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FORECASTING THE COURSE OF CUMULATIVE COST CURVES FOR DIFFERENT CONSTRUCTION PROJECTS

Mariusz SZÓSTAK[1]
Wroclaw University of Science and Technology, Faculty of Civil Engineering,
Department of Building Engineering

Abstract

Planning the course of cumulative cost curves and effectively monitoring the implementation process and the incurred financial outlays are still significant problems in the management of construction projects. This is particularly noticeable during the execution phase of construction works. Therefore, it is worthwhile to correctly determine the shape of the cost curve before starting this stage and to periodically examine its fitting to the scheduled course of the budgeted cost curve, the envelope of cost curves characterised by the best-fit curve. There are many methods of forecasting and estimating the costs of construction works, but they are very often complicated and require the decision-maker to use and elaborate mathematical tools. The aim of the research was to determine the shape and course of the cost curves for selected construction projects. Based on the analysis of the collected data on investment projects in 3 facilities research groups (collective housing, hotels and retail service facilities), an original attempt was made to determine the best fit curve and the area of the curve, which in turn indicates the limits of the correct planning of the cumulative costs of construction projects. The Three Sigma rule was applied, correlations and determinants were determined, and the area of the cost curves was described with a third degree polynomial. The conducted research showed that: 1. the optimal formula for determining the best-fit curve, which allow to determine the cost and time of construction works, is a 3-degree polynomial; 2. cost curves, within a certain bounding box, determine the area of the most likely cash flow; 3. when planning the course of a cost curve, it is advisable to use the bounding box of cost curves rather than a single, model, theoretical, or empirical mathematical expression describing the cost curve.

Keywords: construction project, cumulative cost, Three Sigma rule

1. INTRODUCTION

When forecasting the course of investment projects, a construction manager makes decisions regarding their future state [1]. These decisions can be made hypothetically (using a heuristic search) [2], or can be modelled on the basis of previously acquired knowledge (using empirical distributions of variables

[1] Wroclaw University of Science and Technology, Faculty of Civil Engineering, Department of Building Engineering, 27 Wybrzeże Wyspiańskiego st. 50-370 Wrocław, mariusz.szostak@pwr.edu.pl, +48 72 320 23 45

that are relevant to the actual state) [3, 4]. The manager always has a certain level of probability as regards the accuracy of decisions [5, 6]. They can also be undertaken as a result of the application of specific algorithms which, as long as the actions undertaken are consistent with the algorithms developed for given procedures, ensure certain effects [7, 8].

The latter situation, due to the available knowledge, is very limited in its scope. On the basis of past states, future states can only be programmed within probabilistic terms [9]. An additional limitation is the method of measuring the values of the variables that determine this actual state. Measurements instruments are sometimes accurate and measurements are repeatedly confirmed [10]. However, in some situations the manager has to use measuring instruments that are calibrated poorly standardised, e.g., by defining the state of a quantitative phenomenon in unclear semantic categories: low, medium, high; minimum, low, average, high, maximum; or division into classes [11].

Such a scale should be treated as a model generalisation of the decision situation in which the construction manager is in. Such a manager needs to first assess the actual state in order to design (on this basis) its further fate, e.g. the schedule and cost of planned activities [12]. To sum up, in every factor configuration, managers operate under uncertain conditions. In this case, their task is to improve the decision-making instruments, which - by limiting the scope of uncertainty – allow the implementation costs to be reduced due to undertaken decisions that are better suited to the actual state (facilities and processes). Even in the model of deterministic actions, when the use of algorithms defined by procedures ensures its success (cognitive or decision-making) - various unpredictable circumstances may arise. This is understandable, since no one controls all variables that influence the course of investment processes, anywhere or ever [13, 14]. In the case of a deterministic situation, limiting uncertainty only comes down to specifying algorithms and refining procedures. Such situations are not particularly useful for a construction manager, and activities undertaken within them are secondary to research activities that are related to factors that influence the cost course of investment processes.

2. LITEATURE REVIEW

In every construction project, there are limitations, ie, time parameters, cost parameters, and technical and quality requirements, that must be achieved for the project to be successful [15, 16]. These limitations form the so-called trangle of constraints. An extremely important characteristic is the interdependence of the previously mentioned constraints, meaning that a change in any of the parameters affects the others. For example, the implementation of construction projects in the time planned in the baseline schedule and the reduction of their duration can improve and increase the economic efficiency of an investment [17]. Furthermore, the powerful tool to optimise the process can be the combination of modern technologies, such as BIM technology [18]. Cost planning, from the implementation planning stage, is crucial for the liquidity of the company [19]. This issue is also important for construction companies, which on the basis of these factors make a decision to participate or not in the tender. This decision must be made carefully, as it affects the condition of the company and is an important aspect in its quest for success [20].

Consequently, the correct planning of the work and expenditure schedule is an essential part of planning construction projects. Correct preparation of the work and expenditure schedule can be achieved by:

- correctly determining the start and completion dates of the entire construction project and its individual tasks,
- taking into account the actual links between tasks,
- introduction of the planned costs of the tasks [21].

A correct work plan, characterised by a reasonable work and expenditure schedule, increases the efficiency of the work and allows the contract to be executed at the lowest cost [22]. The final verification of the correctness of the schedule development, i.e. the planned costs and time, takes place only after the completion of the investment, which is why new techniques and tools are constantly being sought to support planning.

2.1. Earned Value Method

Many methods for monitoring construction projects can be found in the literature. One of the most commonly used methods in management practise is the Earned Value Method (EVM) [23, 24]. The Earned Value Method allows for the control of the progress of project implementation in terms of its scope, cost, and duration [25]. This method involves comparing the planned (scheduled) progress of the work (scope) with the planned and actual time (schedule) and cost (budget) [26].

The difficulty of applying the Earned Value Method is due to the problem of correctly determining the earned value (EV, BCWP). The earned value (the budgeted cost of the work performed) is a measure of the actual progress of the work, i.e. the cost of all work progress achieved in the project, or in part of it, which is calculated until the date of the report and which is expressed with regard to budgeted costs. The Errned Value Method allows for the determination of how much, according to the plan, the actual work would cost. An important issue is the correct specification of the actual progress of the work in relation to the plan (schedule) [27]. The correct determination of the earned value is difficult and problematic because it results from the way the value is estimated in engineering practise. The estimation of the value is based on a subjective, experience-based assessment of the percentage progress of the works by the Investor's Supervision Inspector. Many construction projects are based on the estimate of the progress of the works and the progress accepted by the project supervisor and the work contractor. Therefore, the earned value (*EV, BCWP*) often equals the value of the actual costs of work performed (*AC, ACWP*), which means that the **-** cumulative cost curves overlap *(EV = AC)*.

The Earned Value Method is a relatively accurate method, but it has limitations in its practical use and inaccuracies. One problem relates to data quality obtained from a construction site. The method is very sensitive to the input of financial data. This method does not manage with schedule modifications, resulting from, among other, the need for changes occurring during the execution of construction work, random situations and changes in cost classification [28]. In addition, problems arise in the practical application of the Earned Value Method, for example, a difficulty to correctly determine the percentage progress of the work performed or incomplete data on the actual costs. These irregularities may lead to misinterpretation of indicators and erroneous estmitaes, based on them, of the completion dates and costs of the investment project [29, 30].

The described challenges have led researchers to develop alternative methods that build on the Earned Value Method, but attempt to more reliably predict the total cost and completion date of projects. Examples of such methods include estimating cost and time by dividing or packing construction work [31], using statistical methods [32, 33], applying fuzzy Earned Value Management [34-36] or hybrid artificial intelligence [37].

2.2. The cumulative cost curve – the "S" curve

Another tool for measuring the use of financial expenditures in a construction project is the cumulative cost curve [38]. This is as effective as the Earned Value Method, which represents the planned financial flows on a timeline by means of a cumulative cost curve. The cumulative cost curve shows the progress of an investment project from the beginning of construction work to their completion. The cumulative cost is the sum of the costs incurred by all resources assigned to a given task. Graphically, the cumulative cost curve is flattened in the beginning and end phases of the construction project, while it is very steep in the middle phase. Therefore, it is called an 'S' curve [39].

Based on ongoing financial data, it is possible to generate and compare the curves of budgeted and actual costs [40]. Cost curves define, within a certain boundary, the area of cash flows [41]. In order to rationally assess the end date of a project, or its actual costs, the method of correcting (on the current basis) the course of the actual cost curve may be used. For example, in an accessible literature, it is known that higher order polynomials are used to update the course of the above-mentioned curves. One of the challenges in applying the cumulative cost curve is the correct definition of the inflection point of the curve, i.e. to determine the moment in time when the curve changes from a convex to a concave curve. Unfortunately, this point is difficult, or even impossible, to define [42].

The S-curve method has been continuously modified through research. For example, in order to determine the shape of the cost curves, the following technic or method were used: fuzzy sets [43], fuzzy regression [44], artificial intelligence [45], and BIM technology [46]. The traditional method has been extended to include risk analysis [47], and assessing the impact of the key causes of delays [48].

Unexpected changes and inaccuracy in the data are elements that are inherent in cost flow management. Accounting for these elements is possible with modelling using probability theory, such as the use of a fuzzy stochastic model. An alternative approach to determining cost curves is a simulation approach based on stochastic cost curves [49, 50], a probabilistic approach using Bayes models [51], and other probability distributions known from the literature [52].

In the research conducted, empirical methods can also be used to estimate the course of cumulative cost curves in various construction projects. The mathematical formulas obtained for the cost curves are based on real historical data, which concerned, among others, construction projects implemented in the UK [53], Iran [54], Taiwan [55], and the USA [56].

Both tools, the Earned Value Method and the Cumulative Cost Curve, are applicable to monitoring and controlling the progress of construction works. However, it should be noted that the aim of this research is not the monitoring of the implementation of works, rather the planning stage that precedes the execution stage. To develop an effective cumulative cost curve planning model that reflects reality, it is necessary to collect the required numerical data. In addition, the data set must be sufficiently large to provide valuably research material for the generalisation of the issue under study.

Based on a literature review, it was shown that research on the course of the cost curve is a very demanding task and the “search” for a universal solution is still ongoing. The graphical shape of the theoretical cost curves is very regular, fluid, and continuous, while analyzing the actual course of empirical cost curves, it should be noted that their shape deviates significantly from the standard “S” curve. Cost curves, within a certain bounding box, determine the area of cash flows. Therefore, when planning the course of the cost curve, it is advisable to use the space of cost curves rather than one model mathematical expression.

There is a lack of research into the development of the areas of cost curves and the best fit of cumulative cost curves at the planning and performed stages of construction projects. This research is an attempt to fill this research gap.

3. METHODS AND MODELS

3.1. Research approach

The research was based on the result of many years of studies concerning the development of methods and tools to model multicriteria processes in construction engineering [57, 58]. In contrast to real-time decision making, the analysis of past states whose effect will be visible in the future requires a more complex decision-making process. Decisions and actions are taken in this case on a multivariate basis and require precise determination of the cumulative cost curve and continuous monitoring of its alignment with the cost curve planned in the investment budget. The use of two-valued logic is not always sufficient and decisions are no longer deterministic, increasing the need for complex decision management processes. [23, 24]

3.2. Research Methodology

To achieve the set of research goals, an original research methodology consisting of 7 stages was developed. The proposed methodology is presented in figure 1.

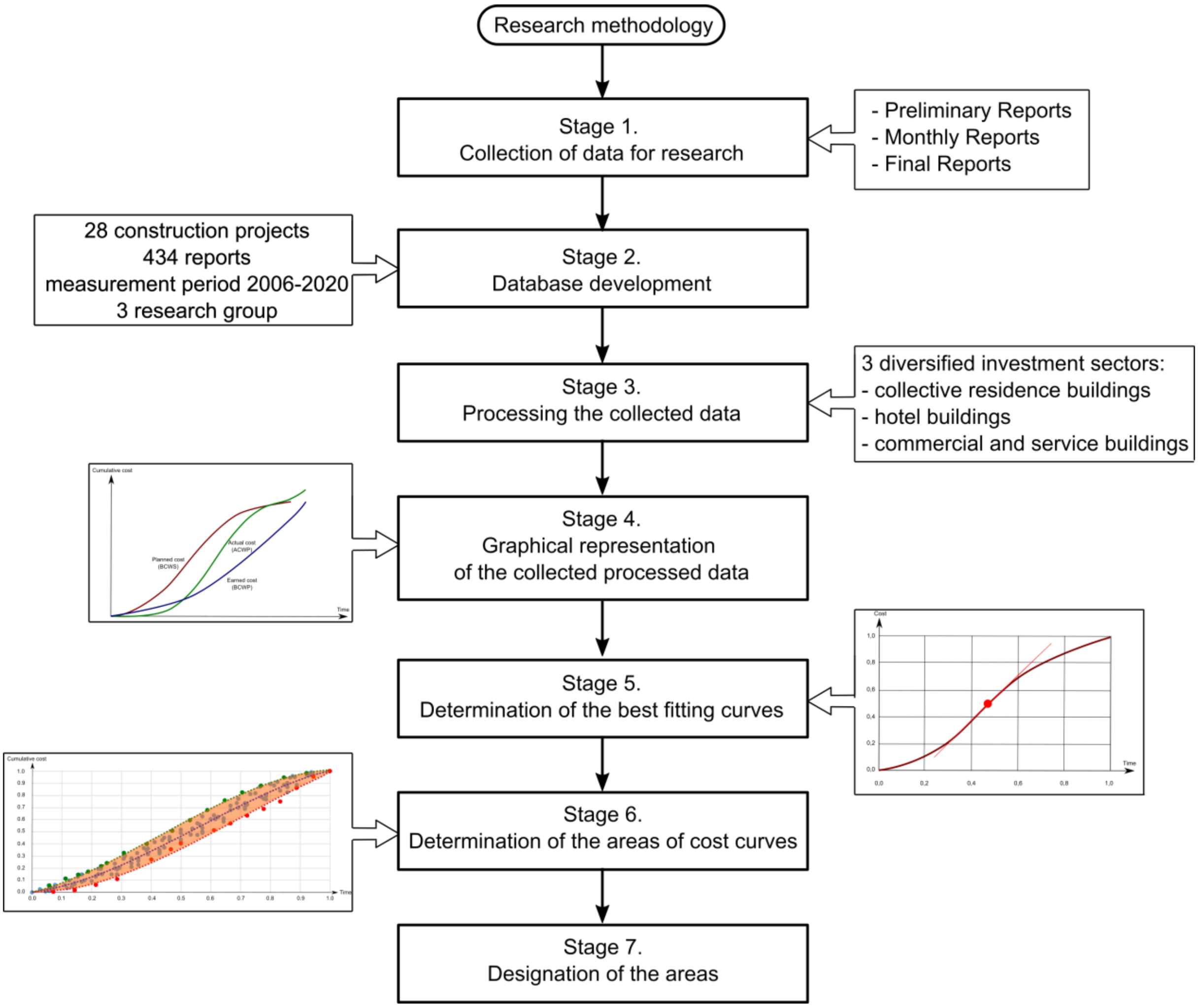

Fig.1. Research methodology

3.2.1. Stage 1. The collection of data for research

To start with, it is necessary to obtain reliable and real data on completed construction projects. The data include:

- the baseline work and expenditure schedule, which involves the budget of the construction works that were planned and contracted by the contractor,
- information on the actual course of the construction process..

On the basis of the basic work and expenditure schedule, it is possible to graphically present the planned course of the cumulative cost curve. On the other hand, on the basis of monthly statements concerning the amount of construction works executed, it is possible to actually recreate the course of the works and develop a curve of actual cumulative costs.

The above information was obtained from the monthly reports of the Bank Investment Supervisor, who was one of the participants in the construction projects analysed. BIS reports were compiled on the basis of analysis of settlements, invoices, and direct technical and financial inspections at the analysed construction sites. The inspections were conducted to measure the actual amount of construction works executed and also to measure the progress of the schedule at the time of the control. The course of the S curves was determined based on the cumulative values of the amount of work performed, which constituted the cumulative values.

As part of the research, data concerning 28 construction projects belonging to 3 research groups were collected, that is, 434 reports prepared by banking investment supervision inspectors that participate in the course of the analysed construction projects. The groups corresponded to 3 different investment sectors. All the collected data presented in Table 1 refer to investments implemented in Poland in the period from 2006 to 2020.

Table 1. Summary of the number of construction projects analysed and prepared reports

Group	**Investment number**	**Number of measurements**	
A. Collective residence buildings	A.1	16	188
	A.2	16	
	A.3	16	
	A.4	8	
	A.5	15	
	A.6	15	
	A.7	22	
	A.8	18	
	A.9	22	
	A.10	22	
	A.11	18	
B. Hotel buildings	B.1	34	125
	B.2	6	
	B.3	17	
	B.4	11	
	B.5	10	
	B.6	10	
	B.7	12	

Group	Investment number	Number of measurements	
	B.8	11	
	B.9	14	
C. Commercial and service buildings	C.1	15	121
	C.2	13	
	C.3	11	
	C.4	24	
	C.5	17	
	C.6	6	
	C.7	16	
	C.8	19	
	TOTAL	434	

3.2.2. Stage 2. Database development

The following assumptions were adopted for the development of the database:

Based on the reports collected in step 1, it is possible to obtain the following information:

- information about the planned schedule and budget of the investment implementation,
- information about the actual course of the investment,
- information on the concluded annexes, or protocols,
- information about the occurrence of delays,
- information about the real cost and time of the completion of the investment.

As a result of the conducted analyses of the reports, a database that characterises individual construction projects was developed (i.e. a collective summary of data in the form of a two-dimensional table in Microsoft Excel). In the table, each subsequent column contains data concerning the subsequent reporting periods, while each successive row contains data regarding the next construction project. Each data set contains the following values:

- the budgeted cost of work scheduled ($BCWS_i$), which is determined for each individual period $i \in (1, \dots, n)$, where n is the planned number of accounting periods,
- the cumulative value of the budgeted cost of work scheduled (C_{BCWS_i}), which is determined for each individual period under study $i \in (1, \dots, n)$. It is obtained by summing up the individual values of the budgeted cost of work scheduled ($BCWS_i$) preceding the analyzed period (including the audited accounting period) according to the following formula:

$$C_{BCWS_i} = BCWS_1 + BCWS_2 + \cdots + BCWS_i + \cdots + BCWS_n = \sum_{i=1}^{n} BCWS_i, \tag{3.1}$$

- the budgeted cost of the work performed ($BCWP_j$), which is determined for each individual period under study $j \in (1, \dots, m)$, where m is the real number of accounting periods.,
- the cumulative value of the budgeted cost of work performed (C_{BCWP_j}), which is determined for each individual examined period $j \in (1, \dots, m)$. It is obtained by summing up the individual values

of the budgeted cost of work performed ($BCWP_j$) preceding the analyzed period (including the audited accounting period) according to the below formula:

$$C_{BCWP_j} = BCWP_1 + BCWP_2 + \cdots + BCWP_j + \cdots + BCWP_m = \sum_{j=1}^{m} BCWP_j, \quad (3.2)$$

- the actual cost of the work performed ($ACWP_j$), which is determined for each individual examined period $j \in (1, \dots, m)$, where m is the real number of accounting periods,
- the cumulative value of the actual cost of work performed (C_{ACWP_j}), which is determined for each individual examined period $j \in (1, \dots, m)$. It is obtained by summing up the individual values of the actual cost of work performed ($ACWP_j$) preceding the analyzed period (including the audited accounting period) according to the below formula:

$$C_{ACWP_j} = ACWP_1 + ACWP_2 + \cdots + ACWP_j + \cdots + ACWP_m = \sum_{j=1}^{m} ACWP_j \quad (3.3)$$

3.2.3. Stage 3. Processing the collected data

The data collected in the database 78ormalized78e individual construction projects. Each project is 78ormalized78ed by a different duration and cost of implementation. To compare the data collected for different construction projects, it is necessary to properly process the collected data, and therefore to perform a comparative analysis, the data was 78ormalized. For this purpose, for each individual analysed construction project, it was necessary to determine the following auxiliary unitless values:

- the unitless value of the budgeted cost of work scheduled (V_{BCWS_i}) for each individual period $i \in (1, \dots, n)$, where n is the planned number of accounting periods, which is calculated in accordance with the following formula:

$$V_{BCWS_i} = \frac{BCWS_i}{C_{BCWS_n}} \quad (3.4)$$

- the unitless value of the planned time of budgeted work (V_{SD_i}) for each individual period under study $i \in (1, \dots, n)$, which is calculated in accordance with the following formula:

$$V_{SD_i} = \frac{SD_i}{C_{SD}} \quad (3.5)$$

where:
C_{SD} – the planned total duration of the analysed construction project,
SD_i – the examined subsequent planned period of the implementation of the analyzed construction project $i \in (1, \dots, n)$,

- the unitless value of the budgeted cost of work performed (V_{BCWP_j}) for each individual examined period $j \in (1, \dots, m)$, where m is the real number of accounting periods, which is calculated in accordance with the following formula:

$$V_{BCWP_j} = \frac{BCWP_j}{C_{BCWP_m}} \quad (3.6)$$

- the unitless value of the planned time of work performed (V_{PD_j}) for each individual examined period $j \in (1, \dots, m)$, which is calculated in accordance with the following formula:

$$V_{PD_j} = \frac{PD_j}{C_{PD}} \quad (3.7)$$

where:
C_{PD} – the actual total duration of the analysed construction project,
PD_j – the examined subsequent actual period of the implementation of the analyzed construction project $j \in (1, \dots, m)$,

- the unitless value of the actual cost of work performed (V_{ACWP_j}) for each individual examined period $j \in (1, \dots, m)$, where m is the real number of accounting periods, which is calculated in accordance with the following formula:

$$V_{ACWP_j} = \frac{ACWP_j}{C_{ACWP_m}} \tag{3.8}$$

- the unitless value of the actual time of work performed (V_{AD_j}) for each individual examined period $j \in (1, \dots, m)$, which is calculated in accordance with the following formula:

$$V_{AD_j} = \frac{AD_j}{C_{AD}} \tag{3.9}$$

where:
C_{AD} – the actual total duration of the analysed construction project,
AD_j – the examined subsequent actual period of the implementation of the analyzed construction project $j \in (1, \dots, m)$,

- the unitless value of the ratio of the actual cost of work performed (V_{ACWP_j}) to the budgeted cost of work scheduled $\left(C_{BCWS_n}\right)$ for each individual examined period j∈ (1, …, m), which was calculated from the formula below, where: m is the real number of settlement periods, which is determined on the basis of the amount of financial works executed and the actual duration of the analyzed construction investment; n is the planned number of settlement periods, which is determined on the basis of the baseline work and expenditure schedule.

$$V_{ACWP_j/BCWS_n} = \frac{V_{ACWP_j}}{C_{BCWS_n}} \tag{3.10}$$

- the unitless value of the ratio of the actual time of work performed (AD_j) to the cumulative value of the planned time of work scheduled (C_{SD_n}) for each individual examined period $j \in (1, \dots, m)$, which was calculated from the formula below:

$$V_{AD_j/SD_n} = \frac{AD_j}{C_{SD_n}} \tag{3.11}$$

3.2.4. Step 4. Graphical representation of the collected processed data

Using the unitless auxiliary values determined in stage 3, it is possible to graphically present the collected data in relation to:

- the budgeted values of work scheduled ($BCWS_i$),
- the actual values of work performed ($ACWP_j$),

- deviations between the budgeted values of the scheduled work and the actual values of the work performed to compare the actual values of work performed ($ACWP_j$) and the budgeted values of work scheduled ($BCWS_i$)

for each individual analysed construction project, and also in relation to homogeneous research groups (which correspond to 3 different investment sectors (A-C)) and a diverse group of various investment sectors.

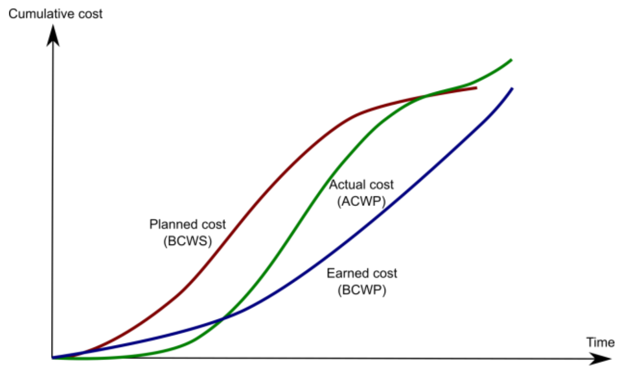

Fig. 2. Graphical representations of cumulative costs curves

3.2.5. Stage 5. Determination of the best fitting curves

Using the cumulative cost curves developed in step 4, it is possible to analytically determine the equation of the curve in the form of a polynomial with the best fit. In order to define and describe the course of the curves, used a third degree polynomial and a curve's inflection point.

The correlation coefficient R and the coefficient of determination R^2 were used as a measure of the fitting of the trend function to the actual values. The coefficient of determination takes values from 0 to 1. The better the fitting of the model, the closer the value of R^2 is to one.

On the basis of the analysis of the shape of the cost curves, it was noticed that the cost curves resemble the shape of the letter "S". From a mathematical point of view, the letter "S" has two convexities (and one inflection point (x_0). In the first period of the implementation of work (first phase), the cost curves are convex (this means that the function graph lies above the tangent graph for each point from interval $< 0, x_0)$. In the middle phase, during the intensified execution of construction works, it can be noticed that the central part of the cost curve is steep, i.e. inclined at a large angle with respect to the time axis. At a certain point, the cost curve for the execution of the construction works reaches the inflection point (x_0), which indicates the moment of transition of the investment to the second stage of implementation. At this stage, the increase in costs begins to slow down. In the second phase of the execution of the work, the cost curve is concave, that is, convex upwards (this means that the graph of the function lies under the graph of the tangent for each point from interval $(x_0, 1 >)$.

The third degree polynomial can be written as follows:

$$f(x) = a_1 \cdot x^3 + a_2 \cdot x^2 + a_3 \cdot x^1 + a_0, \tag{3.12}$$

where:

- the abscissa axis takes values from 0 to 1: $x \in < 0,1 >$
- the ordinate axis takes values from 0 to 1: $y \in < 0,1 >$
- the cost curve begins at a point with coordinates (0,0), which means that the intercept is 0: $a_0 = 0$,
- the polynomial for $x = 1$ always takes the value of 1 ($y = 1$) for the completed investment: $f(1) = 1 = a_1 \cdot x^3 + a_2 \cdot x^2 + a_3 \cdot x$, which means that: $a_1 + a_2 + a_3 = 1$

- the 3rd order polynomial has at most 1 inflection point. To determine the inflection point, a necessary condition must be met, which in this case is the zeroing of the second derivative of function $f''(x) = 0$:

  and from the above, we can obtain the inflection point: $x_0 = \frac{-a_2}{3 \cdot a_1}$.
- the polynomial (cost curve) can be characterized by the inflection point (x_0), which informs about the moment of transition to the "second phase".

3.2.6. Stage 6. Determination of the areas of cost curves

Based on the data collected, it is possible to determine the area of the cost curves in which the best-fit curves are located (figure 3). To determine the area of cost curves, it is necessary to define the following for each data set analysed:

- the best-fit curve (step 5),
- the curve limiting the area of cost curves "from the top",
- the curve limiting the area of cost curves "from the bottom".

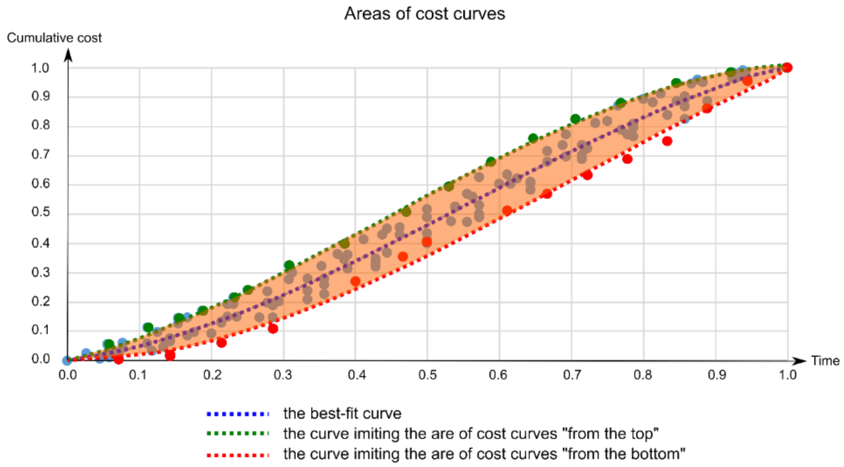

Fig. 3. The areas of cost curves

3.2.7. Stage 7. Designation of the areas

Using the cost curve areas, which were determined at stage 6, it is possible to:

- for planned investments - determine the shape and time course of the budgeted cost of work schedule curve (curve: *BCWS*),
- for implemented investments - monitor the course of the investment and the appropriate response to deviations from the assumed parameters of the implementation of works (cost deviation, deviation from the schedule). Corrective actions should be taken when the actual course of an investment begins to differ from the planned one.

3.3. Process modeling

A construction manager (depending on his role), while preparing for the implementation of the investment, determines certain parameters that characterise the investment project. In turn, an investor, when planning an investment project, determines the available budget for the investment and the date of completion. The contractor of works prepares a work and expenditure schedule that allows the value of the construction works to be planned, and also determines the necessary time for their implementation. Correct planning of cash flows over time has a significant impact on the financial liquidity of the decision maker and the success of the implementation of a given project.

A construction manager, by knowing the budget of a specific investment and its duration, will be able to properly plan financial flows over time using the cost curve of the best fit.

Knowledge of the planned course of cumulative financial outlays over time and the shape of the S curve and its deviations allows rational actions that aim to achieve the intended goal and success in the implementation of a construction project to be undertaken.

During the implementation of the investment, it is important for decision-makers to make appropriate decisions in the case of emerging anomalies and/or changes in the investment at its various stages. Using the cost curve spaces, which were designated at stage 6 for the budgeted costs of the works scheduled (curve: BCWS) and for the actual costs of works performed (curve: ACWP), it is possible to properly monitor the course of the investment, and then react appropriately to the situation. Depending on where the controlled investment is located, it is possible to quickly estimate the expected completion time (an extension of the implementation time) of the investment and budget (an increase in its value).

The Three Sigma Rule was used for the monitoring and controlled analysis of investments. It is well known that in practise, the three-sigma rule is used as a warning system in the case of danger, abnormal behaviour, or something out of the ordinary. By developing an algorithm based on the principle of three sigmas, it is possible to construct a system of "warning" about irregularities.

It was assumed in the research that the analysed data have a normal distribution. For the analysed problem, this assumption is met with a sufficient approximation, and therefore it is justified to say that according to the Three-Sigma rule (figure 4):

- 68.2% of the values lie at a distance of $\leq\sigma$
- 95.5% of the values lie at a distance of $\leq 2\sigma$
- 99.7% of the values lie at a distance of $\leq 3\sigma$

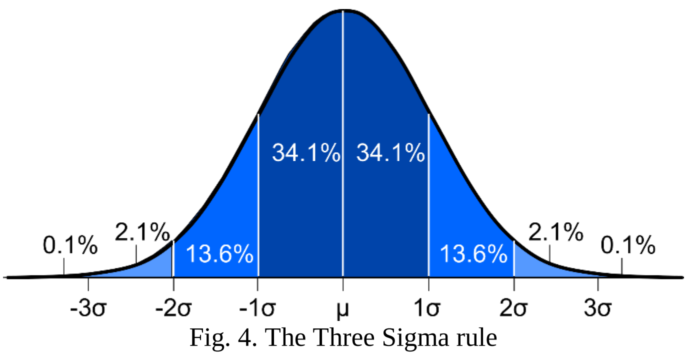

Fig. 4. The Three Sigma rule

The spaces of cost curves defined by 3 curves: the best-fit curve, the curve limiting the space of cost curves "from above", and the curve limiting the space of cost curves "from the bottom" were divided according to the three sigma rule into three areas:

- area within the range <-σ, σ> - acceptable range,
- area within the range <-2σ, σ> ∪ <-σ, 2σ> - tolerable range,
- area within the range <-3σ, 2σ> ∪ <-2σ, 3σ> - unacceptable range.

The monitoring of an investment is carried out in relation to the work and expenditure schedule planned by the decision-maker – the curve of the budgeted cost of work scheduled (BCWS). At the time of inspection of the investment status, three situations may arise for which the following solutions are recommended:

- situation 1: the analyzed value is within the acceptable range, i.e. within $< -\sigma, \sigma >$. This means that the investment is implemented correctly with slight deviations, and it needs to be just monitored in relation to the model curve of the budgeted costs of work scheduled (BCWS).
- situation 2: the analyzed value is within the tolerable range, i.e. within $< -2\sigma, \sigma > \cup < -\sigma, 2\sigma >$. This means that the investment is implemented with deviations that may affect its budget and completion date. It is recommended to compare the investment with the model curve of the actual cost of the work performed (ACWP).
- situation 3: the analyzed value is within an unacceptable range, i.e. within $< -3\sigma, 2\sigma > \cup < -2\sigma, 3\sigma >$. This means that the investment is carried out with large deviations that may have a significant impact on the value of the budget being increased and the investment completion date being extended. It is important to compare the investment with the model curve of the actual cost of the work performed (ACWP).

4. RESEARCH RESULTS

4.1. Shape of the curve of cummulative BCWS

After conducting the calculations in accordance with the proposed research methodology, figure 3 shows the results obtained for all the analysed construction projects, which were divided into 3 research groups.

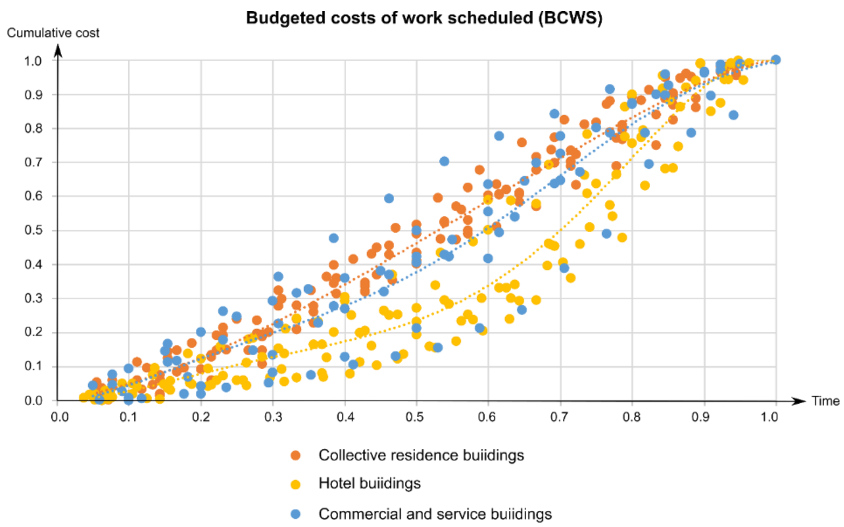

Fig. 5. The curve of cumulative budgeted costs of scheduled work (BCWS)

Next, the best-fit curve and the curves limiting the obtained area "from the top" and "from the bottom" were determined. A third degree polynomial and the curve's inflection point were also determined for each curve. Finally, on the basis of the previous steps, it was possible to determine the envelope for the cost curves and divide the obtained area into the three scenarios analysed, which allow the monitoring and control of construction projects in accordance with the adopted Three-Sigma rule (figure 6).

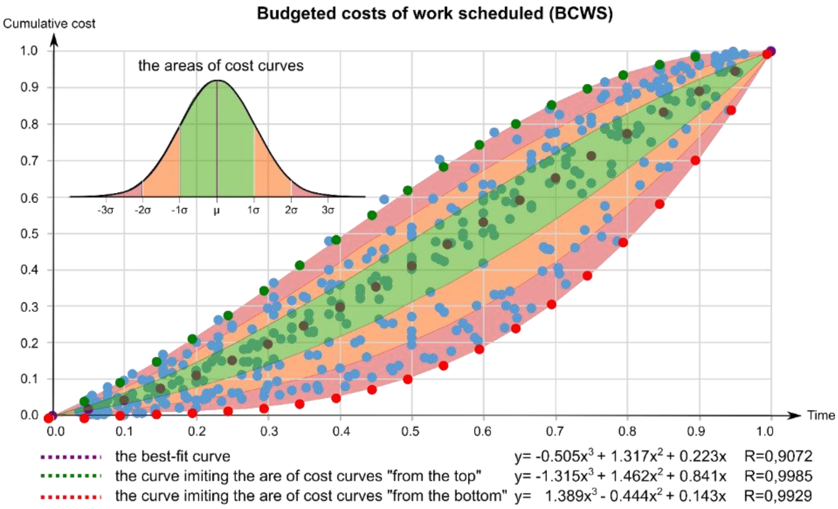

Fig. 6. The best-fit curve of cumulative BCWS and the spaces of cost curves

4.2. The shape of the curve of the cummulative ACWP

Similar calculations were performed for the budgeted cumulative cost curves for the actual cumulative cost curves (figure 7), i.e., based on the properly processed data, the best fit curve, the curves limiting the obtained area, the envelope and the division of the area were determined (figure 8).

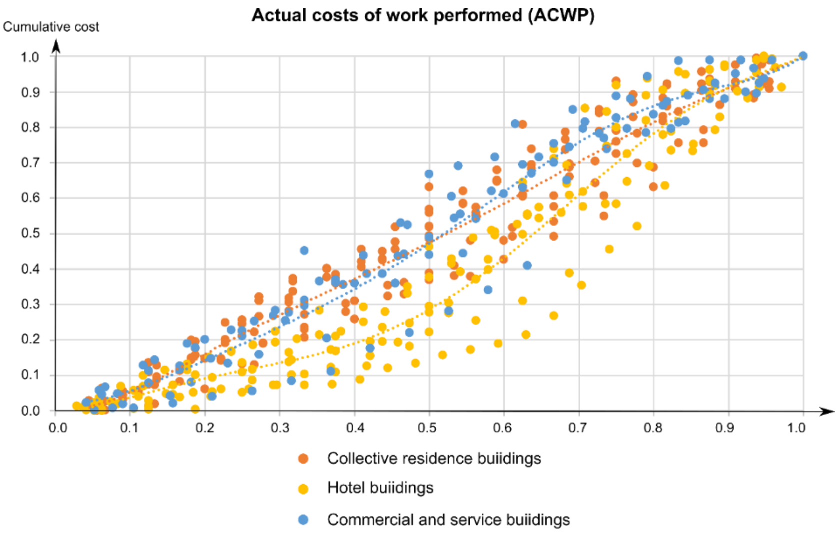

Fig. 7. The curve of the actual cumulative costs of the work performed (ACWP)

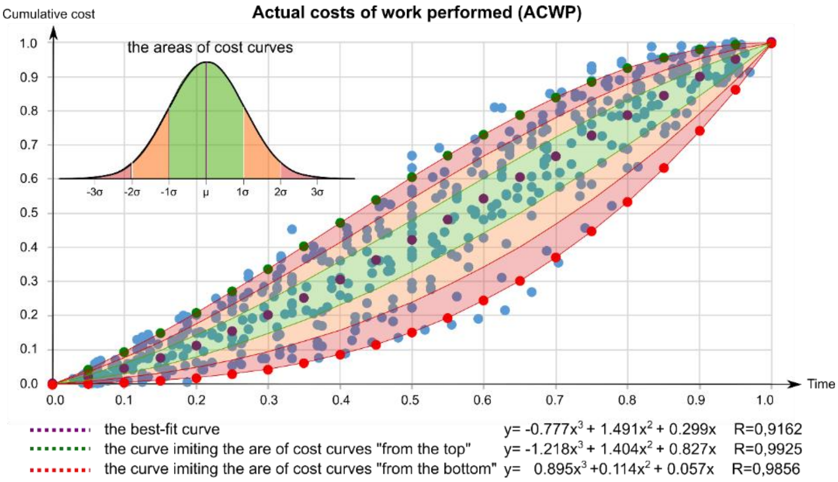

Fig. 8. The best-fit curve of cumulative ACWP and the spaces of cost curves

4.3. Comparison of the obtained results

The last stage of the research led to the comparison of the areas obtained and the analysis of the results obtained. Figure 9 shows the results obtained and their graphic interpretation.

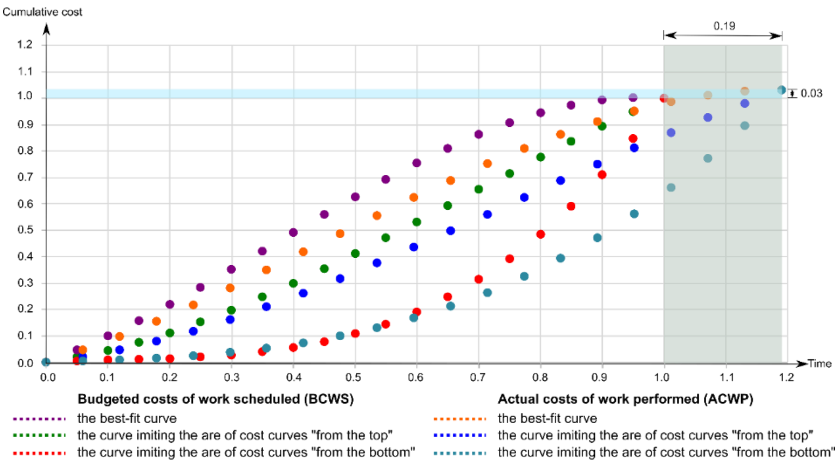

Fig. 9. Comparison of the obtained results

5. DISSCUSSION

The research included an analysis of the baseline work and expenditure schedules and the monthly actual cost values, determined by the measurements of incurred and generated costs / amounts of construction works performed for different construction projects (collective housing, hotels and retail-service facilities). The cumulative values of the amounts of work performed on the construction site constituted the cumulative cost, which when determined cyclically and consistently, points out the course of the S curve that corresponds to a monitored and controlled construction investment. The collected data on the projects can be used to distinguish typological research samples for investments of a similar profile. The measurement results were according to the proposed research methodology by the author.

Actual costs of investment tasks were assessed. The resulting trend of the cumulative cost flow curves can be estimated, with a good fit, when forecasting the construction process. The areas of the S curves of the budgeted costs were determined using the 3rd degree polynomials, which indicate the area of expected costs in a construction project and the estimated values of their deviations.

6. CONCLUSIONS

The research conducted on an attempt to describe the shape of cumulative course of cost curves using mathematical relationships between two variable parameters: cost and time. Previous researchers have proposed various mathematical formulas to describe the shape of the cost curve, and the most common uses have been:

- a 6-degree polynomial [58, 59],
- a 3-degree polynomial [55, 60, 61],
- less frequently a 2-degree polynomial and a linear function [62].

In each situation, the cumulative cost curve is described by one of these models, which allows one to estimate costs as a function of time and progress of the work.

A 1-degree polynomial (linear function) and 2-degree polynomial (square trinomial) are used to describe the relationship between cost and time. However, in order to be able to use a linear function or square trinomial, it is necessary to divide the investment process into shorter periods, e.g. 3 periods, which is difficult to determine universally. Regardless of the approach used, it should be remembered that the cost curves are characterised by a variable slope and inflection point, which makes it difficult to describe them accurately with a single mathematical function; therefore, the use of a linear or square trinomial is a great generalisation. An incorrect division, that is, deviating from reality, into shorter periods may lead to an error and an unreal shape of the cost curve [62, 63].

Based on the analysis of the literature and our own research, it is reasonable to use the curve space (envelope) to describe the cost curve, which should include the cumulative cost curve, while higher-order polynomials i.e. minimum 3-degree polynomials, should be used to describe the best fit of cumulative cost curve.

It should be noted that the use of a polynomial trend, e.g. 6 degrees, although allowing one to obtain a high value of the correlation coefficient and a low value of the coefficient of variation, but its practical application can be difficult and complicated for decision makers and can be considered inappropriate [64]. From the practical point of view, the use of polynomials of higher degree than 4 may be difficult for investors, works contractors.

Therefore, simple mathematical formulas are sought for the cost curves. Analysis of the literature on the subject indicates that the optimal formula for determining the best-fit curve is a 3-degree polynomial, which was proposed in the presented research.

REFERENCES

1. Szafranko, E and Harasymiuk, J 2022. Modelling of Decision Processes in Construction Activity. *Applied Sciences* **12,** 3797.
2. Bozejko, W, Hejducki, Z, Uchroński, M. and Wodecki, M 2014. Solving resource-constrained construction scheduling problems with overlaps by metaheuristic. *Journal of Civil Engineering and Management* **20**, 649–659.
3. Bozejko, W, Hejducki, Z and Wodecki, M 2019. Flowshop scheduling of construction processes with uncertain parameters. *Archives of Civil and Mechanical Engineering* **19**, 194–204.
4. Plebankiewicz, E, Zima, K and Wieczorek, D 2021. Modelling of time, cost and risk of construction with using fuzzy logic. *Journal of Civil Engineering and Management* **27**, 412–426.

5. Kowacka, M, Skorupka, D, Duchaczek, A, Waniewska, A and Dudziak-Gajowiak, D 2019. Risk analysis in surveying works related to roads construction. *Scientific Review Engineering and Environmental Sciences* **28** , 377–382.
6. Radziszewska-Zielina, E, Adamkiewicz, D, Szewczyk, B and Kania, O 2022. Decision-making support for housing projects in post-industrial areas. *Sustainability* **14**, 3573.
7. Hsieh, T-Y, Hsiao-Lung Wang, M and Chen, C-W 2004. A case study of S-curve regression method to project control of construction management via T-S fuzzy model. *Journal of Marine Science and Technology* **12,** 209–216.
8. Sztubecka, M, Skiba, M, Mrówczyńska, M and Mathias M 2020. Noise as a Factor of Green Areas Soundscape Creation. *Sustainability* **12**(3), 999.
9. Konior, J 2019. Fuzziness over randomness in unforeseen works of construction projects. *Civil Engineering and Architecture* **7** , 42–48.
10. Konior, J and Szóstak M 2020. Methodology of planning the course of the cumulative cost curve in construction projects. *Sustainability* **12** , 2347.
11. Mrówczyńska, M, Skiba, M, Bazan-Krzywoszańska, A, Bazuń, D and Kwiatkowski M 2018. Social and infrastructural conditioning of lowering energy costs and improving the energy efficiency of buildings in the context of the local energy policy. *Energies* **11**, 2302.
12. Leśniak, A and Plebankiewicz, E 2015. Modeling the decision-making process concerning participation in construction bidding. *Journal of Management in Engineering* **31**(2).
13. Kasprowicz, T 2017. Quantitative assessment of construction risk. *Archives of Civil Engineering* **63**, 55–62.
14. Kasprowicz, T 2017. Quantitative identification of construction risk. *Archives of Civil Engineering* **63**, 63–75.
15. Kerzner, H 2003 Project Management: A systems approach to planning, scheduling, and controlling, John Wiley&Sons, Inc., New York, USA.
16. Połoński, M 2018. Management of construction investment process, Wydawnictwo SGGW (in Polish).
17. Jaśkowski, P, Biruk, S and Krzemiński, M 2020. Planning repetitive construction processes to improve robustness of schedules in risk environment. *Archives of Civil Engineering* **66**, 643–657.
18. Skrzypczak, I, Oleniacz, G, Leśniak, A, Zima, K, Mrówczyńska, M and Kazak, J. 2022. Scan-to-BIM method in construction: Assessment of the 3D buildings model accuracy in terms inventory measurements. *Building Research & Information* **50**(8), 859-880.
19. Leśniak, A, Kubek, D, Plebankiewicz, E, Zima, K and Belniak, S 2018. Fuzzy AHP application for supporting contractors' bidding decision. *Symmetry* **10**, 642.
20. Leśniak, A 2015. Classification of the Bid/No bid criteria-factor analysis. *Archives of Civil Engineering* **61**(4), 79-90.
21. Konior, J 2015. Enterprise's risk assessment of complex construction projects. *Archives of Civil Engineering* **61**, 63–74.
22. Lo, W and Chen, Y-T 2007. Optimization of contractor's S-curve. 24th International Symposium on Automation & Robotics in Construction (ISARC 2007), 417–420.
23. Project Management Institute 2017. A guide to the project management body of knowledge (PMBOK guide) 6th Edition, Project Management Institute (PMI).
24. IPMA 2015. IPMA Individual Competence Baseline.
25. Chen, H-L, Chen, W-T and Lin, L 2016. Earned value project management: Improving the predictive power of planned value. *International Journal of Project Management* **34**, 22–29.
26. Przywara, D and Rak, A 2021. Monitoring of time and cost variances of schedule using simple Earned Value Method indicators. Applied Sciences **11**, 1357.

27. Konior, J 2019. Monitoring of construction projects feasibility by Bank Investment Supervision approach. *Civil Engineering and Architecture* **7,** 31–35.
28. Dziadosz, A, Kapliński, O and Rejment, M 2014. Usefulness and fields of the application of the Earned Value Management in the implementation of construction projects. *Budownictwo i Architektura* **13**, 357–364.
29. Czarnigowska, A 2008. Earned value method as a tool for project control. *Budownictwo i Architektura* **3**, 15–32.
30. Czarnigowska, A 2009. Monitoring of project progress using the Earned Value. *Przegląd Budowlany* (in Polish) **80,** 50–55.
31. Howes, R 2000. Improving the performance of Earned Value Analysis as a construction project management tool, Engineering. *Construction and Architectural Management* **7**, 399–411.
32. Lipke, W, Zwikael, O, Henderson, K and Anbari, F 2009. Prediction of project outcome. The application of statistical methods to earned value management and earned schedule performance indexes. *International Journal of Project Management* **27**, 400–407.
33. Chen, Z, Demeulemeester, E, Bai, S and Guo, Y 2020. A Bayesian approach to set the tolerance limits for a statistical project control method. *International Journal of Production Research* **58,** 3150–3163.
34. Salari, M, Bagherpour, M and Reihani, M 2015. A time-cost trade-off model by incorporating fuzzy earned value management: A statistical based approach. *Journal of Intelligent & Fuzzy Systems* **28**, 1909–1919.
35. Salari, M and Khamooshi, H 2016. A better project performance prediction model using fuzzy time series and data envelopment analysis. *Journal of the Operational Research Society* **67,** 1274–1287.
36. Hajali-Mohamad, M-T, Mosavi, M-R and Shahanaghi, K 2016. Optimal estimating the project completion time and diagnosing the fault in the project. *DYNA* **83,** 121–127.
37. Yaseen, Z-M, Ali, Z-H, Salih, S-Q and Al-Ansari N 2020 Prediction of risk delay in construction projects using a hybrid artificial intelligence model. *Sustainability* **12**, 1514.
38. Chao, L-C, and Chen, H-T 2015. Predicting project progress via estimation of S-curve's key geometric feature values. *Automation in Construction* **57,** 33–41.
39. Cristóbal, J 2017. The S-curve envelope as a tool for monitoring and control of projects. *Procedia Computer Science* **121,** 756–761.
40. Tijanić, K and Car-Pušić D 2017. Application of S-curve in EVA Method. 13th International Conference "Organization, Technology and Management in Construction, 103–115.
41. Cheng, M-Y, Tsai, H-C, and Liu, C-L 2009. Artificial intelligence approaches to achieve strategic control over project cash flows. *Automation in Construction* **18,** 386–393.
42. Kozień, E 2017. Application of approximation technique to on-line updating of the actual cost curve in the earned value method. *Czasopismo Techniczne* **9,** 181–195.
43. Mohagheghi, V, Meysam Mousavi, S and Vahdani, B 2017. An assessment method for project cash flow under interval-valued fuzzy environment. *Journal of Optimization in Industrial Engineering* **22,** 79–80.
44. Hsieh, T-Y, Wang, M, Chen, C-W, Chen, C-Y, Yu, S-E, Yang, H-C and Chen, T-H 2006. A new viewpoint of s-curve regression model and its application to construction management. *International Journal on Artificial Intelligence Tools* **15,** 131–142.
45. Chao, L-C, and Chien, C-F 2010. A model for updating project S-curve by using neural networks and matching progress. *Automation in Construction* **19**, 84–91.
46. Wang, K-C, Wang, W-C, Wang, H-H, Hsu, P-Y, Wu, W-H and Kung, C-J 2016. Applying building information modeling to integrate schedule and cost for establishing construction progress curves. *Automation in Construction* **72,** 397–410.

47. Maravas, A and Pantouvakis, J-P 2012. Project cash flow analysis in the presence of uncertainty in activity duration and cost. *International Journal of Project Management* **30,** 374–384.
48. Mohamad, H-M, Mohamad, M-I, Saad, I, Bolong, N, Mustazama, J and Razali, S-N-M 2021. A case study of s-curve analysis: Causes, effects, tracing and monitoring project extension of time. *Civil Engineering Journal (Iran)* **7**, 649–661.
49. Barraza, G-A, Back, W-E and Mata, F 2004. Probabilistic forecasting of roject performance using stochastic S curves. *Journal of Construction Engineering and Management* **130**, 25–32.
50. Yao, J-S, Chen, M-S and Lu, H-F 2006. A fuzzy stochastic single-period model for cash management. *European Journal of Operational Research* **170,** 72–90.
51. Kim, B-C and Reinschmidt, K 2007. An S-curve Bayesian model for forecasting probability distributions on project duration and cost at completion. 25th Inaugural Construction Management and Economics: 'Past, Present and Future' Conference, CME 2007 - Reading, United Kingdom, 1449–1459.
52. Kim, B and Reinschmidt, KF 2009. Probabilistic forecasting of project duration using Bayesian inference and the beta distribution. *Journal of Construction Engineering and Management* **135**, 178–186.
53. Blyth, K and Kaka, A 2006. A novel multiple linear regression model for forecasting S-curves, Engineering. *Construction and Architectural Management* **13**, 82–95.
54. Banki, M-T and Esmaeeli, B 2008. Using historical data for forecasting s-curves at construction industry. IEEE International Conference on Industrial Engineering and Engineering Management, 282–286.
55. Chao, L-C and Chien, C-F 2009. Estimating project S-curves using polynomial function and neural networks. *Journal of Construction Engineering and Management* **135,** 169–177.
56. Jiang, A, Issa, R and Malek, M 2011. Construction project cash flow planning using the Pareto optimality efficiency network model. *Journal of Civil Engineering and Management* **17**, 510–519.
57. Konior, J and Szóstak, M 2020. The S-curve as a tool for planning and controlling of construction process-case study. *Applied Sciences* **10**.
58. Szóstak, M 2021. Planning the time and cost of implementing construction projects using an example of residential buildings. *Archives of Civil Engineering* **67,** 243–259.
59. Ostojic-Skomrlj, N and Radujkovic, M 2012. S-curve modelling in early phases of construction projects. *Gradevinar* **64,** 647–654.
60. Peer, S 1982. Application of Cost-Flow Forecasting Models. *Journal of the Construction Division* **108**, 226–232.
61. Miskawi, Z 1989. An S-curve equation for project control. *Construction Management and Economics* **7**(2), 115-124.
62. Boussabaine, A-H and Elhag, T 1999. Applying fuzzy techniques to cash flow analysis. *Construction Management and Economics* **17**(6), 745–755.
63. Cioffi, D 2005. A tool for managing projects: an analytic parameterization of the S-curve. *International Journal of Project Management* **23,** 215–222.
64. Szóstak, M 2023. Best fit of cumulative cost curves at the planning and performed stages of construction projects. *Buildings* **13**, 13.

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