MATHEMATICAL MODELING OF MAGNETO PULSATILE BLOOD FLOW THROUGH A POROUS MEDIUM WITH A HEAT SOURCE

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In the present study a mathematical model for the hydro-magnetic non-Newtonian blood flow in the non-Darcy porous medium with a heat source and Joule effect is proposed. A uniform magnetic field acts perpendicular to the porous surface. The governing non-linear partial differential equations have been solved numerically by applying the explicit finite difference Method (FDM). The effects of various parameters such as the Reynolds number, hydro-magnetic parameter, Forchheimer parameter, Darcian parameter, Prandtl number, Eckert number, heat source parameter, Schmidt number on the velocity, temperature and concentration have been examined with the help of graphs. The present study finds its applications in surgical operations, industrial material processing and various heat transfer operations.

Key words: heat source, blood flow, porous medium and MHD.

1. Introduction

Flow of blood through arteries is an important physiological problem, and is of high interest for biomedical researchers, physiologists, and clinicians. One of the important aspects of this problem that requires special attention is the deviation in the flow characteristics of blood when the artery is exposed to an externally applied magnetic field. This aspect has been the subject of vigorous research in recent years. Mathematical modeling of heat transfer in bio-fluid engineering was initiated in the late 1940s in a seminal paper by Pennes (1984) which laid the foundations for conduction heat transfer in tissue. The pulsatile fluid flow with heat transfer finds its applications in the fields such as mechanical and industrial thermal engineering systems. Baish (1990) studied heat transport by countercurrent blood vessels in the presence of an arbitrary pressure gradient. Consiglieri *et al.* (2003), Davalos *et al.* (2003) discussed the theoretical analysis of the heat convection coefficient in large vessels. An analytical study of heat transfer in finite tissue with two blood vessels and uniform Dirichlet boundary conditions was made by Shrivastava *et al.* (2005).

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These studies were all confined to Newtonian blood flow models. Although many Newtonian models considering Navier-Stokes equations are studied, the rheological nature of bio-fluids such as blood, plasma, bile etc. have also been recognized in a lot many studies. It is required to develop a non-Newtonian model that can present more accurate results in field of study of physiological fluids. Due to the pumping action of the heart in the human circulation system, a pulsatile feature of blood must also be considered. Skalak and Chien (1982) presented an investigation of the non-Newtonian flow of blood, considering erythrocytes as soft tissues. An excellent summary of a number of rheological models for blood was provided by Cokelet (1972).

In mid-1980's engineers became interested in the influence of magnetic fields on blood flows primarily with a view to utilize MHD (magneto-hydrodynamics) in controlling blood flow velocities in surgical procedures and also establishing the effects of magnetic fields on blood flows in astronauts, citizens living in the vicinity of EM (electro-magnetic) towers. The presence of iron oxides in the hemoglobin molecule has been shown (Takeuchi *et al.*, 1995) to produce strong magnetic properties in blood. Under oxygenated conditions blood exhibits diamagnetic properties and under deoxygenated conditions it behaves as a paramagnetic fluid. Several authors have also reported on heat transfer in bio-magnetic fluid flows including (Tzirtzilakis and Tanoudis, 2003) for bio-magnetic convective heat transfer over a stretching surface and Louckopoulos and Tzirtzilakis (2004) for bio-magnetic flow and heat transfer in a parallel-plate system.

The presence of a porous medium during the flow presents a more physically realizable study. This approach can be modeled in the blood vessels and pulmonary systems due to the presence of fatty deposits and artery blockages. Generally, a Darcy model is most widely used for modeling the porous conditions but under higher pressure gradients and in highly porous regimes where inertial effects dominate viscous effects, the Darcian model flow is inadequate. Khaled and Vafai (2003) presented a rigorous review of heat and fluid dynamics applications in porous (biological) media. The effects of temporally-varying wall mass flux on hydromagnetic pulsatile Newtonian blood flow in a Darcian porous model of the cardiovascular system, using a regular perturbation technique were investigated by Ogulu and Amos (2007). Various studies using this model have been presented in the context of porous media heat transfer, as described by Pop and Ingham (2001). Significant works include those by Preziosi and Farina (2002) who studied mass exchange using an extended Darcy model, Vankan *et al.* (1997) who considered non-Darcy transport in blood perfused tissue and Sorek and Sideman (1986) who analyzed blood flow in cardiac vessels using a Darcy-Forchheimer model. Recently, Bhargava *et al.* (2007) used the Darcy-Forchheimer model to analyze pulsating magneto-hydrodynamic blood flow and species diffusion in a porous medium channel. Joule dissipation acts as a volumetric heat source and finds its practical interest due to large industrial applications. Recently, Sharma *et al.* (2013) studied heat and mass transfer in a magnetic bio fluid flow through a non-Darcian porous medium with Joule effect

In the above mentioned studies the effects of heat sources/sinks have not been considered. A recent pilot study, however, indicates that the moisture content of the skin may also alter the response of vascular endothelial cells to local heat (KcLellan *et al.*, 2009). In these studies, if the skin was kept dry during warming, the skin blood flow response was significantly smaller than if moist heat was used as a heating source (KcLellan *et al.*, 2009). There is some evidence in the literature to support this concept. Petrofsky *et al.* (2009) noted that the increased blood flow response observed in the skin could have been due to the moisture in the heat source or the faster rate of rise in skin temperature or both. Kandasamy *et al.* (2005) discussed heat and mass transfer effect along a wedge with a heat source and concentration in the presence of suction/injection taking into account the chemical reaction of first order. Sharma *et al.* (2007; 2007) studied MHD free convective flow of a viscous fluid past an infinite vertical porous plate with heat source/sink effect. Unsteady MHD convective Heat and Mass transfer past a semi-infinite vertical permeable moving plate with heat absorption was discussed by Chambkha (2004). Sharma *et al.* (2007; 2008; 2011) discussed the radiation effect on a free convective flow along a uniform moving porous vertical plate in the presence of a heat source/sink and transverse magnetic field. Recently, Soret and Dufour effects on an unsteady MHD mixed convective flow past an infinite vertical plate with Ohmic dissipation and heat source were studied by Sharma *et al.* (2014). Hence, the main objective of the present investigation is to study a

biological fluid flow with heat and mass transfer considering its pulsatile hydro-magnetic rheological nature under the presence of viscous dissipation, Joule heating and a finite heat source through a Darcian porous medium.

2. Mathematical formulation

Consider an unsteady two-dimensional rheological bio-fluid flow between the narrow channel in the presence of a porous medium with viscous dissipation, Joule dissipation and heat source effects. The plates are kept at a distance of 2H with wall transpiration effects. The pulsatile nature of a bio-fluid like blood is considered. Blood is pumped through the human cardiovascular system by the heart. Here, the source parameter effect has been considered by taking real pistons in action. The transverse magnetic field (B_0) is applied considering the electrical nature of the bio-fluid. The porous medium here is of non-Darcian nature and the wall transpiration is present as injection at (-H) and suction at (H). The Joule dissipation and heat source/sink have also been included in the energy equation. The fluid here is non-Newtonian in nature; therefore the Nakamura-Sawada model especially used for such cases has been employed. The mass conservation for the species is considered by taking its concentration C_1 at the lower plate (-H) and C_2 at the upper plate (H). Thus, under these assumptions, the problem is governed by the following system of equations:

Linear Momentum Equation

$$\frac{\partial u}{\partial \tau} + V_0 \frac{\partial u}{\partial y} + \frac{1}{\rho} \frac{\partial P}{\partial x} = \nu_B \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\nu_B}{k_p} u - bu^2 . \tag{2.1}$$

Energy Equation

$$\frac{\partial T}{\partial \tau} + V_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu_B}{c_p} \left(\frac{\partial u}{\partial y}\right)^2 + s^* (T - T_m) + \frac{\sigma B_0^2}{\rho c_p} u^2 . \tag{2.2}$$

Concentration Equation

$$\frac{\partial C'}{\partial \tau} + V_0 \frac{\partial C'}{\partial y} = D \frac{\partial^2 C'}{\partial y^2} . \tag{2.3}$$

The corresponding boundary conditions are

$$y = -H: \quad u = 0; \quad T = T_1; \quad C' = C_1,$$

$$y = H: \quad u = 0; \quad T = T_2; \quad C' = C_2$$

where μ_B is Newtonian dynamic viscosity, V_o is the wall transpiration velocity ($V = V_o$ at the lower plate and $V = -V_o$ at the upper plate), β denotes the upper limit of the apparent viscosity coefficient, where u is the x-direction (longitudinal velocity), P is the hydrodynamic pressure, k_p is the hydraulic conductivity (permeability) of the porous material, ρ is the density of the fluid, b is a Forchheimer coefficient related to the porous medium geometry, τ is the dimensional time, σ is the electrical conductivity of the bio-fluid, B_0 is the transverse magnetic field strength, α is the thermal diffusivity, c_p is the specific heat capacity of the bio-fluid, s^* is the source parameter, T is the bio-fluid temperature, C is the species concentration, D is the mass diffusivity of the species, $\partial P/\partial x$ denotes the longitudinal pressure gradient.

Introducing the following non-dimensional parameters

$$U = \frac{u}{V_0}, \quad X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad t = \frac{V_0}{H}\tau, \quad P^* = \frac{P}{\rho V_0}, \quad \Phi = \frac{T - T_m}{T_2 - T_m}, \quad \mathrm{Re} = \frac{HV_0}{\nu_B},$$

$$Nm = \frac{\sigma B_0 H}{\rho V_0}, \quad \lambda = \frac{k_p V_0}{\nu_B H}, \quad \mathrm{Nf} = Hb, \quad \mathrm{Pr} = \frac{\nu_B}{\alpha}, \quad \mathrm{Ec} = \frac{V_0^2}{c_p(T_2 - T_m)}, \quad \mathrm{Sc} = \frac{HV_0}{D},$$

$$C = \frac{C' - Cm}{C_2 - C_m}, \quad S = \frac{s^*}{V_0}.$$

Equations (2.1) to (2.3) are reduced to the following non-dimensional form:

Momentum Equation

$$\frac{\partial U}{\partial t} + \frac{\partial U}{\partial Y} + \frac{\partial P}{\partial X} = \frac{1}{\mathrm{Re}}\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 U}{\partial Y^2} - NmU - \frac{1}{\lambda}U - \mathrm{Nf}\,U^2. \tag{2.4}$$

Energy Equation

$$\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial Y} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}}\frac{\partial^2 \theta}{\partial Y^2} + \frac{\mathrm{Ec}}{\mathrm{Re}}\left(\frac{\partial U}{\partial Y}\right)^2 + S\theta + \mathrm{Ec}\,NmU^2. \tag{2.5}$$

Concentration Equation

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial Y} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 C}{\partial Y^2}. \tag{2.6}$$

The transformed boundary conditions become

$$
\begin{aligned}
Y = -1: &\quad U = 0; \quad \Theta = -1; \quad C = -1, \\
Y = 1: &\quad U = 0; \quad \Theta = 1; \quad C = 1
\end{aligned}
\tag{2.7}
$$

where X and Y are dimensionless coordinates parallel and transverse to the bio-fluid flow, respectively, U is the transformed velocity component in the X-direction, $P*$ is the transformed hydrodynamic pressure (* dropped for convenience in analysis), t is the dimensionless time, θ is the dimensionless temperature, Re is a transpiration Reynolds number, Nm is the hydromagnetic parameter, λ is a Darcian (permeability) parameter, Nf is the Forchheimer (quadratic porous drag) parameter, $Tm = (T_1 + T_2)/2$ is the characteristic temperature, $Cm = (C_1 + C_2)/2$ is the characteristic concentration, Pr is the Prandtl number, Ec is the Eckert Number, Sc is the Schmidt number and S is the dimensionless source parameter.

As the present problem of fluid flow is pulsatile in nature, therefore the pressure gradient component is decomposed into a steady component and an oscillatory component as below

$$-\frac{\partial P}{\partial X} = \left(\frac{\partial P}{\partial X}\right)_s + \left(\frac{\partial P}{\partial X}\right)_o e^{i\omega t}. \tag{2.8}$$

First, to solve the above Eqs (2.4)-(2.5) coupled equations, the pressure term is redefined as

$$-\frac{\partial P}{\partial X} = P_s + P_0 \cos(w*t)$$

where P_s = the static pressure component, P_0 = the oscillatory pressure component.

3. Method of solution

The above nonlinear dimensionless partial differential equations with boundary conditions have been solved numerically by applying the Explicit Finite Difference Technique. The finite difference equations corresponding to Eqs (2.4) to (2.6) are as follows

$$\frac{U_i^{j+1} - U_i^{j-1}}{2\Delta t} + \frac{U_{i+1}^j - U_{i-1}^j}{2\Delta Y} + \left(P_s + P_0 \cos \omega t\right) =$$

$$= \frac{1}{\text{Re}}\left(1 + \frac{1}{\beta}\right)\frac{U_{i+1}^j - 2U_i^j + U_{i-1}^j}{\Delta Y^2} - \left(Nm + \frac{1}{\lambda}M\right)U_i^j - \text{Nf}\left(U_i^j\right)^2,$$

(3.1)

$$\frac{\theta_i^{j+1} - \theta_i^{j-1}}{2\Delta t} + \frac{\theta_{i+1}^j - \theta_{i-1}^j}{2\Delta Y} = \frac{1}{\text{Pr}\,\text{Re}}\frac{\theta_{i+1}^j - 2\theta_i^j + \theta_{i-1}^j}{\Delta Y^2} +$$

$$+ S\theta_i^j + \frac{\text{Ec}}{\text{Re}}\left(\frac{U_{i+1}^j - u_{i-1}^j}{2\Delta Y}\right)^2 + Nm\,\text{Ec}\left(U_i^j\right)^2,$$

(3.2)

$$\frac{C_i^{j+1} - C_i^{j-1}}{2\Delta t} + \frac{C_{i+1}^j - C_{i-1}^j}{2\Delta Y} = \frac{1}{\text{Sc}}\frac{C_{i+1}^j - 2C_i^j + C_{i-1}^j}{\Delta Y^2}.$$

(3.3)

To obtain the difference equations, the region of the flow is divided into a gird or mesh of lines parallel to Y and t axes. Solutions of difference equations are obtained at the intersection of these mesh lines called nodes. The finite-difference equations at every internal nodal point on a particular n-level constitute a tri-diagonal system of equations. These equations are solved by using the Thomas algorithm (Hoffman, 1992).

In order to prove the convergence of the finite difference scheme, the computation is carried out for slightly changed values of ΔY and, Δt. Negligible changes are observed in the values of u, T, and C and also after each cycle of iteration the convergence checking is performed i.e., $\left|f^{n+1} - f^n\right| < 10^{-8}$ is satisfied at all points. Thus, due to computational cost and accuracy the above mesh size was considered optimal.

4. Results and discussion

The hydro-magnetic non-Newtonian bio-fluid flow in the non-Darcy porous medium with a heat source and Joule effect has been discussed in preceding sections. In order to get a physical insight into the problem, the numerical calculations for the distribution of the velocity, temperature and concentration for various dimensionless parameters were made. The bio-fluid is blood and the values of various parameters used are taken as in Sharma *et al*. (2013). In order to access the accuracy of our results, comparisons have

been made with Rawat *et al.* (2009) and results are shown in Tab.1. An excellent agreement can be observed between both studies for the dimensionless velocity profile (*U*) with the transverse coordinate (*Y*).

Table 1. Comparison of present study and Rawat *et al.* (2009).

Re=*0.5*, β=*4*, ω=*8*, Ps=10, *Po=7*, λ=*5*, Nf=*0.002*, Nm=*0.3*, Ec=*6*10⁻⁹*, Pr=*21*		
	U(at *t=0.5*)	
Y	Rawat *et al.*(2009)	Present study
-1	*0*	*0*
-0.65854	*0.477402*	*0.477452*
-0.31707	*0.792337*	*0.792369*
0.31707	*0.866090*	*0.866126*
0.658537	*0.572876*	*0.572932*
1	*0*	*0*

Figure 1 depicts the velocity profile for different values of the Reynolds number (Re) at *t=0.5*. As the Reynolds number involves transpiration velocity, hence an increase in Re results in an increase of velocity (*U*) values and generally the peaks are shifted towards the right. We note that the Reynolds number of unity implies a creeping flow regime, i.e., where the inertial core encountered in higher velocity porous flows has not developed, as discussed by Dybbs and Edwards (1984). Figure 2 shows the effects of the non-Newtonian parameter (β) on the velocity. Lower values of β imply higher values of viscosity and thus the lower values of velocity. As β increases the velocity increases and so the peak will be maximum as β approaches infinity. Figure 3 shows the influence of the Darcian parameter (λ) on the velocity profiles. Higher values of λ imply lower fatty deposits and blockages in the fluid flow channel, resulting in lower resistances and higher velocities. The effects of the hydro-magnetic parameter (*Nm*) are depicted in Fig.4. As the values of *Nm* increase, the velocity parabola is lowered. This is due to the retarding forces (Lorentz forces) generated by the magnetic field as bio-fluid's electrical properties have been already confirmed. As such magnetic fields are extremely useful in regulating blood flows in practical applications.
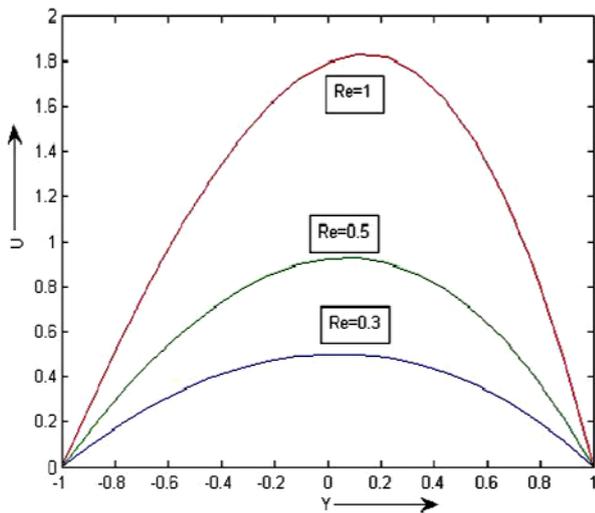

Fig.1. *U* versus *Y* for various transpiration Reynolds numbers (Re) at *t = 0.5*.
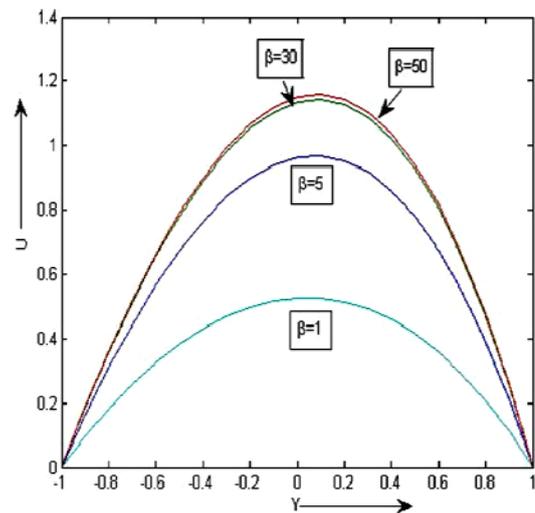

Fig.2. *U* versus *Y* for various non- Newtonian parameter values (β) at *t = 0.5*.
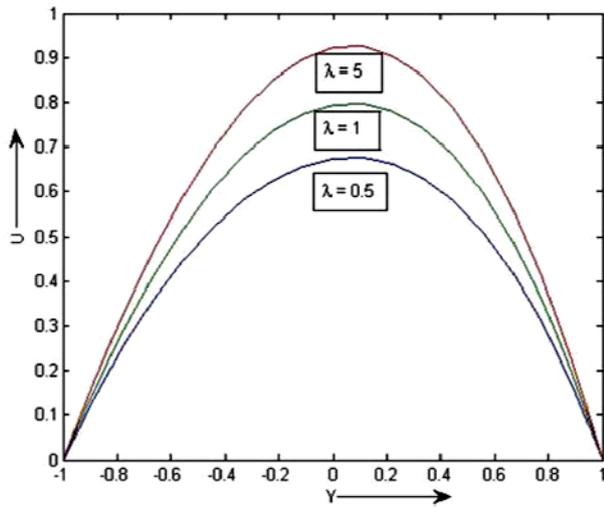
Fig.3. *U* versus *Y* for various Darcian parameter values (λ) at *t = 0.5*.
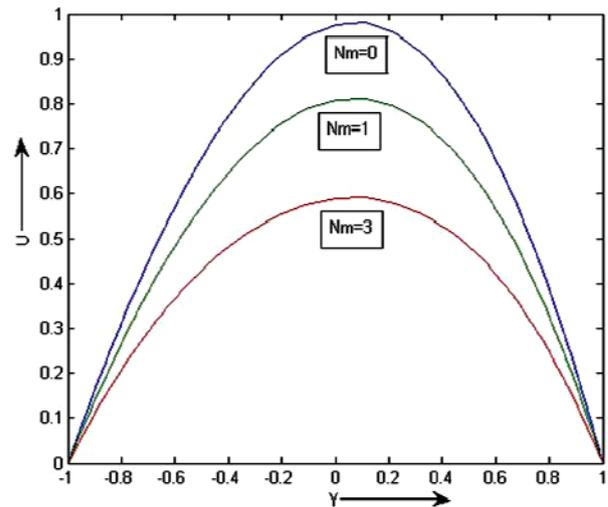
Fig.4. *U* versus *Y* for various hydromagnetic parameter values (*Nm*) at *t = 0.5*.

In Fig.5, the influence of the hydro-magnetic parameter (*Nm*) at two Ec values on the temperature is depicted. As earlier known *Nm* has an inhibiting effect on velocity and thus on temperature due to the Joule dissipation term in Eq.(2.4). The Joule dissipation term acts as a volumetric heat source, which becomes more important at higher Ec values. In Fig.5a, it is noticed that as *Nm* increases the temperature peaks decrease and in Fig.5b higher Ec value results in more heat generation and larger temperature values and the oscillations are increased due to the oscillatory nature of the velocity profile. The effects of the Prandtl number (Pr) for two different values of Ec on temperature distribution are shown in Fig.6. Pr represents the ratio of momentum diffusivity to thermal diffusivity. Larger Pr fluids (Pr >*1*) will diffuse momentum faster than heat. The numerical results show that the temperature decreases with an increase in the Prandtl number. This is due to the fact that a fluid with a high Prandtl number has a relatively low thermal conductivity which results in the reduction of the thermal boundary layer thickness.
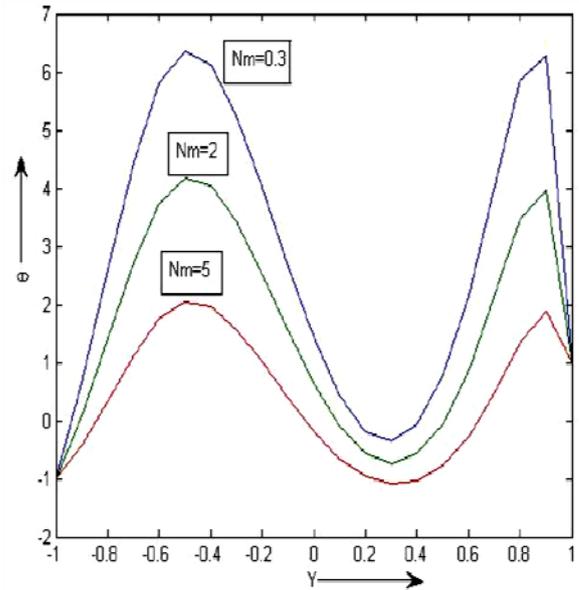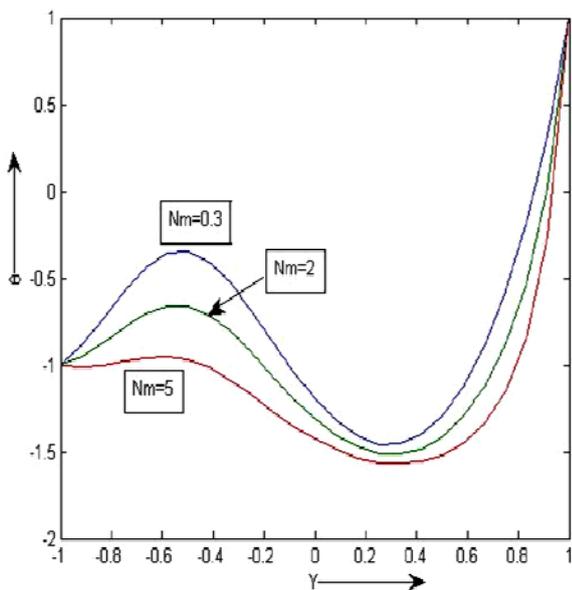

Fig.5. θ versus *Y* for various hydromagnetic parameter values (*Nm*) at *t = 0.5* for two Eckert numbers (a) Ec=*0.0006* (b) Ec=*0.006*.
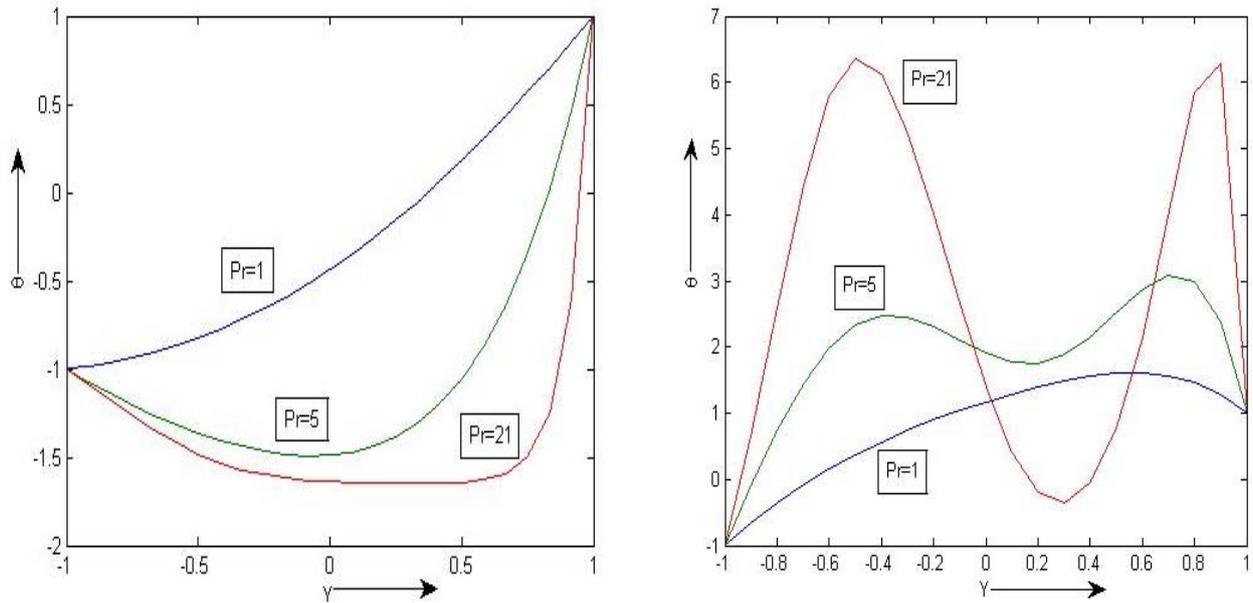
Fig.6. θ versus *Y* for different values of Pr at *t = 0.5* for two Eckert numbers (a) Ec=*6X10⁻⁹* (b) Ec=*0.006*.

In Fig.7 and Fig.8, the influence of the heat source parameter (*S*) taking both positive and negative values at two Ec values is illustrated. In Fig.7a and 7b, the heat sink parameter (*S<0*) effects are shown. It can be observed that effects of *S* are more pronounced at its higher negative values. More negative values result in higher temperatures but at higher Eckert number, larger oscillations result in a reversal of sink parameter effects and higher temperatures are observed at higher Ec and lower *S* value. Figures 8a and 8b depict the effect of the heat source parameter (*S>0*). For Ec=*0.0006*, higher *S* values imply lower temperature due to the cooling between the two plates, but the upper plate boundary condition results in the value of Θ =*1* at the upper plate. Similarly, higher Ec (= *0.006*) results in more oscillations and higher temperatures.
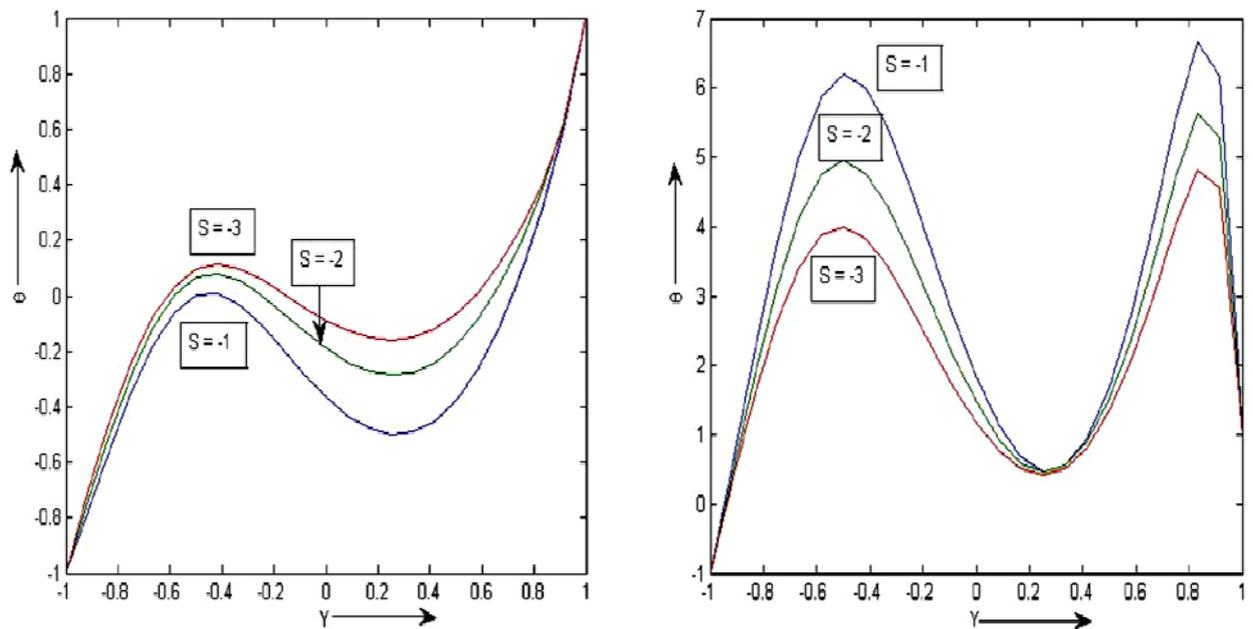

Fig.7.   θ versus *Y* for various sink parameter values(S) at *t = 0.5* for two Eckert numbers (a) Ec=*0.0006* (b) Ec=*0.006*.
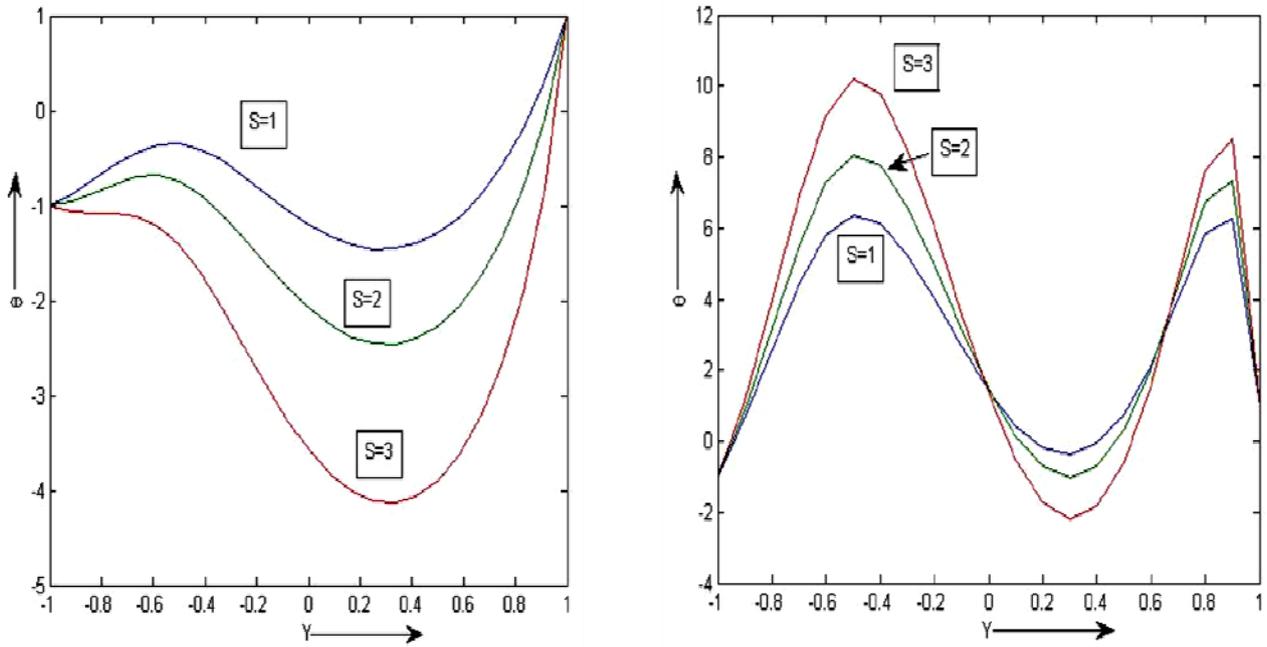
Fig.8. θ versus *Y* for various source parameter values (*S*) at *t = 0.5* for two Eckert numbers Ec=*0.0006* (b) Ec=*0.006*.
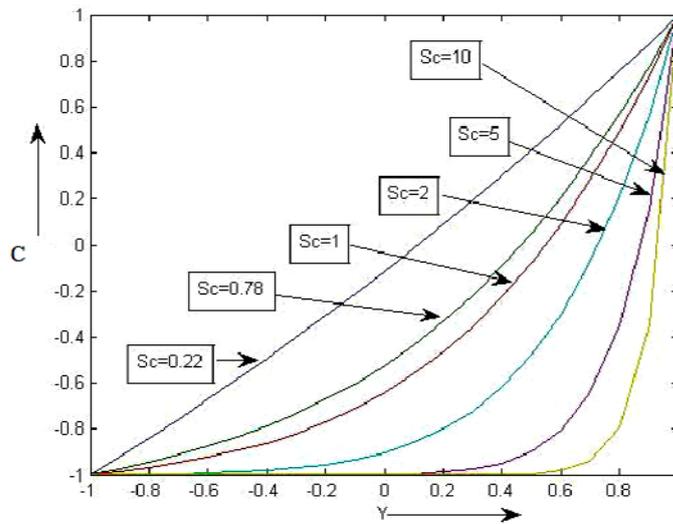

Fig.9. Concentration versus *Y* for various Schmidt numbers (Sc) at *t = 0.5*.

In Fig.9, the effect of the Schmidt number (Sc) over the species concentration in the channel has been shown for its various values. The Schmidt number defines the relative effectiveness of momentum and mass transport by diffusion in the velocity and concentration fields. Lower Sc values physically represent for example in industrial applications, hydrogen gas as the species diffusing (Sc = *0.22*), ammonia (Sc = *0.78*), methanol diffusing in air (Sc=*1.0*) and ethylbenzene in air (Sc=*2.0*). Sc physically represent the relativeness of momentum and mass diffusion. A comparison of curves in the figure shows a decrease in the concentration (*C*) with an increase in the Schmidt number (Sc). Physically, the increase of Sc means a decrease of the molecular diffusivity (*D*), that results in a decrease of the concentration boundary layer. Hence, the concentration of the species is higher for small values of Sc and lower for larger values of Sc.

Nomenclature

A – thermal diffusivity

B_0 – transverse magnetic field strength

b – Forchheimer coefficient related to the porous medium geometry

C – species concentration

$C_m = \dfrac{c_1 + c_2}{2}$ – characteristic concentration

c_p – specific heat capacity of the bio-fluid

D – mass diffusivity of the species

Ec – Eckert number

k_p – hydraulic conductivity (permeability) of the porous material

Nf – Forchheimer (quadratic porous drag) parameter/ number

N_m – hydro-magnetic parameter

P – hydrodynamic pressure

P^* – transformed hydrodynamic pressure (* dropped for convenience in the analysis)

P_s – steady component of pressure gradient

P_0 – oscillatory pressure component

Pr – Prandtl number

Re – transpiration Reynolds number

S – heat source parameter

Sc – Schmidt number

T – bio-fluid temperature

$T_m = \dfrac{T_1 + T_2}{2}$ – characteristic temperature

t – dimensionless time

U – transformed velocity component in the X-direction

u – x - direction (longitudinal velocity)

V_o – wall transpiration velocity ($V = V_o$ at the lower plate and $V = -V_o$ at the upper plate)

X – dimensionless coordinate parallel to the bio-fluid flow

Y – dimensionless coordinate transverse to the bio-fluid flow

β – upper limit of the apparent viscosity coefficient= rheological parameter (β)

θ – dimensionless temperature

λ – Darcian (permeability) parameter

ν_B – Newtonian dynamic viscosity

ρ – density of the fluid

σ – electrical conductivity of the bio-fluid

τ – dimensional time

ω – dimensionless angular frequency

$\partial P / \partial X$ – longitudinal pressure gradient

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