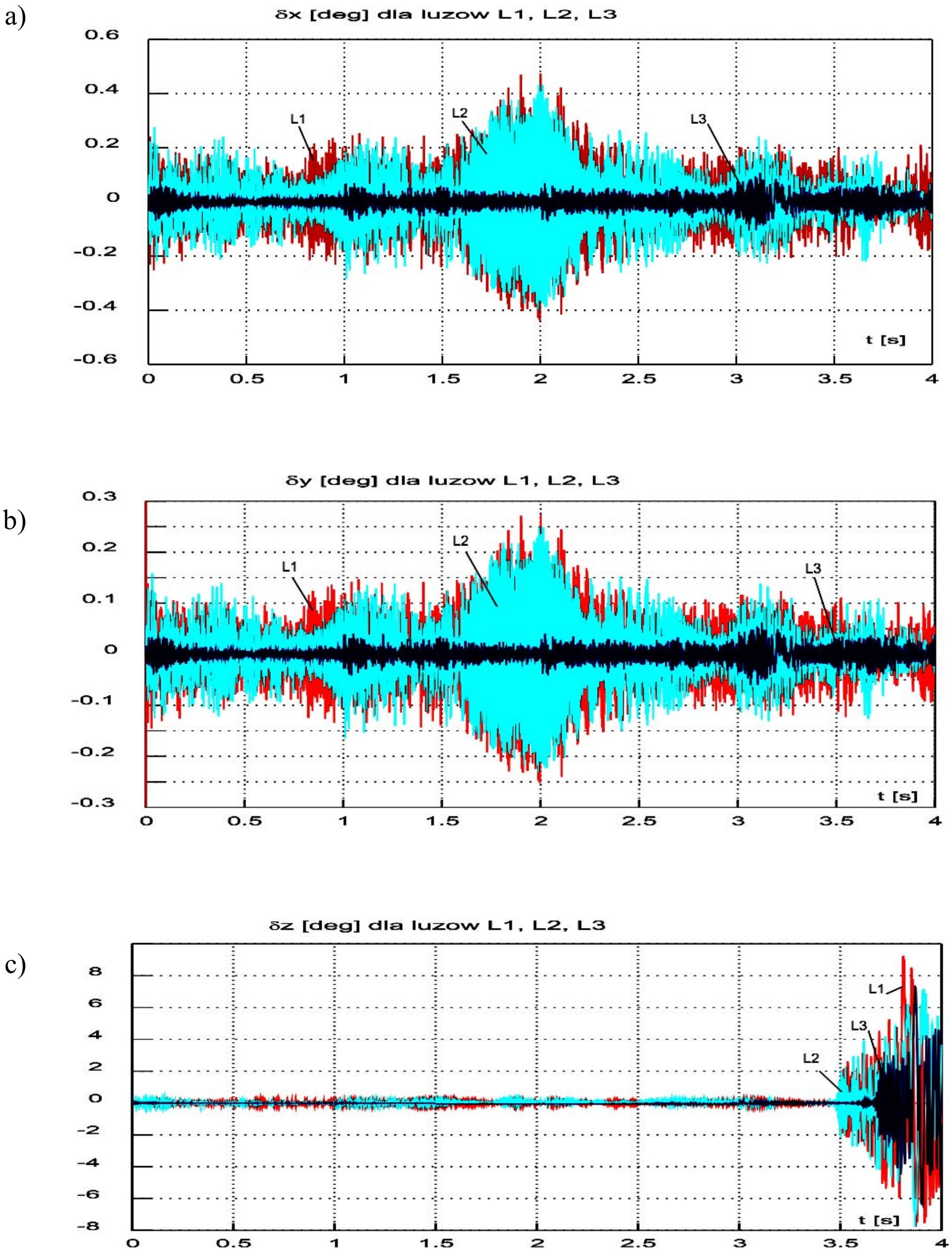

Fig.12. Graphs of a) δ*x*, b) δ*y* and c) δ*z* changes in the angle of orientation, describing system motion along path *MM'M''* with L_1, L_2 and L_3 sets of clearances.

The mechanism without clearances reaches position M'' in time $t = 4s$. Because of the clearances in the mechanism pairs a decrease in the accuracy of the positioning of the links, reflected in an increase in error d_{zM}, is observed already at $t = 3.5\ s$ (Fig.11c), which is due to the fact that the system approaches singular configuration *M'*'. Positioning errors d_{zM}, reach values as high as *20 mm.*

The graphs clearly show that clearances significantly affect the accuracy of motion of the driven member. The clearance values amounting to few hundredths of millimetres in extreme cases result in positioning errors in the order of centimetres.

An analysis of the graphs shows that in all the cases the errors for set L_2 of clearances are greater than the ones for clearances L_3. This means that the system is very sensitive to the presence of radial clearances h_{LR} in the bearings of the universal joints. In order to achieve good positioning accuracy one should minimize this clearance. Axial clearances h_{LW} in the pairs have a much smaller effect on the positioning accuracy of the driven link.

When analyzing the graphs one should bear in mind that the position of the mechanism members during motion along the particular trajectory is dynamically defined under the influence of the external load and the body forces. When the motion conditions are changed, e.g., by arranging the mechanism with its base up, the simulation results assume different values.

4.2. Studies of positioning the driven member of the WUT MR2120 mechanism

The links in a mechanism without clearances assume an exactly defined position for given kinematic excitations (drives) q_1, q_2, q_3. In the case of the system with clearances in its kinematic pairs, under defined kinematic excitations the links can displace around the theoretical position, changing their position and orientation depending on the clearance values and the dynamic equilibrium.

Simulation studies relating to the prediction of driven link positioning accuracy in selected points of the work zone were carried out. Figure 13 shows a schematic of the simulation with the 3 points selected for the analysis of the driven link in the work zone, defined by the successive positions of point M, i.e,. M^1, M^2 and M^3. Positions M^2 and M^3 describe the singular configurations of the analyzed mechanism. In position M^2 the system is in the singular position for the inverse kinematics problem (Figs 3a and d). Table 2 contains the coordinates of the selected points: M^1, M^2 and M^3 and the corresponding kinematic excitations q_1, q_2 and q_3 of the drives.

Table 2. Coordinates of platform positions and corresponding excitations.

Point	M^1	M^2	M^3
x_M [m]	*0.0*	*0.0*	*0.0*
y_M [m]	*0.0*	*0.0*	*0.0*
z_M [m]	*0.5*	*0.7962*	*0.2509*
q_1 [deg]	*59.33*	*112.37*	*46.70*
q_2 [deg]	*59.33*	*112.37*	*46.70*
q_3 [deg]	*59.33*	*112.37*	*46.70*

The simulations consisted in forcing the dynamic motion of the driven link (the platform) around the position of point M determined by assigned quantities q_1, q_2, q_3. At constant excitations the motion of the mechanism links results solely from the clearances in the pairs. The aim of the simulations was to determine the range of possible changes in linear position d_{xM}, d_{yM}, d_{zM} and angular position δx, δy, δz around the position determined for the system without clearances. Table 3 shows the results of computations and simulations for the selected platform positions: M^1, M^2 and M^3 for the successive sets: L_1, L_2 and L_3 of clearances.

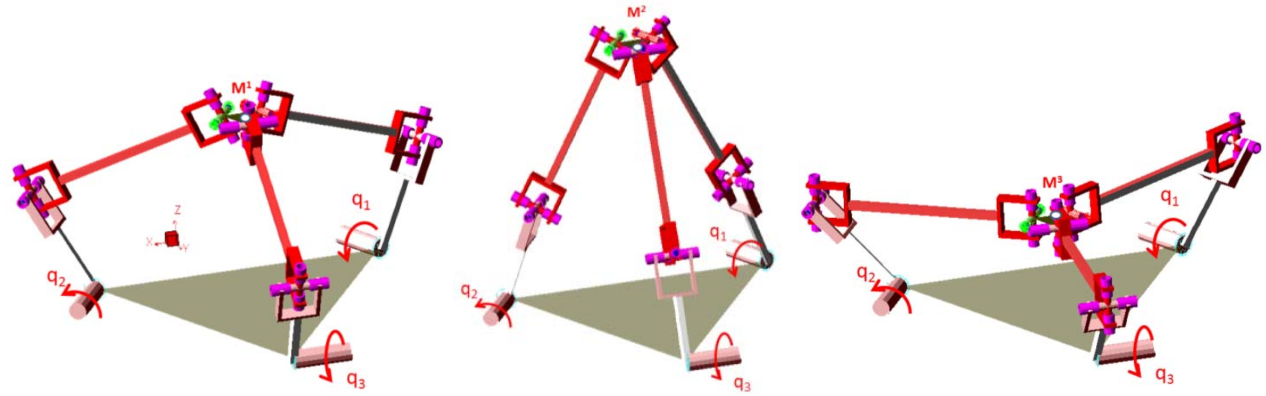

Fig.13. View of the mechanism with platform positions M^1, M^2 and M^3.

Studies of platform positioning in the selected positions confirmed the findings from the simulations of the system motion along trajectory $MM'M''$ (sect. 4.1). The sensitivity of the system to clearances significantly increases as the mechanism members approach the singular positions (points M^2 and M^3). In point M^2 lying on the work zone boundary, which is the singularity of the inverse kinematics problem, positioning errors d_{xM} and d_{yM} fluctuate around ± *1.02 mm* at relatively small error d_{zM} ($|d_{zM}| < 0.22$ *mm*). Position M^3, being the configuration of the forward kinematics problem, is particularly sensitive to positioning errors along axis *Z*. In this position errors d_{zM} relative to axis *Z* reach ± *16.5 mm* (Tab.3, row M^3). In the other positions: M^1, M^2 (Tab.3) errors d_{zM} are significantly smaller.

The positioning studies also indicated a greater sensitivity of the mechanism under analysis to radial clearances h_{LR} than to axial clearances h_{LW}. In each case, the positioning and orientation errors are smaller for set L_3 than for set L_2 (Tab.3). When designing or selecting kinematic pairs for the system one should choose solutions with possibly the lowest radial clearances.

In order to determine system positioning errors close to the singular positions more precisely one should repeat the analyses for a wider range of link positions and clearance sets. Nevertheless, one can formulate a general conclusion that the smaller the clearances in the pairs, the closer can the mechanism approach a singular configuration.

Table 3. Results of computations and simulations of the system for selected platform position M^1, M^2 and M^3 for clearances sets L_1, L_2 and L_3.

Point	**Set of clearance**	d_{xM} **[*mm*]**	d_{yM} **[*mm*]**	d_{zM} **[*mm*]**	**δ*x* [deg]**	**δ*y* [deg]**	**δ*z* [deg]**
M^1	L_1	*-0.12 ÷ 0.11*	*-0.12 ÷ 0.13*	*-0.15÷ 0.16*	*-0.36 ÷ 0.36*	*-0.08 ÷ 0.08*	*-0.12 ÷ 0.12*
	L_2	*-0.08 ÷ 0.09*	*-0.08÷ 0.08*	*-0.11 ÷ 0.10*	*-0.18 ÷ 0.18*	*-0.03 ÷ 0.03*	*-0.06 ÷0.06*
	L_3	*-0.02 ÷ 0.02*	*-0.02 ÷ 0.02*	*-0.02 ÷ 0.02*	*-0.25 ÷ 0.25*	*-0.01 ÷ 0.01*	*-0.08 ÷ 0.08*
M^2	L_1	*-1.01 ÷ 1.0*	*-1.0 ÷ 1.02*	*-0.22 ÷ 0.21*	*-0.86 ÷ 0.84*	*-0.7 ÷ 0.176*	*-0.4 ÷ 0.3*
	L_2	*-0.7 ÷ 0.7*	*-0.70 ÷ 0.72*	*-0.10 ÷ 0.11*	*-0.71 ÷ 0.74*	*-0.57 ÷ 0.6*	*-0.34 ÷ 0.4*
	L_3	*-0.19 ÷ 0.2*	*-0.18 ÷ 0.19*	*-0.01 ÷ 0.01*	*-0.2 ÷ 0.3*	*-0.19 ÷ 0.2*	*-0.16 ÷ 0.24*
M^3	L_1	*-0.15 ÷ 0.16*	*-0.18 ÷ 0.17*	*-16.5 ÷ 16.4*	*-0.7 ÷ 0.65*	*-0.7 ÷ 0.6*	*-8.1 ÷ 8.0*
	L_2	*-0.1 ÷ 0.11*	*-0.10÷ 0.10*	*-13.2 ÷ 13.4*	*-0.5 ÷ 0.45*	*-0.5 ÷ 0.45*	*-7.2 ÷ 7.2*
	L_3	*-0.06 ÷ 0.06*	*-0.04 ÷ 0.05*	*-1.2 ÷ 1.3*	*-0.05 ÷ 0.03*	*-0.06 ÷ 0.04*	*-5.7 ÷ 5.3*

5. Conclusion

The effect of clearances which occur in the kinematic pairs of the **WUT MR2120** parallel mechanism on its motion was examined in order to develop a method which would help to predict errors in the positioning of the system links in order to determine the allowable clearances ensuring a desired positioning accuracy. For this purpose it was necessary to develop a method of modelling the kinematic pairs with clearances taken into account. A numerical method of creating models of pairs with clearances, as shape connections with constraints in the form of contact interactions, was proposed. Using the universal joint as an example it was shown how a model of a pair with radial and axial clearances taken into account is constructed. The presented way of modelling the universal joint with clearances can be easily adapted to build models of other kinematic pairs (rotational, cylindrical, spherical, etc.).

Clearances in the pairs significantly affect the mechanism motion characteristics, causing changes in the positioning and orientation of the driven link. The simulation studies of the mechanism motion along the selected trajectory, and driven link positioning in the selected work zone places showed quantitative and qualitative effects of the clearances, contributing to the deterioration in the positioning of the mechanism links. It was observed that near a singular position the sensitivity of the mechanism to positioning errors increased significantly. The investigated parallel mechanism was found to show an increased sensitivity to positioning errors resulting from radial clearances in the pairs in comparison with errors caused by axial clearances.

The occurrence of singular positions in the work zones of parallel mechanisms is one of the drawbacks of such systems. Therefore analyses of the behaviour of a parallel system with clearances, close to singular positions should not be neglected in the design process.

The results and the methods of modelling presented in this paper can be useful in further research on translational parallel mechanisms aimed at improving their utilitarian properties and extending the range of their applications. The proposed method of modelling parallel mechanisms with clearances in the pairs taken into account can be successfully used to investigate other kinds of mechanisms (lever, cam, etc.).

Nomenclature

d_{xM}, d_{yM}, d_{zM} – displacements of centre M of platform 7 relative to the same point for the system without clearances
h_{LR} - radial clearance in the universal joint
h_{LW} - axial clearance in the universal joint
L_i – the name of set of clearances
M^i – selected positions of point M
q_i – kinematic excitations q_1, q_2, q_3 of drives
x_M, y_M, z_M – the coordinates of point M for the system without clearances
x_M^L, y_M^L, z_M – the coordinates of point M for the system with clearances
$\alpha_x, \alpha_y, \alpha_z$ – the platform orientation angles for the system without clearances
$\alpha_x^L, \alpha_y^L, \alpha_z^L$ – the platform orientation angles for the system with clearances
$\delta x, \delta y, \delta z$ - changes in the angles of platform 7 orientation

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