APPLICATION OF MULTI-VALUED WEIGHTING LOGICAL FUNCTIONS IN THE ANALYSIS OF A DEGREE OF IMPORTANCE OF CONSTRUCTION PARAMETERS ON THE EXAMPLE OF HYDRAULIC VALVES

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In the optimization process, changes in the construction parameters value influence the behaviour of functions depending on time. Weighting logical coefficients for the stabilisation time are taken into consideration here, i.e., a shorter (better) stabilisation time has a more important (bigger) value of the weighting coefficient. An example of applying weighting logical functions in the analysis of a degree of importance of construction parameters of a hydraulic valve is presented in the paper.

Key words: optimization process, weighting logical coefficients, hydraulic valves, pomps.

1. Introduction

Among tools supporting decision making processes, it is possible to differentiate decision tables and trees, dendrites, tree classifiers as well as graphs. These tools are included in the so-called decision support methods based on graphs. A set of decisions (and relations between them) is written in a graphic way out of a mathematical model which is the main basis of the decision process realisation which a decision-maker can use in order to solve problems of any kind. Modelling of the whole process is necessary in the case of supporting decision-making processes in the optimization of mechanical systems. A lot of different feedbacks between elements of internal structure can be differentiated in the objects being analysed (e.g., mechanical systems). It is necessary to use an appropriate graphic model where the cause and effect relationship, which takes place inside an analysed object, occurs. Fluid-flow machines form a vast group of sets used in industry (Francis and Betts, 1997; Giergiel, 1990; Kurowski, 2001). Decision tables and logical functions (Deptuła, 2014; Stępniewski, 1994) can be applied in the issues of modelling machine systems with differential equations (ordinary and partial ones). It results from the fact that non-linear elements can be divided into a finite number of linear elements (parts) what leads to getting several linear systems. Discrete optimization of fluid-flow machines (Suzuki and Urata, 2003; Żak and Stefanowski, 1994) is based on indicating the degree of importance of construction and exploitation parameters. Guidelines concerning the sequence of making decisions result from multi-valued decision trees and taking into consideration the realisation of the assumed purpose function (e.g., the system stabilisation).

2. Multi-valued logical functions with weighting coefficients

Graphs of output data with the indicated stabilisation time as modelling results (e.g., in the programs such as: Matlab, Fluent) depend on given data of construction parameters. Changes in such values (e.g., decreasing, increasing, remaining unchanged), in the process of designing the set for different work

conditions can be written in the form of a code in multi-valued logic, whereas the set of design guidelines can be presented as a sum of multi-valued logical products.

2.1. Weighting coefficients

In the partial multi-valued logical function of n variables $(m_1,...,m_n)$ - valued, the weighting coefficient w_i before the canonical product has the value within this scope $\langle w_1,...,w_n \rangle$, if $w_j = w_{j-1} + w_{j-2} + ... + w_1$, where $j = 2,...,n$. Therefore, function weights in the graph in Fig.1 can be described by the following set of logical equations: $f_2 : w_2(0,1,2)$, which means that the f_0 stabilisation is obtained the fastest and the f_2 stabilisation is obtained the latest, which means $w_0 > w_1 > w_2$ (Deptuła, 2011; Deptuła and Partyka, 2010), that is f_i and f_j, $w_i < w_j$, if $t_t > t_j$.
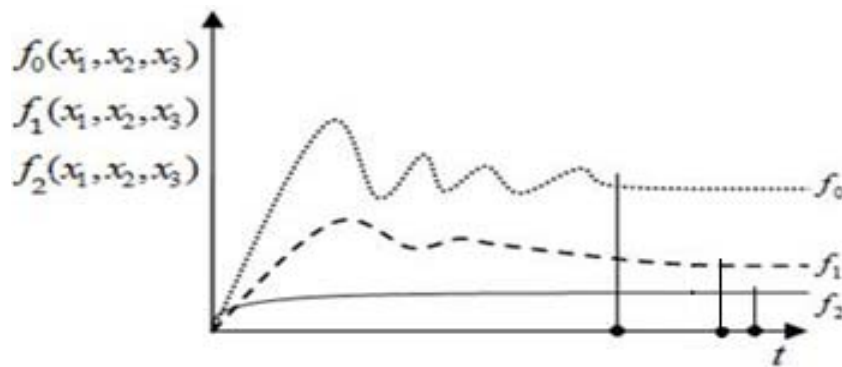

Fig.1.   Graphs of the functions f_0, f_1, f_2 depending on time for the coded version of multi-valued decision variables $x_1 = 0, x_2 = 1, x_3 = 2$.

Then, an alternative, multi-valued normal form is created where a bigger logical weighting coefficient means a shorter stabilisation time (Deptuła and Partyka, 2012).
It is possible to apply the Quine – McCluskey algorithm of multi-valued functions minimization in multi-valued logical functions with weighting products (Deptuła, 2014).

3. Sets of multi-valued logical equations with weighting products

In the case of flow rate calculations (e.g., in positive-displacement pumps or centrifugal pumps) many characteristics are taken into account at the same time and they form the set of functions $f_1, f_2, f_3,...,f_n$.
The set of equations can be defined as Y_S^R

$$Y_S^R : \begin{cases} R_i = r_i + r_{i+j} + ... + r_n, \\[2mm] R_{i+1} = r_i + r_{i+j} + ... + r_n, \\[2mm] R_m = r_i + r_{i+j} + ... + r_n. \end{cases} \tag{3.1}$$

where: R- the set of logical equations- $R : \{R_i, R_{i+1}, ..., R_m\}$, S- the set of canonical products as elements- $S : \{r_i, r_{i+1}, ..., r_n\}$, if: $S \subset R$, $R_i = r_i + r_{i+1} + ,..., + r_n$.

The set of multi-valued logical equations is presented in the form of a morphological table where the number of verses is equal to the number of equations $R_j, j=1, ..., m$.

The set of multi-valued logical equations can be solved using combinatorics in view of morphological analysis with maintaining postulates of the Rosser-Turguette system (Deptuła, 2014; 2011).

Example 1.

Logically presented theoretical possibilities of changes in the numbers of construction parameters have the following form: $x_1=0, 1, x_2=0, 1, 2, x_3=0, 1,$ (where the sign "_" means "to be kept unchanged" and it was obtained out of modelling of the set of two multi-valued logical equations for output data (Deptuła and Partyka, 2012)

$$\begin{cases} y_1(t) = 1(001) + 1(011) + 2(021) + 1(000), \\ \\ y_2(t) = 1(001) + 2(011) + 3(021) + 1(000), \end{cases} \tag{3.2}$$

After minimization, the real solution can be written in the following form

$$y = 1 \cdot 001 + 1 \cdot 011 + 2 \cdot 021 + 1 \cdot 000, \quad \text{what results in:} \quad 1 \cdot (00-) + 1 \cdot (0-1) + 2 \cdot (021).$$

4. Weighting multi-valued logical functions in the analysis of a degree of importance of construction parameters of the overflow valve

The overflow valve is applied in the systems in order to let the excess of fluid flow to container where the pump efficacy is bigger than the need. An example of a drive system of an actuator with an overflow valve is presented in Fig.2 (Deptuła, 2014; Żak and Stefanowski, 1994; Smith, 2003).
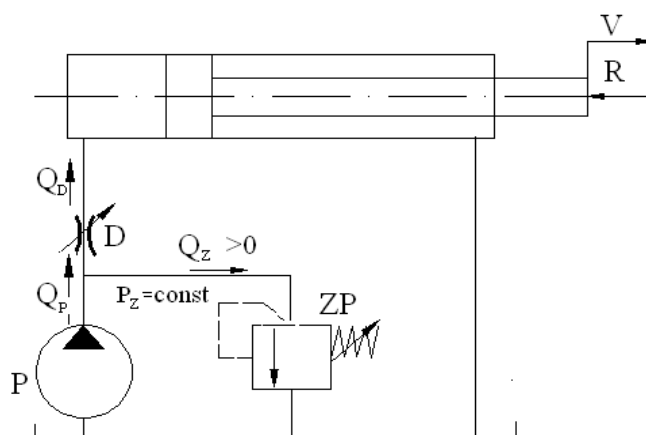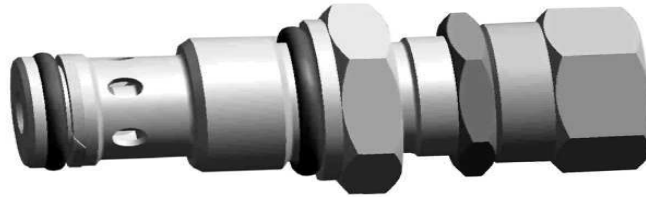
Fig.2. A scheme of an actuator with an overflow valve.

The equation of forces acting on the closing component of a valve is presented in the following way (Deptuła, 2014)

$$\frac{Q_p^{\,2}}{A_l}\rho + \rho \cdot A_2 + \rho \cdot l\frac{dQ_p}{dt} = G_{ap} + S + k \cdot x + f\frac{dx}{dt} + m\frac{d^2 x}{dt^2} + \Phi\sqrt{2 \cdot \rho} \cdot \cos(v) \cdot Q_p \sqrt{p} \,, \qquad (4.1)$$

whereas equations of flows have the following form

$$Q = \mu \cdot K \cdot x\sqrt{p} + A_l\frac{dx}{dt} + \frac{V}{B}\frac{dp}{dt} \,, \qquad (4.2)$$

$$Q_p = \mu \cdot K \cdot x\sqrt{p} + A_l\frac{dx}{dt} \qquad (4.3)$$

where

$$K = \pi \cdot d_m\sqrt{\frac{2}{\rho}} \,. \qquad (4.4)$$

Equations of the valve work in a dimensionless form used to make simulation are presented in the following form

$$\rho\frac{Q_o^{\,2}}{A_l S_o}Q_{pw}^2 + \frac{A_2 p_o}{S_o}p_w + \frac{T_{Qp}}{T_o}\frac{dQ_{pw}}{dt_w} = 1 + \frac{kx_o}{S_o}x_w + \frac{T_f}{T_o}\frac{dx}{dt_w} + \left(\frac{T_{ms}}{T_o}\right)^2\frac{d^2 x}{dt_w^2} +$$

$$+\Phi\frac{\sqrt{2\rho}}{S_o}\cos(v)Q_o Q_{pw}\sqrt{p_o}\sqrt{p_w}\,, \qquad (4.5)$$

$$Q_w = \mu x\sqrt{p_w} + \frac{T_A}{T_o}\frac{dx}{dt_w} + \frac{dp_w}{dt_w} \,, \qquad (4.6)$$

$$Q_{pw} = \mu x\sqrt{p_w} + \frac{T_A}{T_o}\frac{dx}{dt_w} \,. \qquad (4.7)$$

4.1. Weighting coefficients

In order to make a discrete optimization, changes in parameters have been coded in the following way: 0- large decrease, 1- small decrease, 2- without changes, 3- increase, 4- large increase (for m and k) and : 0- small decrease, 1- without changes, 2- increase (for d). For example, a combination of changes **122** means a small decrease in mass m, leaving the spring constant without changes k and an increase of the diameter d. On the other hand, the combination **402** means a large increase in mass m, a large decrease of the spring constant k and an increase of the diameter d in relation to the adopted arithmetic values in the early stage of designing. Depending on the adopted combinations of code changes in parameters m, k and d in canonical products, the behaviour of functions which depend on time is different.

When we look at the behaviour of the functions x, Q and p at the time of stabilization $t_w < 200t_o$ (strict condition), if there are products of code changes of parameters m, k and d the following values of weighting parameters have been adopted:

- w_i=4, $t_w \leq 50t_o$; w_i=3, $50t_o < t_w \leq 100t_o$; w_i=2, $100t_o < t_w \leq 150t_o$; w_i=1, $150t_o < t_w \leq 200t_o$.

When we increase the stabilisation time to $t_w < 800t_o$ (liberal work conditions), it has been assumed that:

- w_i=4, $t_w \leq 200t_o$; w_i=3, $200t_o < t_w \leq 400t_o$; w_i=2, $400t_o < t_w \leq 600t_o$; w_i=1, $600t_o < t_w \leq 800t_o$.

The weighting coefficient w_i in the case of code indications depends on the stabilisation time t, but $l_i < l_j$ if $t_i > t_j$. Figure 3 shows exemplary plots of functions x, Q and p, when appropriate changes of code combinations m, k and d occur. Appropriate products of combinations in code changes have the following weighting coefficients:

- in the case of a limitation $t_w < 200t_o$: x: $2 \cdot (222) + 1 \cdot (212) + 1 \cdot (121)$; Q: $2 \cdot (222) + 3 \cdot (212)$; p: $2 \cdot (222) + 2 \cdot (212)$;

- in the case of a limitation $t_w < 800t_o$: x: $4 \cdot (222) + 4 \cdot (212) + 4 \cdot (121)$; Q: $4 \cdot (222) + 4 \cdot (212) + 3 \cdot (212)$; p: $4 \cdot (222) + 4 \cdot (212) + 3 \cdot (212)$.
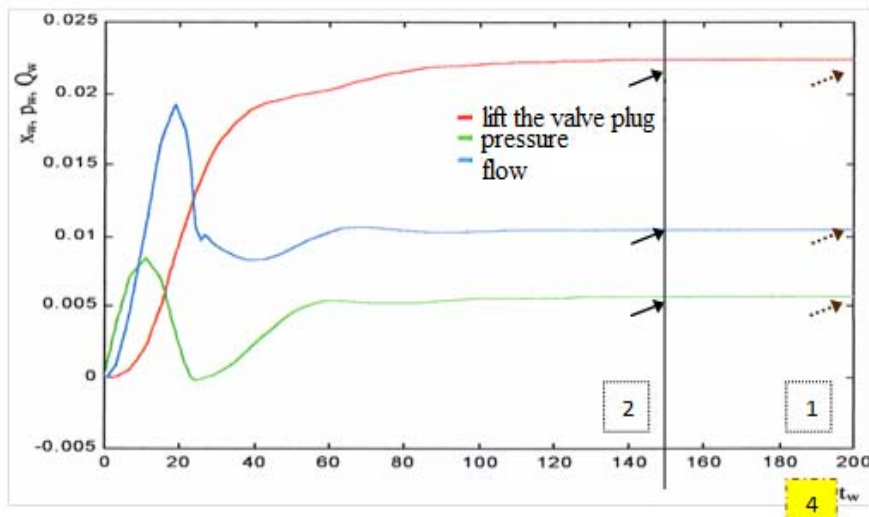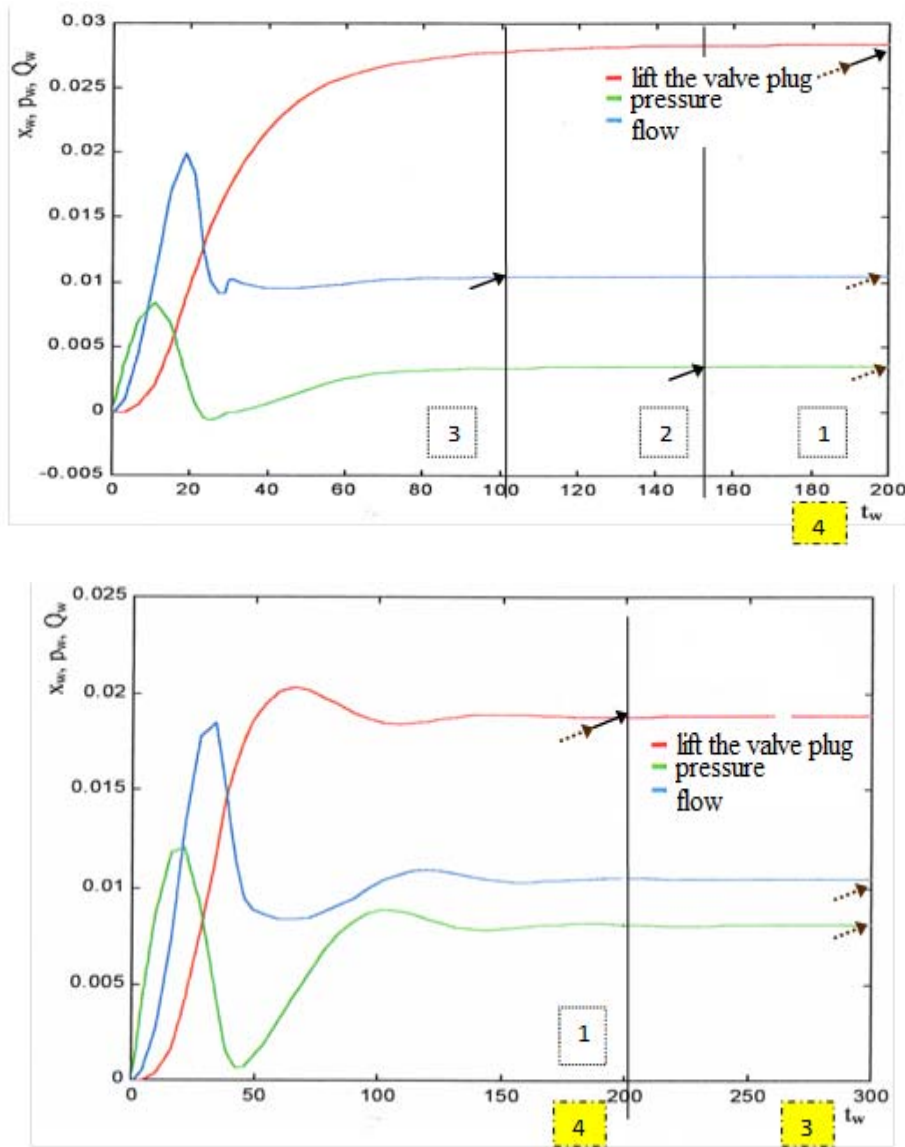
Fig.3. Time function plots *x*, *Q*, *p* with the indicated stabilisation time and weighting coefficients in the case of limitations: $t_w < 200\ t_o$ ⤷ → and $t_w < 800\ t_o$ ⤷ ⋯► of code changes in parameters: *m, k* and *d*: *222*; *212*, *121* (Deptuła, 2014).

Table 1. KAPN of changes in parameter values *m*, *k* and *d*($t_w < 200\ t_{o,}\ \dfrac{w_{max}}{w_{stab.}} < 3.6$).

m	k	d	m	k	d	m	k	d
2	2	2	1	0	2	0	1	2
2	1	2	0	2	1	0	0	1
1	2	1	0	2	2	0	0	2
1	2	2	0	1	1	0	3	2
1	1	1	1	1	2	1	3	2
2	0	2						

In the case of the overflow valve, a set of three multi-valued logical equations of output data x, Q, p was obtained out of modelling (Deptuła, 2014) respectively for a limiting condition

a) $t_w < 200t_o$

$$Y_{t_w<200t_o} = \begin{cases} x = 2\cdot222+1\cdot212+1\cdot121+2\cdot122+1\cdot111+2\cdot112+1\cdot132+2\cdot202+2\cdot102+ \\ +2\cdot021+3\cdot022+3\cdot032+2\cdot011+3\cdot012+1\cdot001+3\cdot002 \\ \\ Q = 2\cdot222+3\cdot212+3\cdot122+1\cdot111+4\cdot112+3\cdot132+3\cdot202+4\cdot102+3\cdot021+4\cdot022+ \\ +3\cdot032+4\cdot011+4\cdot012+4\cdot001+4\cdot002+2\cdot312+3\cdot300+3\cdot101+2\cdot000+2\cdot100 \\ \\ p = 2\cdot222+2\cdot212+3\cdot122+1\cdot111+3\cdot112+2\cdot132+2\cdot202+3\cdot102+ \\ +2\cdot021+3\cdot022+3\cdot032+2\cdot011+3\cdot012+1\cdot001+3\cdot002+1\cdot101 \end{cases}$$

b) $t_w < 800t_o$

$$Y_{t_w<800t_o} = \begin{cases} x = 4\cdot222+4\cdot212+4\cdot121+4\cdot122+4\cdot111+4\cdot112+4\cdot132+ \\ +4\cdot202+4\cdot102+4\cdot021+4\cdot022+4\cdot032+4\cdot011+4\cdot012+4\cdot001+ \\ +4\cdot002+3\cdot322+3\cdot312+3\cdot031+3\cdot010+3\cdot232+3\cdot300+3\cdot020+ \\ +2\cdot221+2\cdot332+1\cdot211+1\cdot201+3\cdot101+1\cdot422+3\cdot000 \\ \\ Q = 4\cdot222+4\cdot212+3\cdot121+4\cdot122+4\cdot111+4\cdot112+4\cdot132+ \\ +4\cdot202+4\cdot102+4\cdot021+4\cdot022+4\cdot032+4\cdot011+4\cdot012+4\cdot001+ \\ +4\cdot002+3\cdot322+3\cdot312+3\cdot031+3\cdot010+3\cdot232+3\cdot300+3\cdot020+ \\ +2\cdot221+2\cdot332+1\cdot211+1\cdot201+4\cdot101+1\cdot422+3\cdot000+1\cdot402 \\ \\ p = 4\cdot222+4\cdot212+3\cdot121+4\cdot122+4\cdot111+4\cdot112+4\cdot132+ \\ +4\cdot202+4\cdot102+4\cdot021+4\cdot022+4\cdot032+4\cdot011+4\cdot012+4\cdot001+ \\ +4\cdot002+3\cdot322+4\cdot312+3\cdot031+3\cdot010+3\cdot232+4\cdot300+3\cdot020+ \\ +2\cdot221+2\cdot332+2\cdot211+2\cdot201+4\cdot101+1\cdot422+1\cdot412+4\cdot000+1\cdot100+3\cdot402 \end{cases}$$

A solution of the set of equations $Y_{t_w<200t_o}$ is 5120 (*16(x) 20(Q) 16(p)*) of theoretical versions of solutions. A solution true for the set $Y_{t_w<200t_o}$ obtains the following form

$$f(x,Q,p)^{t_w<200t_o} = 2\cdot222+1\cdot212+2\cdot122+1\cdot111+2\cdot112+1\cdot132+2\cdot202+$$
$$+2\cdot102+2\cdot021+3\cdot022+3\cdot032+2\cdot011+3\cdot012+1\cdot001+3\cdot002 =$$
$$= 1\cdot(001)+3\cdot(0-2)+2\cdot(011)+2\cdot(021)+1\cdot(1-2)+2\cdot(122)+$$
$$+2\cdot(112)++2\cdot(102)+(1\cdot111)+2\cdot(202)+1\cdot(212)+2\cdot(222).$$

In the case of the limiting condition $t_w < 800t_o$, there are *30690 (30(x) 33(Q) 31(p))* real versions of solutions. Solutions can be obtained in the following form

$$f\left(x,Q,p\right)^{t_w<800t_o} = 4\cdot222+4\cdot212+3\cdot121+4\cdot122+4\cdot111+4\cdot112+4\cdot132+$$
$$+4\cdot202+4\cdot102+4\cdot021+4\cdot022+4\cdot032+4\cdot011+4\cdot012+$$
$$+4\cdot001+4\cdot002+3\cdot322+3\cdot312+3\cdot031+3\cdot010+3\cdot232+3\cdot300+$$
$$+3\cdot020+2\cdot221+2\cdot332+1\cdot211+1\cdot201+3\cdot101+1\cdot422+3\cdot000=$$
$$= 3(00-)+4(001)+4(002)+3(01-)+4(011)+4(012)+3(02-)+$$
$$+4(021)+4(022)+3(0-1)+4(001)+4(011)+4(021)+4(0-2)+$$
$$+4(1-2)+1(-22)+3(322)+4(222)+4(122)+4(022)+4(032)+$$
$$+3(101)+4(111)+3(121)+1(201)+1(211)+4(212)+2(221)+$$
$$+3(232)+3(300)+3(312)+2(332).$$

If we adopted a very strict condition in the graph of functions x, Q and p: stabilisation time $t_w < 100t_o$ and weighting coefficients values: $w_i=4$, $t_w \le 25t_o$; $w_i=3$, $25t_o < t_w \le 50t_o$; $w_i=2$, $50t_o < t_w \le 75t_o$; $w_i=1$, $75t_o < t_w \le 100t_o$, then a weighting multi-valued set of equations for x, Q and p would have the following form

$$Y_{t_w<100t_o} = \begin{cases} x=1\cdot022+1\cdot032+1\cdot012+1\cdot002 \\ Q=1\cdot222+1\cdot212+1\cdot122+3\cdot112+2\cdot132+1\cdot202+3\cdot102+1\cdot021+3\cdot022+1\cdot032+ \\ \quad +3\cdot011+4\cdot012+4\cdot001+4\cdot002+1\cdot312+1\cdot300+1\cdot101+1\cdot000+1\cdot100, \\ p=1\cdot122+1\cdot112+1\cdot102+1\cdot022+1\cdot032+1\cdot012+1\cdot002. \end{cases}$$

The real solution would have the following form

$$f(x,Q,p)^{t_w<100t_o} = 1\cdot022+1\cdot032+1\cdot012+1\cdot002 = 1\cdot(0-2).$$

Conclusions

The article is about a procedure of combinatorial solving of weighting multi-valued sets of logical equations describing guidelines of designing in view of the morphological analysis with keeping Rosser-Turguette's postulates.

Weighting multi-valued set of logical equations describing guidelines of designing can be minimized separately or together with keeping the logical equivalence. In this way, we can also keep individual properties of each function.

It has been proved that in a general case, minimization of logical functions with weighting coefficients can be the same as without weighting coefficients. However, a better reflection of physical models of hydraulic sweep systems has been obtained in their mathematical models for example in overflow valves or proportional ones.

Increasing, decreasing or keeping such values unchanged in the readjusting process of the system to different work conditions can be coded in the multi-valued logic, whereas designing guidelines can be presented as a sum of multi-valued logical products. Solutions to the system of weighting multi-valued logical equations are versions of code changes in parameters m, k and d of all functions x, Q and p depending on time t. In the case of such assumptions, it is also possible to introduce conditions of uncertainty for appropriate logical products of designing guidelines, which means a partly described function in the automata theory.

Nomenclature

d – the valve diameter [m]
d_1 – inlet manifold diameter [m]
d_2 – valve seat diameter [m]
F – hydrodynamic reaction force [N]
K – spring constant, [N/m]
m – valve head mass [kg]
$(m_1,...,m_n)$ – multi-valued logical function of n variables $(m_1,...,m_n)$ - valued
n – the number of different letters in the Boolean function,
P – flow intensity [m^3/s]
$p_{otw.}$ – valve opening pressure [MPa]
p_p – pressure in the inlet part in a particular position [MPa]
p_z – pressure over the valve in a particular position [MPa]
Q_1 – flow rate of the pumped liquid [m^3/s]
Q_2 – flow rate of the liquid coming out of the valve [m^3/s]
Q_{m1} – massive flow rate of the liquid going into the valve [m^3/s]
Q_{m2} – massive flow rate of the liquid coming out of the valve [m^3/s]
S – spring bias force [N]
t – time [s]
V – the valve volume $\left[m^3 \right]$
x – spring deflection [m]
x_o – free vibrations amplitude [m]
(x_1, x_2, x_3) – decision variables
β_p – coefficient of compressibility [m^3/s]
η – viscous friction coefficient [Ns/m]
ρ – liquid density $\left[kg/m^3 \right]$

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