IJME

Int. J. of Applied Mechanics and Engineering, 2023, vol.28, No.3, pp.127-158
DOI: 10.59441/ijame/172902

MAGNETO-THERMO-PIEZO-ELASTIC WAVE IN AN INITIALLY STRESSED ROTATING MONOCLINIC CRYSTAL IN A TWO-TEMPERATURE THEORY

Anand Kumar YADAV
Shishu Niketan Model Senior Secondary School, Sector 22-D, Chandigarh, INDIA
E-mail: yadavanand977@gmail.com

This research problem is an investigation of wave propagation in a rotating initially stressed monoclinic piezoelectric thermo-elastic medium under with the effect of a magnetic field.

A two-temperature generalized theory of thermo-elasticity in the context of Lord-Shulman's theory is applied to study the waves under the magnetic field.

The governing equations of a rotating initially stressed monoclinic piezoelectric thermo-elastic medium with a magnetic field are formulated. This research problem is solved analytically, for a two-dimensional model of the piezo-electric monoclinic solid, and concluded that there must be four piezo-thermoelastic waves, three coupled quasi waves (*qP* (quasi-P), *qT* (quasi-thermal), and *qSV* (quasi-SV)) and one piezoelectric potential (PE) wave propagating at different speeds. It is found that at least one of these waves is evanescent (an evanescent wave is a non-propagating wave that exists) and that there are therefore no more than three bulk waves.

The speeds of different waves are calculated and the influence of the piezoelectric effect, two-temperature parameter, frequency, rotation, and magnetic field on phase velocity, attenuation coefficient, and specific loss is shown graphically.

This model may be used in various fields, e.g. wireless communications, signal processing, and military defense equipment are all pertinent to this study.

Key words: piezo-electricity, monoclinic, rotation, magnetic field, phase velocity, attenuation coefficient, specific loss, two-temperature.

1. Introduction

The classical theory of elasticity is one of the most significant sections of continuum mechanics, which was extended to include thermal effects. This new theory was called 'Thermo-elasticity'. Lord and Shulman [1] generalized the thermo-elasticity theory introducing one relaxation time. Dhaliwal and Sherief [2] gave a theory of generalized thermos-elasticity for anisotropic materials. Youssef [3, 4] developed a new theory of two-temperature hyperbolic thermo-elasticity for entropy due to heat supply and heat conduction. Ignaczak and Ostoja-Starzewski [5], and Hetnarski and Ignaczak [6] provided a detailed study of different types of generalized theories of thermo-elasticity. Chandrasekharaih [7, 8] elaborated mathematical characteristics of the generalized thermo-elasticity Chen and Gurtin [9, 10] studied a heat conduction problem in two-temperature theory. A thermo-piezoelectricity theory was first proposed by Mindlin [11] who gave governing equations. Nowacki [12, 13, 14] and Chandrasekharaiah [15, 16] explored piezo-thermo-elasticity further and generalized Mindlin' theory of piezo-thermo-elasticity. The "piezoelectric effect" is the internal creation of an electrical charge resulting from a mechanical force being applied. Also, conversely when an electric field is applied, internal mechanical strain is produced in the piezoelectric medium. The theory of thermo-piezo-electricity deals with mechanical, thermoelastic and electric fields. Because of this electromechanical coupling properties and decreased acoustic impedance, piezoelectric materials find use in a variety of devices, including underwater sonar detectors, accelerometers, contact microphones, echo sounders, and ultrasonic imaging systems, and many others. Propagation of acoustic energy at boundary surface is widely utilized in fields such as frequency control, transduction, and signal processing. Piezoelectric materials are frequently used in applications for smart structures like actuators and sensors because when piezoelectric ceramics and piezoelectric polymers undergo mechanical stresses such as compression, stretching and bending, electric potential is produced. These materials are used in aerospace, mechanical, civil and bio-engineering. Every and Neiman [17] investigated the reflection of plane electroacoustic waves at the boundary of a piezoelectric half-space and showed that that at least one of these waves is evanescent. In the literature there is a variety of issues pertaining to the phenomena of plane wave reflection and refraction for piezoelectric materials. Othman and Ahmed [18] examined the deformation in a piezo-thermoelastic rotating medium under different thermo-elasticity theories. The reflection and transmission of plane waves from a fluid-piezo-thermoelastic solid interface was studied by Vashishth and Sukhija [19]. Singh [20] explored the waves in a prestressed piezoelectric solid half-space. Many researchers have published papers on waves in piezo-thermo-elastic media e. g., Jain *et al.* [21], Othman et al. [22], Guha *et al.* [23], Kumar and Harsha [24], Abdulaziz *et al.* [25], Deswal *et al.* [26], Lotfy *et al.* [27]. Ye [28] in his book introduced piezoelectric materials to the world and showed that piezoelectricity being a property of certain types of crystals enabling them to convert mechanical stress to electrical charge and vice versa. Some other problems on wave propagation in anisotropic and isotropic thermo-elastic media with various parameters are studied by many investigators such as Keith and Crampin [29], Singh and Tomar [30] and Singh and Yadav [31-32]. Chattopadhyay and Choudhary [33], Chattopadhyay et al. [34], Singh and Khurana [35], Sahar et al. [36], Montanaro [37], Gupta and Vashishth [38], Singh and Yadav [39, 40], Yadav [41, 42] Marin and Marinescu, [44] Yadav [46, 47], Maity *et al.* [48], Marin and his co-workers [51-52], and Carrera *et al.* [53]. Yadav et al. [54] investigated piezo electric waves in orthotropic medium under hygro-thermal conditions. Piezoelectric materials have many applications in engineering as well as medical devices for example piezoelectric sensors, resonant devices and sensor instrumentation, signal, piezoelectric micropumps, in pediatric devices and pediatric cardiovascular devices. Monoclinic crystalline materials such as beta-sulfur, gypsum, borax, orthoclase, kaolin, muscovite, clinoamphibole, clinopyroxene, jadeite, azurite, and spodumene crystals are available.

2. Basic equations

Consider a homogenous initially stressed piezoelectric monoclinic magneto-thermoelastic medium which is permeated by a primary magnetic field $\boldsymbol{B}$, permeates it such that $\boldsymbol{B}=\mu_e\boldsymbol{H}$ which rotates with an angular velocity $\boldsymbol{\Omega}=\Omega\boldsymbol{n}$, where $\boldsymbol{n}$ is the unit vector denoting the direction of the rotational axis about the x-axis as a unit vector with $\boldsymbol{\Omega}=(\Omega,0,0)$ and magnetic field $\boldsymbol{H}=(H,0,0)$ at reference temperature T_0. Following Lord and Shulman [1], Schoenberg and Censor [49], Willson [50], Biot [45], Montanaro [43], and Mindlin [11] the basic equation and constitutive relations are as follows:
Equation of motion:

$$\bar{\sigma}_{ij,j}+\left(u_{i,k}\bar{\sigma}^0_{kj}\right)_{,j}-\bar{g}_{ijk}E_k+a_{ij}T+(\boldsymbol{J}\times\boldsymbol{B})_i=\rho\left\{\ddot{\boldsymbol{u}}_i+\left(\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u})\right)_i+\left(2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}}\right)_i\right\}. \tag{2.1}$$

Gauss equation:

$$D_{i,i}=0. \tag{2.2}$$

Strain-displacement-relation:

$$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}). \tag{2.3}$$

Stress-strain-temperature and electric field relation:

$$\bar{\sigma}_{ij} = \bar{C}_{ijkl}e_{kl} - \bar{g}_{ijk}E_k + a_{ij}T,$$
$$E_i = -\psi_{,i}. \tag{2.4}$$

Electric displacement, strain, electric field and temperature relation:

$$D_k = \bar{g}_{kij}e_{ij} + \bar{\gamma}_{ki}E_i - p_k T. \tag{2.5}$$

Entropy equation:

$$\rho \dot{S} T_0 = -q_{i,i}. \tag{2.6}$$

Generalized Fourier's transform:

$$\left(1 + \tau_0^t \frac{\partial}{\partial t}\right) q_i = -K_{ij} T_{,i}. \tag{2.7}$$

Entropy, temperature and electric field relation:

$$\rho S = a_{ij}\left(1 + \tau_0^t \frac{\partial}{\partial t}\right) e_{ij} + \frac{\rho C_E}{T_0}\left(1 + \tau_0^t \frac{\partial}{\partial t}\right) T - p_k\left(1 + \tau_0^t \frac{\partial}{\partial t}\right) E_k,$$
$$\rho S T_0 = a_{ij}T_0\left(1 + \tau_0^t \frac{\partial}{\partial t}\right) e_{ij} + \rho C_E\left(1 + \tau_0^t \frac{\partial}{\partial t}\right) T + p_k T_0\left(1 + \tau_0^t \frac{\partial}{\partial t}\right) \psi_{,k}. \tag{2.8}$$

The Maxwell equations:

$$Curl\ \boldsymbol{H} = \boldsymbol{J}, \qquad Curl\ \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \qquad \mathrm{div}\ \boldsymbol{B} = 0, \qquad \boldsymbol{B} = \mu_e \boldsymbol{H}. \tag{2.9}$$

The generalized Ohm's law in deformable continua is:

$$\boldsymbol{J} = \sigma[\boldsymbol{E} + (\dot{\boldsymbol{u}} \times \boldsymbol{B})], \tag{2.10}$$

The linearized Maxwell's stress tensor due to the magnetic field is given by:

$$\bar{\tau}_{ij} = \mu_e\left[H_i h_j + H_j h_i - H_k h_k \delta_{ij}\right],$$
$$(\boldsymbol{J} \times \boldsymbol{B})_i = \mu_e\left(Curl\ \boldsymbol{h} \times \boldsymbol{H}\right). \tag{2.11}$$

Two-temperature relation for an anisotropic medium:

$$T = \Phi_{CT} - \left(a_1^* \frac{\partial^2 \Phi_{CT}}{\partial x^2} + a_2^* \frac{\partial^2 \Phi_{CT}}{\partial y^2} + a_3^* \frac{\partial^2 \Phi_{CT}}{\partial z^2}\right), \tag{2.12}$$

$$\bar{C}_{ijkl} = \bar{C}_{klij} = \bar{C}_{jikl} = \bar{C}_{ijlk}, \quad \bar{g}_{ijk} = \bar{g}_{kij} = \bar{g}_{kji}, \quad a_{ij} = a_{ji}, \quad \bar{\gamma}_{ij} = \bar{\gamma}_{ji}, \quad K_{ij} = K_{ji},$$

$$a_{ij} = -\beta_i^t \delta_{ij}, \quad K_{ij} = K_{ii}\delta_{ij}, \; q_i = K_{ij} T_{,j},$$

$$\bar{\sigma}_{kj}^{0} = P_{ii}\delta_{ij}, \quad 1 \Leftrightarrow 11, \quad 2 \Leftrightarrow 22, \quad 3 \Leftrightarrow 33, \quad 4 \Leftrightarrow 23, \quad 5 \Leftrightarrow 13, \quad 6 \Leftrightarrow 12,$$

$$\bar{g}_{14} = \bar{g}_{123} = \bar{g}_{132}, \quad \bar{g}_{16} = \bar{g}_{112} = \bar{g}_{121}, \quad \bar{g}_{21} = \bar{g}_{211}, \quad \bar{g}_{22} = \bar{g}_{222},$$

$$\bar{g}_{23} = \bar{g}_{233}, \quad \bar{g}_{25} = \bar{g}_{213} = \bar{g}_{231}, \quad \bar{g}_{34} = \bar{g}_{323} = \bar{g}_{332}, \quad \bar{g}_{36} = \bar{g}_{312} = \bar{g}_{321},$$

$$\bar{\gamma}_{11} = \bar{\gamma}_{111}, \quad \bar{\gamma}_{22} = \bar{\gamma}_{222}, \quad \bar{\gamma}_{33} = \bar{\gamma}_{333}, \quad \bar{\gamma}_{13} = \bar{\gamma}_{133},$$

where a_i^* ($a_i^* > 0$, $(i = 1, 2, 3)$,) are two-temperature parameters and Φ_{CT} is the conductive temperature, $T = T^* - T_0$, is the thermodynamical temperature such that $\left|\frac{T}{T_0}\right| << 1$, T^* is the absolute temperature of the medium, $\bar{\sigma}_{ij}$ is stress tensor, e_{ij} is strain tensor, S is entropy, $\bar{\sigma}_{kj}^{0}\left(P_{22}, P_{33}\right)$ is the normal stress tensor referring to initial stress, β_2^t, β_3^t, $\left(\beta_2^t = \left(\bar{C}_{22} + \bar{C}_{23}\right)\alpha_{2t} + \left(\bar{C}_{23} + \bar{g}_{34}\right)\alpha_{3t}\right)$, $\left(\beta_3^t = 2\bar{C}_{23}\alpha_{1t} + \left(\bar{C}_{33} + \bar{g}_{23}\right)\alpha_{3t}\right)$, are thermal coefficients, α_{2t}, α_{3t}, are coefficients of linear thermal expansion.

The material matrix for the double symmetry axis parallel to the *Y*-axis of class 2 monoclinic crystal is considered

$$\begin{bmatrix} \bar{\sigma}_{xx} \\ \bar{\sigma}_{yy} \\ \bar{\sigma}_{zz} \\ \bar{\sigma}_{yz} \\ \bar{\sigma}_{zx} \\ \bar{\sigma}_{xy} \end{bmatrix} = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & 0 & \bar{C}_{15} & 0 \\ \bar{C}_{12} & \bar{C}_{22} & \bar{C}_{23} & 0 & \bar{C}_{25} & 0 \\ \bar{C}_{13} & \bar{C}_{23} & \bar{C}_{33} & 0 & \bar{C}_{35} & 0 \\ 0 & 0 & 0 & \bar{C}_{44} & 0 & \bar{C}_{46} \\ \bar{C}_{15} & \bar{C}_{25} & \bar{C}_{35} & 0 & \bar{C}_{55} & 0 \\ 0 & 0 & 0 & \bar{C}_{46} & 0 & \bar{C}_{66} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \\ 2e_{yz} \\ 2e_{zx} \\ 2e_{xy} \end{bmatrix} - \begin{bmatrix} 0 & \bar{g}_{21} & 0 \\ 0 & \bar{g}_{22} & 0 \\ 0 & \bar{g}_{23} & 0 \\ \bar{g}_{14} & 0 & \bar{g}_{34} \\ 0 & \bar{g}_{25} & 0 \\ \bar{g}_{16} & 0 & \bar{g}_{36} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix},$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \bar{g}_{14} & 0 & \bar{g}_{16} \\ \bar{g}_{21} & \bar{g}_{22} & \bar{g}_{23} & 0 & \bar{g}_{25} & 0 \\ 0 & 0 & 0 & \bar{g}_{34} & 0 & \bar{g}_{36} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \\ 2e_{yz} \\ 2e_{zx} \\ 2e_{xy} \end{bmatrix} + \begin{bmatrix} \bar{\gamma}_{11} & 0 & \bar{\gamma}_{13} \\ 0 & \bar{\gamma}_{22} & 0 \\ \bar{\gamma}_{13} & 0 & \bar{\gamma}_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix},$$

$$\bar{\sigma}_{xx} = \bar{C}_{11}e_{xx} + \bar{C}_{12}e_{yy} + \bar{C}_{13}e_{zz} + \bar{C}_{15}e_{zx} + \bar{g}_{21}\psi_{,y},$$

$$\bar{\sigma}_{yy} = \bar{C}_{12}e_{xx} + \bar{C}_{22}e_{yy} + \bar{C}_{13}e_{zz} + \bar{C}_{25}e_{zx} + \bar{g}_{22}\psi_{,y},$$

$$\bar{\sigma}_{zz} = \bar{C}_{13}e_{xx} + \bar{C}_{13}e_{yy} + \bar{C}_{33}e_{zz} + \bar{C}_{35}e_{zx} + \bar{g}_{23}\psi_{,y},$$

$$\bar{\sigma}_{yz} = \bar{C}_{44}e_{yz} + \bar{C}_{46}e_{xy} + \bar{g}_{14}\psi_{,x} + \bar{g}_{34}\psi_{,z},$$

$$\bar{\sigma}_{zx} = \bar{C}_{15}e_{xx} + \bar{C}_{25}e_{yy} + \bar{C}_{35}e_{zz} + \bar{C}_{55}e_{zx} + \bar{g}_{25}\psi_{,y},$$

$$\bar{\sigma}_{xy} = \bar{C}_{46}e_{yz} + \bar{C}_{66}e_{xy} + \bar{g}_{16}\psi_{,x} + \bar{g}_{36}\psi_{,z},$$

$$D_x = \bar{g}_{14}2e_{yz} + \bar{g}_{16}2e_{xy} + \bar{\gamma}_{11}E_x + \bar{\gamma}_{13}E_z,$$

$$D_y = \bar{g}_{21}e_{xx} + \bar{g}_{22}e_{yy} + \bar{g}_{23}e_{zz} + \bar{g}_{25}2e_{zx} + \bar{\gamma}_{22}E_y,$$

$$D_z = \bar{g}_{34}2e_{yz} + \bar{g}_{36}2e_{xy} + \bar{\gamma}_{13}E_x + \bar{\gamma}_{33}E_z.$$

3. Formulation of the problem and solution

We consider a homogeneous initially stressed piezoelectric monoclinic (class 2) magneto-thermo-elastic medium in two-temperature theory when the magnetic field is interpreted as $\boldsymbol{H} = \boldsymbol{H_0} + \boldsymbol{h}$, $\boldsymbol{H_0} = (H_0, 0, 0)$, $\boldsymbol{h}(h_x, h_y, h_z)$ is a shift in the magnetic field, a generated magnetic field $\boldsymbol{h} = (h, 0, 0)$ and an generated electric field $\boldsymbol{E}$, created as a result of the introduction of an original magnetic field and rotation about for X-axis. Centripetal and Coriolis acceleration $(\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u}))$ and $\left(2\boldsymbol{\Omega}\times\frac{\partial \boldsymbol{u}}{\partial t}\right)$ are created when the medium which is flawless conducting rotates, $\sigma \to \infty$, with displacement vector $\boldsymbol{u} = (0, v, w)$ and $\frac{\partial}{\partial x} = 0$. The applied magnetic field will be affected in directions along all three coordinate axes, if the medium is not perfectly conducting, and attenuations will occur due to the medium's resistivity. The governing equations for the monoclinic crystal belonging to class 2 with the double symmetry axis parallel to the y-axis in the yz-plane following Montanaro [37] and Mindlin [11] become:
Equation of motion;

$$\begin{gathered}(\bar{C}_{22} + P_{22})\frac{\partial^2 v}{\partial y^2} + (\bar{C}_{44} + P_{33})\frac{\partial^2 v}{\partial z^2} + (\bar{C}_{23} + \bar{C}_{44})\frac{\partial^2 w}{\partial y \partial z} + \bar{g}_{22}\frac{\partial^2 \psi}{\partial y^2} + \\ + \bar{g}_{34}\frac{\partial^2 \psi}{\partial z^2} - \beta_2'\frac{\partial T}{\partial y} + (\boldsymbol{J}\times\boldsymbol{B})_2 = \rho\left(\frac{\partial^2 v}{\partial t^2} - \Omega^2 v - 2\Omega\frac{\partial w}{\partial t}\right),\end{gathered} \tag{3.1}$$

$$(\bar{C}_{44}+P_{22})\frac{\partial^2 w}{\partial y^2}+(\bar{C}_{33}+P_{33})\frac{\partial^2 w}{\partial z^2}+(\bar{C}_{23}+\bar{C}_{44})\frac{\partial^2 v}{\partial y\partial z}+$$
$$+(\bar{g}_{34}+\bar{g}_{23})\frac{\partial^2\psi}{\partial y\partial z}-\beta_3^t\frac{\partial T}{\partial z}+(\boldsymbol{J}\times\boldsymbol{B})_3=\rho\left(\frac{\partial^2 w}{\partial t^2}-\Omega^2 w+2\Omega\frac{\partial v}{\partial t}\right), \tag{3.2}$$

Gauss equation;

$$\bar{g}_{34}\frac{\partial^2 v}{\partial z^2}+\bar{g}_{22}\frac{\partial^2 v}{\partial y^2}+(\bar{g}_{23}+\bar{g}_{34})\frac{\partial^2 w}{\partial y\partial z}-\bar{\gamma}_{22}\frac{\partial^2\psi}{\partial y^2}-\bar{\gamma}_{33}\frac{\partial^2\psi}{\partial z^2}-p_3\frac{\partial T}{\partial z}=0. \tag{3.3}$$

Following Lord and Shulman [1], the heat energy equation is:

$$K_2\frac{\partial^2 T}{\partial y^2}+K_3\frac{\partial^2 T}{\partial z^2}=\left(1+\tau_0^t\frac{\partial}{\partial t}\right)\left\{T_0(\beta_2^t\frac{\partial^2 v}{\partial y\partial t}+\beta_3^t\frac{\partial^2 w}{\partial z\partial t})+\rho C_E\dot{T}+p_3T_0\frac{\partial\dot{\psi}}{\partial z}\right\}. \tag{3.4}$$

From Eqs (2.9), (2.10) and relation $(\boldsymbol{J}\times\boldsymbol{B})_i=\mu_e\left(Curl\ \boldsymbol{h}\times\boldsymbol{H}\right)$, we obtain:

$$(\boldsymbol{J}\times\boldsymbol{B})_1=0,\quad (\boldsymbol{J}\times\boldsymbol{B})_2=\mu_e H_0^2\left\{\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z}\right\},\quad (\boldsymbol{J}\times\boldsymbol{B})_3=\mu_e H_0^2\left\{\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial^2 w}{\partial z^2}\right\}. \tag{3.5}$$

Using Eqs (2.12) and (3.5) in Eq.(3.1)-(3.4), we get:
Equation of motion;

$$(\bar{C}_{22}+P_{22})\frac{\partial^2 v}{\partial y^2}+(\bar{C}_{44}+P_{33})\frac{\partial^2 v}{\partial z^2}+(\bar{C}_{23}+\bar{C}_{44})\frac{\partial^2 w}{\partial y\partial z}+\mu_e H_0^2\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z}\right)+$$
$$-\beta_2^t\frac{\partial}{\partial y}\left\{\Phi_{CT}-\left(a_2^*\frac{\partial^2\Phi_{CT}}{\partial y^2}+a_3^*\frac{\partial^2\Phi_{CT}}{\partial z^2}\right)\right\}+\bar{g}_{22}\frac{\partial^2\psi}{\partial y^2}+\bar{g}_{34}\frac{\partial^2\psi}{\partial z^2}= \tag{3.6}$$
$$=\rho\left(\frac{\partial^2 v}{\partial t^2}-\Omega^2 v-2\Omega\frac{\partial w}{\partial t}\right),$$

$$(\bar{C}_{44}+P_{22})\frac{\partial^2 w}{\partial y^2}+(\bar{C}_{33}+P_{33})\frac{\partial^2 w}{\partial z^2}+(\bar{C}_{23}+\bar{C}_{44})\frac{\partial^2 v}{\partial y\partial z}+\mu_e H_0^2\left(\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial^2 w}{\partial z^2}\right)+$$
$$-\beta_3^t\frac{\partial}{\partial z}\left\{\Phi_{CT}-\left(a_2^*\frac{\partial^2\Phi_{CT}}{\partial y^2}+a_3^*\frac{\partial^2\Phi_{CT}}{\partial z^2}\right)\right\}+(\bar{g}_{34}+\bar{g}_{23})\frac{\partial^2\psi}{\partial y\partial z}= \tag{3.7}$$
$$=\rho\left(\frac{\partial^2 w}{\partial t^2}-\Omega^2 w+2\Omega\frac{\partial v}{\partial t}\right),$$

Gauss equation;

$$\bar{g}_{22}\frac{\partial^2 v}{\partial y^2}+\bar{g}_{34}\frac{\partial^2 v}{\partial z^2}+(\bar{g}_{23}+\bar{g}_{34})\frac{\partial^2 w}{\partial y\partial z}-p_3\frac{\partial}{\partial z}\left\{\Phi_{CT}-\left(a_2^*\frac{\partial^2\Phi_{CT}}{\partial y^2}+a_3^*\frac{\partial^2\Phi_{CT}}{\partial z^2}\right)\right\}+$$
$$-\bar{\gamma}_{22}\frac{\partial^2\psi}{\partial y^2}-\bar{\gamma}_{33}\frac{\partial^2\psi}{\partial z^2}=0. \tag{3.8}$$

The conduction equation in the two-temperature theory is [2]:

$$K_2\frac{\partial^2\Phi_{CT}}{\partial y^2}+K_3\frac{\partial^2\Phi_{CT}}{\partial z^2}=\left(1+\tau_0^t\frac{\partial}{\partial t}\right)\left\{T_0\left(\beta_2^t\frac{\partial^2 v}{\partial y\partial t}+\beta_3^t\frac{\partial^2 w}{\partial z\partial t}\right)+\right.$$
$$\left.+\rho C_E\left\{\dot{\Phi}_{CT}-\left(a_2^*\frac{\partial^2\dot{\Phi}_{CT}}{\partial y^2}+a_3^*\frac{\partial^2\dot{\Phi}_{CT}}{\partial z^2}\right)\right\}+p_3T_0\frac{\partial\dot{\psi}}{\partial z}\right\}. \tag{3.9}$$

4. Plane-wave solution of the problem

Consider plane waves moving in the direction of the positive Y-axis in a homogeneous, initially strained rotating magneto-thermo-monoclinic solid half-space. The solution of Eqs (3.6) to (3.9) are sought in the following form:

$$\left.\begin{aligned} v&=\tilde{A}\exp\{ik(y\sin\theta\pm z\cos\theta-Vt)\},\\ w&=\tilde{B}\exp\{ik(y\sin\theta\pm z\cos\theta-Vt)\},\\ T&=\tilde{C}\exp\{ik(y\sin\theta\pm z\cos\theta-Vt)\},\\ \psi&=\tilde{D}\exp\{ik(y\sin\theta\pm z\cos\theta-Vt)\}.\end{aligned}\right\} \tag{4.1}$$

where, $\tilde{A},\tilde{B},\tilde{C},\tilde{D}$ are the constants, $\pm$, upper positive sign is for incidence wave, lower negative sign is for reflected wave, $(\sin\theta,\cos\theta)$ is the projection of wave normal onto the yz-plane, V is the phase velocity and k is the wave number. Using Eq.(4.1) in Eqs (3.6) to (3.9), we get:

$$(T_1-\Omega*\zeta)V^2\tilde{A}+\left(T_2+2i\frac{\Omega}{\omega}\zeta\right)V^2\tilde{B}+i\frac{\bar{\beta}_2^t}{k}\left(\zeta+\ell^*\right)\sin\theta\,\tilde{C}+g_1^*V^2\tilde{D}=0, \tag{4.2}$$

$$\left(T_2-2i\frac{\Omega}{\omega}\zeta\right)V^2\tilde{A}+\left(T_3-\tilde{\Omega}*\zeta\right)V^2\tilde{B}\pm\frac{i\bar{\beta}_3^t}{k}\left(\zeta+\ell^*\right)\cos\theta\,\tilde{C}+g_2^*V^2\tilde{D}=0, \tag{4.3}$$

$$g_1^*V^2\tilde{A}+g_2^*V^2\tilde{B}\pm i\frac{\bar{p}_3}{k}\left(\zeta+\ell^*\right)\cos\theta\tilde{C}-\gamma_1^*V^2\tilde{D}=0, \tag{4.4}$$

$$\zeta\varepsilon\sin\theta V^2\tilde{A} \pm \zeta\varepsilon\bar{\beta}^t\cos\theta V^2\tilde{B} + i\frac{\bar{\beta}_2^t}{k}\zeta\left\{T_5 - (\zeta+\ell^*)\right\}\tilde{C} \pm \zeta\varepsilon^p\cos\theta V^2\tilde{D} = 0. \tag{4.5}$$

For the non-trivial solution of Eqs (4.2) to (4.5), the determinant of the coefficients of $\tilde{A},\tilde{B},\tilde{C},\tilde{D}$ must vanish, i.e.,

$$\begin{vmatrix} T_1 - \Omega^*\zeta & T_2 + 2i\dfrac{\Omega}{\omega}\zeta & \bar{\beta}_2^t(\zeta+\ell^*)\sin\theta & g_1^* \\ T_2 - 2i\dfrac{\Omega}{\omega}\zeta & T_3 - \Omega^*\zeta & \pm\bar{\beta}_3^t(\zeta+\ell^*)\cos\theta & g_2^* \\ g_1^* & g_2^* & \pm\bar{p}_3(\zeta+\ell^*)\cos\theta & -\gamma_1^* \\ \zeta\varepsilon\sin\theta & \pm\zeta\varepsilon\bar{\beta}^t\cos\theta & \bar{\beta}_2^t\zeta\left\{T_5-(\zeta+\ell^*)\right\} & \pm\zeta\varepsilon^p\cos\theta \end{vmatrix} = 0,$$

which can be written in terms of power of ζ as:

$$Y_0\zeta^4 + Y_1\zeta^3 + Y_2\zeta^2 + Y_3\zeta = 0. \tag{4.6}$$

Equation (4.6) can be written as:

$$\zeta = 0, \tag{4.7}$$

and

$$Y_0\zeta^3 + Y_1\zeta^2 + Y_2\zeta + Y_3 = 0 \tag{4.8}$$

where

$$Y_0 = \left\{1 - \left(\frac{\Omega}{\omega}\right)^2\right\}^2\left(\bar{p}_3\varepsilon^p\cos^2\theta - \bar{\beta}_2^t\gamma_1^*\right),$$

$$\begin{aligned} Y_1 = {} & \Omega^*(T_1+T_3)\left(\bar{\beta}_2^t\gamma_1^* - \bar{p}_3\varepsilon^p\cos^2\theta\right) + 2\Omega^* g_2^*\bar{\beta}_3^t\varepsilon^p\cos^2\theta + \Omega^*\bar{\beta}_2^t\left(g_1^{*2}+g_2^{*2}\right) + \\ & +\Omega^*\varepsilon\gamma_1^*\left(\bar{\beta}_3^t\bar{\beta}^t\cos^2\theta + \bar{\beta}_2^t\sin^2\theta\right) + 2\Omega^* g_1^*\bar{\beta}_2^t\varepsilon^p\sin\theta\cos\theta + \\ & +\left\{1-\left(\frac{\Omega}{\omega}\right)^2\right\}^2\left(\bar{p}_3\varepsilon^p\ell^*\cos^2\theta + \bar{\beta}_2^t\gamma_1^*T_5 - \bar{\beta}_2^t\gamma_1^*\ell^*\right), \end{aligned}$$

$$\begin{aligned} Y_2 = {} & \Omega^*(T_1+T_3)\left(\gamma_1^*\ell^*\bar{\beta}_2^t - \gamma_1^*T_5\bar{\beta}_2^t - \ell^*\bar{p}_3\varepsilon^p\cos^2\theta\right) + \gamma_1^*\bar{\beta}_2^t\left(T_2^2 - T_1T_3\right) - T_2\bar{p}_3\varepsilon^p\cos^2\theta + \\ & +T_1T_3\bar{p}_3\varepsilon^p\cos^2\theta + \Omega^*\bar{\beta}_2^t\left(g_1^{*2}+g_2^{*2}\right)\left(\ell^*-T_5\right) - T_1\bar{p}_3\varepsilon g_2^*\left(\bar{\beta}_2^t+\bar{\beta}^t\right)\cos^2\theta - T_1\bar{\beta}^t g_2^{*2} + \\ & -T_1\bar{\beta}_3^t\bar{\beta}^t\varepsilon\gamma_1^*\cos^2\theta - T_3\bar{\beta}_2^t g_1^* + 2T_2\bar{\beta}_2^t g_1^* g_2^* + T_2\bar{\beta}_3^t\varepsilon^p g_1^*\cos^2\theta - T_3\bar{\beta}_2^t\varepsilon\gamma_1^*\sin^2\theta + \\ & +\varepsilon\sin 2\theta\left(T_2\bar{\beta}_3^t\gamma_1^* - T_3\bar{p}_3 g_1^* + T_2\bar{p}_3 g_2^* + g_1^* g_2^*\bar{\beta}_3^t\right) + T_2\bar{\beta}^t\varepsilon\bar{p}_3 g_1^*\cos^2\theta - \varepsilon g_2^*\bar{\beta}^t\sin^2\theta + \\ & +2\Omega^*\varepsilon^p\ell^*\cos\theta\left(\bar{\beta}_3^t g_2^*\cos\theta + \bar{\beta}_2^t g_1^*\sin\theta\right) + \Omega^*\varepsilon\gamma_1^*\ell^*\left(\bar{\beta}_3^t\bar{\beta}^t\cos^2\theta + \bar{\beta}_2^t\sin^2\theta\right) - \\ & +\varepsilon g_1^*\bar{\beta}_3^t\bar{\beta}^t\cos^2\theta, \end{aligned}$$

$$Y_3=\left(T_2^2-T_1T_3\right)\left(\gamma_1^*\ell^*\bar{\beta}_2^t-\gamma_1^*T_5\bar{\beta}_2^t-\ell^*\bar{p}_3\varepsilon^p\cos^2\theta\right)-T_1\varepsilon\bar{\beta}^t\ell^*\cos^2\theta\left(2\bar{p}_3g_2^*+\gamma_1^*\bar{\beta}_3^t\right)+$$
$$+g_2^*\bar{\beta}_2^t\left(T_1g_2^*-T_2g_1^*\right)(T_5-\ell^*)-g_1^*\bar{\beta}_2^t\left(T_2g_2^*-T_3g_1^*\right)(T_5-\ell^*)+T_2\bar{\beta}_2^tg_2^*\varepsilon^p\ell^*\sin 2\theta+$$
$$-T_3\bar{\beta}_2^t\ell^*\sin\theta\cos\theta\left(g_1^*\varepsilon^p+\varepsilon\gamma_1^*\right)+T_2\bar{\beta}_3^t\ell^*\cos\theta\left(g_1^*\varepsilon^p\cos\theta+\varepsilon\gamma_1^*\sin\theta\right)+$$
$$+T_2\bar{\beta}_2^t\bar{\beta}^t\varepsilon\gamma_1^*\ell^*\sin\theta\cos\theta+\varepsilon\ell^*\left(g_2^*\bar{\beta}_2^t\sin\theta-\bar{\beta}_3^tg_1^*\cos\theta\right)\left(\bar{\beta}^tg_1^*\cos\theta-g_2^*\sin\theta\right)+$$
$$+\bar{p}_3g_1^*\varepsilon\ell^*\cos\theta\left(T_2\bar{\beta}^t\cos\theta-T_3\sin\theta\right),$$

$$\zeta=\rho V^2,\quad T_1=\bar{C}_{22}\sin^2\theta+P_{22}\sin^2\theta+\mu_eH_0^2\sin^2\theta+\bar{C}_{44}\cos^2\theta+P_{33}\cos^2\theta,$$

$$T_2=\pm(\bar{C}_{23}+\bar{C}_{44}+\mu_eH_0^2)\sin\theta\cos\theta,$$

$$T_3=\bar{C}_{44}\sin^2\theta+P_{22}\sin^2\theta+\bar{C}_{33}\cos^2\theta+P_{33}\cos^2\theta+\mu_eH_0^2\cos^2\theta,$$

$$T_4=K_2\sin^2\theta+K_3\cos^2\theta,\quad T_5=\frac{T_4}{\tau^*C_E},\qquad \ell^*=\rho\omega^2(a_2^*\sin^2\theta+a_3^*\cos^2\theta),$$

$$g_1^*=\bar{g}_{22}\sin^2\theta+\bar{g}_{34}\cos^2\theta,\quad g_2^*=\pm(\bar{g}_{23}+\bar{g}_{34})\sin\theta\cos\theta,$$

$$\gamma_1^*=\bar{\gamma}_{22}\sin^2\theta+\bar{\gamma}_{33}\cos^2\theta,\quad \bar{\beta}_2^t=\frac{\beta_2^t}{\rho},\quad \bar{\beta}_3^t=\frac{\beta_3^t}{\rho},\quad \bar{p}_3=\frac{p_3}{\rho},$$

$$\tau^*=\tau_0^t+\frac{i}{\omega},\quad \Omega^*=1+\left(\frac{\Omega}{\omega}\right)^2,\quad \varepsilon=\frac{\beta_2^{t2}T_0}{\rho C_E},\quad \varepsilon^p=\frac{\beta_2^tp_3T_0}{\rho C_E},$$

$$\beta_2^{t2}=\left(\beta_2^t\right)^2,\quad \bar{\beta}^t=\frac{\beta_3^t}{\beta_2^t},\quad \omega=kV.$$

The three roots $\zeta_j=\rho V_j^2,\ (j=1,2,3,)$ of Eq.(4.8) relate to the complex phase velocities $V_j,(j=1,\ 2,\ 3,)$ of three quasi waves, namely, coupled quasi-thermoelastic qP, qT, qSV waves, respectively. The root of Eq.(4.7) $\zeta_j=\rho V_4^2=0\Rightarrow V_4^2=0,$ represents an evanescent wave ($PE-wave$), an evanescent wave is the non-propagating wave that exists. The evanescent wave, also known as an evanescent field, is an electromagnetic oscillating electric or magnetic field whose energy is spatially concentrated close to the source but which does not spread as an electromagnetic wave. Evanescent waves do not propagate as they do not possess energy and the exponential decay in its amplitude, therefore without any dissipation of energy. There are four physiologically acceptable partial waves in the reflected field if the incidence wave is configured such that the component of slowness (inverse phase velocity) is parallel to the reflecting surface that at least one of these waves is evanescent and for certain reflection geometries, these waves are homogeneous bulk waves, when the longitudinal sheet of the slowness surface possesses negatively curved regions. The shear waves propagating along the axis of symmetry are independent of thermal and piezoelectric effects, whereas longitudinal waves, thermal waves are dependent on these effects. Longitudinal waves, thermal waves, and *SH* (shear horizontal) waves propagating perpendicular to the axis of symmetry are independent of the piezoelectric effect. SV (shear vertical) wave depends upon the piezoelectric effect.

The four roots V_j $(j=1,2,.,4)$ of the Eqs (4.7) and (4.8) correlate to complex phase velocity values of coupled quasi-thermoelastic systems of qP, qT, qSV and $(PE-wave)$ waves. If $V_j^{-1}=c_j^{-1}+i\omega^{-1}Q_j,(i=1,2,.,4)$, then phase velocity V, wave number k are complex. Real parts, Re $(V)\geq 0$, of the four roots of the Eqs (4.7) and (4.8) exemplify the propagation rate of qP,qT,qSV and $\left(PE-wave, V=0\right)$ waves, $\mathrm{Im}\,g(V)\leq 0$, refers to damped wave. Phase velocity, attenuation coefficient, specific loss and penetration depth are defined as:

Phase velocities, $$V_j=\frac{\left\{\mathrm{Re}\left(V_j\right)\right\}^2+\left\{\mathrm{Im}\,g\left(V_j\right)\right\}^2}{\mathrm{Re}\left(V_j\right)}. \tag{4.9}$$

Attenuation coefficient, $$Q_j=\frac{-\omega\,\mathrm{Im}\,g\left(V_j\right)}{\left\{\mathrm{Re}\left(V_j\right)\right\}^2+\left\{\mathrm{Im}\,g\left(V_j\right)\right\}^2}. \tag{4.10}$$

Specific loss $$S_j==4\pi\left|\frac{VQ}{\omega}\right|, \qquad (j=1,2,3,) \tag{4.11}$$

where $\mathrm{Re}(.)$ denotes real part of phase velocity and $\mathrm{Im}\,g(.)$ denotes imaginary part of phase velocity V_j. V_1,V_2, V_3, are phase velocities, Q_1,Q_2,Q_3, are constants of attenuation, S_1,S_2,S_3, are constants of specific loss, k_1,k_2,k_3, are wave numbers of coupled quasi -thermoelastic qP,qT,qSV and one wave with V_4, is zero $(PE-wave, V=0)$.

5. Special case: Initially stressed magneto-thermo-piezoelectric rotating monoclinic case (In the absence of two temperatures)

Equation of motion;

$$\begin{gathered}(\bar{C}_{22}+P_{22})\frac{\partial^2 v}{\partial y^2}+(\bar{C}_{44}+P_{33})\frac{\partial^2 v}{\partial z^2}+(\bar{C}_{23}+\bar{C}_{44})\frac{\partial^2 w}{\partial y\partial z}+\\ +\bar{g}_{22}\frac{\partial^2\psi}{\partial y^2}+\bar{g}_{34}\frac{\partial^2\psi}{\partial z^2}-\beta_2^t\frac{\partial T}{\partial y}+(\boldsymbol{J}\times\boldsymbol{B})_2=\rho\left(\frac{\partial^2 v}{\partial t^2}-\Omega^2 v-2\Omega\frac{\partial w}{\partial t}\right),\end{gathered} \tag{5.1}$$

$$\begin{gathered}(\bar{C}_{44}+P_{22})\frac{\partial^2 w}{\partial y^2}+(\bar{C}_{33}+P_{33})\frac{\partial^2 w}{\partial z^2}+(\bar{C}_{23}+\bar{C}_{44})\frac{\partial^2 v}{\partial y\partial z}+(\bar{g}_{34}+\bar{g}_{23})\frac{\partial^2\psi}{\partial y\partial z}+\\ -\beta_3^t\frac{\partial T}{\partial z}+(\boldsymbol{J}\times\boldsymbol{B})_3=\rho\left(\frac{\partial^2 w}{\partial t^2}-\Omega^2 w+2\Omega\frac{\partial v}{\partial t}\right).\end{gathered} \tag{5.2}$$

Gauss equation;

$$\bar{g}_{34}\frac{\partial^2 v}{\partial z^2}+\bar{g}_{22}\frac{\partial^2 v}{\partial y^2}+(\bar{g}_{23}+\bar{g}_{34})\frac{\partial^2 w}{\partial y\partial z}-\bar{\gamma}_{22}\frac{\partial^2\psi}{\partial y^2}-\bar{\gamma}_{33}\frac{\partial^2\psi}{\partial z^2}-p_3\frac{\partial T}{\partial z}=0. \tag{5.3}$$

The heat equation is given by Lord and Shulman [2] as follows:

$$K_2 \frac{\partial^2 T}{\partial y^2} + K_3 \frac{\partial^2 T}{\partial z^2} = \left(1 + \tau_0^t \frac{\partial}{\partial t}\right)\left\{T_0\left(\beta_2^t \frac{\partial^2 v}{\partial y \partial t} + \beta_3^t \frac{\partial^2 w}{\partial z \partial t}\right) + \rho C_E \dot{T} + p_3 T_0 \frac{\partial \psi}{\partial z}\right\}, \tag{5.4}$$

which can be written as:

$$(T_1 - \Omega^* \zeta)A + \left(T_2 + 2i\frac{\Omega}{\omega}\zeta\right)B + i\frac{\beta_2^t}{k}\sin\theta\, C + g_1^* D = 0, \tag{5.5}$$

$$\left(T_2 - 2i\frac{\Omega}{\omega}\zeta\right)A + (T_3 - \Omega^* \zeta)B \pm \frac{i\beta_3^t}{k}\cos\theta\, C + g_2^* D = 0, \tag{5.6}$$

$$g_1^* A + g_2^* B \pm i\frac{p_3}{k}\cos\theta C - \gamma_1^* D = 0, \tag{5.7}$$

$$\zeta\varepsilon \sin\theta\, A \pm \zeta\varepsilon\,\overline{\beta}^t \cos\theta B + i\frac{\beta_2^t}{k}\{T_5 - \zeta\}C \pm \zeta\varepsilon^p \cos\theta D = 0, \tag{5.8}$$

$$\begin{vmatrix} T_1 - \Omega^*\zeta & T_2 + 2i\frac{\Omega}{\omega}\zeta & \beta_2^t \sin\theta & g_1^* \\ T_2 - 2i\frac{\Omega}{\omega}\zeta & T_3 - \Omega^*\zeta & \pm\beta_3^t \cos\theta & g_2^* \\ g_1^* & g_2^* & \pm p_3 \cos\theta & -\gamma_1^* \\ \zeta\varepsilon\sin\theta & \pm\zeta\varepsilon\,\overline{\beta}^t\cos\theta & \beta_2^t(T_5 - \zeta) & \pm\zeta\varepsilon^p\cos\theta \end{vmatrix} = 0,$$

which is cubic in ζ and can be written:

$$X_0\,\zeta^3 + X_1\zeta^2 + X_2\zeta + X_3 = 0, \tag{5.9}$$

$$X_0 = \left\{\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right\}\left(\overline{p}_3\varepsilon^p\cos^2\theta - \overline{\beta}_2^t\gamma_1^*\right),$$

$$X_1 = \Omega^*(T_1 + T_3)\left(\overline{\beta}_2^t\gamma_1^* - \overline{p}_3\varepsilon^p\cos^2\theta\right) + 2\Omega^* g_2^*\overline{\beta}_3^t\varepsilon^p\cos^2\theta + \Omega^*\overline{\beta}_2^t\left(g_1^{*2} + g_2^{*2}\right) +$$
$$+\Omega^*\varepsilon\gamma_1^*\left(\overline{\beta}_3^t\overline{\beta}^t\cos^2\theta + \overline{\beta}_2^t\sin^2\theta\right) + 2\Omega^* g_1^*\overline{\beta}_2^t\varepsilon^p\sin\theta\cos\theta + \overline{\beta}_2^t\gamma_1^* T_5\left[\Omega^{*2} - 4\left(\frac{\Omega}{\omega}\right)^2\right],$$

$$X_2 = \Omega^* \gamma_1^* T_5 \overline{\beta}_2^t\, (T_1 + T_3) + \gamma_1^* \overline{\beta}_2^t (T_2^2 - T_1 T_3) - T_2 \overline{p}_3 \varepsilon^p \cos^2\theta + T_1 T_3 \overline{p}_3 \varepsilon^p \cos^2\theta +$$
$$- T_5 \Omega^* \overline{\beta}_2^t \left(g_1^{*2} + g_2^{*2}\right) - T_1 \overline{p}_3 \varepsilon g_2^* \left(\overline{\beta}_2^t + \overline{\beta}^t\right)\cos^2\theta - T_1 \overline{\beta}^t g_2^{*2} - T_1 \overline{\beta}_3^t \overline{\beta}^t \varepsilon \gamma_1^* \cos^2\theta - T_3 \overline{\beta}_2^t g_1^* +$$
$$+ 2T_2 \overline{\beta}_2^t g_1^* g_2^* + T_2 \overline{\beta}_3^t \varepsilon^p g_1^* \cos^2\theta - T_3 \overline{\beta}_2^t \varepsilon \gamma_1^* \sin^2\theta + \varepsilon \sin 2\theta \Big(T_2 \overline{\beta}_3^t \gamma_1^* - T_3 \overline{p}_3 g_1^* + T_2 \overline{p}_3 g_2^* +$$
$$g_1^* g_2^* \overline{\beta}_3^t \Big) + T_2 \overline{\beta}^t \varepsilon \overline{p}_3 g_1^* \cos^2\theta - \varepsilon g_2^* \overline{\beta}^t \sin^2\theta - \varepsilon g_1^* \overline{\beta}_3^t \overline{\beta}^t \cos^2\theta,$$

$$X_3 = \gamma_1^* T_5 \overline{\beta}_2^t \left(T_1 T_3 - T_2^2\right) + T_5 \overline{\beta}_2^t \left(T_1 g_2^{*2} - 2T_2 g_1^* g_2^* + T_3 g_1^{*2}\right).$$

In a monoclinic medium the angle of the incidence is not equal to the angle of reflection. For analyzing reflection problem, the procedure to compute reflected angles was discussed by Singh and Khurana [35] for a monoclinic medium. Singh and Yadav [40] formulated the procedure to compute reflected angles for a rotating magneto thermo-monoclinic medium.

6. Particular cases

- **(i)** For an isotropic case $\overline{C}_{22} = \overline{C}_{33} = \lambda + 2\mu$, $\overline{C}_{13} = \overline{C}_{23} = \overline{C}_{12} = \lambda$, $\overline{C}_{44} = \overline{C}_{55} = \overline{C}_{66} = \mu$, $\overline{C}_{56} = 0$, $\overline{C}_{24} = 0$, $\overline{C}_{14} = 0$, $\overline{C}_{34} = 0$, $\beta_2^t = \beta_3^t = \beta^t$, $\overline{\beta}^t = 1$, $K_2 = K_3 = K$, the Eq.(4.8) reduces for the rotating isotropic magneto-thermoelastic case.
- **(ii)** For $\Omega = 0$, $\Omega^* = 1$, Eq.(4.8) reduces for the monoclinic magneto-piezo-thermoelastic case.
- **(iii)** For $H_0 = 0$, $\mu_e = 0$, The Eq.(4.8) reduces for the rotating monoclinic piezo-thermoelastic case.
- **(iv)** For $\varepsilon = 0$, $D_4 = 0$, $\overline{g}_{22} = \overline{g}_{23} = \overline{g}_{34} = \overline{\gamma}_{22} = \overline{\gamma}_{33} = p_3 = 0$, the Eq.(4.8) reduces for the rotating monoclinic magneto-elastic case.
- **(v)** For $H_0 = 0$, $\mu_e = 0$, $\Omega = 0$, $D_4 = 0$, $\varepsilon = 0$, $\Omega^* = 1$, $\overline{g}_{22} = \overline{g}_{23} = \overline{g}_{34} = \overline{\gamma}_{22} = \overline{\gamma}_{33} = p_3 = 0$, the Eq.(4.8) reduces for the monoclinic elastic case.

7. Numerical verification

The following are relevant considerations for numerical calculations of quasi-plane wave speeds [38]. Monoclinic parameters:

$$\overline{C}_{33} = 145.5\,GPa = 1.455 \times 10^{11}\,N\,m^{-2}, \qquad \overline{C}_{22} = 160.4\,GPa = 1.6 \times 10^{11}\,N\,m^{-2},$$

$$\overline{C}_{44} = 40.86\,GPa = 0.4 \times 10^{11}\,N\,m^{-2}, \qquad \overline{C}_{23} = 75.94\,GPa = 0.7594 \times 10^{11}\,N\,m^{-2},$$

$$\overline{C}_{34} = \overline{C}_{24} = 0, \qquad \rho = 7.45 \times 10^3\,Kg\,m^{-3}.$$

Thermoelastic parameters:

$$C_E = 3.9\times10^2\ J\,Kg^{-1}\deg^{-1},\quad K_2 = 1.24\times10^2\ W\,m^{-1}\deg^{-1},$$

$$K_3 = 1.34\times10^2\ W\,m^{-1}\deg^{-1},\ \ \beta_2 = 5.75\times10^6\ N\,m^{-2}\deg^{-1},\ \beta_3 = 5.17\times10^6\ N\,m^{-2}\deg^{-1},$$

$$T_o = 296\,K,\quad \tau_o = 0.05\,s,\quad a_2^* = 0.4,\quad a_3^* = 0.5,\quad \omega = 5\,Hz.$$

Isotropic parameters:

$$\lambda = 0.3\times10^{11}\,NM^{-2},\quad \mu = 0.25\times10^{11}\,NM^{-2},\quad \rho = 2.7\times10^3\,kg/m^3.$$

Piezoelectric parameters:

$$\bar{g}_{22} = 17.41\,C/m^2,\quad \bar{g}_{34} = 15.4\,C/m^2,\quad \bar{g}_{23} = -6.32\,C/m^2,$$

$$\bar{\gamma}_{22} = 8.29\times10^{-11}\,NK^{-1}m^{-2},\ \bar{\gamma}_{33} = 9.07\times10^{-11}\,NK^{-1}m^{-2},\quad p_3 = 7.6\times10^{-6}\,C\,m^{-2}K^{-1}.$$

With the help of MATLAB Eq.(4.8) is solved numerically to obtain the phase velocity, attenuation coefficient and specific loss of coupled quasi-thermoelastic qP, qT, qSV waves in a piezo-electric rotating monoclinic magneto-thermoelastic medium. The phase velocity, attenuation coefficient and specific loss of coupled quasi-thermoelastic qP, qT, qSV waves are plotted against frequency $20 \le \omega \le 100$, at different values of rotation frequency $\Omega = 0, 10, 50$, when $T_o = 296\,K$, $\theta = 30°$, $\tau_o = 0.05\,s$, $a_2^* = 0.4$, $a_3^* = 0.5$, $H_0 = 10$, and are shown in Fig.1(a-c) to 3(a-c). It is clear from the graph that the phase velocity of coupled quasi-thermoelastic waves qP, qT, qSV increases as frequency changes $20 \le \omega \le 100$, at different values of rotation $\Omega = 0, 10, 50$. Similarly, the alteration of the phase velocity, constant of attenuation and rate of specific loss of coupled quasi-thermoelastic qP, qT, qSV waves are plotted against frequency $20 \le \omega \le 100$, at varying magnetic field values $H_0 = 0, 50, 100$, when $T_o = 296\,K$, $\theta = 30°$, $\tau_o = 0.05\,s$, $a_2^* = 0.4$, $a_3^* = 0.5$, $\Omega = 10$, and are shown in Fig.4(a-c) to 6(a-c). It is clear from the graph that the phase velocity and specific loss of coupled quasi-thermoelastic qP, qT, qSV waves increases as frequency changes $20 \le \omega \le 100$, at various magnetic field intensity parameters $H_0 = 0, 50, 100$.

The variation of phase velocity, attenuation coefficient and specific loss of coupled quasi-thermoelastic qP, qT, qSV waves are plotted against frequency $20 \le \omega \le 100$, at different values of the dielectric constant $\bar{\gamma}_{22} = 8, 10, 12$, when $T_o = 296\,K$, $\tau_o = 0.05\,s$, $\theta = 30°$, $a_2^* = 0.4$, $a_3^* = 0.5$, $\Omega = 10$, and are shown in Fig.7(a-c) to 9(a-c). It is clear from the graph that the phase velocity and specific loss of coupled quasi-thermoelastic qP, qT, qSV waves increase as frequency changes $20 \le \omega \le 100$, at different values of the dielectric constant $\bar{\gamma}_{22} = 8, 10, 12$ at $H_0 = 10$.

The variations of phase velocity, attenuation coefficient and specific loss of coupled quasi-thermoelastic qP, qT, qSV waves are plotted against frequency $20 \le \omega \le 100$, at different value of two-temperature parameter $a_2^* = 0.04, 0.08, 0.12$, when $T_o = 296\,K$, $\tau_o = 0.05\,s$, $\theta = 30°$, $a_3^* = 0.5$, $\Omega = 10$, and are shown in Fig.10(a-c) to 12(a-c). It is clear from the graph that the phase velocity and specific loss of coupled quasi-thermoelastic qP, qT, qSV increases as frequency changes $20 \le \omega \le 100$, at different values of the two-temperature parameter $a_2^* = 0.04, 0.08, 0.12$.

a) b) c)

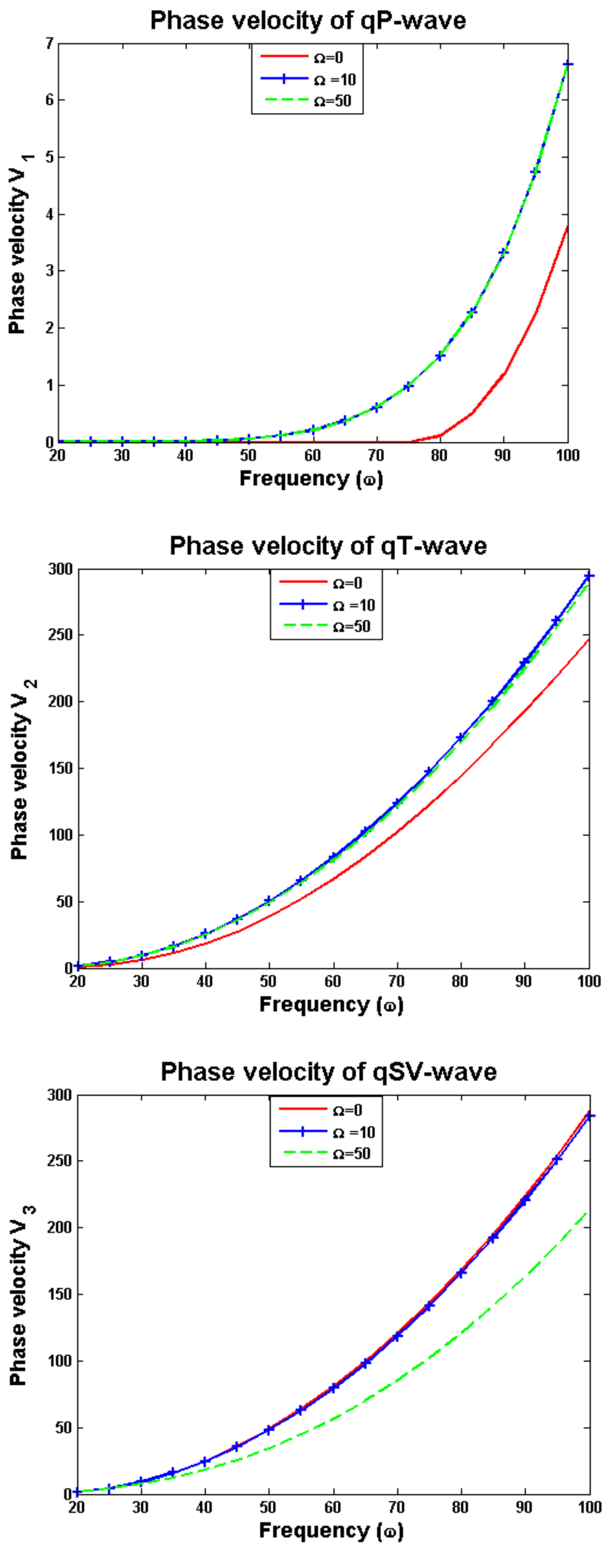


Fig.1(a-c). Variations of phase velocities of qP, qT, qSV waves against frequency $20 \leq \omega \leq 100$, at different values of the rotation $\Omega = 0, 10, 50$.

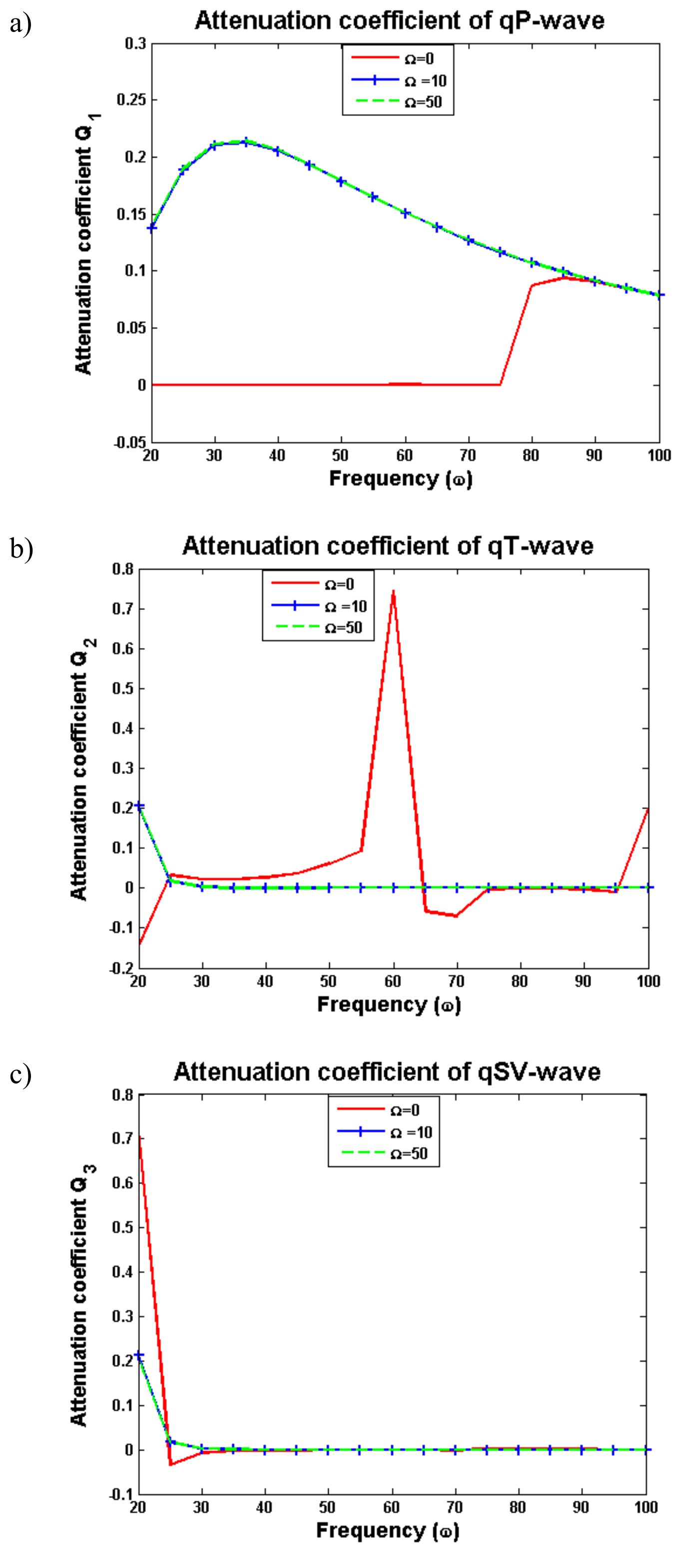
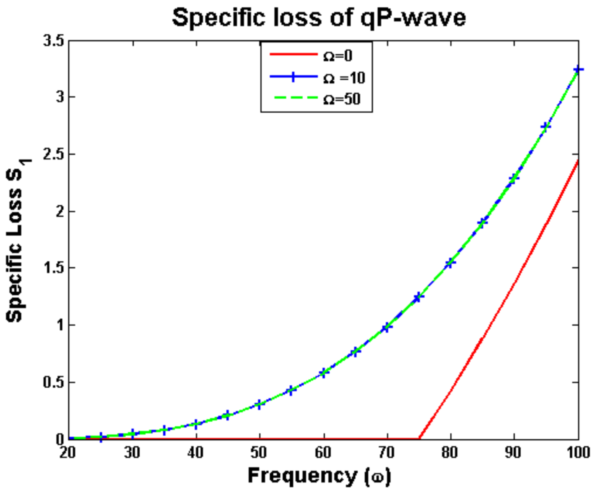
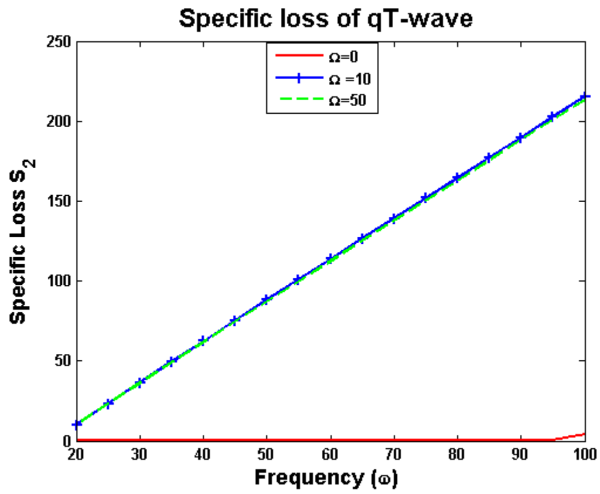


Fig.2(a-c). Variations of the attenuation coefficient of qP, qT, qSV waves against frequency $20 \leq \omega \leq 100$, at different values of rotation $\Omega = 0, 10, 50$.

a)



b)



c)

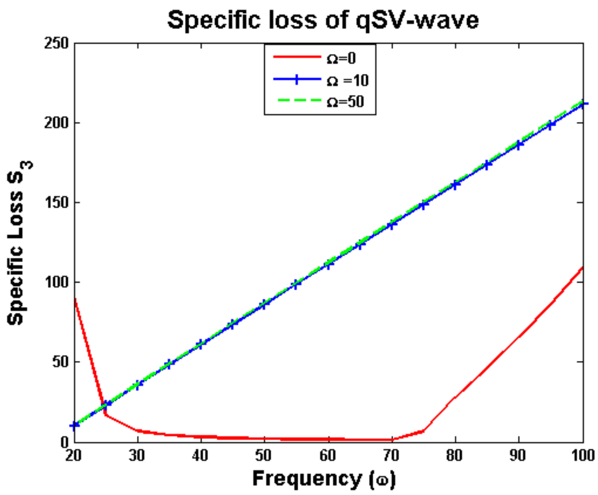


Fig.3 (a-c).Variations of the specific loss of qP, qT, qSV waves against frequency $20 \le \omega \le 100$, at different values of rotation $\Omega = 0, 10, 50$.

a) b) c)

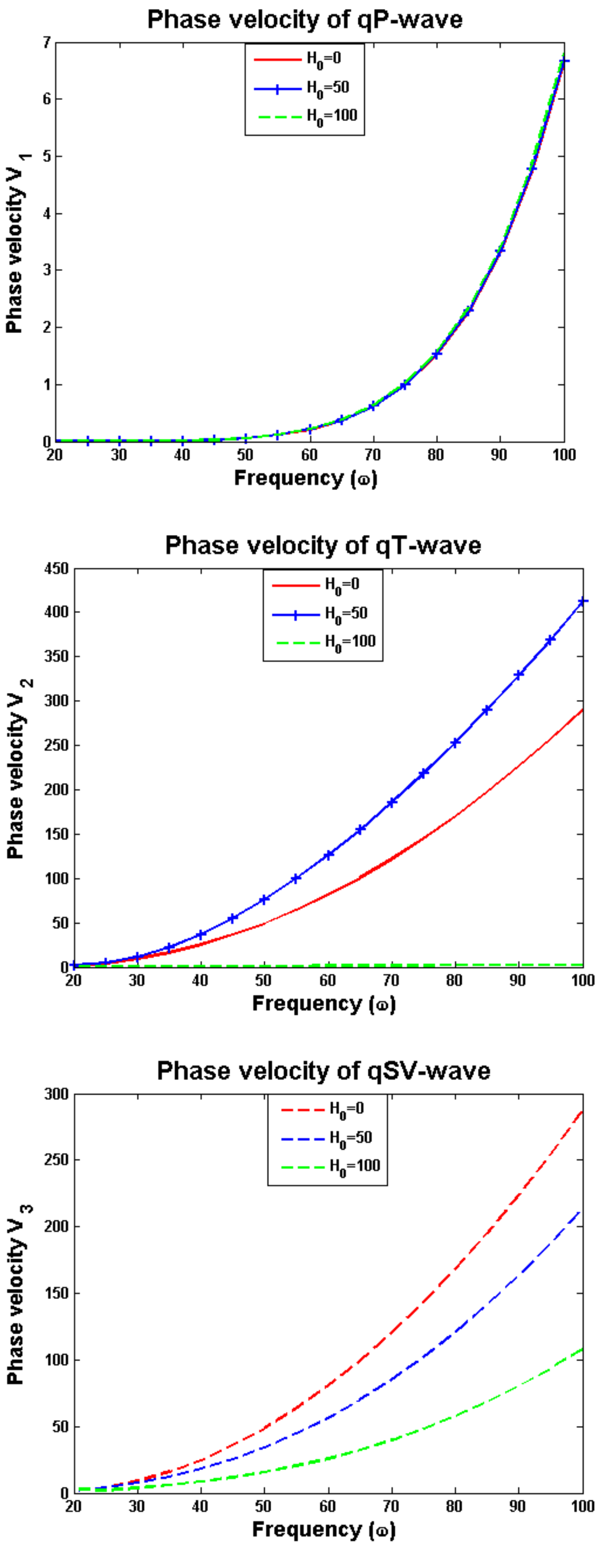
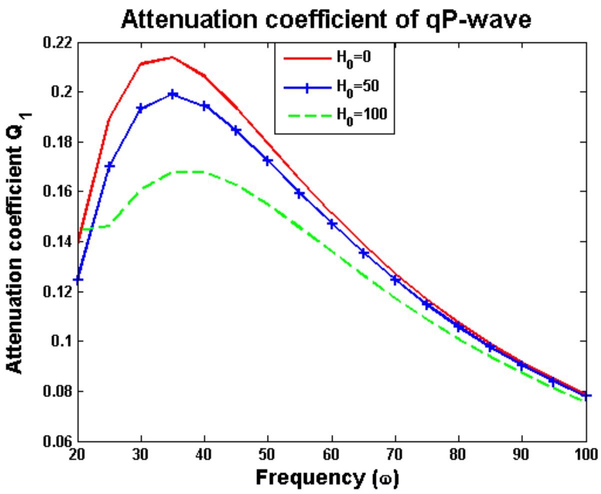
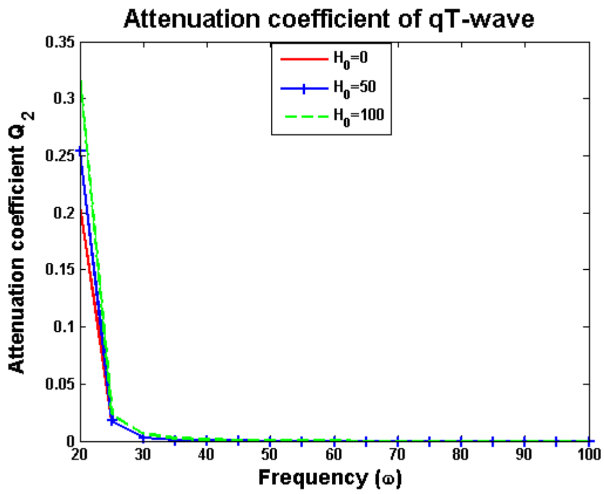


Fig.4(a-c). Variations of phase velocity of qP, qT, qSV waves against frequency $20 \le \omega \le 100$, at different values of the magnetic field $H_0 = 0, 50, 100$.

a)



b)



c)

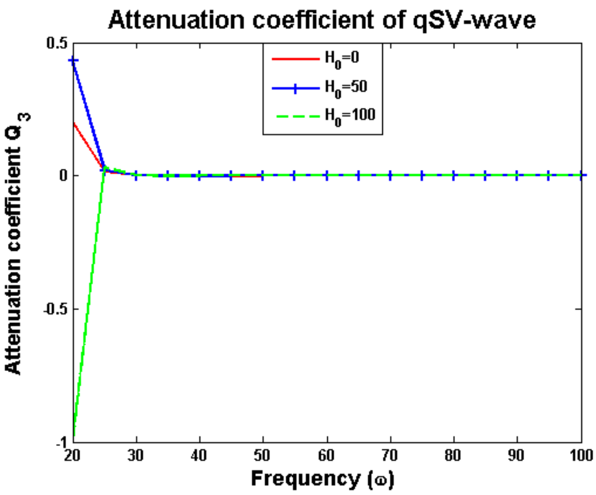


Fig.5c. Variations of the attenuation coefficient of qP, qT, qSV waves against frequency $20 \le \omega \le 100$, at different values of the magnetic field $H_0 = 0, 50, 100$.

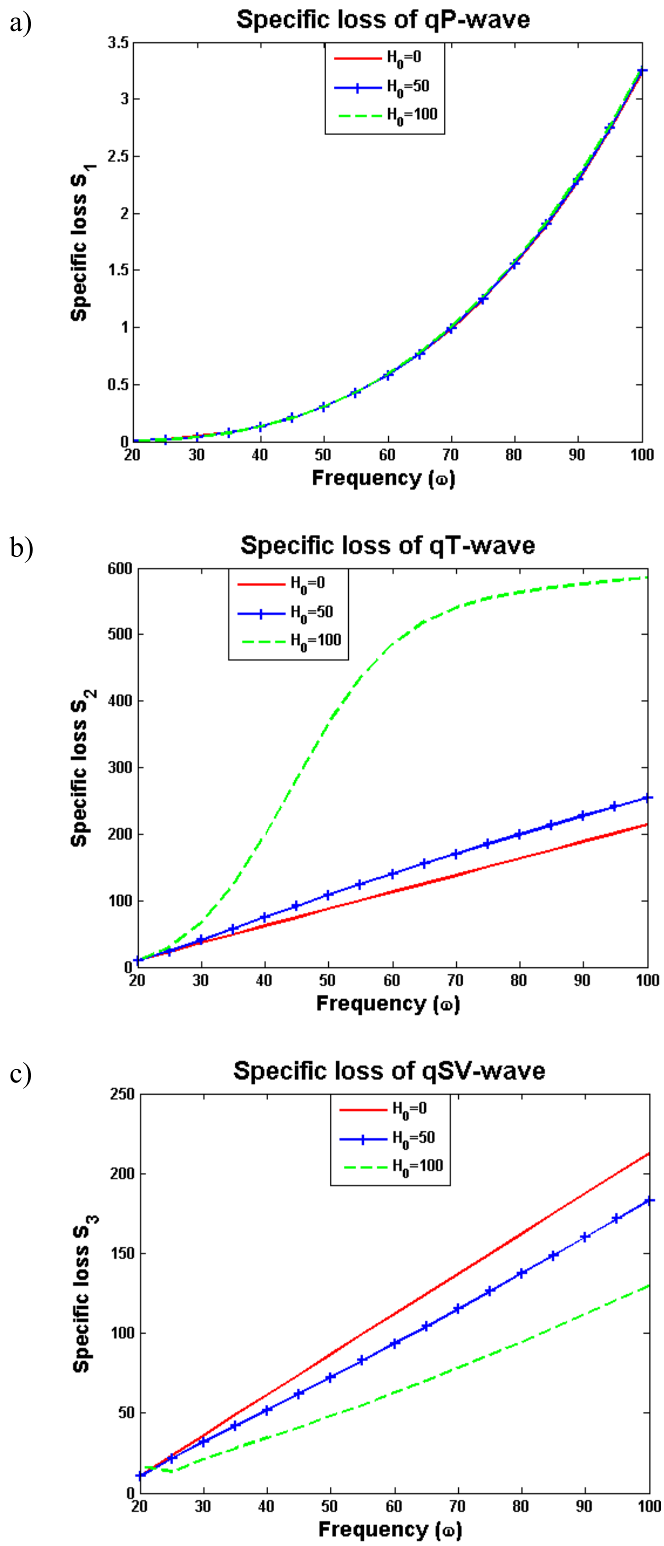


Fig.6(a-c). Variations of specific loss of qP, qT, qSV waves against frequency $20 \le \omega \le 100$, at different values of the magnetic field $H_0 = 0, 50, 100$.

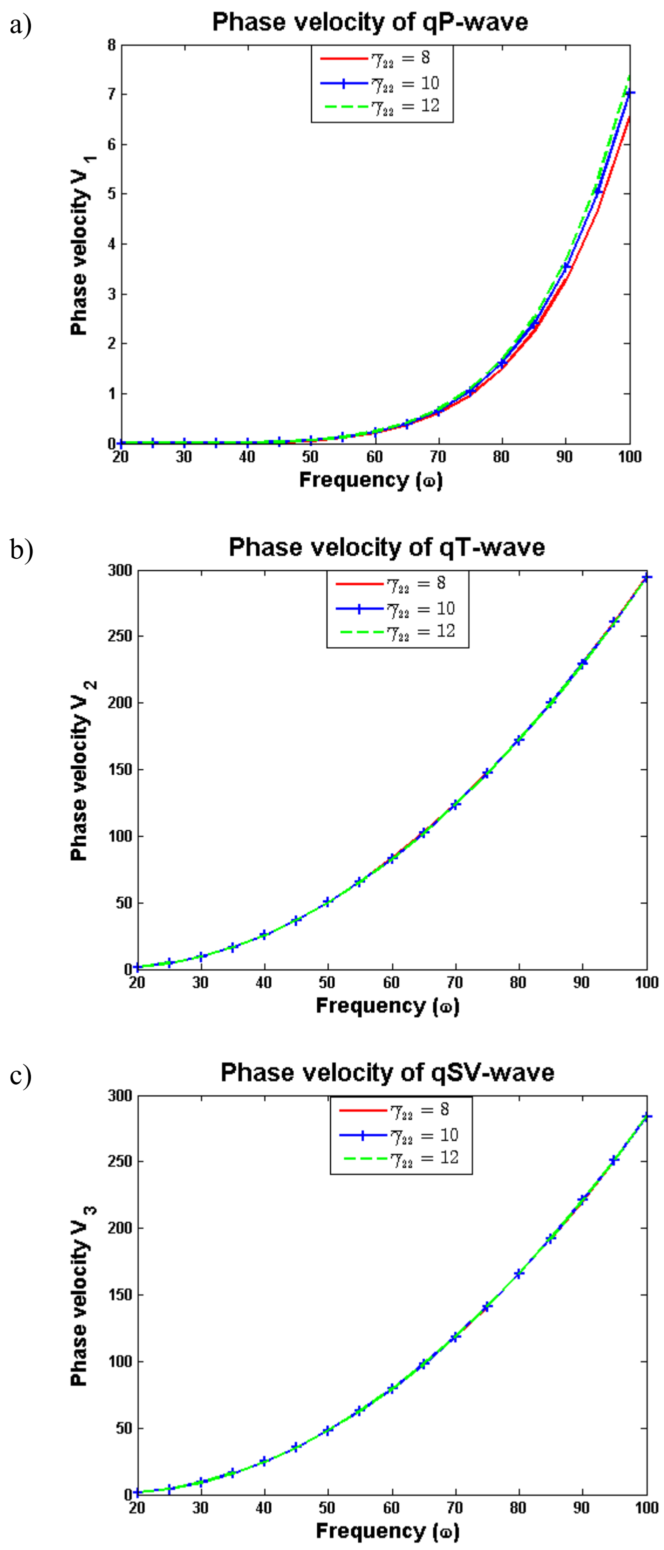


Fig.7(a-c). Variations of phase velocity of qP, qT, qSV waves against frequency $20 \leq \omega \leq 100$, at different values of the dielectric constant $\overline{\gamma}_{22} = 8, 10, 12$.

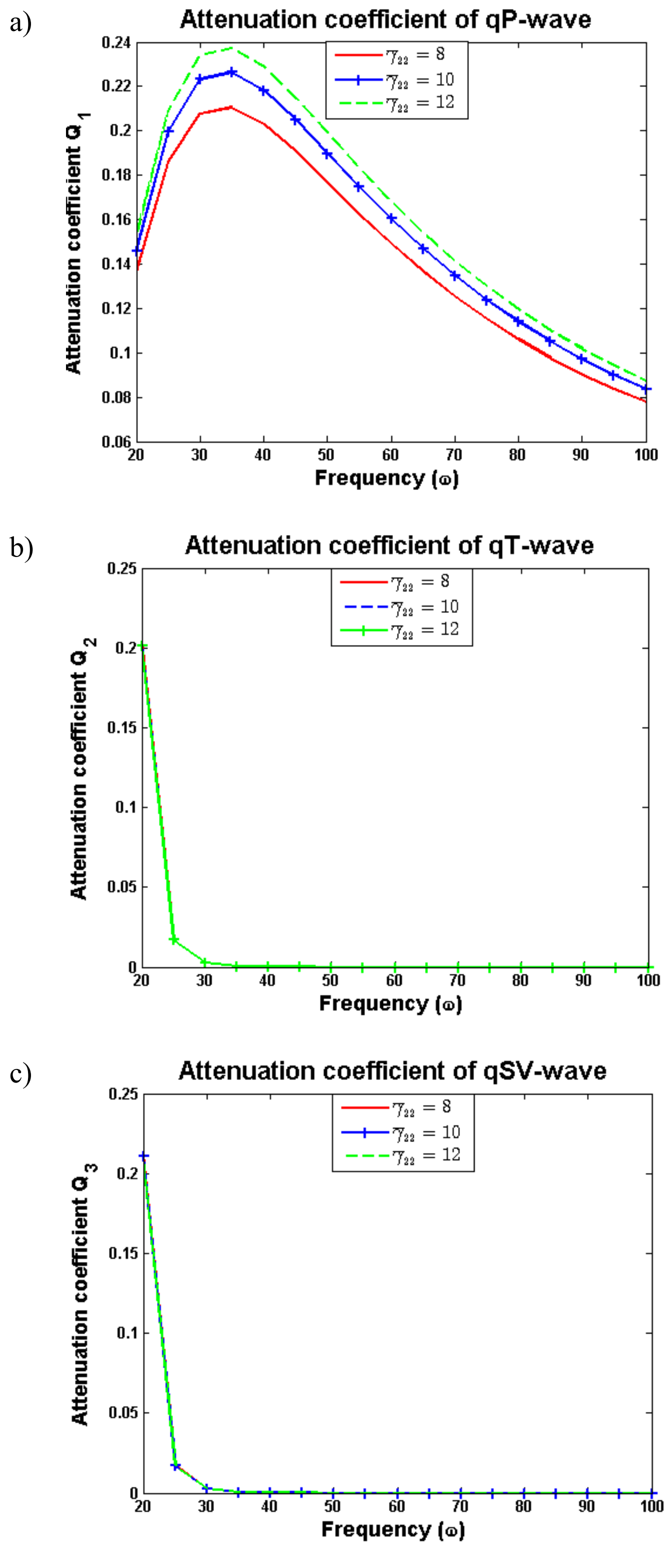


Fig.8(a-c). Variations of the attenuation coefficient of qP, qT, qSV waves against frequency $20 \leq \omega \leq 100$, at different values of the dielectric constant $\overline{\gamma}_{22} = 8, 10, 12$.

a)

b)

c)

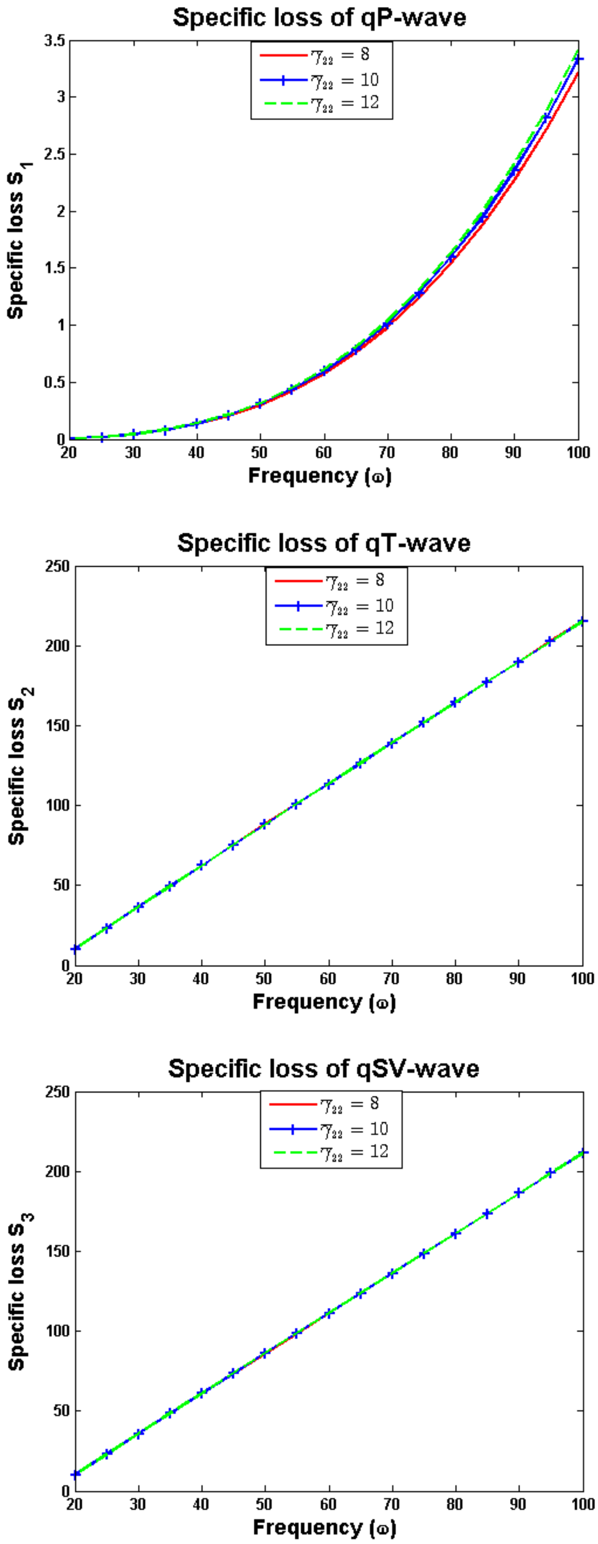


Fig.9(a-c). Variations of specific loss of qP, qT, qSV waves against frequency $20 \le \omega \le 100$, at different values of the dielectric constant $\bar{\gamma}_{22} = 8, 10, 12$.

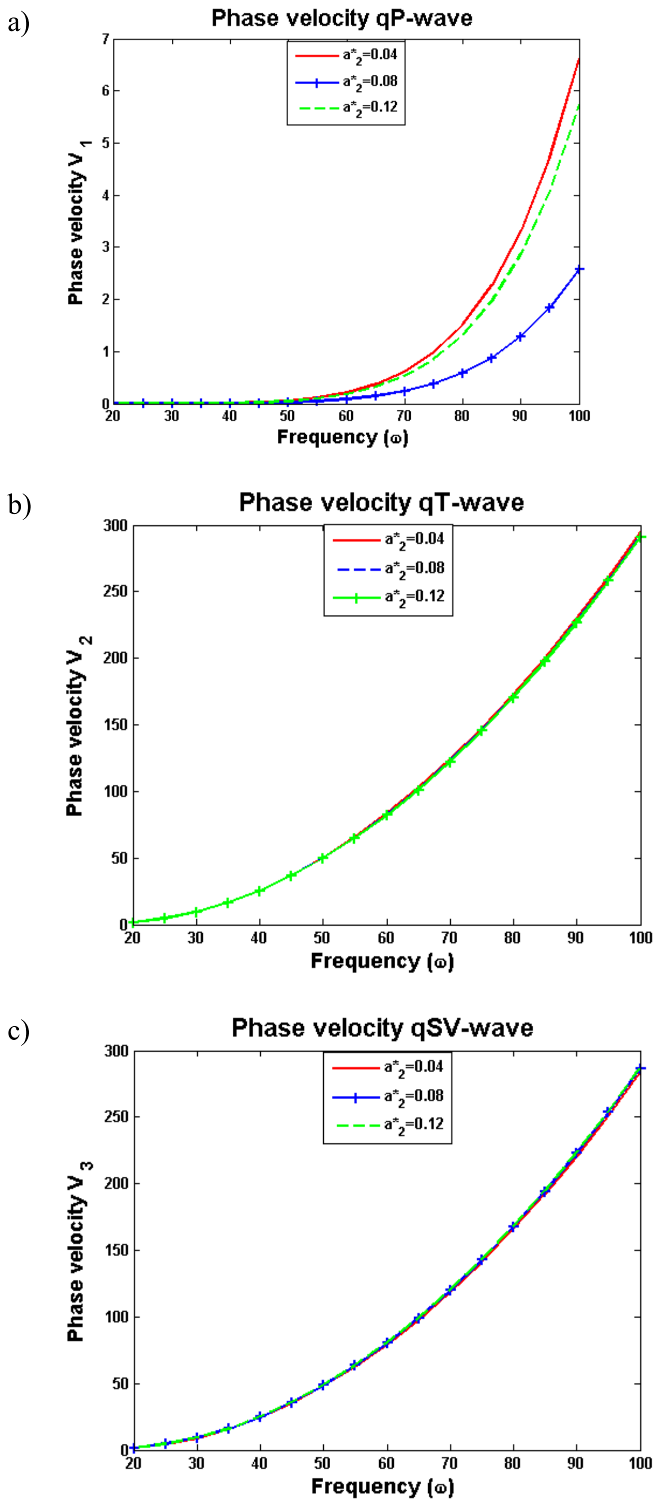


Fig.10(a-c). Variations of phase velocity of qP, qT, qSV waves against frequency $20 \le \omega \le 100$, at different values of $a_2^* = 0.04,\ 0.08,\ 0.12$.

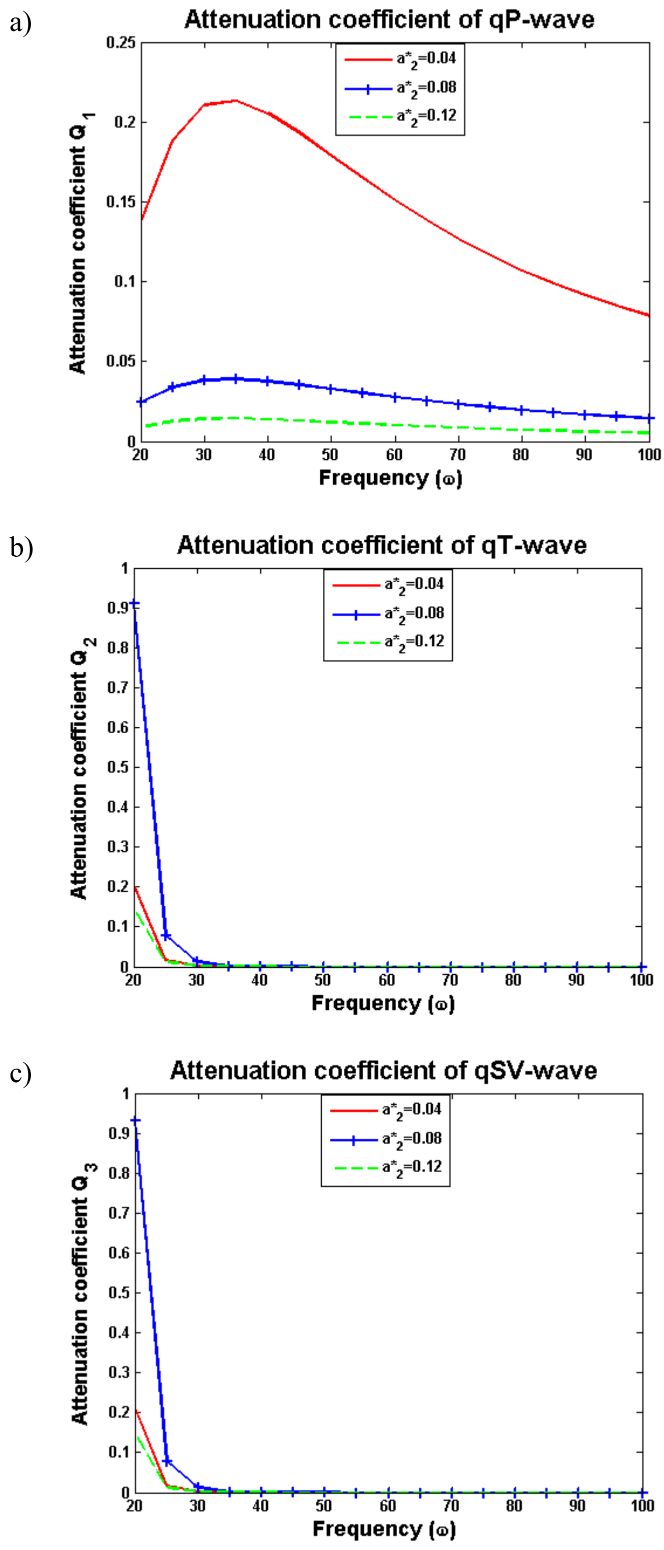
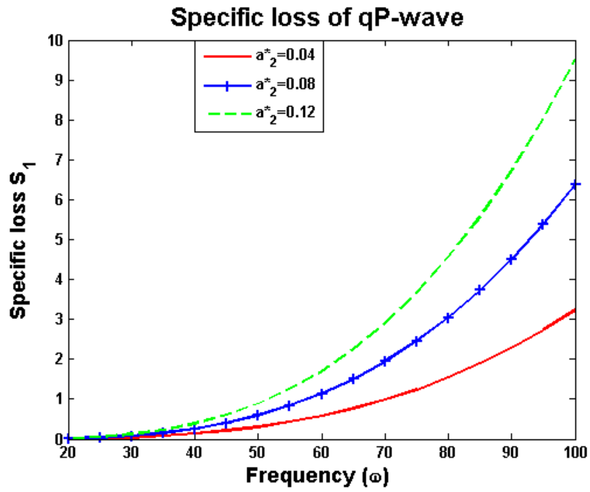
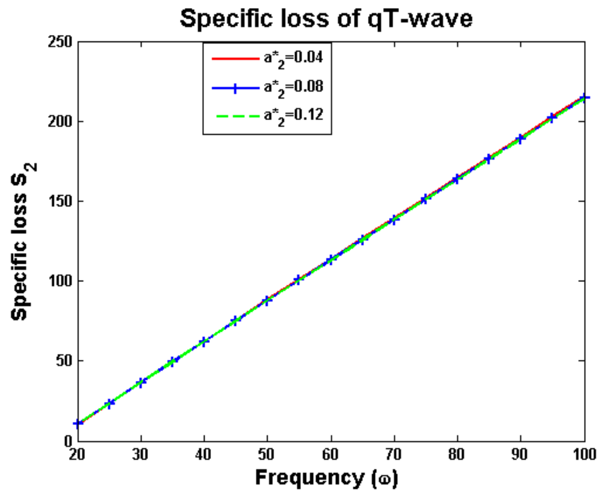


Fig.11(a-c). Variations of the attenuation coefficient of qP, qT, qSV waves against frequency $20 \leq \omega \leq 100$, at different values of $a_2^* = 0.04,\ 0.08,\ 0.12$.

a)



b)



c)

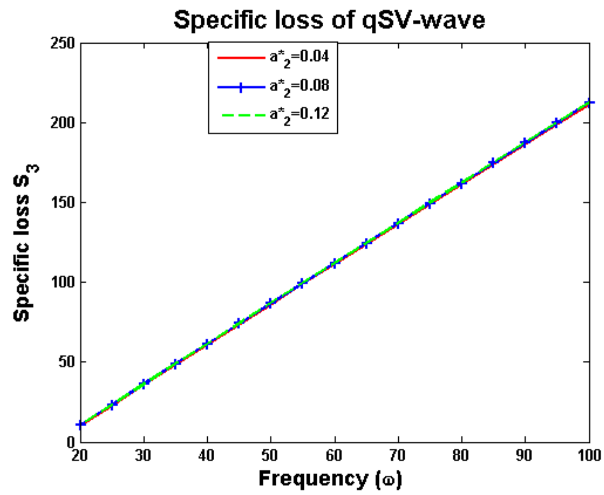


Fig.12(a-c). Variations of specific loss of qP, qT, qSV waves against frequency $20 \leq \omega \leq 100$, at a varying value of $a_2^* = 0.04, 0.08, 0.12$.

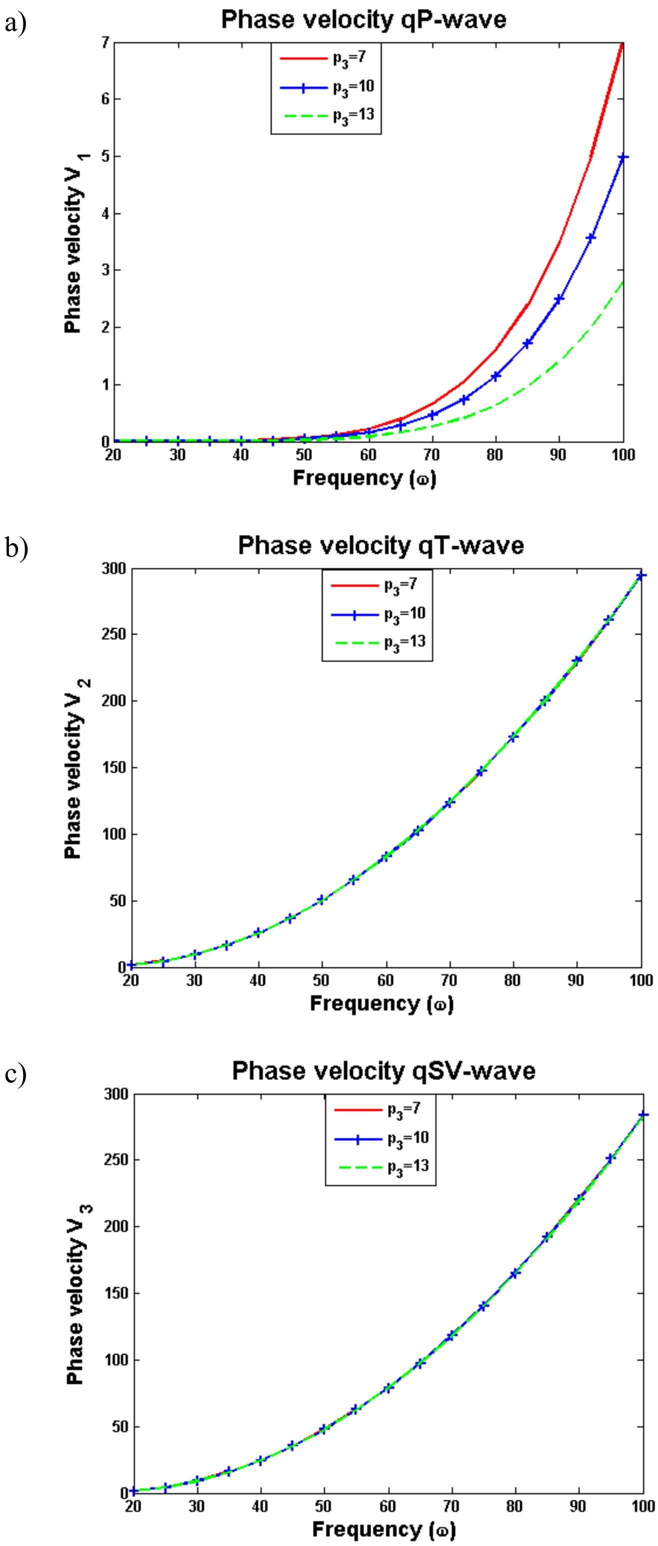


Fig.13(a-c). Variations of phase velocity of qP, qT, qSV waves against frequency $20 \leq \omega \leq 100$, at different values of the pyroelectric constant $p_3 = 7, 10, 13$.

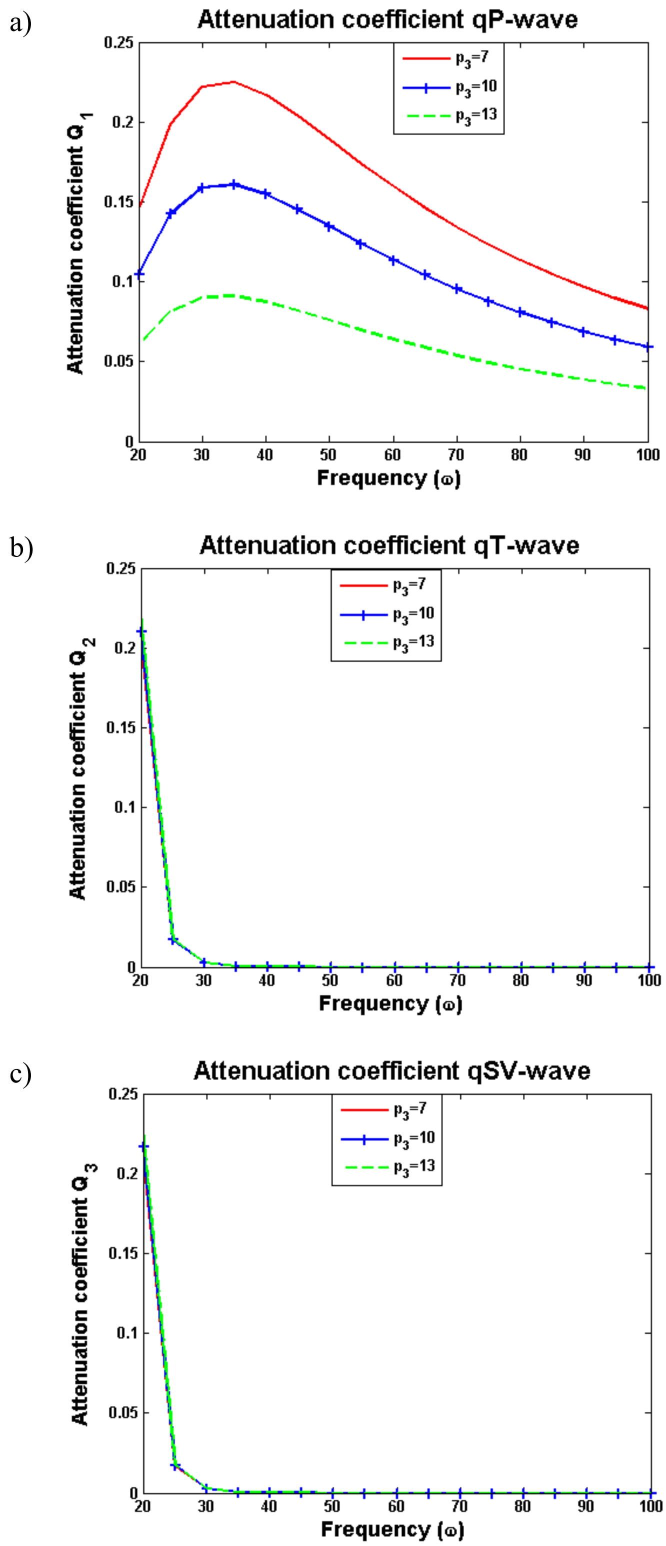


Fig.14(a-c). Variations of the attenuation coefficient of qP, qT, qSV waves against frequency $20 \leq \omega \leq 100$, at different values of the pyroelectric constant $p_3 = 7, 10, 13$.

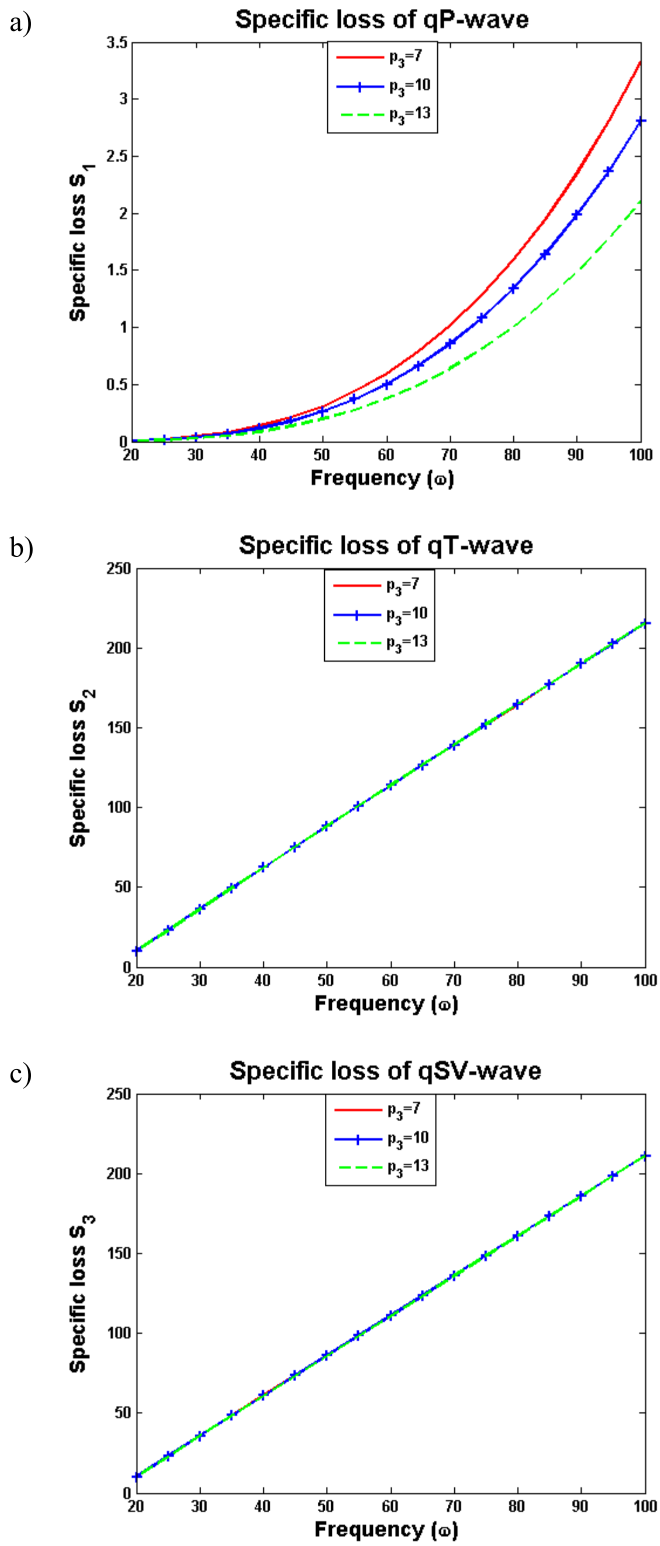


Fig.15(a-c). Variations of specific loss of qP, qT, qSV waves against frequency $20 \leq \omega \leq 100$, at different values of the pyroelectric constant $p_3 = 7, 10, 13$.

The change of phase velocity, coefficient of attenuation and specific loss of coupled quasi-thermoelastic qP, qT, qSV waves are plotted against frequency $20 \leq \omega \leq 100$, at different values of the pyroelectric constant $p_3 = 7, 10, 13$, when $T_o = 296\,K$, $\tau_o = 0.05\,s$, $\theta = 30°$, $a_2^* = 0.04$, $a_3^* = 0.5$, $\Omega = 10$, and are shown in Fig.13(a-c) to 15(a-c). It is clear from the graph phase velocity and specific loss of coupled quasi-thermoelastic qP, qT, qSV waves increase as frequency changes $20 \leq \omega \leq 100$, at different values of the pyroelectric constant $p_3 = 7, 10, 13$.

8. Conclusions

The plane wave solutions of equations governing the rotating monoclinic piezo-magneto-thermoelastic medium are obtained. There exist four piezo-thermoelastic waves, three coupled quasi -thermoelastic plane waves, namely: qP, qT, qSV and ($PE - wave$) waves. The phase velocity, attenuation coefficient and specific loss of these waves are computed for a particular material using MATLAB programming. From numerical results, it is observed that the phase velocity, attenuation coefficient and specific loss of these plane waves are significantly affected by rotation and magnetic and piezo-electric field. The influence of piezoelectric effect, two-temperature parameter, frequency, rotation and magnetic field on the phase velocity, attenuation coefficient and specific loss is shown graphically. The phase velocity and specific loss of coupled piezo-thermoelastic qP, qT, qSV waves increase while the attenuation coefficient fluctuates as frequency changes $20 \leq \omega \leq 100$.

Acknowledgements

I am thankful to Lorentz Centre, Leiden University, Leiden, Netherlands for providing me with office space and a research environment that served as an inspiration for this research work during the Workshop on Coherent Structures: Current Developments and Future Challenges from July 4-8, 2022, Lorentz Centre at Oort in Leiden, Leiden University, Leiden, The Netherlands.

Nomenclature

$a_2^*, a_3^*,$ – two-temperature parameters
$\boldsymbol{B}$ – magnetic induction
$\bar{C}_{ij}$ – elastic constants
C_E – specific heat at constant strain
$\boldsymbol{D}$ – electric displacement vector
$\boldsymbol{E}$ – electric field strength
$\bar{g}_{22}, \bar{g}_{34}, \bar{g}_{23},$ – piezoelectric constants
$\boldsymbol{H}$ – magnetic field strength
$\boldsymbol{h}$ – perturbation of magnetic field strength
$\boldsymbol{J}$ – current
K_1, K_3 – thermal conductivities
k – wave number
$\boldsymbol{n}$ – unit vector
$P_{22}, P_{33},$ – initial stress
$p_3,$ – pyroelectric constant
T – change in temperature variable

T_0 – uniform temperature
t – time
$\boldsymbol{u}$ – displacement vector
v, w – components of the displacement vector
V – wave speed
x, y, z – coordinates
$\beta_2^t, \beta_3^t,$ – thermal coefficients
$\overline{\gamma}_{22}, \overline{\gamma}_{33},$ – dielectric constants
λ, μ – Lame's constants
$\mu_e,$ – magnetic permeability
τ_0^t – thermal relaxation time
ρ – density
σ – electric conductivity of the medium
θ – angle of propagation measured from normal to the half-space
ψ – electric potential
ω – circular frequency
$\boldsymbol{\Omega}$ – rotation vector
$\in$ – electric permeability

References

[1] Lord H.W. and Shulman Y. (1967): *A generalized dynamical theory of thermo-elasticity*.– J. Mech. Phys. Solid, vol.15, pp.209-309.

[2] Dhaliwal R.S. and Sherief H.H. (1980): *Generalized thermos-elasticity for anisotropic media*.– Quart. Appl. Math., vol.33, pp.1-8.

[3] Youssef H.M. (2006): *Theory of two-temperature generalized thermoelasticity*.– IMA Journal of Applied Mathematics, vol.71, No.3, pp.383-390.

[4] Youssef H.M. (2006): *Theory of two-temperature thermoelasticity without energy dissipation. journal of thermal stresses*.– vol.34, No.2, pp.38-146.

[5] Ignaczak J. and Ostoja-Starzewski M. (2009): *Thermoelasticity with Finite Wave Speeds.* Oxford University Press.

[6] Hetnarski R.B. and Ignaczak J. (1999): *Generalized thermoelasticity*.– Journal of Thermal Stresses, vol.22, pp.451-476.

[7] Chandrasekharaih D.S. (1986): *Thermoelasticity with second sound: a review*.– Appl. Mech. Rev., vol.39, No.3, pp.355-376.

[8] Chandrasekharaih D.S. (1998): *Hyperbolic thermoelasticity: a review of recent literature*.– Appl Mech Rev., vol.51, No.12, pp.705-729.

[9] Chen P.J. and Gurtin M.E. (1968): *On a theory of heat conduction involving two temperatures*.– Zeitschrift für Angewandte Mathematik und Physik, vol.19, pp.614-627.

[10] Chen P.J., Gurtin M.E. and Williams W.O. (1969): *On the thermodynamics of non-simple elastic material with two temperatures*.– Zeitschrift fur angewandte Mathematik und Physik, vol.20, pp.107-112.

[11] Mindlin R.D. (1961): *On the equations of motion of piezoelectric crystals*.– Problems of Continuum Mechanics, N.I. Muskelishvili's Birthday 70, SIAM, Philadelphia, pp.282-290.

[12] Nowacki W. (1978): *Some general theorems of thermo-piezoelectricity*. Journal of Thermal Stresses, vol.1, pp.171-182.

[13] Nowacki W. (1979): *Foundations of Linear Piezoelectricity*.– Parkus H (Ed.), Electromagnetic Interactions in Elastic Solids, Springer, Wien, Chapter 1.

[14] Nowacki W. (1983): *Mathematical models of phenomenological piezoelectricity. New problems in mechanics of continua*.– University of Waterloo Press, Ontario, pp.29-49.

[15] Chandrasekharaiah D.S. (1984): *A temperature rate dependent theory of piezoelectricity*.– Journal of Thermal Stresses, vol.7, pp.293-306.

[16] Chandrasekharaiah D.S. (1988): *A generalized linear thermoelasticity theory of piezoelectric media*. Acta Mechanica, vol.71, pp.39-49.

[17] Every A.G. and Neiman VI (1992): *Reflection of electroacoustic waves in piezoelectric solids: Mode conversion into four bulk waves*.– J. Appl. Phys., vol.71, pp.6018, doi: 10.1063/1.350457.

[18] Othman M.I.A. and Ahmed E.A.A. (2019): *Exact analytical solution of a homogeneous anisotropic piezo-thermoelasic half-space of a hexagonal type under different fields with three theories*.– Microsystem Technologies, vol.25, pp.1423-1435.

[19] Vashishth A.K. and Sukhija H. (2015): *Reflection and transmission of plane waves from fluid- piezothermo- elastic solid interface*.– Applied Mathematics and Mechanics, vol.36, pp.11-36.

[20] Singh B. (2010): *Wave propagation in a prestressed piezoelectric half-space*. Acta Mechanica, vol.211, No.3-4, pp.337-344.

[21] Jain K., Kumar S. and Deswal S. (2020): A*nalysis of reflection phenomena in a fiber reinforced piezo-thermoelastic half space with diffusion and two-temperature*.– Journal of Solid Mechanics, vol.12, No.4, pp.912-934, DOI: 10.22034/jsm.2020.1897751.1579

[22] Othman M.I.A., Elmaklizi Y.D. and Ahmed E.A.A. (2017): *Influence of magnetic field on generalized piezo-thermoelastic rotating medium with two relaxation times*.– Microsyst. Technol., vol.23, pp.5599-5612, DOI 10.1007/s00542-017-3513-7.

[23] Guha S., Singh A.K. and Das A. (2019): *Analysis on different types of imperfect interfaces between two dissimilar piezothermoelastic half-spaces on reflection and refraction phenomenon of plane waves*.– Waves in Random and Complex Media, https://doi.org/ 10.1080/ 17455030.2019.1610198.

[24] Kumar P. and Harsha S.P. (2022): *Dynamic analysis of porosity dependent functionally graded sigmoid piezoelectric (FGSP) plate*.– Structures, vol.46, pp.1737-1752.

[25] Abdulaziz Alenazi, Abdelaala Ahmed, El-Bary A.A., Tantawi R.S. and Khaled Lotfy (2023): *Moisture photo-thermoelasticity diffusivity in semiconductor materials: a novel stochastic model*, Crystals, vol.13, pp.42, https://doi.org/10.3390/cryst13010042.

[26] Deswal S., Sheokand S.K. and Kalkal K.K. (2019): *Reflection at the free surface of fiber-reinforced thermoelastic rotating medium with two-temperature and phase-lag*.– Applied Mathematical Modelling, vol.65, pp.106-119.

[27] Lotfy Kh., Elidy E.S. and Tantawi R.S. (2021): *Piezo-photo-thermoelasticity transport process for hyperbolic two-temperature theory of semiconductor material*.– International Journal of Modern Physics C, vol.32, No.7, pp.2150088.

[28] Ye Z.G. (2008): *Handbook of Dielectric, Piezoelectric and Ferroelectric Materials Synthesis, Properties and Applications*.– Woodhead Publishing Limited, CRC Press, New York.

[29] Keith C.M. and Crampin S. (1977): *Seismic body waves in anisotropic media: Reflection and refraction at a plane interface*.– Geophys. J.R. Astr. Soc., vol.49, pp.181-208.

[30] Singh S.S. and Tomar S.K. (2007): *Quasi-P waves at a corrugated interface between two dissimilar monoclinic elastic half-space*.– Int. J. Solids Struct., vol.44, pp.197-228.

[31] Singh B. and Yadav A.K. (2012): *Plane waves in a transversely isotropic rotating magneto-thermo-elastic medium*.– J. Eng. Phys. Thermophys., vol.8, pp.1226-1232.

[32] Singh B. and Yadav A.K. (2012): *Reflection of plane waves in a rotating transversely isotropic magneto-thermoelastic solid half-space*.– Theor. Appl. Mech., Sofia, vol.42, pp.33-60.

[33] Chattopadhyay A. and Choudhary S. (1995): *The reflection phenomena of P- waves in a medium of monoclinic type*.– Int. J. Eng. Sci., vol.33, pp.199-207.

[34] Chattopadhyay A., Saha S. and Chakraboty M. (1996): *The reflection of SV-waves in a monoclinic medium*.– Indian. J. pure Appl. Math., vol.27, pp.1029-1042.

[35] Singh S.J. and Khurana S. (2002): *Reflection of P and SV wave at the free surface of a monoclinic elastic half-space*. Proceedings of Indian Academy of Science, (Earth Planet Science), vol.111, pp.401-412.

[36] Sahar S., Chattopadhyay A. and Banerjee D. (2001): *Reflection and refraction of quasi-p wave due to a sandwich isotropic layer between two monoclinic half-spaces (March 26, 2001)*.– International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics. Paper 21. http://scholarsmine.mst.edu/icrageesd/04icrageesd/session02/21.

[37] Montanaro A. (2015): *On the propagation of plane waves in piezo-electromagnetic monoclinic crystals*.– Arch. Mech., Warsaw, vol.67, No.3, pp.116.

[38] Gupta V. and Vashishth A.K.: *Effect of piezoelectricity on the bulk waves in monoclinic poro-elastic material*.– Journal of Theoretical and Applied Mechanics, Warsaw, vol.54, No.2, pp.571-577. DOI: 10.15632/jtam-pl.54.2.571.

[39] Singh B. and Yadav A.K. (2013): *Reflection of plane waves in an initially stressed perfectly conducting transversely isotropic solid half-space*.– J. Earth Syst. Sci., vol.122. No.4, pp.1045-1053.

[40] Singh B. and Yadav A.K. (2016): *Plane waves in a rotating monoclinic magneto-thermoelastic medium*.– Journal of Engineering Physics and Thermophysics, vol.89. No.2, pp.428-440.

[41] Yadav A.K. (2021): *Reflection of plane waves from the free surface of a rotating orthotropic magneto-thermoelastic solid half-space with diffusion*.– Journal of Thermal Stresses, vol.44, No.1, pp.86-106, DOI: 10.1080/01495739.2020.1842273.

[42] Yadav A.K. (2021): *Reflection of plane waves in a fraction-order generalized magneto-thermo-elasticity in a rotating triclinic solid half-space*.– Mechanics of Advanced Materials and Structures, vol.29, No.18, pp.1-18 DOI: 10.1080/15376494.2021.1926017.

[43] Montanaro A. (1999): *On singular surfaces in isotropic linear thermoelasticity with initial stress*.– Journal of the Acoustical Society of America, vol.106, No.3, pp.1586-1588.

[44] Marin M. and Marinescu C. (1998): *Thermoelasticity of initially stressed bodies, asymptotic equipartition of energies*.– International Journal of Engineering Science, vol.36, No.1, pp.73-86.

[45] Biot M.A. (1965): *Mechanics of Incremental Deformations*.– New York: Wiley.

[46] Yadav A.K. (2021): *Effect of impedance boundary on the reflection of plane waves in fraction-order thermoelasticity in an initially stressed rotating half-space with a magnetic field*.– Int. J. Thermophys., vol.42, pp.3. https://doi.org/10.1007/s10765-020-02753-1.

[47] Yadav A.K. (2021): *Thermoelastic waves in a fractional-order initially stressed micropolar diffusive porous medium*.– Journal of Ocean Engineering and Science, vol.6, pp.376-388, doi: https://doi.org/10.1016/j.joes.2021.04.001

[48] Maity N., Barik S.P. and Chaudhuri P.K. (2013): *Reflection of plane waves in elastic half space with void pores under the action of uniform magnetic field*.– International Journal of Engineering Science, vol.71, pp.62-73.

[49] Schoenberg M. and Censor D. (1973): *Elastic waves in rotating media*.– Quart. Appl. Math., vol.31, pp.115-125.

[50] Willson A.J. (1963): *The propagation of magneto-thermoelastic plane waves*.– Proc. Camb. Phil. Soc., vol.59, pp.483-488.

[51] Alzahrani F., Hobiny A., Abbas I. and Marin M. (2020): *An eigenvalues approach for a two-dimensional porous medium based upon weak, normal and strong thermal conductivities*.– Symmetry, vol.12, No.5, Art. No.848, p.15, doi:10.3390/sym12050848.

[52] Abo-Dahab S.M., Abouelregal A.E. and Marin M. (2020): *Generalized thermoelastic functionally graded on a thin slim strip non-gaussian laser beam*.– Symmetry, vol.12, No.7, Art. No.1094, DOI:10.3390/sym12071094.

[53] Carrera E., Pagani A. and Augello R. (2022): *Large deflection of composite beams by finite elements with node-dependent kinematics*.– Computational Mechanics, vol.69, No.6, pp.1481-1500.

[54] Yadav A.K., Barak M.S. and Gupta V. (2023), "Reflection at the free surface of the orthotropic piezo-hygro-thermo-elastic medium", International Journal of Numerical Methods for Heat & Fluid Flow, Vol. 33 No. 10, pp. 3535-3560. https://doi.org/10.1108/HFF-04-2023-0208.

Received: April 6, 2023
Revised: July 3, 2023