Improved LMI-based conditions for designing of PD-type ILC laws for linear batch processes over two-dimensional setting

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Abstract

This paper considers the problem of designing of iterative learning control (ILC) laws for linear batch processes. Unlike the majority of existing results about ILC law design for linear batch processes over repetitive/two-dimensional setting where Lyapunov theory is applied, this study is focused on formulating the ILC law design procedures by transforming it into an equivalent problem of (structural) stability analysis for a linear Roesser model for two-dimensional (2D) systems. Then, based on a non-conservative version of stability and stabilization conditions for linear 2D systems, suitable PD-type ILC laws are derived by the application of the linear matrix inequality (LMI) approach. Finally, a numerical example is given to show the validity of the proposed design procedure and some advantages are emphasized when compared to the existing alternatives.

1 Introduction

Iterative learning control (ILC) is a specialized method for systems or processes that execute repetitive operations over a finite duration, known as a trial or a batch. The learning mechanism of this control method is the utilization of the historical trial or batch data to update the control input for the next trial and thereby improve the transient responses and tracking performance of subsequent trials [1, 6]. The advantages of ILC is indicated by its simple controller form and remarkable performance, which only requires less prior knowledge of the controlled system and can be easily realized. A literature review indicates that ILC has attracted considerable

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research attention since it has been used for improving the control performance in many practical problems as industrial robotics, see, e.g., [11] and wafer stage motion systems, see, for example, [8]. Additionally, as indicated in [17, 9], a version of ILC can be directly applied in the chemical process industries.

The developed ILC designs for industrial batch processes are mostly based on application so-called lifting technique for discrete-time ILC [1, 6] where the interest is rather restricted to batch domain convergence only. An alternative approach to ILC design is to focus on the inherent two-dimensional (2D) structure of the resulting dynamics [15, 13]. This allows us to consider the interaction between the batch-to-batch error and transient response along the batches. Unfortunately, a direct application of 2D system models and their stability conditions are computationally cumbersome, hence many conservative (necessity is not reached) but trackable relaxations are applied - see [15].

Differently from the most popular approaches to ILC laws design, the aim of this paper is to use some known less conservative stability and stabilization conditions for 2D Roesser models, as these in [5, 2, 12]. Clearly, these results can lead to LMI-based conditions for ILC control law design applied to discrete-time linear batch processes and, as it is done in this paper, design procedures for ILC law are obtained by exploring the fact that structural stability of 2D Roesser model imposes tracking error convergence of the resulting ILC scheme. In particular, this article proposes improved LMI-based conditions for designing of PD-type ILC laws for linear batch processes over two-dimensional setting. These new and less conservative conditions are established by utilization of the results on stability Roesser model with a state feedback in order to guarantee stability of the resulting ILC system along both the time and batch directions. Additionally, the reduced conservatism may be achieved and hence improve the applicability of developed results. Additionally, some simple modification applied to the proposed design procedures allows to assign the poles of so-called intertrial transfer function in open disc centered at the origin with radius r_1 satisfying $0 < r_1 \leq 1$ and hence the speed of batch-to-batch error convergence is potentially increased. The numerical example illustrates the benefits of the approach and show that the proposed LMI conditions are less conservative than the ones available in the literature. Also, the tracking performance of the controlled dynamics is compared with some known results to indicate the potential interest in this paper outcomes.

Throughout this paper, the following notations are used: The null and identity matrices with the required dimensions are denoted by 0 and I, respectively, and the notation $[\,]_{n,0}$ (respectively $[\,]_{0,n}$) denotes an empty matrix with n rows and 0 column (respectively n columns and 0 rows). $\rho(\cdot)$ denotes the spectral radius of its matrix argument, i.e., if $\lambda_n, 1 \leq n \leq m$, denotes the eigenvalues of a $m \times m$ matrix, say L, $\rho(L) = \max\limits_{1 \leq n \leq m} |\lambda_n|$. Also sym$\{N\}$ denotes $N + N^*$ where N^* is the transpose conjugate of a matrix N, $\overline{\mathbb{C}}$ stands for $\mathbb{C} \cup \{\infty\}$ where $\mathbb{C}$ is a set of complex numbers. For $0 < r_1 \leq 1$, let $O(0, r_1)$ be a disk centered at the origin with radius r_1. Finally, for a matrix M and block matrix $\hat{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$ with appropriate dimensions, $M \star \hat{M} = M_{22} + M_{21}(I - MM_{11})^{-1} MM_{12}$ stands for the linear fractional transformation (LFR) of $\hat{M}$ with respect to M.

2 Preliminaries

Let $t \in [0, N-1]$ be the discrete-time index where N is the fixed number of time steps for each batch and $k \geq 0$ be the batch index.

Consider a class of discrete-time batch processes over a finite time interval $t \in [0, N-1]$ represented by the following state-space model

$$
\begin{aligned}
x_k(t+1) &= Ax_k(t) + Bu_k(t), \\
y_k(t) &= Cx_k(t),
\end{aligned}
\tag{1}
$$

where $x_k(t) \in \mathbb{R}^{n_x}$ is the state vector, $y_k(t) \in \mathbb{R}^{n_y}$ is the output vector, $u_k(t) \in \mathbb{R}^{n_u}$ is the control input vector. $\{A, B, C\}$ consists of batch process matrices with appropriate dimensions. Regarding the batch process model of (1) the below assumption is made.

Assumption 1 *The matrix pairs (A, B) and (A, C) are assumed to be controllable and observable respectively, and $CB \neq 0$.*

In the sequel, we assume the repetitive behaviour of a process (1), i.e., it is required to track a desired output trajectory denoted as y_d over a finite time interval $t \in [0, N-1]$. Additionally, the batch process state $x_k(0)$ resets to the same initial value x_0 at the end of each batch, and this value can be set as $x_0 = 0$ without loss of generality.

Our objective in this work is to provide to a control sequence $\{u_k\}_{k \geq 0}$ such that the output y_k follows the desired trajectory y_d as precisely as possible as the batch index k approaches infinity. Therefore the tracking error

$$
e_k(t) = y_d(t) - y_k(t), \ t \in [0, N-1],
$$

converges to zero and hence the tracking performance is improved in the batch-to-batch domain. hese requirements are represented mathematically as

$$
\begin{aligned}
\lim_{k \to \infty} \|e_k(\cdot)\| &= 0, \\
\lim_{k \to \infty} \|u_k(\cdot) - u_\infty(\cdot)\| &= 0,
\end{aligned}
$$

where $\|\cdot\|$ denotes the norm on the underlying function and $u_\infty(\cdot)$ is termed the learned control.

As we look for the tracking error convergence conditions for the batch processes of (1) under repetitive framework, let us apply a standard form of ILC law (i.e. the way of updating the control vector from batch-to-batch) given as an update to the control input (and denoted as $\Delta u_k(t)$) from the current batch, i.e. u_k, to a new input u_{k+1} for the next batch. Therefore, it is fairly obvious that a general iterative control is applied here

$$
u_{k+1}(t) = u_k(t) + \Delta u_k(t),
\tag{2}
$$

where the update $\Delta u_k(t)$ is calculated using the previous batch data. Next, define the following notations

$$
\begin{aligned}
\delta x_{k+1}(t) &= x_{k+1}(t) - x_k(t), \\
\delta u_{k+1}(t) &= u_{k+1}(t) - u_k(t), \\
\delta e_{k+1}(t) &= e_{k+1}(t) - e_k(t)
\end{aligned}
\tag{3}
$$

and let us define a new change of variables by introducing

$$\overline{x}_k(t) = \delta x(p-1, k+1),$$
$$\overline{u}_k(t) = \delta u(p-1, k+1). \tag{4}$$

Then, it is concluded that

$$\overline{x}_k(t+1) = A\overline{x}_k(t) + B\overline{u}_k(t). \tag{5}$$

Next, assume that the batch process is subject to PD-type control law, i.e. the update item in ILC law (2) takes the form

$$\Delta u_k(t) = K_1 \delta x_{k+1}(t) + K_2 e_k(t+1)$$
$$- K_3(e_{k+1}(t) - e_k(t)), \tag{6}$$

where K_1, K_2 and K_3 are matrices of compatible dimensions to be found. Clearly, the above control law uses the current batch data to generate state feedback and the PD-type learning items are generated with the previous batch data. Anyway, by simple derivations, one can obtain the controlled dynamics model as

$$\begin{bmatrix} \overline{x}_k(t+1) \\ e_k(t) \end{bmatrix} = A_{11} \begin{bmatrix} \overline{x}_k(t) \\ e_k(t-1) \end{bmatrix} + A_{12} e_k(t),$$
$$e_{k+1}(t) = A_{21} \begin{bmatrix} \overline{x}_k(t) \\ e_k(t-1) \end{bmatrix} + A_{22} e_k(t), \tag{7}$$

where

$$A_{11} = \begin{bmatrix} A+BK_1 & BK_3 \\ 0 & 0 \end{bmatrix}, \ A_{12} = \begin{bmatrix} B(K_2-K_3) \\ I \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} -C(A+BK_1) & -CBK_3 \end{bmatrix}, A_{22} = I - CB(K_2-K_3).$$

It is worth to note that the dynamics in the model (7) propagates along two independent directions and (7) is in Roesser model [14] structure. Accordingly, the structural stability [5] of the equivalent 2D Roesser model implies that the required batch-to-batch error convergence to zero - see [2]. It is evident that the matrices A_{11}, A_{12}, A_{21} and A_{22} defined in (7) can be rewritten as

$$A_{11} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \begin{bmatrix} K_1 & K_3 \end{bmatrix} = \overline{A} + \overline{B}\overline{K}_1,$$
$$A_{21} = \begin{bmatrix} -CA & 0 \end{bmatrix} - CB \begin{bmatrix} K_1 & K_3 \end{bmatrix} = \overline{C} + CB\overline{K}_1,$$
$$A_{12} = \begin{bmatrix} 0 \\ I \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} (K_2-K_3) = \overline{B}_1 + \overline{B}\overline{K}_2,$$
$$A_{22} = I - CB(K_2-K_3) = I - CB\overline{K}_2, \tag{8}$$

where

$$\overline{K}_1 = \begin{bmatrix} K_1 & K_3 \end{bmatrix}, \overline{K}_2 = (K_2-K_3),$$
$$\overline{B}_1 = \begin{bmatrix} 0 \\ I \end{bmatrix}, \overline{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \overline{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}.$$

2.1 Structural stability of a linear 2D Roesser model

As discussed earlier, 2D system model of (7) allows to represent the behavior of controlled process under ILC law of (2) and (6). Consequently, the representation (7) can facilitate stability analysis and control synthesis.

In particular, utilizing the results presented in [5, 2], the structural stability of Roesser model in (7) along the notation (8) is characterized by the following lemma.

Lemma 1 *(see [5, 2] and references therein) An equivalent 2D Roesser model of the form (7) and (8) is structurally stable if and only if the following conditions hold*

i) $\forall \lambda \in \overline{\mathbb{D}}$, $\det(\lambda I - A_{22}) \neq 0$,

ii) $\forall \lambda \in \partial\overline{\mathbb{D}}$, $\det(G(\lambda)) \neq 0$,

where $\overline{\mathbb{D}} = \left\{ z \in \overline{\mathbb{C}}, |z| \geq 1 \right\}$ (it is just the outside of the open unit disc), $\partial\overline{\mathbb{D}} = \left\{ z \in \overline{\mathbb{C}}, |z| = 1 \right\}$ and

$$G(\lambda) = A_{21}(\lambda I - A_{11})A_{12} + A_{22}.$$

It is fairly obvious that the condition $i)$, is just standard stability condition for discrete-time systems and can be easily checked with LMI conditions. Unfortunately, the main difficulty, which arises, is the computational cost associated with the condition $ii)$. As it is seen it requires computations for all $\forall \lambda \in \partial\overline{\mathbb{D}}$ and clearly the number of computations goes to infinity so the LMI-based formulation to condition $ii)$ cannot be directly and easily provided. Anyway, based on the developments presented in [5] it follows immediately that the conditions $i)$ and $ii)$ in Lemma 1 can be replaced by the following inequalities

$$A_{22}^T P A_{22} - P \prec 0 \tag{9}$$

and

$$G(\lambda)^* P(\lambda) G(\lambda) - P(\lambda) \prec 0, \tag{10}$$

where the matrices P and $P(\lambda)$ satisfy $P \succ 0$ and $P(\lambda) \succ 0$ $\forall \lambda \in \partial\overline{\mathbb{D}}$. Equivalently, the inequality (9) implies that $\rho(A_{22}) \leq 1$ and (10) can be transformed into $\rho(G(\lambda)) \leq 1$ $\forall \lambda \in \partial\overline{\mathbb{D}}$. However, in practice it is desirable to increase the speed of batch-to-batch error convergence. Therefore, we are interested in placing the eigenvalues of A_{22} and $G(\lambda)$ inside the open disc centered at the origin with radius r_1 satisfying $0 < r_1 \leq 1$. Hence, the required versions of (9) and (10) are

$$r_1^{-2} A_{22}^T P A_{22} - P \prec 0 \tag{11}$$

and

$$r_1^{-2} G(\lambda)^* P(\lambda) G(\lambda) - P(\lambda) \prec 0. \tag{12}$$

Furthermore, introduce the notation

$$\begin{aligned}
\mathcal{A}_{21} &= r_1^{-1}\overline{C} - r_1^{-1}CB\overline{K}_1, \\
\mathcal{A}_{22} &= r_1^{-1}I - r_1^{-1}CB\overline{K}_2
\end{aligned} \tag{13}$$

and then the inequalities (11) and (12) can be expressed as

$$\begin{bmatrix} \mathcal{A}_{22} \\ I \end{bmatrix}^T R \otimes P \begin{bmatrix} \mathcal{A}_{22} \\ I \end{bmatrix} \prec 0 \tag{14}$$

and

$$\left[\begin{array}{c} \mathcal{G}(\lambda) \\ I \end{array}\right]^{*} R \otimes P(\lambda) \left[\begin{array}{c} \mathcal{G}(\lambda) \\ I \end{array}\right] \prec 0, \tag{15}$$

where $R = \text{diag}\{1, -1\}$ and

$$\mathcal{G}(\lambda) = \mathcal{A}_{21}(\lambda I - A_{11})A_{12} + \mathcal{A}_{22}. \tag{16}$$

Our concern now is the inequality (15). As $\mathcal{G}(\lambda)$ and $P(\lambda)$ depend on λ, then we need a sequence of transformation that leads to LMI formulation of (15). To proceed, note that the existence of a matrix $P(\lambda) \succ 0$ implies that there exists a matrix $Q(\lambda)$ satisfying $P(\lambda) = \text{sym}\{Q(\lambda)\}$. Consequently, the inequality (15) can be converted into

$$\mathcal{G}(\lambda)^{*}(Q(\lambda) + Q^{*}(\lambda))\mathcal{G}(\lambda) - (Q(\lambda) + Q^{*}(\lambda)) \prec 0$$

or

$$\left[\begin{array}{c} M(\lambda) \\ I \end{array}\right]^{*} \left[\begin{array}{cccc} 0 & I & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & -I \\ 0 & 0 & -I & 0 \end{array}\right] \left[\begin{array}{c} M(\lambda) \\ I \end{array}\right] \prec 0, \tag{17}$$

where

$$M(\lambda) = [\mathcal{G}^{*}(\lambda)Q(\lambda)\ \mathcal{G}^{*}(\lambda)\ Q^{*}(\lambda)]^{*}.$$

The remaining problem is the dependence of $Q(\lambda)$ on the parameter λ. This complex dependence does not allow us to find the feasible solution to inequalities (15) and (17). Fortunately, according to the results of [3], we state the following theorem.

Theorem 1 *Let n_x and n_y be given dimension of the state and output vectors in (1) respectively. Also, assume that (15) has feasible solution for some $P(\lambda)$. This means that there exists $\alpha \in \left[0; b = \dfrac{n_y}{2}((n_x + n_y)^2 + (n_x + n_y) - 2)\right]$ such that $P(\lambda)$ can be taken to have the form*

$$P(\lambda) = \text{sym}\left\{\sum_{h=0}^{\alpha} Q_h \lambda^h\right\} = \Upsilon^{*}(\lambda)\mathbf{Q}\Upsilon(\lambda),$$

with

$$\mathbf{Q} = \left[\begin{array}{c|c} \text{sym}\{Q_0\} & Q_1 \dots Q_\alpha \\ \hline Q_1^{*} & \\ \vdots & 0 \\ Q_\alpha^{*} & \end{array}\right], \Upsilon(\lambda) = \left[\begin{array}{c} \lambda^0 I_{k_1} \\ \lambda^1 I_{k_1} \\ \vdots \\ \lambda^\alpha I_{k_1} \end{array}\right] \tag{18}$$

and $Q_h \in \mathbb{R}^{(n_x + n_y) \times (n_x + n_y)}$, $h = 0, \dots, \alpha$.

Proof 1 *The proof of this theorem was already achieved in [3] - see Theorem 2 from that paper. The only difference is the dimension of A_{11} which is $(n_x + n_y) \times (n_x + n_y)$ instead of n_x.*

Now, with this last result, our immediate concern is to convert the inequality (17) into the LMI-based condition to allow us to compute the required ILC law matrices. Firstly, it is evident that $Q(\lambda)$ can be rewritten as

$$Q(\lambda) = \lambda I \star \left[\begin{array}{c|c} A_{\Upsilon} & B_{\Upsilon} \\ \hline C_{\Upsilon} & D_{\Upsilon} \end{array}\right],$$

where the matrices A_Υ, B_Υ, C_Υ and D_Υ dependent on the parameter $\alpha \geq 0$. By letting α equal to zero (this requires the lowest computational burden), it is easy to reach the following result

$$\left[\begin{array}{c|c} A_\Upsilon & B_\Upsilon \\ \hline C_\Upsilon & D_\Upsilon \end{array}\right] = \left[\begin{array}{c|c} []_{0,0} & []_{0,n_x} \\ \hline []_{n_x,0} & I_{n_x} \end{array}\right].$$

Next, letting $\alpha > 0$ (increasing α leads to higher computational burden), one can obtain

$$A_\Upsilon = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & \vdots \\ \vdots & 0 & \ddots & 1 & 0 \\ \vdots & \vdots & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix} \otimes I_{n_x}, \quad B_\Upsilon = \begin{bmatrix} 0_{(\alpha-1)n_x,n_x} \\ I_{n_x} \end{bmatrix}, \tag{19}$$

$$C_\Upsilon = \begin{bmatrix} 0_{n_x,\alpha n_x} \\ J_\alpha \otimes I_{n_x} \end{bmatrix}, \quad D_\Upsilon = \begin{bmatrix} I_{n_x} \\ 0_{\alpha n_x,n_x} \end{bmatrix},$$

where J_α denotes the $\alpha \times \alpha$ matrix of the special form as

$$\begin{bmatrix} 0 & & 1 \\ & \iddots & \\ 1 & & 0 \end{bmatrix}.$$

Secondly, consider the matrix $M(\lambda)$ in (17) and it follows that

$$M(\lambda) = \lambda I \star \left[\begin{array}{c|c} A_M & B_M \\ \hline C_M & D_M \end{array}\right],$$

where

$$A_M = \begin{bmatrix} A_\Upsilon & 0 & B_\Upsilon A_{12} \\ 0 & A_\Upsilon & 0 \\ 0 & 0 & r_1^{-1}(I-CB\overline{K}_2) \end{bmatrix},$$

$$B_M = \begin{bmatrix} B_\Upsilon A_{11} \\ B_\Upsilon \\ r_1^{-1}(\overline{C}-CB\overline{K}_1) \end{bmatrix},$$

$$C_M = \begin{bmatrix} C_\Upsilon & 0 & D_\Upsilon A_{12} \\ 0 & C_\Upsilon & 0 \end{bmatrix}, D_M = \begin{bmatrix} D_\Upsilon A_{11} \\ D_\Upsilon \end{bmatrix}.$$

Along the above notation, we have the following result on ILC law design by providing an equivalent formulation in terms of matrix inequalities to conditions i) and ii) of Lemma 1.

Theorem 2 *Let n_x and n_y be given dimension of the state and output vectors in (1) respectively. Also, let r_1 be a given positive scalar satisfying $0 < r_1 \leq 1$. Assume that an ILC law (6) is applied to the system (1). Then the resulting ILC dynamics described as 2D Roesser model of the form (7) is structurally stable, and hence batch-to-batch error convergence occurs, if and only if there exist*

an integer $\alpha \in \left[0; b = \dfrac{n_y}{2}((n_x + n_y)^2 + (n_x + n_y) - 2)\right]$, Q_h, $h = 0, \ldots, \alpha$, $X \succ 0$ *and* $Y \succ 0$ *such that*

$$\begin{bmatrix} I & 0 \\ A_M & B_M \\ C_M & D_M \end{bmatrix}^T \begin{bmatrix} \hat{R} \otimes X & 0 \\ 0 & R \otimes \mathbf{Q} \end{bmatrix} \begin{bmatrix} I & 0 \\ A_M & B_M \\ C_M & D_M \end{bmatrix} \prec 0 \tag{20}$$

and

$$\begin{bmatrix} I & 0 \\ A_\Upsilon & B_\Upsilon \\ C_\Upsilon & D_\Upsilon \end{bmatrix}^T \begin{bmatrix} \hat{R} \otimes Y & 0 \\ 0 & -\mathbf{Q} \end{bmatrix} \begin{bmatrix} I & 0 \\ A_\Upsilon & B_\Upsilon \\ C_\Upsilon & D_\Upsilon \end{bmatrix} \prec 0 \tag{21}$$

hold and where

$$R = \mathrm{diag}\{1, -1\}, \ \hat{R} = \mathrm{diag}\{-1, 1\}. \tag{22}$$

Proof 2 *Theorem 2 can be proven easily to be equivalent to the condition obtained by [4, 5] based on a pure linear 2D model and hence the details are omitted.*

The interesting point to note is that Theorem 2 result provides relaxation to condition $ii)$ of Lemma 1 through the S-procedure described in [16]. In what follows, condition $ii)$ of Lemma 1 is implied by condition (20).

Unfortunately, the inequalities provided in Theorem 2 are coupling between unknown matrix variables (note the coupling among X, Y and the control law matrices K_1, K_2 and K_3) that are not LMIs and cannot be directly transformed to ILC law design procedures. This problem can be resolved by the result developed in the next section and provides LMI characterizations for calculating the ILC law matrices in (6) to achieve batch-to-batch error convergence.

3 Main results

In this section, the aim is to develop a new ILC scheme design procedure for systems modeled with (1) where recently developed results on structural stability of 2D systems are used. As mentioned, the inequalities (20) and (21) cannot be directly applied for computation of the ILC control law matrices K_1, K_2 and K_3 of (6). This is mainly due to the fact that the inequalities in this result are bilinear in X and the matrices K_1, K_2 and K_3 (the same problem arises for the matrix Y). However, by using a particular set of transformations, the required LMI condition for computation K_1, K_2 and K_3 can be obtained.

In the sequel, we use the following decomposition

$$\begin{bmatrix} A_{11} & A_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{bmatrix} = \begin{bmatrix} \overline{A} & \overline{B}_1 \\ r_1^{-1}\overline{C} & r_1^{-1}I \end{bmatrix} + \begin{bmatrix} \overline{B} \\ -r_1^{-1}CB \end{bmatrix} \begin{bmatrix} \overline{K}_1 & \overline{K}_2 \end{bmatrix}.$$

Next, redefine the matrices A_{11}, A_{12}, $\mathcal{A}_{21}$ and $\mathcal{A}_{22}$ introduced in (8) and (13) respectively as

$$A_{11} = \overline{A}, \ A_{12} = \overline{B}_1, \ \mathcal{A}_{21} = r_1^{-1}\overline{C}, \ \mathcal{A}_{22} = r_1^{-1}I. \tag{23}$$

Then the next theorem gives novel and possibly less conservative (than known alternatives) condition for existence of the control law matrices K_1, K_2 and K_3.

Theorem 3 *Let n_x and n_y be given dimension of the state and output vectors in (1) respectively. Also, let r_1 be a given positive scalar satisfying $0 < r_1 \leq 1$. Assume that an ILC law (6) is applied to the system (1). Then the resulting ILC scheme described as a 2D Roesser model of the form (7) is structurally stable, and hence batch-to-batch error convergence occurs, if an integer $\alpha \in \left[0; b = \frac{n_y}{2}((n_x + n_y)^2 + (n_x + n_y) - 2)\right]$ can be found such that there exist matrices Q_h, $h = 0, \ldots, \alpha$, $P \succ 0$ and matrices M and S of compatible dimensions such that the LMI*

$$\Lambda^T \begin{bmatrix} \hat{R} \otimes P & 0 \\ 0 & R \otimes \mathbf{Q} \end{bmatrix} \Lambda + \mathrm{sym}\left\{ (\mathcal{A}M + \mathcal{B}S)L_\beta \right\} \prec 0 \tag{24}$$

holds and where R and $\hat{R}$ are as in (22) and

$$\Lambda = \begin{bmatrix} I_\nu & 0 & 0 & 0 & 0 & 0 \\ 0 & I_\nu & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{n_y} & 0 & 0 & 0 \\ A_\Upsilon & 0 & 0 & 0 & 0 & B_\Upsilon \\ 0 & A_\Upsilon & 0 & B_\Upsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{n_y} & 0 \\ C_\Upsilon & 0 & 0 & 0 & 0 & D_\Upsilon \\ 0 & C_\Upsilon & 0 & D_\Upsilon & 0 & 0 \end{bmatrix},$$

$$\mathcal{A} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \mathcal{A}_{22} & \mathcal{A}_{21} \\ A_{12} & A_{11} \\ -I_{n_y} & 0 \\ 0 & -I_{n_x+n_y} \end{bmatrix},$$

$$\mathcal{B}^T = \begin{bmatrix} 0 & 0 & -r_1^{-1}B^T C^T & \overline{B}_1^T & 0 & 0 \end{bmatrix},$$

$$L_\beta = \begin{bmatrix} 0 & 0 & \beta_2 I_{n_y} & 0 & -I_{n_y} & 0 \\ 0 & 0 & 0 & \beta_1 I_{n_x+n_y} & 0 & -I_{n_x+n_y} \end{bmatrix}.$$

Additionally, the scalar parameters β_1 and β_2 can freely be chosen from the set $\mathbb{D} = \{z \in \mathbb{C}, |z| < 1\}$ (it is just the inside of the open unit disc). Also, to have compatibly dimensioned matrices, ν (which appears as dimensions of some blocks in Λ) must be equal to the dimension of the matrix A_Υ defined in (19).

Moreover, if the LMI (24) is feasible, the required control law matrices K_1, K_2 and K_3 are computed as

$$\begin{bmatrix} K_1\ K_3 \,\big|\, \overline{K}_2 \end{bmatrix} = SM^{-1}, \quad K_2 = \overline{K}_2 + K_3. \tag{25}$$

Proof 3 *It is a straightforward consequence of identical steps to those in the proof of Theorem 5 in [5] and hence the details are omitted for the sake of brevity and due to space limitations. Anyway, it has to be emphasized that some slight differences are present due to specific form and dimensions of A_{11}, A_{12}, $\mathcal{A}_{21}$, $\mathcal{A}_{22}$ as given in (23).*

4   Numerical example

In order to illustrate the applicability and effectiveness of our proposed method, one numerical example is given to illustrate the feasibility and demonstrate the effectiveness of the method for the synthesis of the PD-type ILC law for linear batch processes.

The example considers the linearized dynamics of injection molding process. The details of the model can be found in [9, 7]. Following the literature we know that a key process variable to be controlled is the nozzle pressure. When considering the nozzle pressure response to the hydraulic control valve opening the following state-space model is provided - again see [9, 7] for details,

$$x_k(t+1) = \begin{bmatrix} 1.607 & 1 \\ -0.6086 & 0 \end{bmatrix} x_k(t) + \begin{bmatrix} 1.239 \\ -0.9282 \end{bmatrix} u_k(t),$$
$$y_k(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k(t).$$

For illustration, the desired trajectory for the nozzle pressure, which is shown in Figure 1, takes the following form
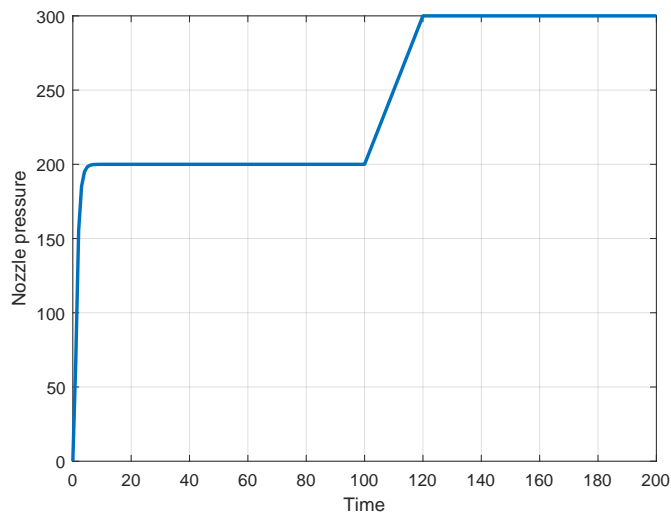

Figure 1: The desired profile.

$$y_d(t) = \begin{cases} 200, & 1 \leq t < 100; \\ 200 + 5(t - 100), & 100 \leq t < 120; \\ 300, & 120 \leq t \leq N = 200. \end{cases}$$

For practical implementation, the initial part of $y_d(t)$ is pre-filtered by $G_f = (z^{-1} + z^{-2})/(3 - z^{-1})$. Furthermore, RMSE (Root Mean Square Error) value of the tracking error is taken as an index to evaluate the tracking performance of the batch processes and it will be computed along each batch.

To demonstrate the effectiveness of the proposed results, the design procedure given in Theorem 3 is executed for $\alpha = 1$, $\beta_1 = 0$, $\beta_2 = 0$, $r_1 = 0.3$ and ILC law matrices in (6) are derived as follows

$$K_1 = [-1.2819 - 0.8021], \ K_2 = 0.7741,$$
$$K_3 = -3.8406 \cdot 10^{-6}.$$

The resulting controlled system represented as Roesser model is structurally stable and hence batch-to-batch error convergence occurs. This can be verified in Figure 2 where the RMSE of the tracking error is shown and compared to the previously presented results in [7, 10]. From comparison in Figure 2, the RMSE of the proposed method is lower when comparing with [7, 10]. Obviously, the effectiveness of the presented ILC design is apparent. Additionally, Figure 3 shows
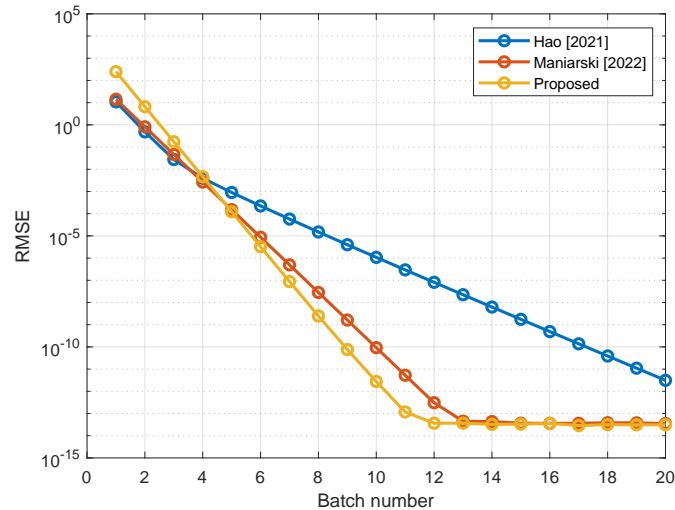

Figure 2: RMSE values obtained in the simulation study for different methods

the spectral radius of the transfer function $\mathcal{G}(\lambda) \ \forall \lambda \in \partial\overline{\mathbb{D}}$. From this comparison of the three ILC law designs in Figure 3, we see that the proposed approach can produce ILC law with lowest level for spectral radius and hence can deliver faster batch-to-batch error convergence.
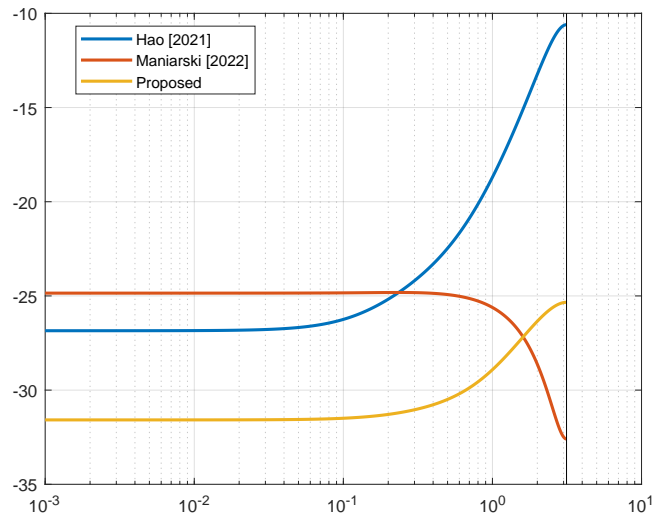

Figure 3: Plot of spectral radius of $\mathcal{G}(\lambda)$

5   Conclusions

In this paper, we have dealt with the design problem of PD-type ILC laws for a class of linear batch processes. Our new results have been obtained by via transforming initial problem into an equivalent one of designing stabilizing state feedback gains for linear Roesser model of 2D systems. After providing the 2D setting of the ILC scheme, we have employed an LMI approach to which derives learning gains directly. It has been shown that the proposed approach can be applicable to a class of linear batch processes. A simulation based case study is given to demonstrate the utility of the proposed design approach and its the effectiveness. It should be noticing that the results developed in this paper are of a nominal model only. Therefore, the results should be directly generalized to batch process models with uncertainty and disturbances, which is a interesting problem to be discussed in the future.

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