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FINITE ELEMENT STUDIES ON FREE VIBRATION OF LAMINATED COMPOSITE CYLINDRICAL SKEW PANELS

C.V. SRINIVASA*
Department of Mechanical Engineering
GM Institute of Technology
Davangere, Karnataka, INDIA-577006
E-mail: srinikanadamani@gmail.com

Y.J. SURESH
Department of Mechanical Engineering
J.N.N. College of Engineering
Shivamogga, Karnataka, INDIA-577204
E-mail: yjsuresh06@gmail.com

W.P. PREMA KUMAR
Department of Civil Engineering
Reva Institute of Technology and Management
Bangalore, Karnataka, INDIA-560064
E-mail: wppremakumar@yahoo.co.in

This paper presents the finite element studies on free vibration of isotropic and laminated composite cylindrical skew panels. The analysis is performed using CQUAD4 and CQUAD8 elements of MSC/NASTRAN. The effects of the panel angle, skew angle, aspect ratio and length-to-thickness-ratio on fundamental frequency of isotropic cylindrical skew panels are studied. The effects of additional parameters such as the fiber orientation angle, numbers of layers and stacking sequence on the fundamental frequency of antisymmetric composite laminates are also studied. It is found that the CQUAD8 element yields better results than the CQUAD4 element in the validation and convergence studies. The CQUAD8 element is employed for the remaining part of the studies. The fundamental frequencies are found to increase with the panel angle and skew angle. When the number of layers in the laminate is large, the variation of the fundamental frequency with the number of layers is not appreciable. The boundary conditions are found to have a significant influence on the fundamental frequency.

Key words: cylindrical skew panel, panel angle, skew angle, antisymmetric laminates, free vibration, finite element, fundamental frequency.

1. Introduction

The skew or oblique cylindrical panels find wide application in the aircraft and spaceship industry. Few studies have been made on such panels. A great need exists for an extensive study on free vibration of skew cylindrical panels and the present work is one attempt in that direction. Kandasamy and Singh (2006) presented a numerical investigation of free vibration of skewed open cylindrical isotropic shells. First-order shear deformation and rotary inertia were included in the formulation. Thin and moderately thick shells were studied. Salil Haldar (2008) used a triangular shallow shell element for the free vibration analysis of laminated composite skewed cylindrical shell panels. Gulshan Taj and Chakrabarti (2013) studied the

* To whom correspondence should be addressed

dynamic response of a functionally graded skew shell using a C^o finite element formulation. Numerical results were presented for cylindrical, spherical and hyper shells for different boundary conditions and skew angles. The present investigation deals with the free vibration studies on isotropic and laminated composite cylindrical skew panels using the CQUAD8 finite element of MSC/NASTRAN. The effects of the skew angle, panel angle, fiber orientation angle, number of layers in the laminate, laminate sequence and boundary conditions on the fundamental frequencies of cylindrical skew panels are investigated.

2. Validation and convergence studies

The geometries of the regular cylindrical panel and skew cylindrical panel with dimensions are shown in Fig.1. Figure 2 illustrates a cylindrical skew panel with global and local coordinate systems and u and v are the displacement components in the x and y directions, respectively. Since u and v are inclined to the skew edges, the displacement boundary conditions cannot be applied directly. In order to overcome this, a local coordinate system (x', y') normal and tangential to the skew edges is chosen.

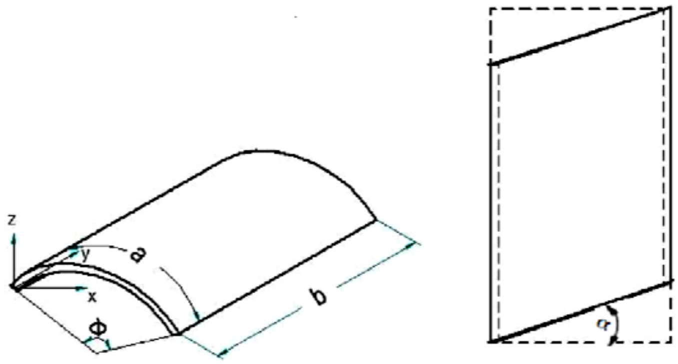

Fig.1. The geometry of skew cylindrical panels with dimensions.

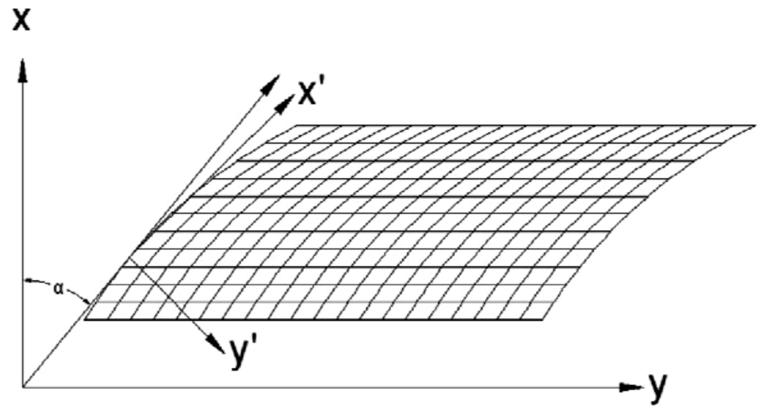

Fig.2. Global and local coordinate systems for cylindrical skew panels used in the finite element analysis.

2.1. Validation check

The validation for the CQUAD4 and CQUAD8 finite elements of MSC/NASTRAN has been made by comparing the non-dimensional fundamental frequency coefficient (K_f) values obtained with those of the literature. The same is presented in Tab.1 for the isotropic cylindrical skew panel. It can be seen from Tab.1

that the results obtained using the present elements are in good agreement with the literature values. In a similar manner a validation check has been made on antisymmetric graphite/epoxy angle-ply laminates. The material properties used are: $E_l / E_t = 40$, $G_{lt} / E_t = 0.5$ and $\nu_{lt} = 0.25$. The values of fundamental frequencies obtained are compared with those available in the literature and the same are presented in Tab.2. The K_f values obtained are in good agreement with those available in the literature. From Tabs 1 and 2 it can be seen that CQUAD8 element yields better results when compared with the CQUAD4 element. Hence the CQUAD8 element is employed in the present work for further study.

Table 1. Fundamental frequency coefficients (K_f) for isotropic cylindrical skew panels fixed at both the curved edges ($\alpha=45^o$, $\text{Ø}=60^o$, $L/R=4.0$).

t/R	Authors		Non dimensional frequency parameter (K_f)
0.01	Salil Haldar (2008)		*0.0240*
	Kandasamy *et al*. (2006)		*0.0240*
	Present	CQUAD4	*0.0245*
		CQUAD8	*0.0241*
0.1	Salil Haldar (2008)		*0.0840*
	Present	CQUAD4	*0.0849*
		CQUAD8	*0.0841*

Table 2. Fundamental frequency coefficients (K_f) for cantilever cylindrical skew composite panels fixed at one of the curved edge ($\alpha=45^o$, $\text{Ø}=60^o$, $L/R=4.0$, $t/R=0.01$).

Antisymmetric Laminate sequence	Researchers		Non dimensional frequency parameter (K_f)
0/90	Salil Haldar (2008)		*1.433*
	Present	CQUAD4	*1.445*
		CQUAD8	*1.435*
0/90/0/90	Salil Haldar (2008)		*1.484*
	Present	CQUAD4	*1.492*
		CQUAD8	*1.488*

2.2. Convergence study

To arrive at the optimum number of elements to be used in the finite element mesh for analysis, a convergence study has been undertaken. It has been performed on simply supported (S-S-S-S) (S3) (Jones, 1975) and clamped (C-C-C-C) (C2) (Jones, 1975) isotropic cylindrical skew panels having the aspect ratio (*=a/b*) of *1.0*, panel angle of 30^o and skew angles 0^o, 15^o, 30^o and 45^o using the CQUAD4 (four-node) and CQUAD8 (eight-node isoparametric curved shell element) elements. The convergence details are furnished in Tab.3.

Table 3. Convergence study for fundamental frequency coefficient (K_f) for isotropic cylindrical skew panels ($a/b=1$, $a/t=100$, panel angle (Ø) $=30^o$).

Finite Element Mesh	Finite Element Type	S-S-S-S				C-C-C-C			
		Skew Angle (α)				Skew Angle (α)			
		0^o	15^o	30^o	45^o	0^o	15^o	30^o	45^o
(20X20)	CQUAD4	0.3827	0.4186	0.5428	0.8151	0.5753	0.6143	0.7506	1.0674
	CQUAD8	0.3815	0.4176	0.5423	0.8171	0.5766	0.6161	0.7543	1.0775
(24X24)	CQUAD4	0.3829	0.4188	0.5432	0.8164	0.5761	0.6152	0.7523	1.0719
	CQUAD8	0.3812	0.4173	0.5419	0.8161	0.5764	0.6157	0.7537	1.0762
(28X28)	CQUAD4	0.3830	0.4190	0.5435	0.8172	0.5765	0.6158	0.7534	1.0746
	CQUAD8	0.3808	0.4169	0.5414	0.8151	0.5760	0.6153	0.7530	1.0748
(32X32)	CQUAD4	0.3830	0.4191	0.5436	0.8176	0.5769	0.6162	0.7541	1.0764
	CQUAD8	0.3803	0.4164	0.5409	0.8140	0.5756	0.6148	0.7523	1.0732
(36X36)	CQUAD4	0.3831	0.4191	0.5437	0.8178	0.5771	0.6165	0.7546	1.0777
	CQUAD8	0.3798	0.4159	0.5403	0.8130	0.5751	0.6142	0.7514	1.0715
(40X40)	CQUAD4	0.3832	0.4192	0.5438	0.8180	0.5772	0.6167	0.7550	1.0786
	CQUAD8	0.3797	0.4157	0.5400	0.8128	0.5750	0.6140	0.7510	1.0710

3. Results and discussion

The results of analysis are presented in the form of a non-dimensional fundamental frequency coefficient K_f defined as $K_f = \omega R \sqrt{\frac{\rho}{E}}$ for isotropic and $K_f = \omega R \sqrt{\frac{\rho}{E_t \ t^2}}$ for laminated composite cylindrical skew panels for the following cases:

a) simply supported and clamped isotropic skew panels and
b) simply supported and clamped antisymmetric angle-ply laminates.

3.1. Simply supported and clamped isotropic cylindrical skew panels

Numerical studies have been made for a number of skew panels with different aspect ratios, skew angles, and length-thickness ratio with simply supported and clamped boundary conditions. The results obtained are tabulated in Tab.4 and Tab.5 in the form of non-dimensional frequency coefficients (K_f) for the panel angle 15^o and 30^o, respectively. The following conclusions can be made from Tabs 4 and 5:

- K_f varies as the aspect ratio (a/b) varies for constant values of the skew angle (α) and a/t ratio
- K_f increases as the skew angle α increases for constant values of a/b and a/t.
- K_f increases as a/t decreases for constant values of the a/b ratio and skew angle.
- The panel angle is seen to have a considerable influence on K_f value.

Table 4. Free vibration of simply supported and clamped isotropic skew curved panels (panel angle =15^0).

a/b	a/t	Non-dimensional fundamental frequency coefficient (K_f)							
		Skew angle (α)							
		0^o	15^o	30^o	45^o	0^o	15^o	30^o	45^o
		S-S-S-S				C-C-C-C			
0.5	1000	0.2382	0.2553	0.3253	0.4832	0.2943	0.3152	0.3903	0.5735
	500	0.3006	0.3240	0.4346	0.6960	0.4122	0.4451	0.5637	0.8577
	100	0.5778	0.6349	0.8452	1.3887	0.9737	1.0489	1.3213	2.0063
	50	0.9517	1.0208	1.2704	1.8951	1.4456	1.5377	1.8726	2.7229
	20	1.6267	1.7139	2.0362	2.8830	2.8765	3.0319	3.6044	5.1022
1.0	1000	0.1972	0.2111	0.2583	0.3647	0.2507	0.2650	0.3154	0.4368
	500	0.2544	0.2835	0.3756	0.5569	0.3553	0.3797	0.4670	0.6810
	100	0.6267	0.6756	0.8525	1.2922	0.9288	0.9934	1.2265	1.8078
	50	1.0163	1.0821	1.3244	1.9514	1.1827	1.2591	1.5361	2.2358
	20	1.3356	1.4254	1.7468	2.5425	2.0885	2.2103	2.6528	3.7626
1.5	1000	0.1828	0.2027	0.2454	0.3505	0.2385	0.2535	0.3075	0.4409
	500	0.2456	0.2691	0.3440	0.4894	0.3565	0.3776	0.4512	0.6253
	100	0.7745	0.8277	1.0194	1.4940	0.9698	1.0369	1.2809	1.8988
	50	1.0289	1.1001	1.3590	2.0149	1.2421	1.3250	1.6261	2.3865
	20	1.3969	1.4945	1.8419	2.6970	2.2859	2.4291	2.9449	4.2166
2.0	1000	0.1755	0.1913	0.2386	0.3219	0.2488	0.2620	0.3085	0.4190
	500	0.2535	0.2730	0.3367	0.4721	0.3595	0.3822	0.4630	0.6605
	100	0.9579	1.0226	1.2467	1.8099	1.0222	1.0936	1.3529	2.0103
	50	1.0633	1.1379	1.4081	2.0889	1.3862	1.4807	1.8237	2.6890
	20	1.5512	1.6593	2.0453	2.9994	2.6855	2.8570	3.4699	4.9566
2.5	1000	0.1738	0.1867	0.2262	0.3054	0.2430	0.2571	0.3074	0.4306
	500	0.2734	0.2923	0.3566	0.5070	0.3970	0.4234	0.5180	0.7538
	100	0.9751	1.0439	1.2830	1.8712	1.0891	1.1654	1.4431	2.1468
	50	1.1116	1.1898	1.4729	2.1854	1.5652	1.6729	2.0631	3.0432
	20	1.7429	1.8636	2.2955	3.3633	3.1146	3.3112	4.0086	5.6675

Table 5. Free vibration of simply supported and clamped isotropic skew cylindrical panels (skew angle = 30^0).

a/b	a/t	Non-dimensional fundamental frequency coefficient (K_f)							
		Skew angle (α)							
		0^o	15^o	30^o	45^o	0^o	15^o	30^o	45^o
		S-S-S-S				C-C-C-C			
0.5	1000	0.1828	0.1934	0.2345	0.3272	0.2098	0.2234	0.2698	0.3801
	500	0.2364	0.2530	0.3224	0.4760	0.2926	0.3132	0.3869	0.5657
	100	0.4011	0.4428	0.6038	1.0139	0.6542	0.7091	0.9086	1.4108
	50	0.5583	0.6131	0.8124	1.3113	0.9591	1.0317	1.2940	1.9467
	20	1.0550	1.1322	1.3991	1.9890	1.6323	1.7244	2.0621	2.9347
1.0	1000	0.1467	0.1527	0.1794	0.2439	0.1694	0.1784	0.2104	0.2875
	500	0.1940	0.2088	0.2550	0.3571	0.2489	0.2628	0.3118	0.4283
	100	0.3791	0.4152	0.5396	0.8120	0.5746	0.6137	0.7505	1.0699
	50	0.5952	0.6421	0.8091	1.2077	0.8962	0.9574	1.1762	1.7099
	20	1.0351	1.1025	1.3480	1.9690	1.2770	1.3556	1.6407	2.3531
1.5	1000	0.1333	0.1380	0.1591	0.2114	0.1598	0.1677	0.1958	0.2629
	500	0.1791	0.1985	0.2407	0.3402	0.2352	0.2498	0.3018	0.4285
	100	0.4203	0.4517	0.5596	0.8009	0.6160	0.6566	0.8000	1.1425
	50	0.7364	0.7867	0.9646	1.3873	0.9522	1.0173	1.2530	1.8434
	20	1.0542	1.1268	1.3882	2.0354	1.3504	1.4370	1.7487	2.5153
2.0	1000	0.1276	0.1323	0.1548	0.2149	0.1557	0.1647	0.1970	0.2768
	500	0.1718	0.1871	0.2325	0.3091	0.2458	0.2584	0.3024	0.4043
	100	0.4917	0.5251	0.6419	0.9163	0.7355	0.7843	0.9586	1.3821
	50	0.9163	0.9757	1.1865	1.6791	1.0027	1.0717	1.3214	1.9457
	20	1.1045	1.1809	1.4547	2.1292	1.5164	1.6141	1.9638	2.8117
2.5	1000	0.1248	0.1353	0.1642	0.2083	0.1711	0.1820	0.2126	0.2789
	500	0.1700	0.1824	0.2198	0.2915	0.2390	0.2524	0.2998	0.4128
	100	0.5795	0.6181	0.7551	1.0853	0.8822	0.9412	1.1528	1.6691
	50	0.9610	1.0221	1.2394	1.7200	1.0662	1.1397	1.4054	2.0676
	20	1.1709	1.2514	1.5399	2.1383	1.7030	1.8113	2.1957	3.1122

3.2. Antisymmetric cylindrical skew panels

a) Simply supported boundary condition

The variations of the fundamental frequency coefficient (K_f) with the skew angle (α), fiber orientation angle (θ) and number of layers (NL) for antisymmetric angle-ply cylindrical skew laminates under S3 boundary conditions [Refer (Jones, 1975) for definition of S3 boundary conditions] are presented in Fig.3 through 6 for panel angle=15^o and Figs 7 through 10 for the panel angle=30^o, respectively. The increase in K_f with an increase in NL beyond NL= *10* is not appreciable in all the cases. There is considerable difference in K_f between NL=*2* and other values of NL. K_f is observed to peak at a value of fibre orientation angle between 25^o and 30^o for skew angles up to 30^o.

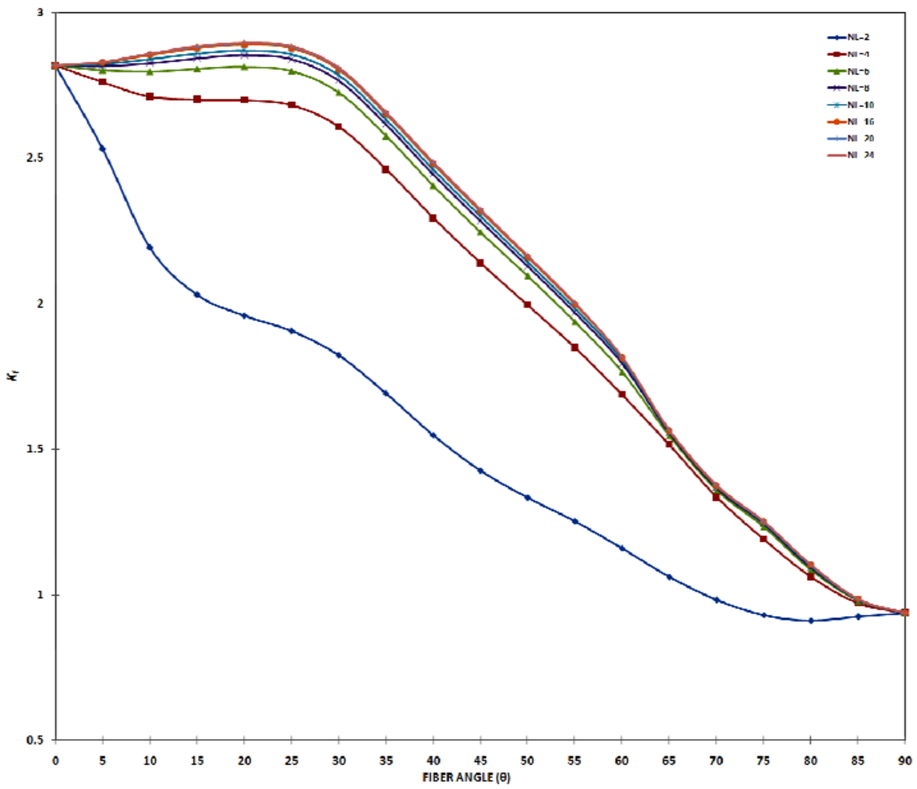

Fig.3. Values of K_f for simply supported antisymmetric angle-ply cylindrical skew panels ($\alpha=0^o$, $Ø=15^o$, S3, $a/b=1$, Graphite/Epoxy).

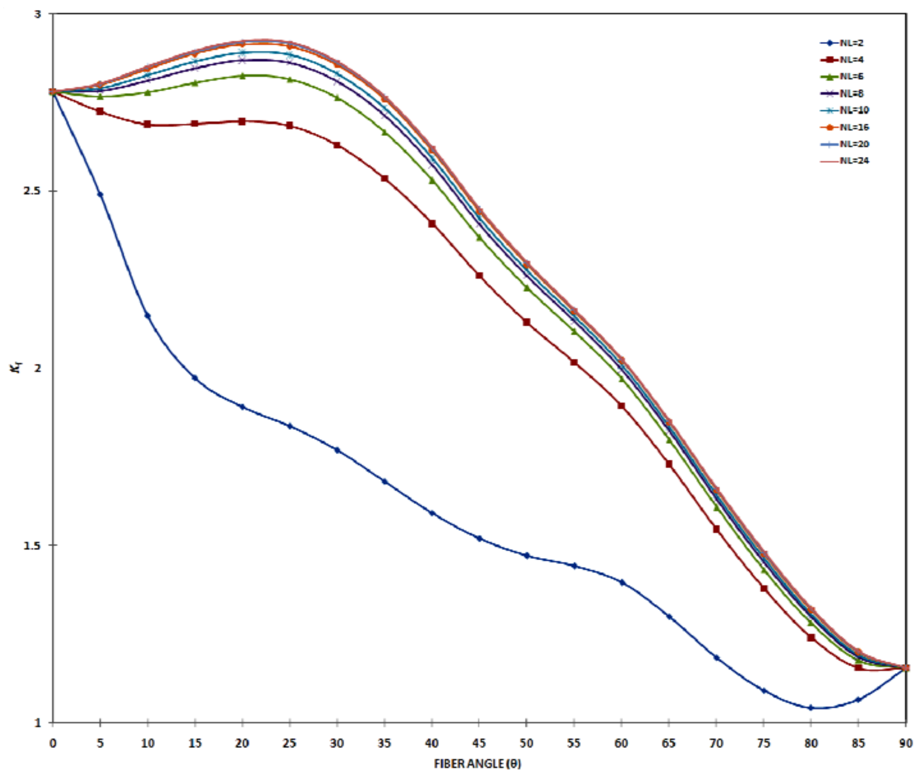

Fig.4. Values of K_f for simply supported antisymmetric angle-ply cylindrical skew panels ($\alpha=15^o$, $Ø=15^o$, S3, $a/b=1$, Graphite/Epoxy).

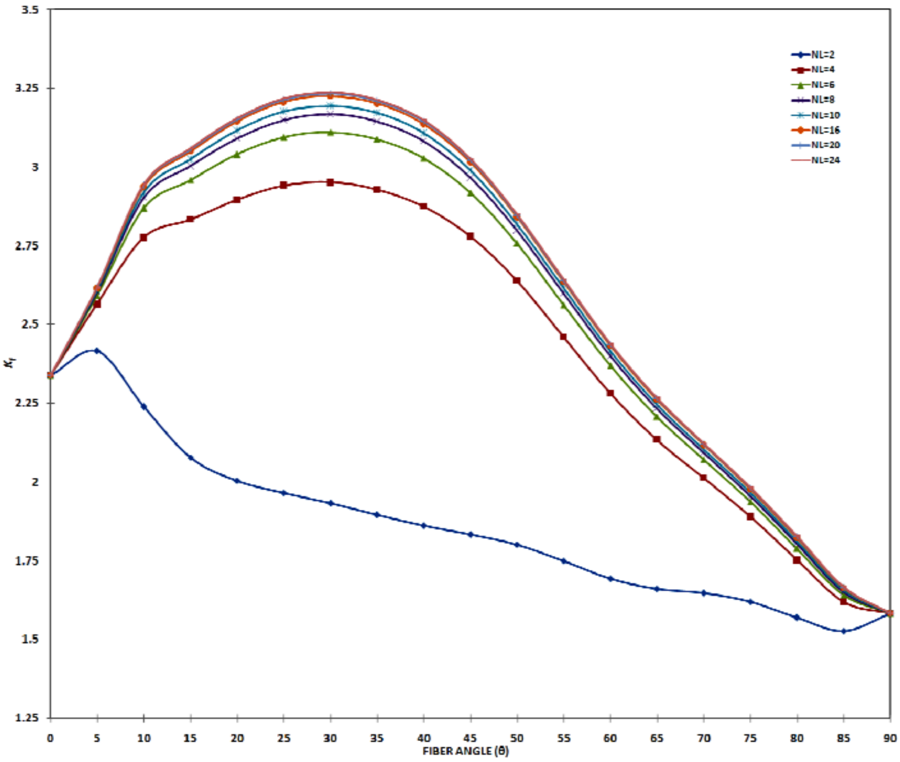

Fig.5. Values of K_f for simply supported antisymmetric angle-ply cylindrical skew panels ($\alpha=30^o$, $Ø=15^o$, S3, $a/b=1$, Graphite/Epoxy).

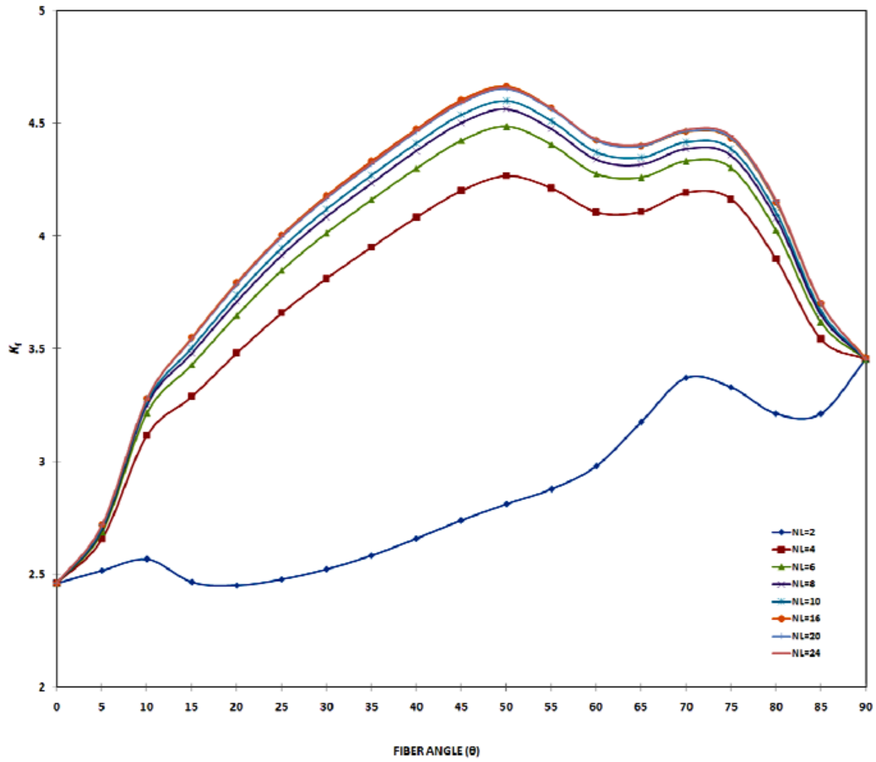

Fig.6. Values of K_f for simply supported antisymmetric angle-ply cylindrical skew panels($\alpha=45^o$, $Ø=15^o$, S3, $a/b=1$, Graphite/Epoxy).

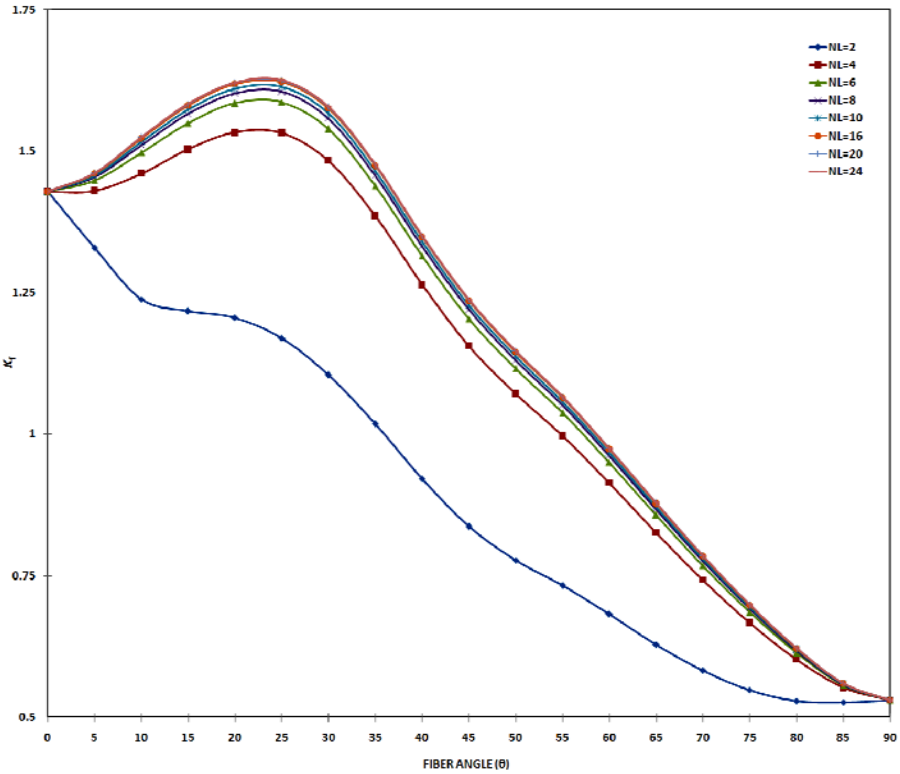

Fig.7. Values of K_f for simply supported antisymmetric angle-ply cylindrical skew panels($\alpha=0^o$, $Ø=30^o$, S3, $a/b=1$, Graphite/Epoxy).

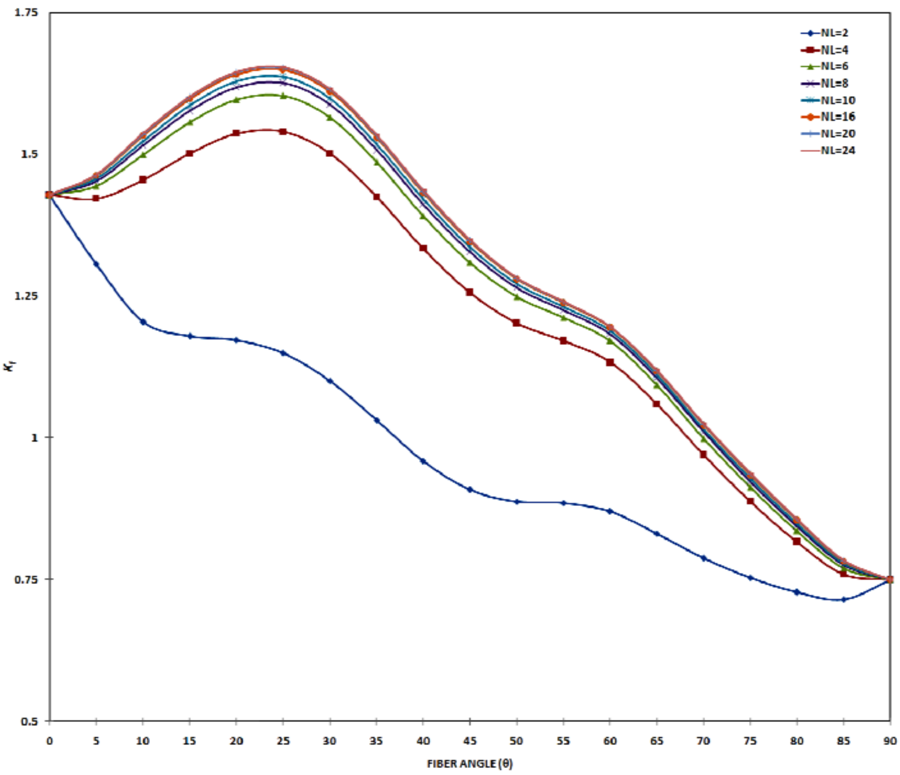

Fig.8. Values of K_f for simply supported antisymmetric angle-ply cylindrical skew panels ($\alpha=15^o$, $Ø=30^o$, S3, $a/b=1$, Graphite/Epoxy).

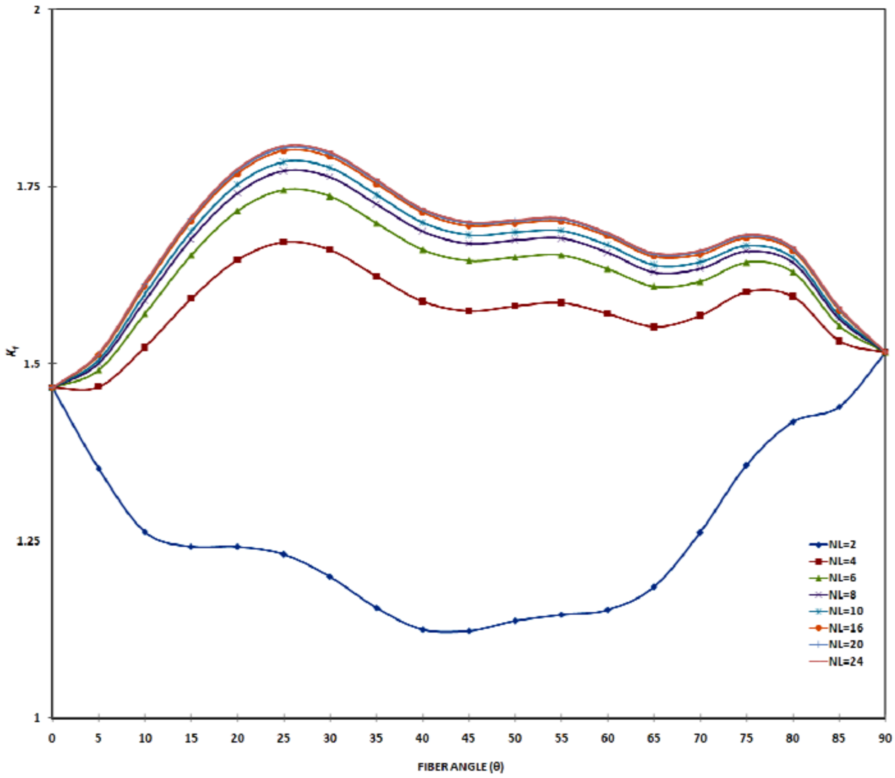

Fig.9. Values of K_f for simply supported antisymmetric angle-ply cylindrical skew panels (α=*30°*, Ø=*30°*, S*3*, *a/b=1*, Graphite/Epoxy).

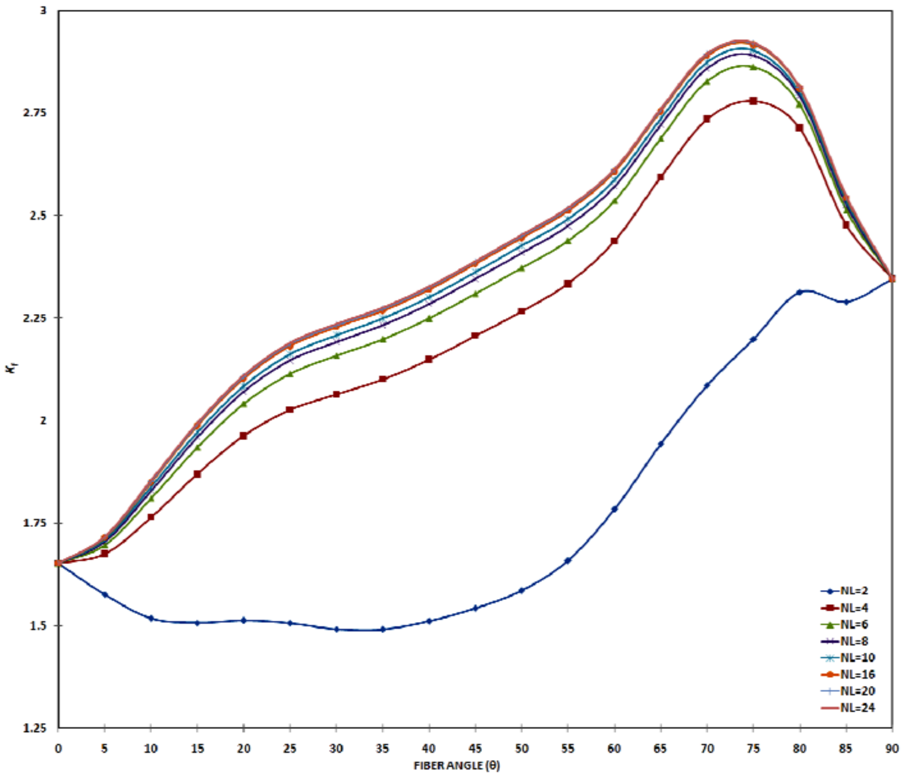

Fig.10. Values of K_f for simply supported antisymmetric angle-ply cylindrical skew panels (α=*45°*, Ø=*30°*, S*3*, *a/b=1*, Graphite/Epoxy).

b) Clamped boundary condition

For antisymmetric angle-ply cylindrical skew panels with skew angles of α=0^o,15^o, 30^o and 45^o, the variations of K_f with θ and NL are presented in Figs 11 through 14 for the panel angle=15^o and in Figs 15 through 18 for the panel angle= 30^o. For NL = *2*, K_f initially decreases and varies later as shown in the figures. The increase in K_f with NL for NL greater than 10 is not appreciable. The boundary condition has a significant influence on the natural frequency, the clamped condition yielding a higher value compared with the simply supported condition.

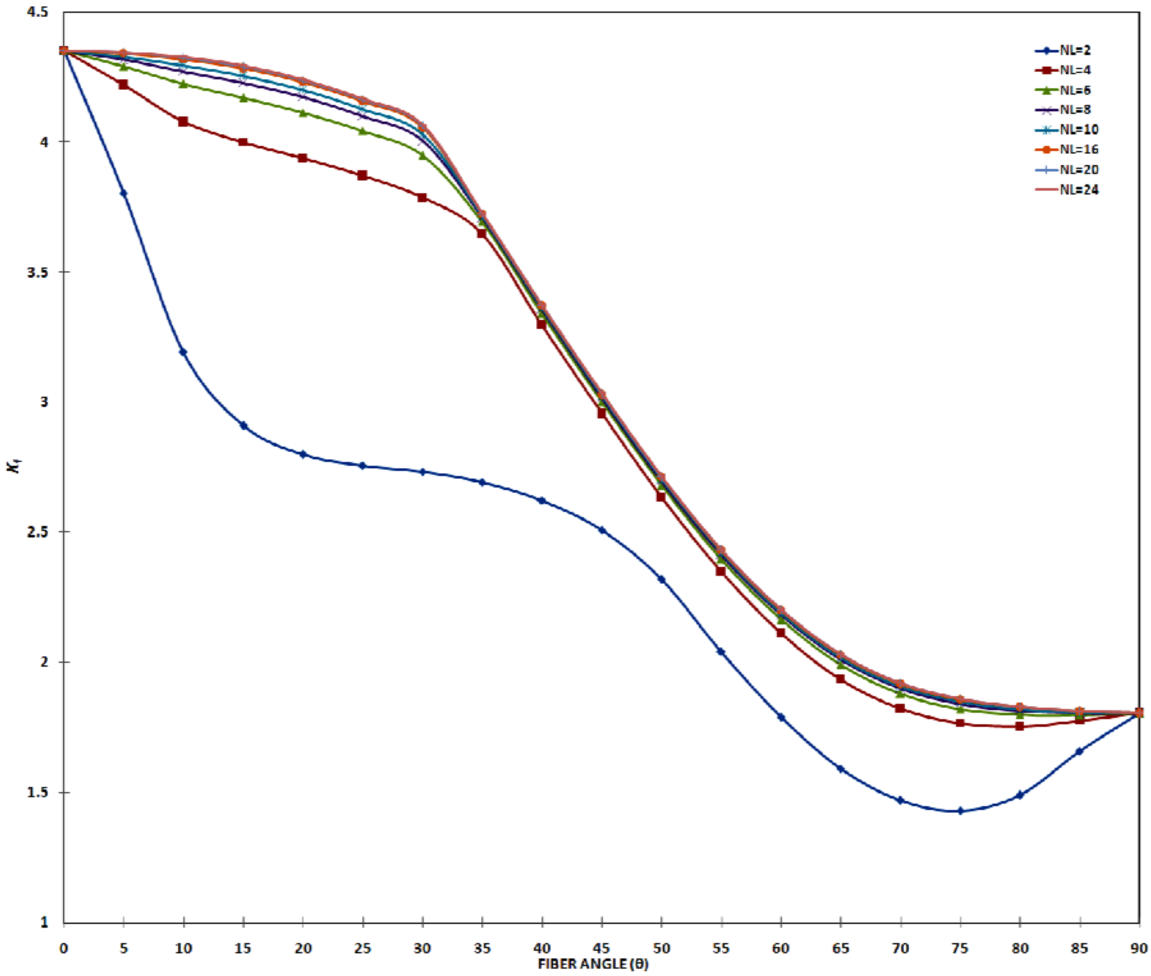

Fig.11. Values of K_f for clamped supported antisymmetric angle-ply skew panels (α=0^o, Ø=15^o C2, *a/b=1*, Graphite/Epoxy).

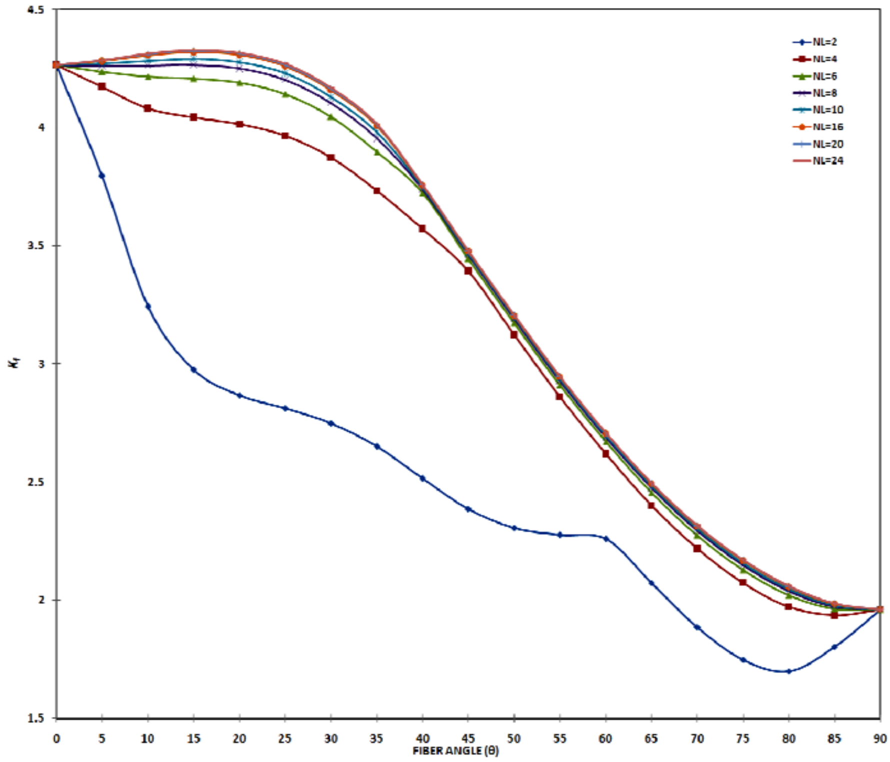

Fig.12. Values of K_f for clamped supported antisymmetric angle-ply skew panels (α=15^o, Ø=15^o C2, $a/b=1$, Graphite/Epoxy).

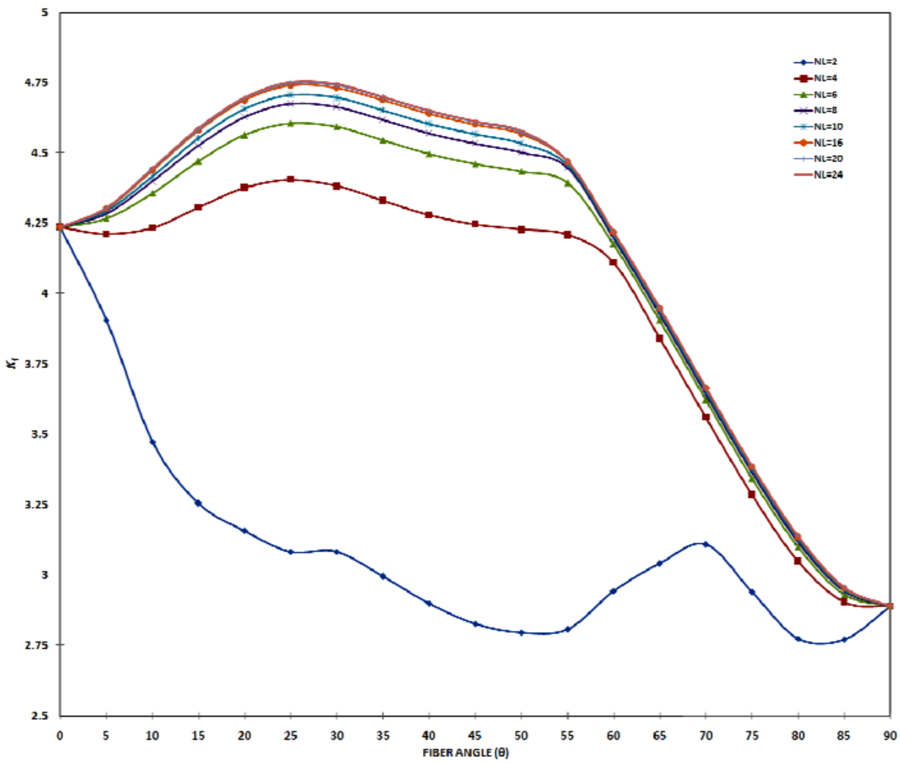

Fig.13. Values of K_f for clamped supported antisymmetric angle-ply skew panels (α=30^o, Ø=15^o C2, $a/b=1$, Graphite/Epoxy).

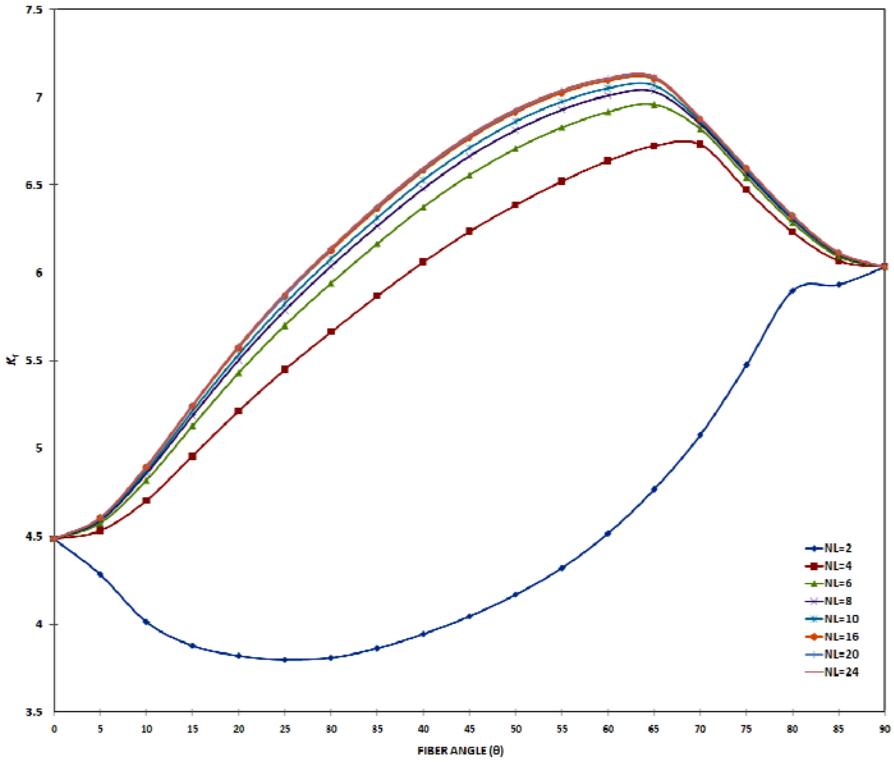

Fig.14. Values of K_f for clamped supported antisymmetric angle-ply skew panels (α=45^o, Ø=15^o C2, *a/b=1*, Graphite/Epoxy).

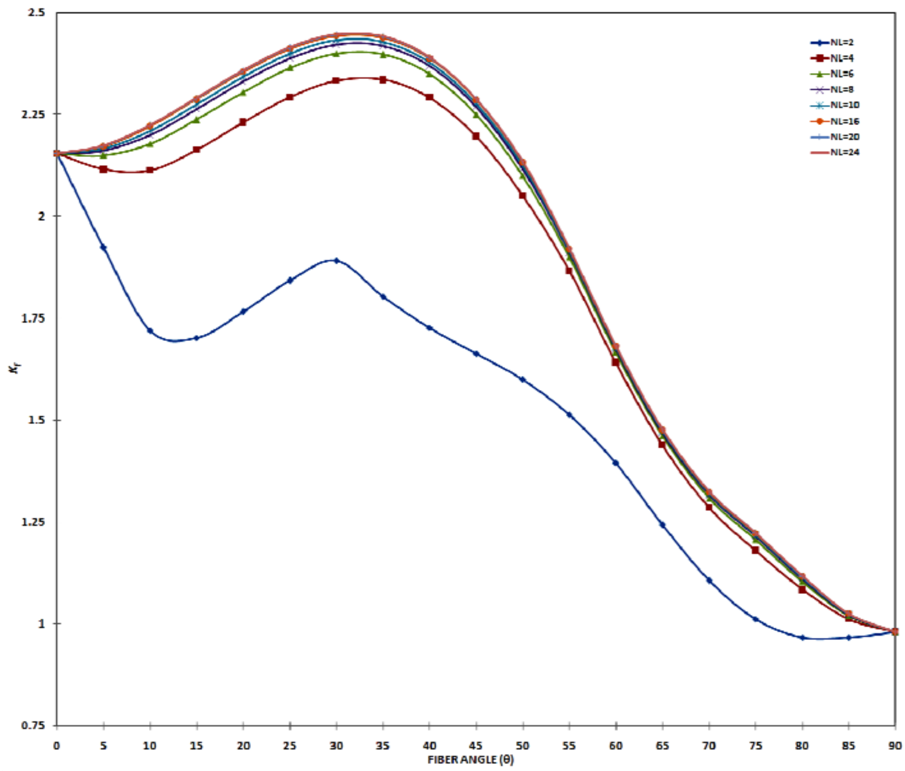

Fig.15. Values of K_f for clamped supported antisymmetric angle-ply skew panels (α=0^o, Ø=30^o C2, *a/b=1*, Graphite/Epoxy).

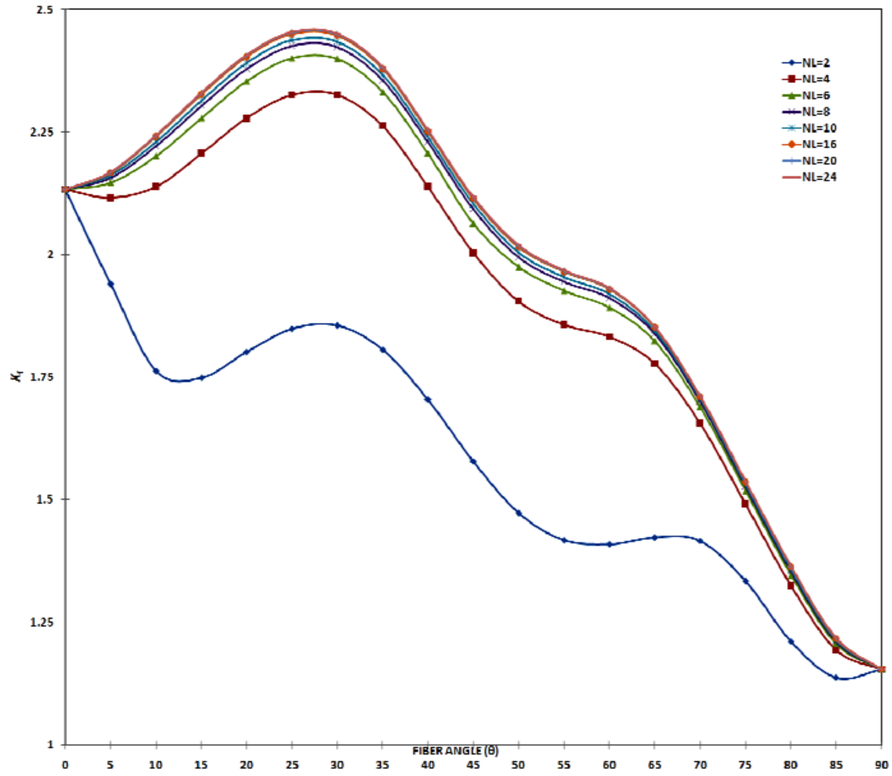

Fig.16. Values of K_f for clamped supported antisymmetric angle-ply skew panels ($\alpha=15^o$, $Ø=30^o$ C2, $a/b=1$, Graphite/Epoxy).

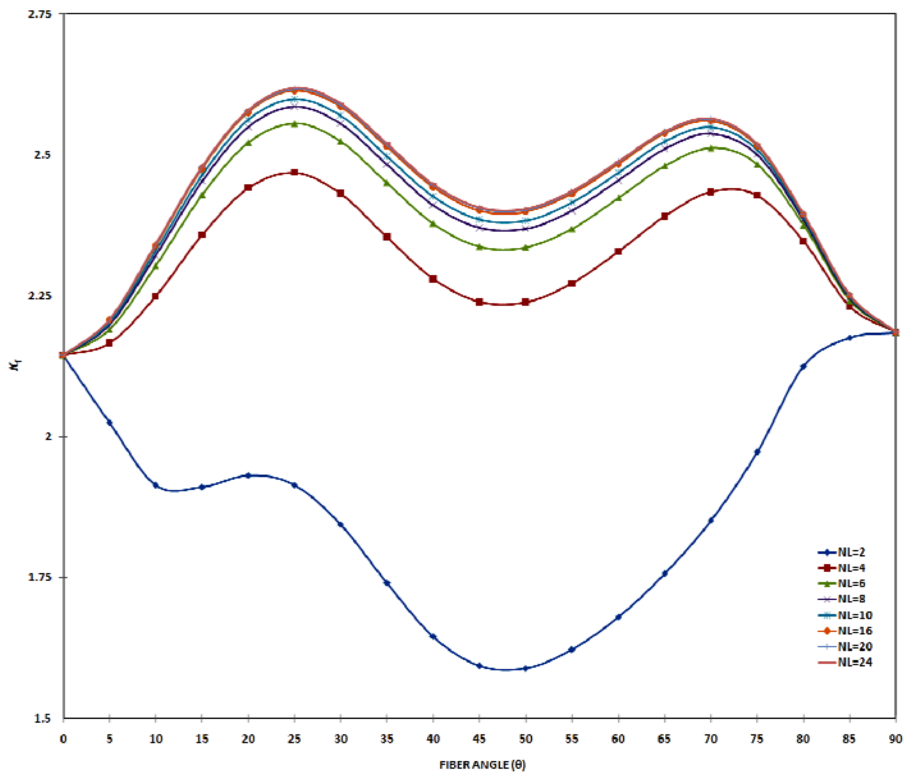

Fig.17. Values of K_f for clamped supported antisymmetric angle-ply skew panels ($\alpha=30^o$, $Ø=30^o$ C2, $a/b=1$, Graphite/Epoxy).

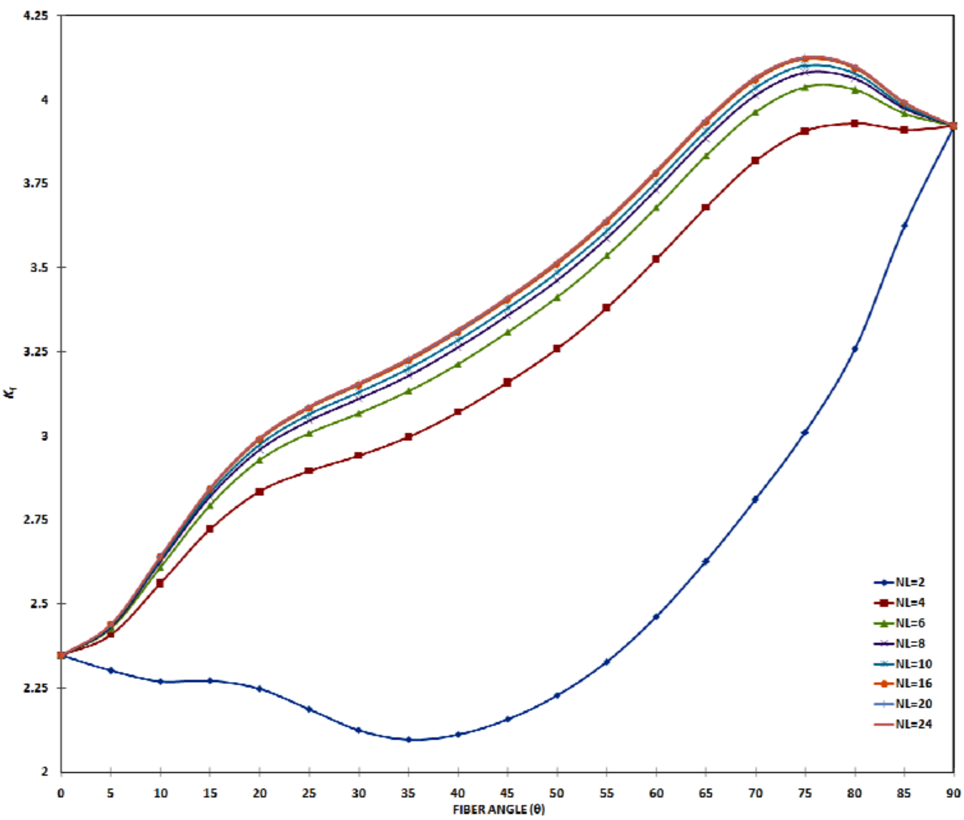

Fig.18. Values of K_f for clamped supported antisymmetric angle-ply skew panels (α=45^o, Ø=30^o C2, $a/b=1$, Graphite/Epoxy).

4. Conclusions

The following conclusions are made based on the above study of skew panels with simply supported and clamped boundary conditions.

4.1. Isotropic skew cylindrical panels

The aspect ratio, skew angle, panel angle, *a/t* ratio and boundary conditions have a considerable bearing on the value of the first natural frequency of the panel. K_f increases as the skew angle α increases for constant values of *a/b* and *a/t*. K_f increases as *a/t* decreases for constant values of the *a/b* ratio and skew angle.

4.2. Antisymmetric skew cylindrical panels

The skew angles, panel angle, fiber orientation angle, number of layers and boundary conditions have a bearing on the value of the first natural frequency of the panel. The increase in K_f with an increase in NL beyond NL= *10* is not appreciable in all the cases. There is considerable difference in K_f when NL=*2* and NL ≥*4*. K_f is observed to peak at a value of fibre orientation angle between 25^o and 30^o for skew angles up to 30^o (simply supported boundary condition). The boundary condition has a significant influence on the natural frequency, the clamped condition yielding a higher value than the simply supported condition.

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Nomenclature

a – projected width of panel
b – length of panel
E – modulus of elasticity of the material of isotropic panel
E_t – Young's modulus of the lamina in the transverse direction
K_f – non-dimensional frequency coefficient (defined later)
NL – number of layers in the laminate
t – panel thickness
α – skew angle of the panel
θ – fiber orientation angle of the lamina
ρ – mass density of the material of the panel
ω – natural angular frequency of panel in radian/sec
Ø – panel angle at the centre of curvature

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