

EFFECT OF DRILL STRING ROTATION ON THE DYNAMIC RESPONSE OF DRILLING RISERS

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The effect of the rotation of a drill string on the response of a drilling riser has been studied. A governing equation for the flexural response that incorporates the effect of the drill string rotation is developed from first principles, and the resulting differential equation is found to have a variable coefficient, which is a function of the drill string rotational speed. Results simulated for the free vibration response show that the drill string rotation reduces the natural frequency and increases the amplitude of vibration of the drilling riser. The implication of these findings is that neglecting the effect of rotation of the drill string leads to under-estimation of the deflection and over-estimation of the natural frequency. Further analysis reveals that for a drilling riser of given dimensions, a drill string rotational speed exists at which the natural frequency of the drilling riser is theoretical equal to zero, and this rotational speed is the threshold rotational speed.

Key words: drilling riser, drill string, drill string rotational speed, free vibration, threshold rotational speed.

1. Introduction

As the exploration of underwater crude and gas resources becomes increasingly needful, offshore technology for deep water drilling systems comes into closer focus. The fact that the aim of underwater crude and gas exploration is to access and drill such resources, with the aid of drilling systems, underscores the importance of a drilling riser, which is the direct component of the drilling system by which this is made possible. A drilling marine riser, part of the blowout system, houses the drill bit and drill string. It is designed to be flexible and sturdy enough to withstand any movement from the drilling platform situated above the sea.

The analysis of general risers has a historical span of over half a century, and a lot has been discovered in this area to the extent that it is well established. Charkrabarti and Frampton (1982) have presented a state-of-the-art review of riser analysis, and they developed the governing equation of motion of general risers from first principles. Ghadimi (1988) used the lumped mass discretization method to derive the equations of motion of flexible risers in three-dimensional space. Spanos *et al.* (1990) developed a model for a marine riser system that is appropriate for the study of its dynamic behaviour in deepwater conditions. The model was used to estimate the steady-state response of the riser to harmonic excitation. Kaewunruen *et al.* (2005) investigated the nonlinear free vibration of marine risers to determine the nonlinear natural frequencies and their corresponding mode shapes. Based on the virtual work-energy function of marine risers, they developed the structural model from the strain energy due to bending, the virtual works due to

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effective tension and external forces, and the kinetic energies due to the marine riser and the internal motions. Kerber and Wiercigroch (2088) studied the effect of weak structural nonlinearity on the dynamic behaviour of a vertical marine riser subjected to vortex-induced vibration, whereas Chen *et al.* (2009) re-examined the vibration characteristics of marine risers using the method of the differential transformation.

Some devotion has been given to the study of drilling risers in particular, but all such studies have been mainly focussed on the drill string as can be observed from the following studies: Vaz and Patel (1995), Sampaio *et al.* (2007), Gulyayev *et al.* (2009), Gulyayev and Borshch (2011), etc. These studies have treated the drill string as an independent structure whose motion does not influence any other structure or vice versa. This is not always the case in reality as the drill string is housed by the drilling riser to which it is attached at various points along its span. The implication is that the connection between the drilling riser and the drill string is such that there is a negligible relative translational motion between them, but relative rotational motion of the drill string is permitted. The focus of this study is on the response of drilling risers during the production of crudeoil, at which time the drill string is rotating. In this paper, the effect of the rotation of the drill string on the behaviour of the drilling riser has been investigated. First, a modified riser model that applies specifically to the drilling riser was developed, and then the effect of the rotational speed of the drill string on the free vibration response of the drilling riser was examined by means of codes written in Mathematica™.

2. Mathematical model for flexural motion of drilling risers

Starting from first principles as in Chakrabarti and Frampton (1982), the force exerted by the rotation of the drill string, $F_R\theta$, on the drilling riser can be represented on the riser segment as shown in Fig.1.

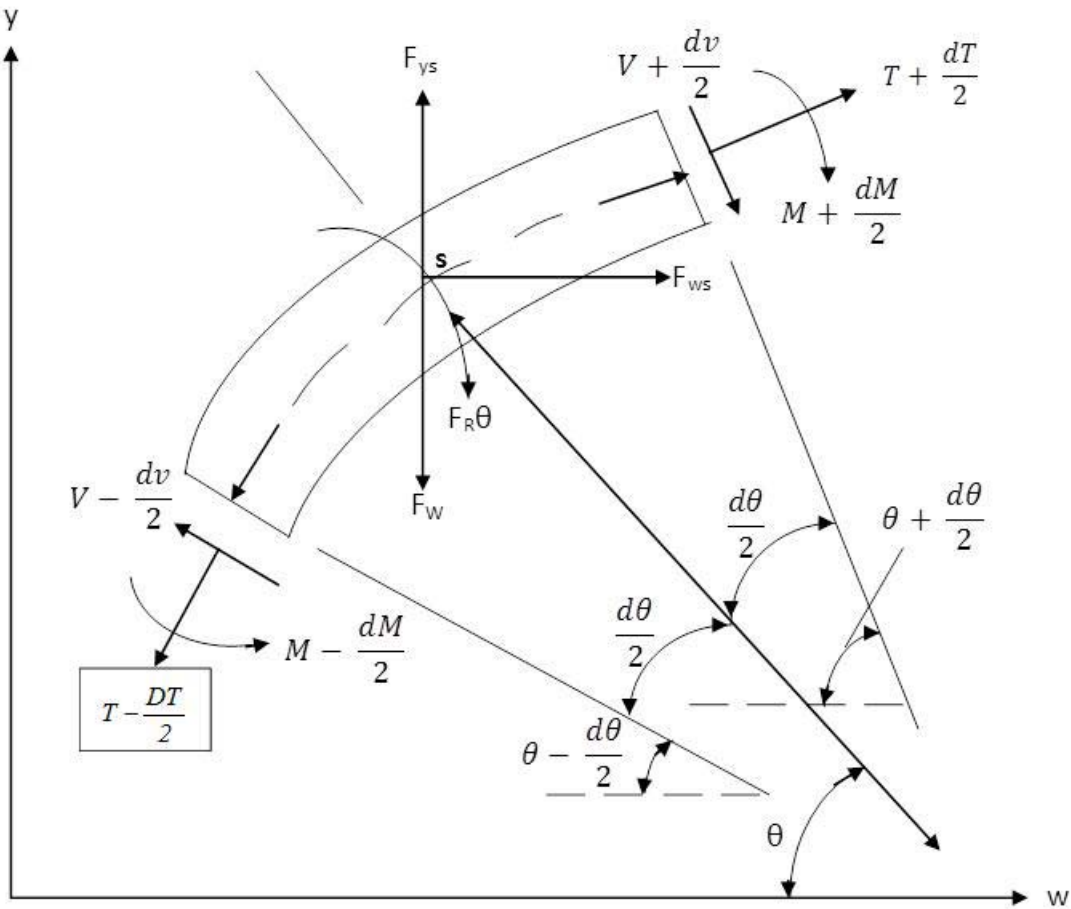


Fig.1. Free body diagram of a bent tubular segment.

The following mathematical assumptions have been used

(A) The length of the segment is small so that

$$\cos d\theta \approx 1 \quad \text{and} \quad \sin d\theta \approx d\theta \quad (2.1)$$

(B) The small deflection beam theory is applicable. Therefore,

$$\sin\theta \approx \frac{\partial w}{\partial s} \quad \text{and} \quad \cos\theta \approx \frac{dy}{ds}. \quad (2.2)$$

(C) The angle of deflection, θ , is small, i.e.,

$$ds \approx dy, \quad \cos\theta \approx 1, \quad \sin\theta \approx \theta \approx \frac{\partial w}{\partial y} \quad \text{and} \quad \frac{d\theta}{dy} \approx \frac{\partial^2 w}{\partial y^2}. \quad (2.3)$$

2.1. Vertical equilibrium equation

Considering the equilibrium of vertical forces, we have

$$\begin{aligned} & \left(V - \frac{dV}{2} \right) \sin \left(\theta - \frac{d\theta}{2} \right) - \left(V + \frac{dV}{2} \right) \sin \left(\theta + \frac{d\theta}{2} \right) + \\ & - \left(T - \frac{dT}{2} \right) \cos \left(\theta - \frac{d\theta}{2} \right) + \left(T + \frac{dT}{2} \right) \cos \left(\theta + \frac{d\theta}{2} \right) - F_w + F_{ys} = 0. \end{aligned} \quad (2.4)$$

Expanding the double-angle trigonometric ratios in Eq.(2.4) and simplifying the resulting equation yields

$$A \cos\theta - B \sin\theta - F_w + F_{ys} = 0 \quad (2.5)$$

where

$$A = dT \cos \frac{d\theta}{2} - 2V \sin \frac{d\theta}{2},$$

$$B = dV \cos \frac{d\theta}{2} + 2T \sin \frac{d\theta}{2}.$$

2.2. Horizontal equilibrium equation

In a similar manner, the equilibrium of horizontal forces would yield

$$A \sin\theta + B \cos\theta + F_{ws} - m_w \ddot{w} ds = 0 \quad (2.6)$$

where m_w = mass of the segment including added mass per unit length acting in the w direction; \ddot{w} = acceleration of point s in the w -direction.

2.3. Moment equilibrium equation

Taking moments about point s gives the moment equilibrium equation as

$$dT \left(1 - \cos \frac{d\theta}{2} \right) + dM + 2V \sin \frac{d\theta}{2} + \left(F_R \cos \frac{d\theta}{2} \right) \theta = 0 . \quad (2.7)$$

Considering that the length of the segment is small, Eq.(2.1), the three equilibrium equations can be written as

$$\begin{aligned} A_I \cos \theta - B_I \sin \theta - f_w + f_{ys} &= 0 , \\ A_I \sin \theta + B_I \cos \theta + f_{ws} - m_w \ddot{w} &= 0 , \\ \frac{dM}{ds} + V + F_R \theta &= 0 \end{aligned} \quad (2.8)$$

where

$$\begin{aligned} A_I &= \frac{dT}{ds} - V \frac{d\theta}{ds} , \\ B_I &= \frac{dV}{ds} + T \frac{d\theta}{ds} \end{aligned} \quad (2.9)$$

f_w, f_{ys} and f_{ws} are the weight and force intensities (i.e., force per unit length).

To develop the equation of motion for the horizontal displacement of the riser, A_I in Eq.(2.8a) is substituted into Eq.(2.8b) to get

$$B_I \sec \theta + (f_w - f_{ys}) \tan \theta + f_{ws} - m_w \ddot{w} = 0 . \quad (2.10)$$

By making use of Eqs (2.8c) and (2.9b), the expression for B_I is obtained as

$$B_I = \frac{d}{ds} \left[- \frac{d}{ds} \left(EI \frac{d\theta}{ds} \right) - F_R \theta \right] + T \frac{d\theta}{ds} . \quad (2.11)$$

The following expressions are applicable to all risers (Chakrabarti and Frampton, 1982).

$$\begin{aligned} f_w &= \gamma_s (A_o - A_i) , \\ f_{wp} &= f_{np} \cos \theta , \quad \text{as part of } f_{ws} , \\ f_{yp} &= -f_{np} \sin \theta \quad \text{as part of } f_{ys} , \\ f_{np} &= (A_o \bar{P}_o - A_i \bar{P}_i) \frac{d\theta}{ds} - (A_o \gamma_o - A_i \gamma_i) \sin \theta . \end{aligned} \quad (2.12)$$

Equations (2.11) and (2.12) are substituted into Eq.(2.10) and the equation of motion for horizontal displacement of the marine risers is derived as

$$\frac{d^2}{ds^2} \left(EI \frac{\partial^2 w}{\partial y^2} \right) - (T + A_o \bar{P}_o - F_R - A_i \bar{P}_i) \frac{\partial^2 w}{\partial y^2} +$$

$$- \left[\gamma_s (A_o - A_i) - f_{ys} - A_o \gamma_o + A_i \gamma_i \right] \frac{\partial w}{\partial y} + m_w \ddot{w} = f_{ws}. \quad (2.13)$$

In the literature, the effect of the axial force induced by rotation is normally neglected i.e., $F_R=0$. This condition is okay for modelling the dynamic response of general risers. However, a special situation occurs with drilling risers where the axial force induced by rotation of the drill string can significantly affect the response of the riser. Then F_R cannot be ignored and the expression for determining this force is considered next.

A detailed derivation of the axial force induced by a rotating hollow cylindrical shaft can be found in Behzad and Bastami (2004). They expressed this force as

$$F = \rho v I_p \Omega^2 \quad (2.14)$$

where $I_p = \pi(b^4 - a^4)/2$ is the moment of inertia of the shaft and Ω is the rotational speed of the shaft in rev/min.

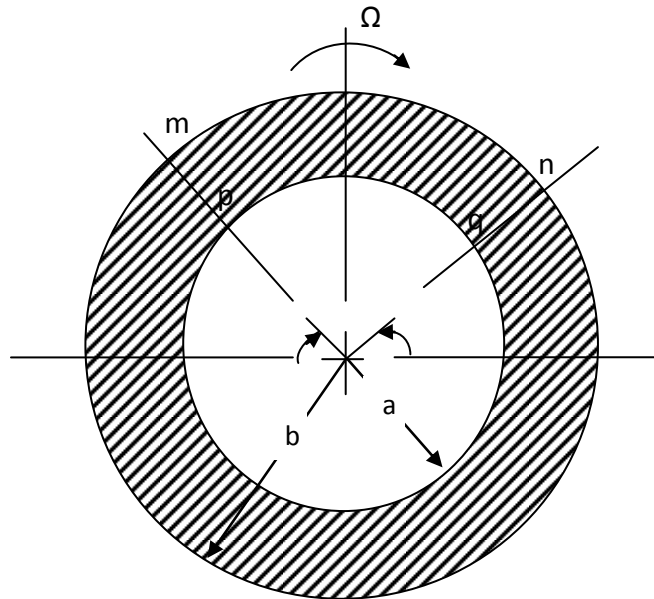


Fig.2. Section of a rotating shaft.

Here it is considered that the rotation of the drill string of a drilling riser will induce an axial force on the drilling riser that can be expressed by Eq.(2.14). Therefore, Eq.(2.14) is substituted into Eq.(2.13) to include the effect of the rotation of the drill string on the response of the drilling riser as follows

$$EI \frac{\partial^4 w}{\partial y^4} - \mu \frac{\partial^2 w}{\partial y^2} - \beta \frac{\partial w}{\partial y} + m_w \frac{\partial^2 w}{\partial t^2} = f_{ws} \quad (2.15)$$

where $\mu = T + A_o \bar{P}_o - A_i \bar{P}_i - \nu \rho I_P \Omega^2$; $\beta = \gamma_s (A_o - A_i) - f_{ys} - A_o \gamma_o + A_i \gamma_i$.

From Eq.(2.15), the effect of the rotation of the drill string on the flexural motion of the drilling riser is introduced through the coefficient μ .

3. Free vibration analysis

An analytical solution to Eq.(2.15) can be written as

$$w(y, t) = \sum_{m=1}^N W_m \sin\left(\frac{m\pi y}{l}\right) f(t) \quad (3.1)$$

where W_m is the amplitude of the response of the riser, and $f(t)$ is the temporal response of the riser and the modal parameter of interest in this study.

3.1. Temporal response analysis

For the free vibration, the excitation force f_{ws} is zero so that Eq.(2.15) becomes

$$EI \frac{\partial^4 w}{\partial y^4} - \mu \frac{\partial^2 w}{\partial y^2} - \beta \frac{\partial w}{\partial y} + m_w \frac{\partial^2 w}{\partial t^2} = 0. \quad (3.2)$$

After substituting the solution into Eq.(3.2) we have

$$\lambda \ddot{f}(t) + \alpha f(t) = 0 \quad (3.3)$$

where $\lambda = m_w \sum_{m=1}^N W_m \sin\left(\frac{m\pi y}{l}\right)$ and

$$\alpha = EI \sum_{m=1}^N \left(\frac{m\pi}{l}\right)^4 W_m \sin\left(\frac{m\pi y}{l}\right) + \mu \sum_{m=1}^N \left(\frac{m\pi}{l}\right)^2 W_m \sin\left(\frac{m\pi y}{l}\right) + \beta \sum_{m=1}^N \left(\frac{m\pi}{l}\right) W_m \cos\left(\frac{m\pi y}{l}\right).$$

For the fundamental mode ($m = 1$) and at mid-span ($y = l/2$)

$$\lambda = m_w W_1 \quad \text{and} \quad \alpha = \left[EI \left(\frac{\pi}{l}\right)^4 + \mu \left(\frac{\pi}{l}\right)^2 \right] W_1.$$

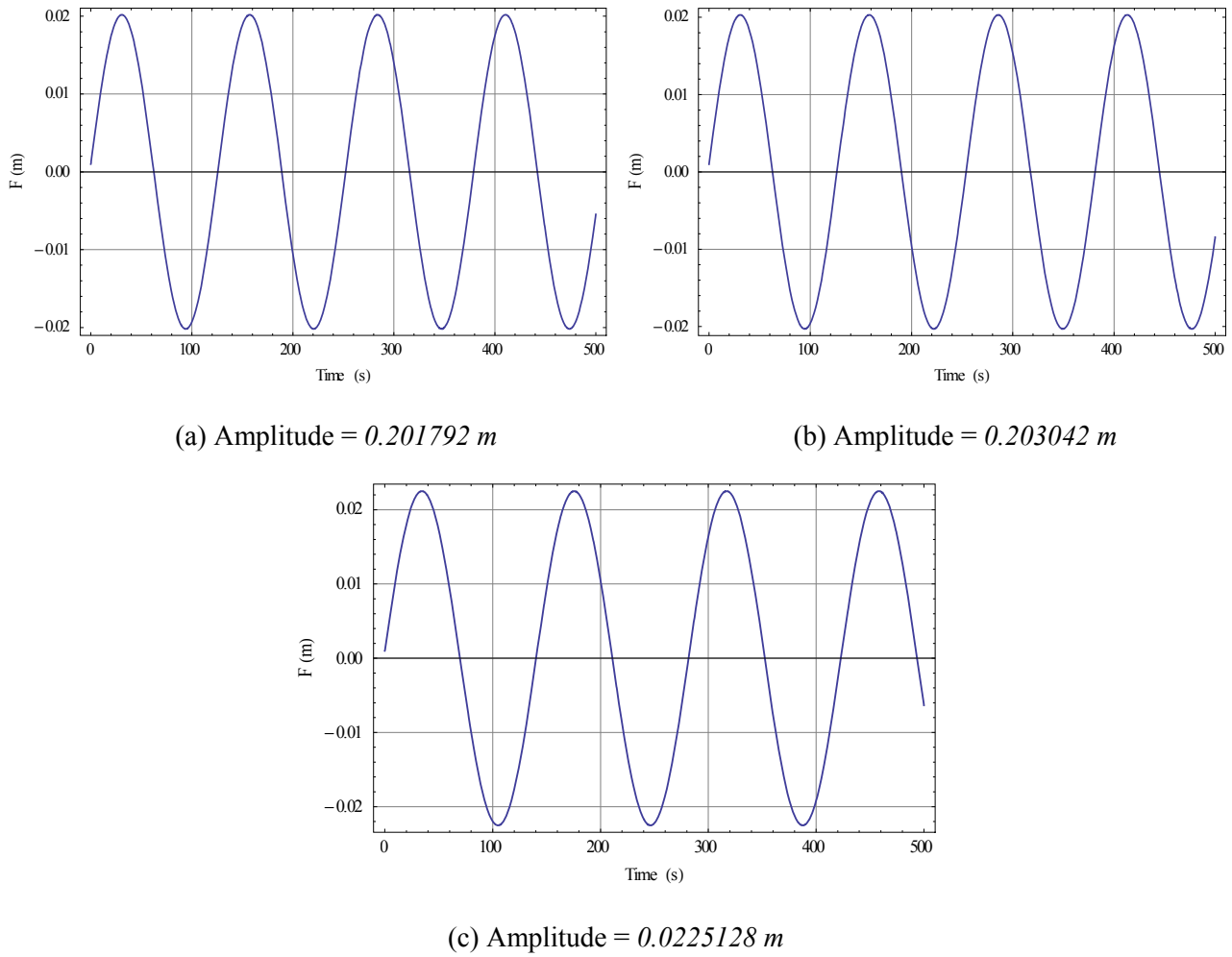


Fig.1. Time-domain response of the drilling riser for (a) 0 rev/min (b) 500 rev/min (c) 2000 rev/min.

3.2. Natural frequencies

Assuming $f(t)$ is a sinusoidal function, then the solution can be written as

$$w(y,t) = \sum_{m=1}^N W_m \sin\left(\frac{m\pi y}{l}\right) e^{i\omega_n t} \tag{3.4}$$

where ω_n is the natural frequency of the riser.

Substituting Eq.(3.4) into Eq.(3.2) we arrive at

$$\sum_{m=1}^N W_m \left[EI \left(\frac{m\pi}{l}\right)^4 \sin\left(\frac{m\pi y}{l}\right) + \mu \left(\frac{m\pi}{l}\right)^2 \sin\left(\frac{m\pi y}{l}\right) + \right. \\ \left. - \beta \left(\frac{m\pi}{l}\right) \cos\left(\frac{m\pi y}{l}\right) - m_w \omega_n^2 \sin\left(\frac{m\pi y}{l}\right) \right] = 0 . \tag{3.5}$$

To solve Eq.(3.5), we multiply through by $\sin\left(\frac{m\pi y}{l}\right)$ and integrate over the span of the riser. When this is done, we arrive at the following equation.

$$\sum_{m=1}^N A_m \left[EI \left(\frac{m\pi}{l} \right)^4 + \mu \left(\frac{m\pi}{l} \right)^2 - m_w \omega_n^2 \right] = 0. \tag{3.6}$$

Equation (3.6) is the *characteristic* or *frequency equation* and can be used to obtain the natural frequency of the m^{th} mode of vibration of the riser as shown in Eq.(3.7).

$$\omega_n^2 = \frac{I}{m_w} \left[EI \left(\frac{m\pi}{l} \right)^4 + \mu \left(\frac{m\pi}{l} \right)^2 \right].$$

4. Results and discussion

The natural frequencies for the first ten modes of vibration for different lengths of the drilling riser are shown Tab.1.

Table 1 (a – c). Natural frequencies of the drilling riser for different rotational speeds of the drill string.

(a) Riser length = 500 m.

| Rotational speed (rev/min) | | $\Omega = 0$ | $\Omega = 100$ | $\Omega = 200$ | $\Omega = 300$ | $\Omega = 400$ | $\Omega = 500$ |
|----------------------------|----|--------------|----------------|----------------|----------------|----------------|----------------|
| Mode of vibration | 1 | 0.0994976 | 0.0994428 | 0.0992781 | 0.099003 | 0.0986165 | 0.0981174 |
| | 2 | 0.262348 | 0.262265 | 0.262015 | 0.261598 | 0.261014 | 0.26026 |
| | 3 | 0.514259 | 0.514164 | 0.513877 | 0.513399 | 0.512729 | 0.511867 |
| | 4 | 0.861942 | 0.861841 | 0.861537 | 0.861031 | 0.860321 | 0.859407 |
| | 5 | 1.30724 | 1.30713 | 1.30682 | 1.3063 | 1.30557 | 1.30462 |
| | 6 | 1.85075 | 1.85064 | 1.85032 | 1.84979 | 1.84905 | 1.84809 |
| | 7 | 2.49272 | 2.49261 | 2.49229 | 2.49176 | 2.49101 | 2.49004 |
| | 8 | 3.23326 | 3.23316 | 3.23283 | 3.23229 | 3.23154 | 3.23056 |
| | 9 | 4.07243 | 4.07232 | 4.072 | 4.07145 | 4.07069 | 4.06972 |
| | 10 | 5.01025 | 5.01014 | 5.00981 | 5.00927 | 5.00851 | 5.00753 |

(b) Riser length = 1000 m.

| Rotational speed (rev/min) | | $\Omega = 0$ | $\Omega = 100$ | $\Omega = 200$ | $\Omega = 300$ | $\Omega = 400$ | $\Omega = 500$ |
|----------------------------|----|--------------|----------------|----------------|----------------|----------------|----------------|
| Mode of vibration | 1 | 0.0618142 | 0.0617922 | 0.0617259 | 0.0616154 | 0.0614602 | 0.0612602 |
| | 2 | 0.130808 | 0.130766 | 0.130641 | 0.130432 | 0.130139 | 0.129761 |
| | 3 | 0.212954 | 0.212896 | 0.212723 | 0.212434 | 0.212029 | 0.211508 |
| | 4 | 0.312522 | 0.312453 | 0.312243 | 0.311893 | 0.311403 | 0.310772 |
| | 5 | 0.432274 | 0.432195 | 0.431958 | 0.431563 | 0.43101 | 0.430297 |
| | 6 | 0.5739 | 0.573814 | 0.573557 | 0.573129 | 0.572529 | 0.571757 |
| | 7 | 0.73842 | 0.73833 | 0.738058 | 0.737605 | 0.736971 | 0.736155 |
| | 8 | 0.926455 | 0.926361 | 0.926078 | 0.925607 | 0.924947 | 0.924097 |
| | 9 | 1.13839 | 1.13829 | 1.138 | 1.13751 | 1.13683 | 1.13596 |
| | 10 | 1.37446 | 1.37436 | 1.37406 | 1.37357 | 1.37287 | 1.37198 |

(c) Riser length = 1500 m.

| Rotational speed (rev/min) | | $\Omega = 0$ | $\Omega = 100$ | $\Omega = 200$ | $\Omega = 300$ | $\Omega = 400$ | $\Omega = 500$ |
|----------------------------|----|--------------|----------------|----------------|----------------|----------------|----------------|
| Mode of vibration | 1 | 0.049617 | 0.0496048 | 0.0495681 | 0.049507 | 0.0494212 | 0.0493107 |
| | 2 | 0.101036 | 0.101012 | 0.10094 | 0.10082 | 0.100651 | 0.100434 |
| | 3 | 0.155954 | 0.155919 | 0.155814 | 0.155639 | 0.155393 | 0.155077 |
| | 4 | 0.215885 | 0.21584 | 0.215705 | 0.21548 | 0.215165 | 0.214759 |
| | 5 | 0.282113 | 0.282059 | 0.281898 | 0.281629 | 0.281252 | 0.280767 |
| | 6 | 0.355688 | 0.355627 | 0.355443 | 0.355135 | 0.354705 | 0.354151 |
| | 7 | 0.43744 | 0.437372 | 0.437168 | 0.436829 | 0.436352 | 0.435739 |
| | 8 | 0.528013 | 0.52794 | 0.527719 | 0.527352 | 0.526836 | 0.526173 |
| | 9 | 0.627901 | 0.627822 | 0.627588 | 0.627197 | 0.626648 | 0.625943 |
| | 10 | 0.737476 | 0.737394 | 0.737148 | 0.736736 | 0.73616 | 0.735419 |

Numerical results were simulated from Mathematica™ codes using the input values in Tab.2. Tables 1(a) – (c) show the natural frequencies of the drilling riser for different rotational speeds of the drill string. The second column in these tables represents the natural frequency without the rotation of the drill string. The results reveal that the rotation of the drill string reduces the stiffness of the drilling riser and by extension its natural frequency.

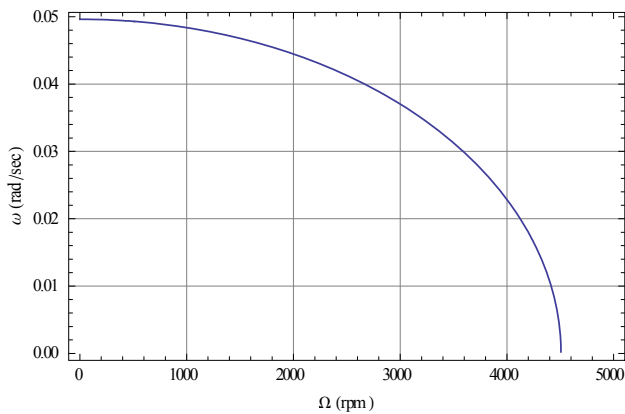
Table 2. Properties and input parameters of model riser and waves.

| S/No | Quantity | Symbol | S.I. Units | Value |
|------|--------------------------------|----------|------------|--------|
| 1 | Top tension | T_l | N | 3000 |
| 2 | Riser length | L | m | 1500 |
| 3 | Density of steel | ρ_s | Kg/m^3 | 7850 |
| 4 | Density of water | ρ_w | Kg/m^3 | 1000 |
| 5 | Density of mud | ρ_m | Kg/m^3 | 1410 |
| 6 | Outer diameter of riser | d_o | m | 0.3934 |
| 7 | Inner diameter of riser | d_i | m | 0.2809 |
| 8 | Outer diameter of drill string | b | m | 0.1683 |
| 9 | Inner diameter of drill string | a | N/m | 0.1483 |
| 10 | Modulus of elasticity | E | GN/m^2 | 210 |
| 11 | Drag coefficient | C_D | - | 0.7 |
| 12 | Coefficient of added mass | C_m | - | 1.0 |

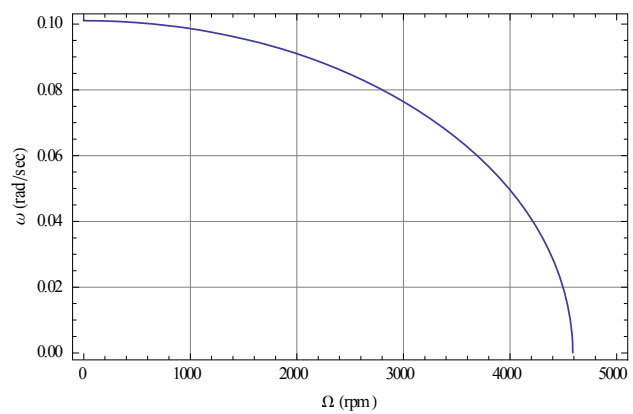
Figures 2(a) – (e) are plots showing the variation of the natural frequency of the drilling riser with rotational speed of the drill string. For each mode of vibration, the plots reveal a rotational speed at which the natural frequency of the drilling riser is theoretically zero. The value of the critical rotational speed hereafter referred to as the threshold rotational speed, increases with increasing mode of vibration and can be determined from the following expression

$$\Omega_{thres} = \sqrt{\frac{I}{\rho v I_P} \left[EI \left(\frac{m\pi}{l} \right)^2 + T + A_o \bar{P}_o - A_i \bar{P}_i \right]}$$

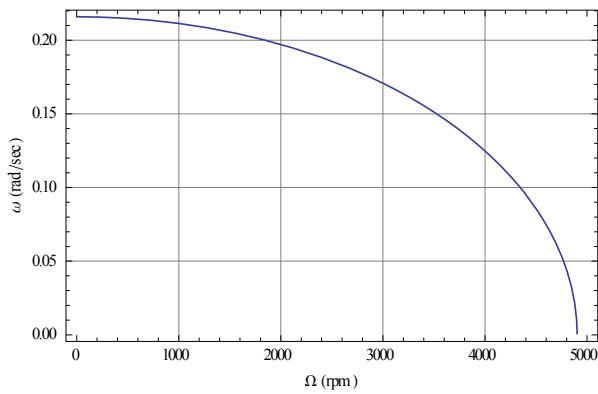
where Ω_{thres} is the threshold rotational speed of the drill string.



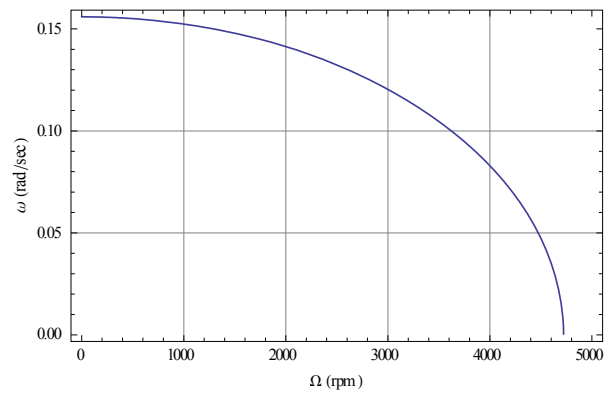
(a) Mode 1



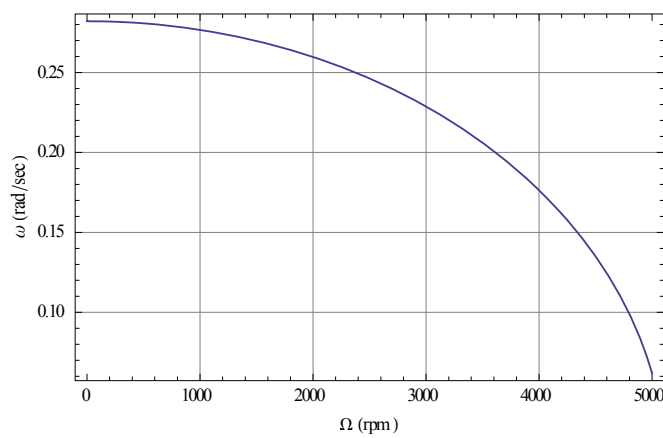
(b) Mode 2



(c) Mode 3



(d) Mode 4



(e) Mode 5

Fig.2. Variation of riser natural frequency with drill string rotational speed for different modes of vibration.

The physical significance of this is that there is a threshold rotational speed of the drill string for a given drilling riser configuration, at which there is no meaningful response of the drilling riser. Although the values of the threshold rotational speed as shown in Figs 2(a) – (e) are well beyond practical drilling speed range (i.e., less than 200 rev/min), the existence of such a phenomenon is noteworthy. Additionally, it can be inferred that before reaching the threshold rotational speed, the drill string would have failed in operation.

The time-domain responses for drill string rotational speeds of 0 rev/min , 500 rev/min , and 2000 rev/min are shown in Figs 1(a) – (c) respectively. It was assumed that the initial displacement and velocity of the riser are very small, and so, the initial conditions used were 1 mm and 1 mm/s respectively. The time-domain responses are all sinusoidal as is expected for an undamped free vibration. The amplitude of the temporal responses reveals that the rotation of the drill string increases the maximum displacement of the riser. The plots in Figs 3(a) – (c) corroborate this observation. Additionally, the plots in Figs 3(a) – (c) reveal that for the same rotational speed of the drill string, the amplitude of vibration reduces with increasing mode of vibration. This can be explained by the fact that the riser becomes stiffer at higher modes and therefore offers a better resistance to displacements.

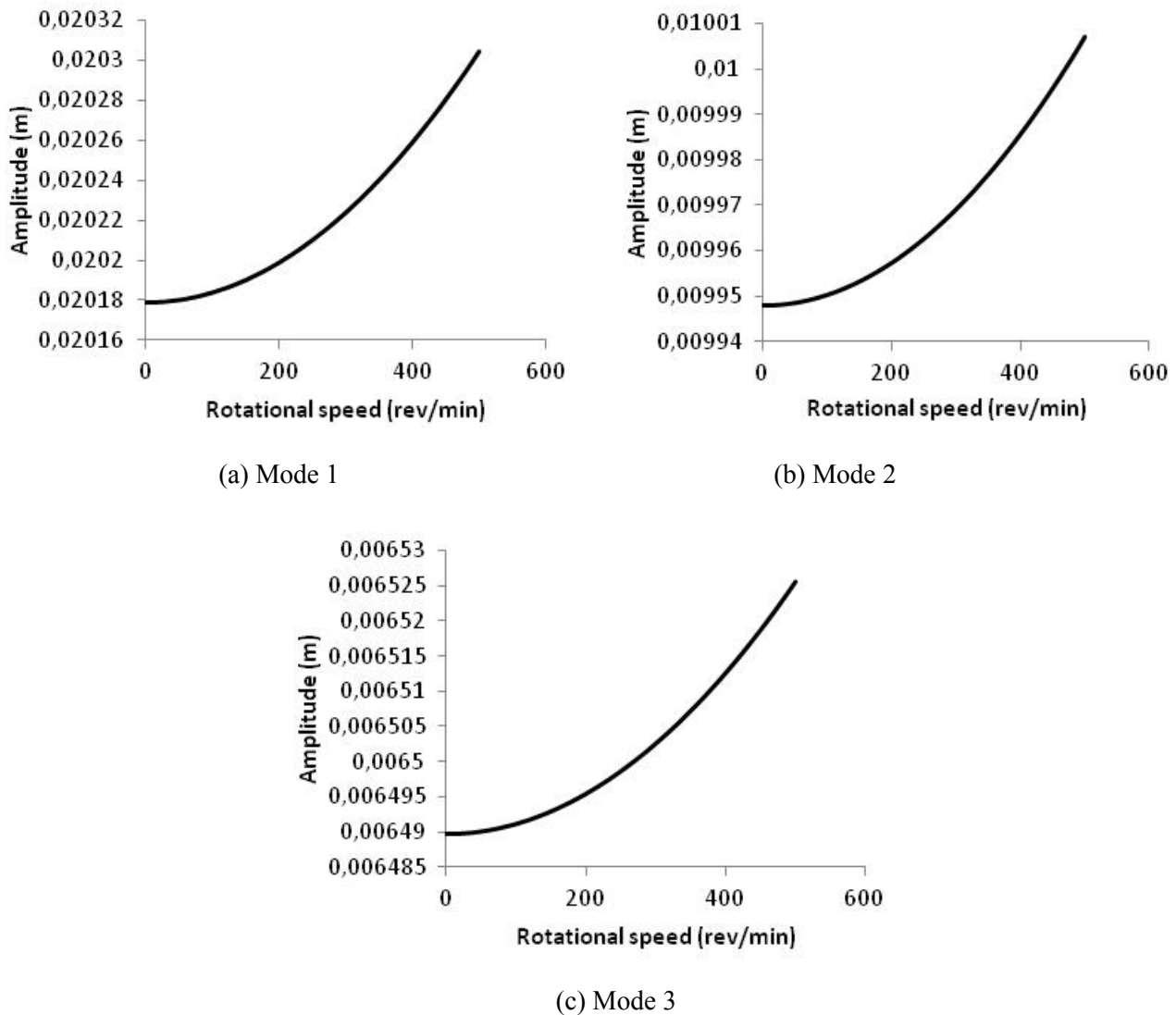


Fig.3. Time-domain response amplitude versus drill string rotational speed for different modes of vibration.

5. Conclusion

The effect of the rotational speed of the drill string of a drilling riser on the free vibration of the drilling riser has been investigated in this paper. The rotation of the drill string induces an axial force on the riser, which affects its response. This concept has been incorporated in developing the governing equation of the drilling riser, and the resulting differential equation has a variable coefficient, which is a function of the drill string rotational speed. The governing equation was solved, and results were simulated for the natural frequencies and temporal displacements by means of bespoke Mathematica™ codes. The results reveal that the effect of the rotation of the drill string is to reduce the natural frequency and increase the amplitude of vibration of the riser. The implication of these findings is that neglecting the effect of rotation of the drill string leads to under-estimation of the deflection and over-estimation of the natural frequency. It was also discovered that for a drilling riser with given dimensions, there exists a

rotational speed of the drill string at which the natural frequency of the riser is theoretically zero. This rotational speed has been referred to as the threshold rotational speed of the drill string. In the present simulations, the threshold rotational speeds for different modes of vibration were found to be way beyond practical drilling speeds. This study shows that the dynamics of a drilling riser during production is significantly different from that of general risers because of the rotation of the drill string, which influences the behaviour of the drilling riser.

Nomenclature

| | |
|-------------|---|
| A_O | – cross-sectional area of riser (external) |
| A_i | – cross-sectional area of riser (internal) |
| C_D | – drag coefficient |
| C_m | – inertia coefficient |
| D_O | – outer diameter of riser |
| D_i | – inner diameter of riser |
| EI | – flexural rigidity |
| F | – external force on riser |
| f | – force density |
| M | – moment of force |
| m_w | – mass of segment including added mass per unit length acting in the w direction |
| \bar{P}_o | – external pressure at the elevation of the centre of the cross-section (point s) |
| \bar{P}_i | – internal pressure at the elevation of the centre of the cross-section (point s) |
| T | – tension (axial force) |
| t | – time |
| U | – current velocity |
| V | – shear force |
| w | – horizontal axis |
| y | – vertical axis |
| γ_o | – weight density of outer fluid |
| γ_i | – weight density of inner fluid |
| γ_s | – weight density of riser material |
| ρ_w | – mass density of water |

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