

EFFECTS OF VISCOUS DISSIPATION OVER AN UNSTEADY STRETCHING SURFACE EMBEDDED IN A POROUS MEDIUM WITH HEAT GENERATION AND THERMAL RADIATION

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This work analyzes the impact of viscous dissipation on an unstable stretching surface in a porous medium with heat generation and thermal radiation-an important factor for numerous engineering applications like cooling baths and plastic sheets. Using MATLAB's Runge-Kutta fourth-order approach, the controlling partial differential equations are converted into highly nonlinear ordinary differential equations that can be solved numerically. The findings show that a decrease in the skin friction coefficient, temperature profiles, velocity, and Nusselt number occurs when the unsteadiness parameter is increased. In contrast to the Prandtl number, which rises with temperature profile and reduced Nusselt number, the Eckert number rises with a dimensionless temperature profile and reduced Nusselt number. Reduced Nusselt number and temperature profile affect the heat generation parameter; a decrease in skin friction coefficient and velocity profile correlate with the porosity parameter. Furthermore, the radiation parameter rises as the temperature distribution and Nusselt number decrease.

Key words: heat generation, porous medium, thermal radiation, unsteady stretching surface, viscous dissipation.

1. Introduction

Examining fluid dynamics on over-stretching surfaces is crucial in numerous scientific and engineering domains, ranging from manufacturing processes to environmental systems. An essential part of this domain is investigating the impacts of viscous dissipation on an unstable stretched surface with heat generation and thermal radiation, while the surface is embedded in a porous medium. As technology advances spur innovation in material processing, understanding these phenomena is essential to enhancing production procedures, ensuring product quality, and addressing environmental problems.

In many technical processes, including cooling baths, plastic sheets, aerodynamic extrusion, metallurgical operations, and glass blowing, boundary layer flow is applied on a stretching surface. Continuous stretching of the sheet is necessary in manufacturing to reach a certain thickness, and both the stretching rate and the sheet's cooling rate have an impact on the finished product. Thermal radiation has emerged as a critical component of engineering sciences with applications in many different engineering disciplines. The effects of radiation and dissipation of the thermal boundary layer over a nonlinear stretching sheet were examined by the authors of [1].

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According to [2], they numerically analyzed two-dimensional fluid motion over a continuously stretched surface using similarity transformation. By examining different impacts of this flow, the authors of [3-5] expanded on this work. Although earlier research concentrated on linearly extending sheets, Kumaran and Ramanaiah [6] investigated a quadratic stretching sheet with viscous flow and observed that sheet velocity may be quadratic or exponential rather than always having a linear relationship. In boundary layer flow, Ali and Magyari [7] looked into the properties of mass and heat transmission over a stretching sheet. While every research study mentioned above worked with a steady stretching sheet, a flat sheet's impulsive motion can cause the stretching sheet to become unstable, which in turn causes instability in the flow field, heat transfer, and mass transfer. An abrupt shift in wall velocity, free stream, wall temperature, etc. is what causes this unsteadiness. No attempt has been made to investigate the embedding of an unstable stretching surface in a porous material, based on the literature. This research examines the impacts of heat generation, porosity, viscous dissipation, and thermal radiation on a surface with unstable stretching after first studying its applications.

2. Mathematical modelling

Assuming a flow of incompressible fluid on a stretching surface that is continuously moving in two dimensions as an unstable laminar flow. The problem is governed by the linear momentum, energy, and continuity equations.

Continuity equation [8]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (2.1)$$

The linear momentum conservation equation is [7,9]:

$$\frac{\partial u}{\partial t} + \frac{u\partial u}{\partial x} + \frac{v\partial u}{\partial y} = \frac{\nu\partial^2 u}{\partial y^2} - \frac{\nu}{K} u. \quad (2.2)$$

Conservation of energy equation [10]:

$$\frac{\partial T}{\partial t} + \frac{u\partial T}{\partial x} + \frac{v\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{Q}{\rho C_p} (T - T_\infty) + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2, \quad (2.3)$$

$$q_r = \frac{4\sigma}{3a} \frac{\partial T^4}{\partial y}. \quad (2.4)$$

In this equation, the velocities in the x and y directions is denoted by u and v , respectively; C_p is the specific heat at constant pressure; t is the time; ν is kinematic viscosity; K is permeability; κ is thermal conductivity; ρ is fluid density; T is the boundary layer temperature; Q is the heat source (if $Q > 0$) or heat sink (if $Q < 0$); T_w is the surface temperature, and T_∞ is the free stream temperature. Furthermore, σ and α stand for the mean absorption coefficient and the Stefan-Boltzmann constant, respectively. The formulation of the quantity T^4 as a linear function of temperature is made possible by the temperature variations in the flow. Therefore, using Taylor series, and neglecting higher-order terms, the expansion of T^4 in a Taylor series about T_∞ , the expression can be simplified as:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \quad (2.5)$$

Equation (2.3) on the energy becomes: Using Eqs (2.4) and (2.5):

$$\frac{\partial T}{\partial t} + \frac{u\partial T}{\partial x} + \frac{v\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3\bar{\alpha}\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p} (T - T_\infty) + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2. \quad (2.6)$$

The velocity of the stretched surface is [8],

$$u_w(x,t) = \frac{\alpha x}{1 - \gamma t}, \quad (2.7)$$

with these boundary conditions:

$$y=0, \quad u = U_\infty(x,t), \quad v=0, \quad T = T_w(x,t), \quad (2.8)$$

$$y \rightarrow \infty, \quad u = 0, \quad T = T_\infty. \quad (2.9)$$

Using the stream function $\psi(x, y)$, the equation of continuity is satisfied such that:

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}. \quad (2.10)$$

2.1. Mathematical analysis

The following dimensionless coordinates are introduced to simplify the problem's mathematical analysis so that [6,10]:

$$\eta = \sqrt{\frac{\alpha}{\nu(I - \gamma t)}} y, \quad (2.11)$$

$$\psi(x, y) = \sqrt{\frac{\alpha \nu x^2}{(I - \gamma t)}} f(\eta), \quad (2.12)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (2.13)$$

$$T_w - T_\infty = \frac{\alpha}{2\nu x^2} (I - \gamma t)^{-3/2}. \quad (2.14)$$

The transformed conservation of linear momentum equation:

$$f''' + ff'' - f'^2 - A \left(f' + \frac{1}{2} \eta f'' \right) - \lambda f' = 0. \quad (2.15)$$

The transformed conservation of energy equation:

$$\left(1 + \frac{4}{3R}\right)\theta'' + Pr \left[2f'\theta + f\theta' + Ecf'^{n_2} - \frac{A}{2}(3\theta + \eta\theta') + \delta\theta\right] = 0, \quad (2.16)$$

$$\delta = \frac{Qx}{\rho C_p U_w}, \quad (2.17)$$

$$R = \frac{\kappa\alpha}{4\sigma T_\infty^3}, \quad (2.18)$$

$$\lambda = \lambda = \frac{\nu(1 - \gamma t)}{\kappa\alpha}. \quad (2.19)$$

3. Parameters of engineering interest

Here, quantitative analysis of the heat transfer parameters within the fluid is expressed mathematically.

3.1. Heat transfer coefficient

This is a quantitative analysis of convective heat transfer between the wall of the fluid and the fluid medium itself [11].

$$-\theta'(0) = \frac{Nu_x}{\sqrt{Re_x}}. \quad (3.1)$$

3.2. Coefficient of skin friction

This is a dimensionless drag coefficient that is used to express the relationship between the shearing stress that the wind exerts at the surface of the earth and the frictional force per unit area [12].

$$f''(0) = C_f \sqrt{Re_x}. \quad (3.2)$$

3.3. Prandtl number

The dimensionless Prandtl number (Pr), named after the famous German scientist Ludwig Prandtl, expresses the ratio of momentum diffusivity to heat diffusivity. The Prandtl number is stated mathematically as [13,14]:

$$Pr = \frac{\mu C_p}{k}. \quad (3.3)$$

3.4. Eckert number

Viscous dissipation in natural convection flow, as indicated by the Eckert number, assumes significance particularly in extensive flow fields or under the influence of a strong gravitational field [9].

$$Ec = \frac{U_w^2}{C_p (T_w - T_\infty)}. \quad (3.4)$$

3.5. Boundary value problem solver for ODEs (BVP4C)

One numerical method for solving boundary value problems (BVPs) related to ordinary differential equations (ODEs) is the BVP4C method. This method is particularly useful when dealing with problems that involve differential equations subject to boundary conditions. The flowchart for the BVP4C is shown in Fig.1.

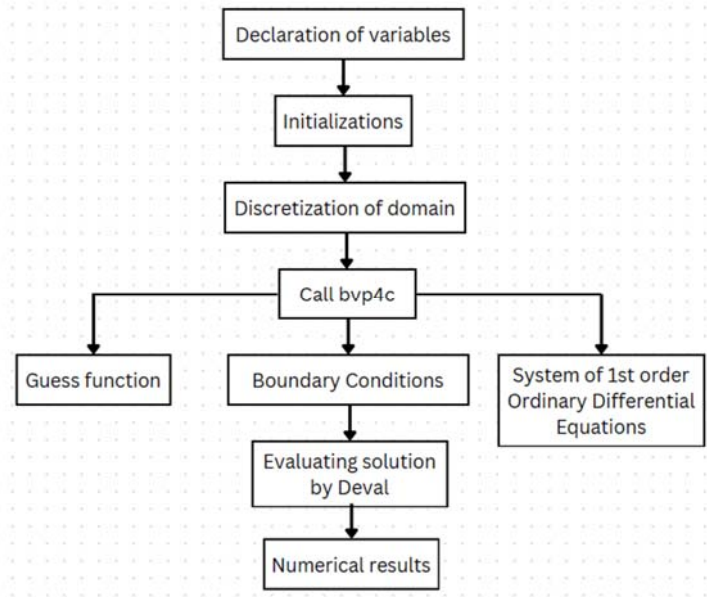


Fig.1. The BVP4C flowchart.

4. Results and discussions

Equations 2.15 and 2.16, which were transformed, were numerically solved in accordance with the given boundary conditions using MATLAB's BVP4C function. Key parameters, such as the heat generation parameter (δ), radiation parameter (R), Prandtl number (Pr), Eckert number (Ec), unsteadiness parameter (A), porous parameter (λ), and radiation parameter (R) were computed with accuracy to the fourth decimal place, guaranteeing satisfactory convergence results.

$f'(0)$ - is the velocity profile,

$f''(0)$ - is the coefficient of skin friction,

$\theta(0)$ - is the temperature profile,

$-\theta'(0)$ - is the reduced Nusselt number.

4.1. Code validation

To affirm the numerical methodology employed in this research, the outcomes for the reduced Nusselt number at the surface, denoted as $-\theta'(0)$ were juxtaposed with findings from previous studies by [10-12] Tab.1 presents a comparative analysis, revealing a substantial agreement between the current study and the existing work.

Table 1. Comparison of reduced Nusselt number $-\theta'(0)$ for $A=0$ (steady state) $Ec = \lambda = \delta = R = 0$ and $Pr = 1$.

present work	Freidoonimehr and Rahimi [15]	Elbashbeshy <i>et al.</i> [16]	Ali [17]	Ishak <i>et al.</i> [18]
1.0000	1.0000	0.9999	1.0054	1.0000

Figure 2 shows how the unsteadiness parameter affects the dimensionless velocity profile and shows a constant decline with increasing unsteadiness, in line with research by [19].

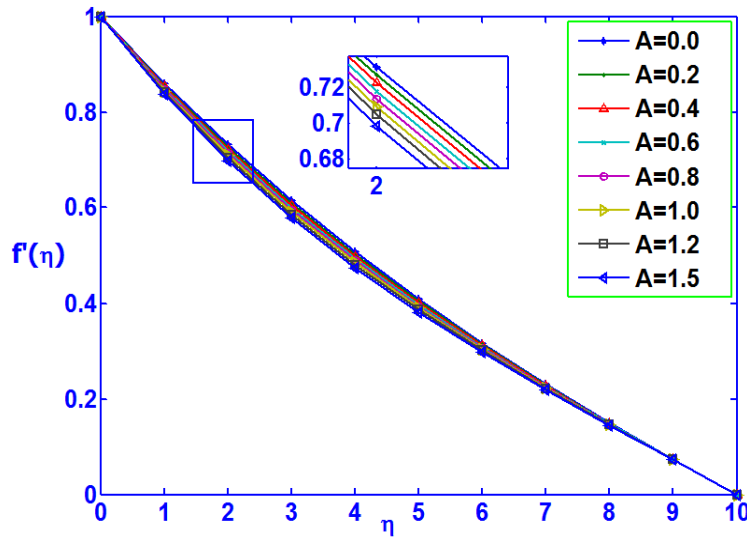


Fig.2. Effect of unsteadiness parameter A over velocity profile $f'(0)$ when $Pr = 0.7$, $Ec = 0.1$, $\delta = 0.1$, $\lambda = 0.5$, $R = 0.5$.

In Fig.3, it is clear that the unsteadiness parameter affects the skin friction coefficient, as the coefficient decreases as the unsteadiness parameter increases.

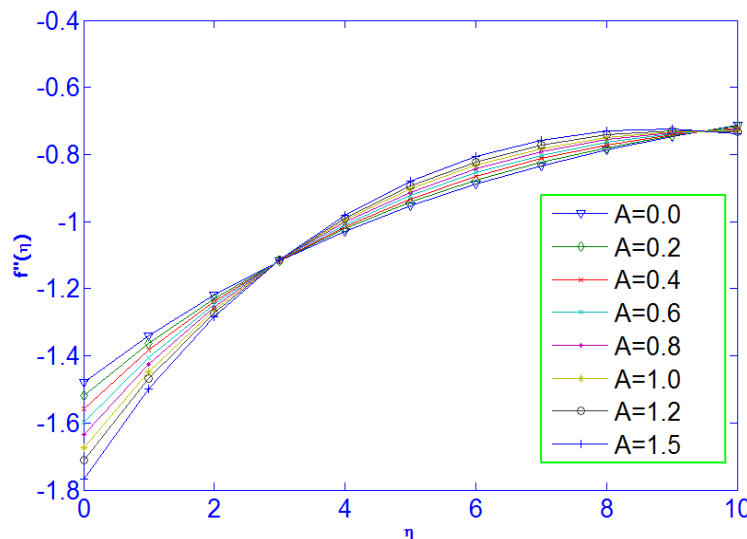


Fig.3. Unsteadiness parameter A effect on skin friction $-f''(0)$ when $Pr = 0.7$, $Ec = 0.1$, $\delta = 0.1$, $\lambda = 0.5$, $R = 0.5$.

Figures 4 and 5 demonstrate that the unsteadiness parameter correlates with a reduction in the dimensionless temperature profile and a reduced Nusselt number, respectively.

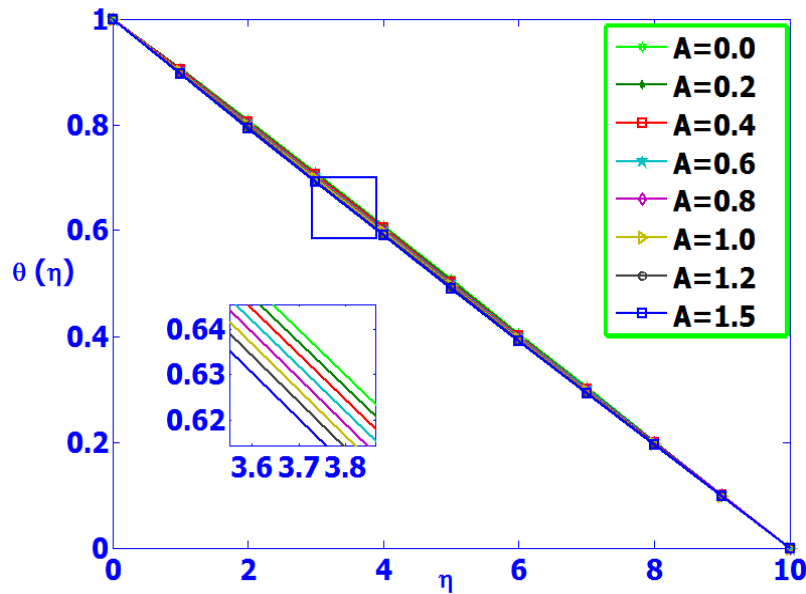


Fig.4. Unsteadiness parameter effect on temperature profile $\theta(0)$ when $Pr = 0.7$, $Ec = 0.1$, $\delta = 0.1$, $\lambda = 0.5$, $R = 0.5$.

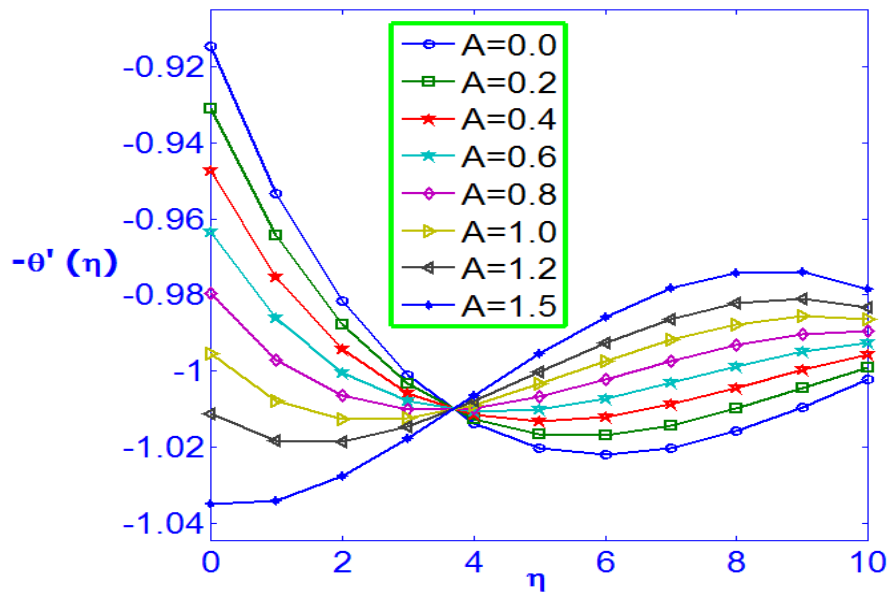


Fig.5. Effect of unsteadiness parameter A over reduced Nusselt number $-\theta'(0)$ when $Pr = 0.7$, $Ec = 0.1$, $\delta = 0.1$, $\lambda = 0.5$, $R = 0.5$.

The influence of the viscous dissipation parameter, represented by the Eckert number, is presented in Figs 6 and 7, indicating its increasing impact on both the dimensionless temperature profile and reduced Nusselt number.

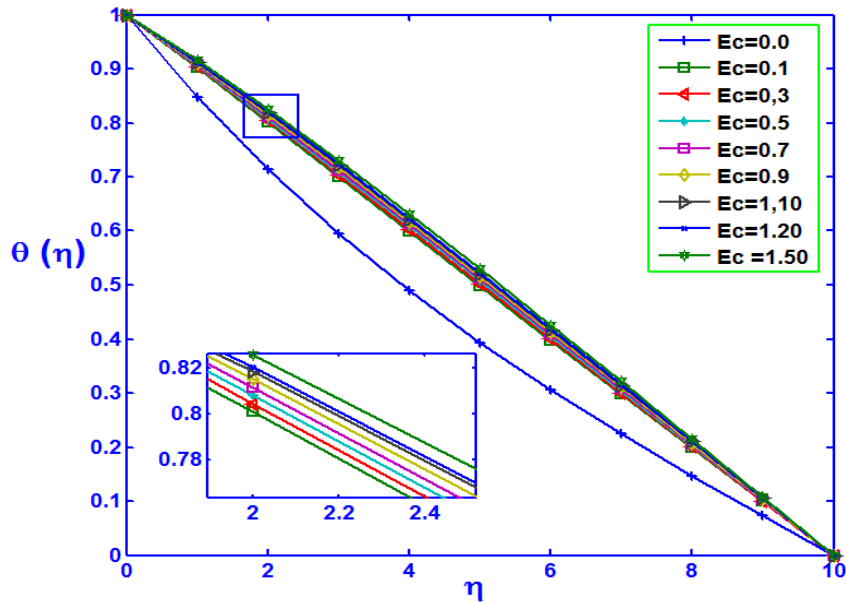


Fig.6. Effect of Eckert number Ec over temperature profile $\theta(\eta)$ when $Pr = 0.7$, $\delta = 0.1$, $\lambda = 0.5$, $R = 0.5$, $A = 0.8$.

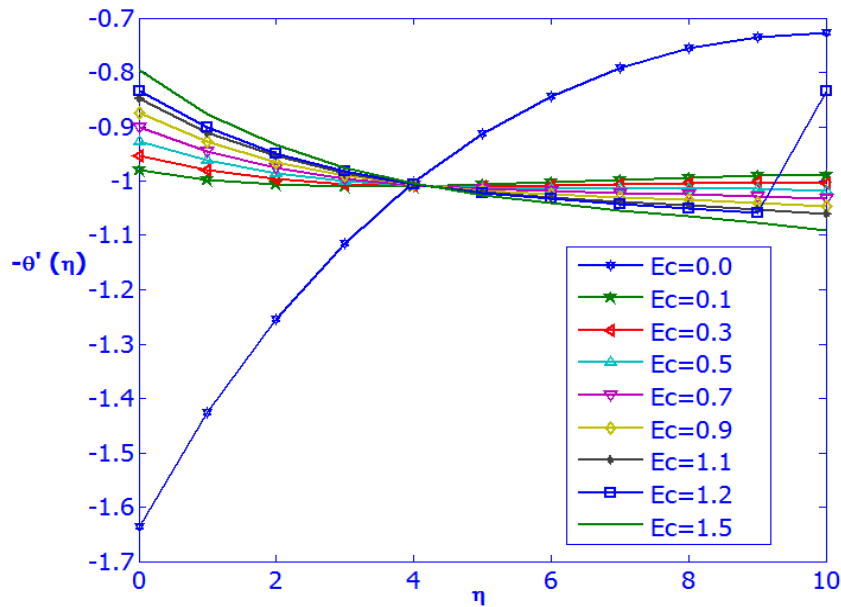


Fig.7. Effect of Eckert number over reduced Nusselt number $-\theta'(0)$ when $Pr = 0.7$, $\delta = 0.1$, $\lambda = 0.5$, $R = 0.5$, $A = 0.8$.

Figure 8 illustrates the Prandtl number's effect on the dimensionless temperature profile, showing an increase with rising temperature.

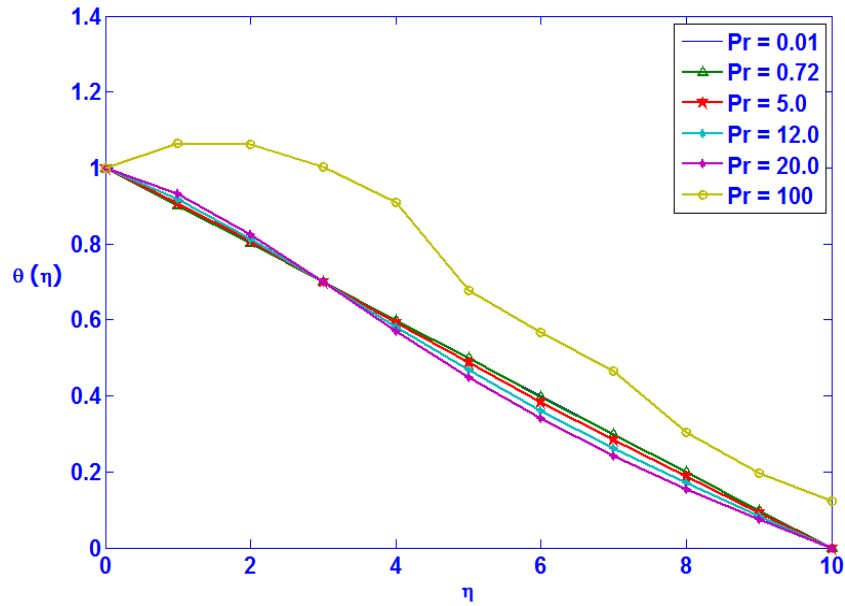


Fig.8. Effect of Prandtl number Pr over temperature profile $\theta(0)$ when $\delta=0.1$, $\lambda=0.5$, $R=0.5$, $A=0.8$ $Ec=0.1$.

Similarly, Fig.9 displays the increasing effect of the Prandtl number on the reduced Nusselt number.

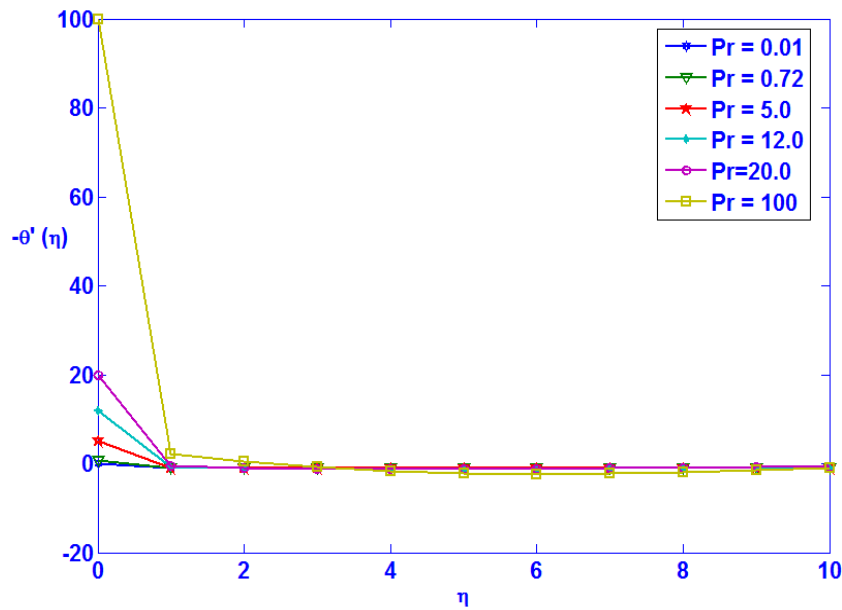


Fig.9. Effect of Prandtl number Pr over reduced Nusselt number $-\theta'(0)$ $\delta=0.1$, $\lambda=0.5$, $R=0.5$, $A=0.8$ $Ec=0.1$.

Figures 10 and 11 showcase the dimensionless temperature profile and reduced Nusselt number for varying heat generation values, indicating an increase in both parameters.

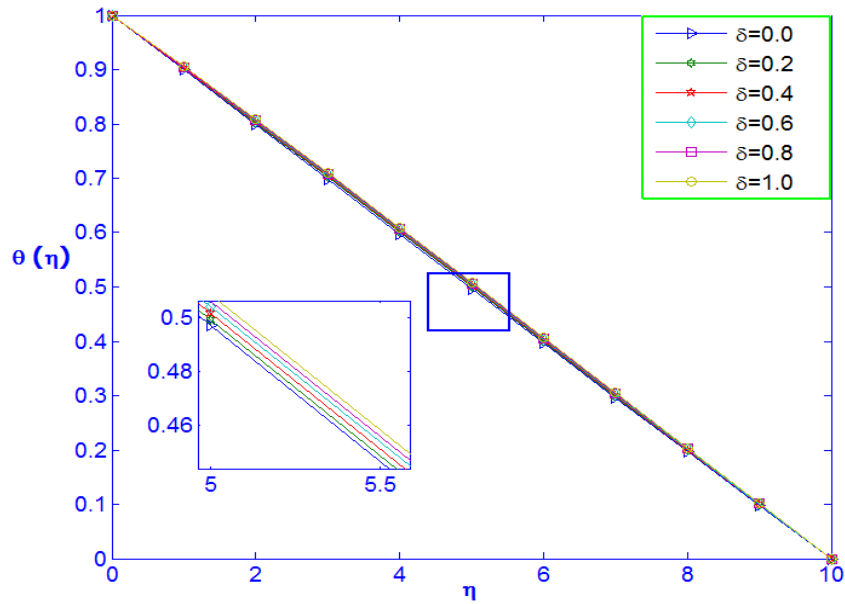


Fig.10. Impact of temperature profile on heat generation parameter $\delta\theta(0)$ when $\lambda = 0.5$, $R = 0.5$, $A = 0.8$, $Ec = 0.1$, $Pr = 0.7$.

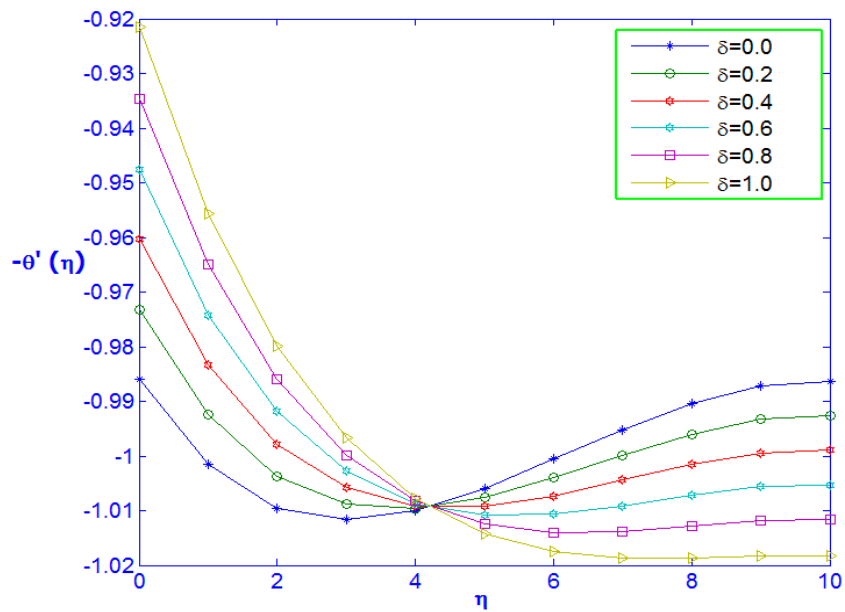


Fig.11. Effect of heat generation parameter δ over reduced Russell number $-\theta'(0)$ when $\lambda = 0.5$, $R = 0.5$, $A = 0.8$, $Ec = 0.1$, $Pr = 0.7$.

Figures 12 and 13 illustrate how the porosity parameter affects the skin friction coefficient and the velocity profile, respectively. An increase in the porosity parameter is connected with a decrease in the velocity profile and an increase in the skin friction coefficient.

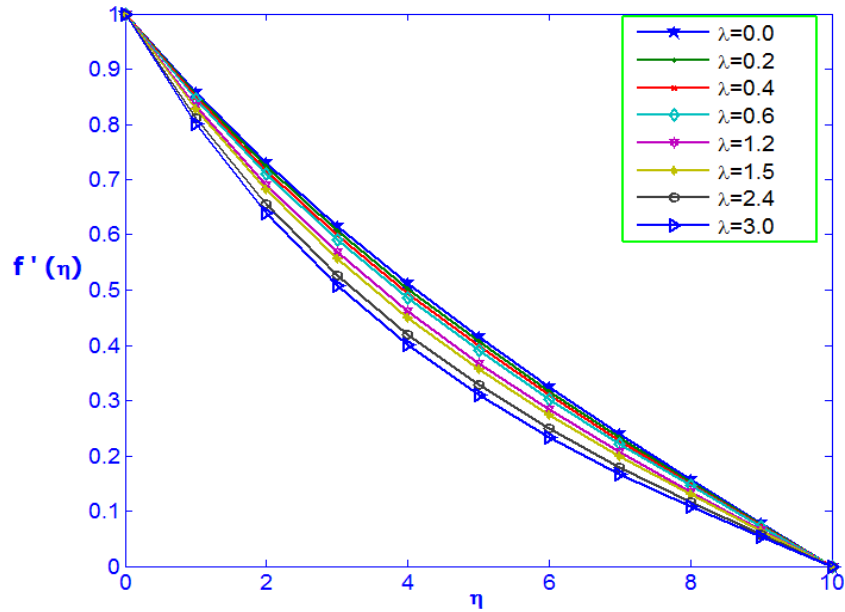


Fig.12. Porous parameter λ effect over velocity profile $f'(0)$ when $\delta=0.1$, $R=0.5$, $A=0.8$, $Ec=0.1$, $Pr=0.7$.

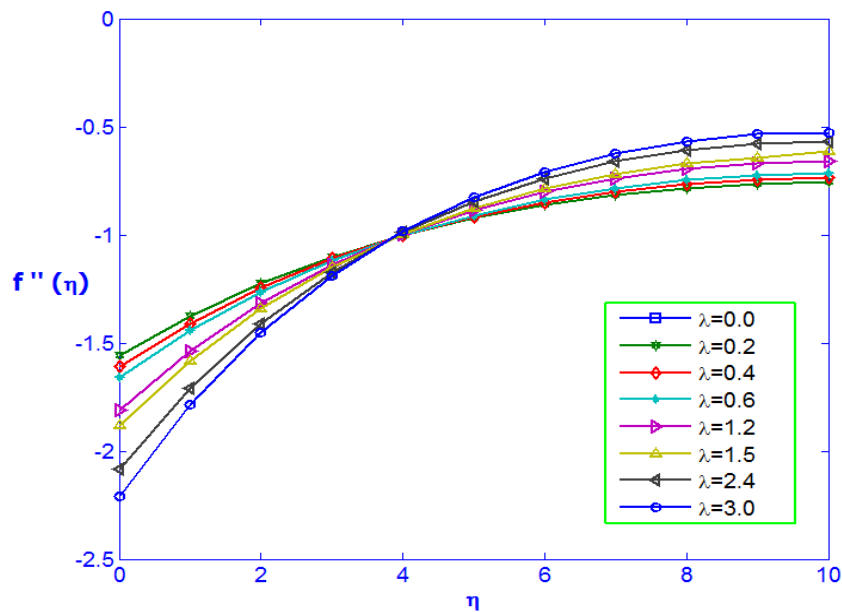


Fig.13. Effect of the porous parameter λ over skin friction $-f''(0)$ when $\delta=0.1$, $R=0.5$, $A=0.8$, $Ec=0.1$, $Pr=0.7$.

Lastly, Figs 14 and 15 depict the impacts of radiation factors on both the reduced Nusselt number and dimensionless temperature profile respectively, showing an increase with rising values of radiation parameters.

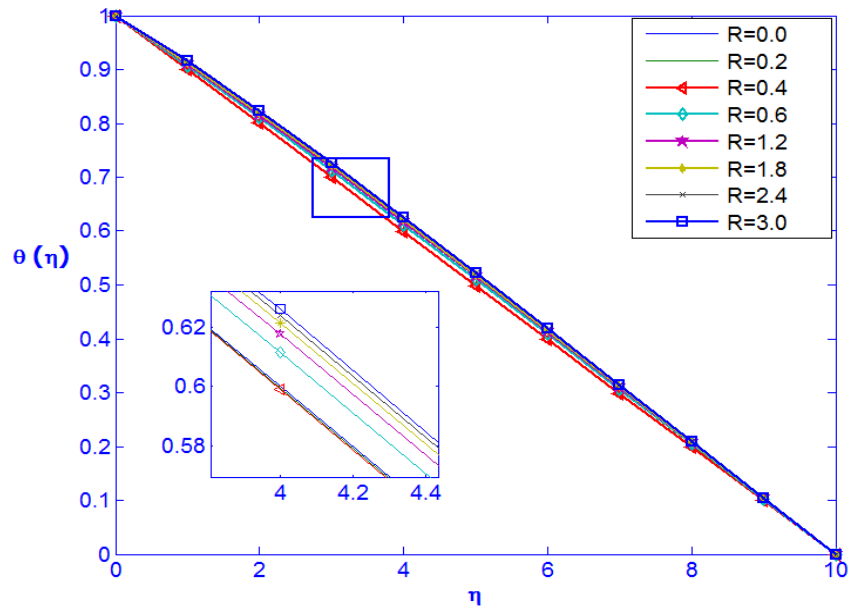


Fig.14. Radiation parameter effect on temperature profile $\theta(0)$ when $Pr = 0.7$, $Ec = 0.1$, $\delta = 0.1$, $\lambda = 0.5$, $A = 0.8$.

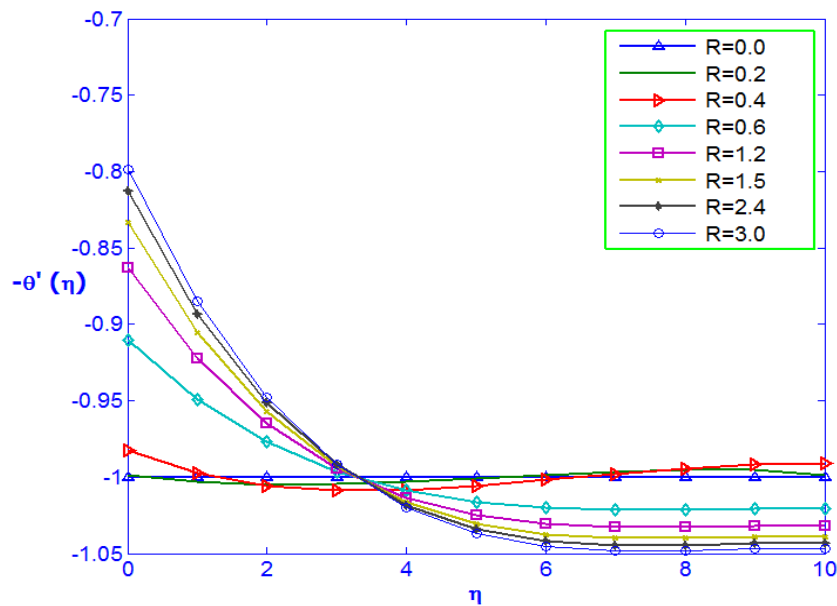


Fig.15. Effect of radiation parameter R over reduced Nusselt number $-\theta'(0)$ when $Pr = 0.7$, $Ec = 0.1$, $\delta = 0.1$, $\lambda = 0.5$, $A = 0.8$.

5. Conclusion

This work analyses the effects of viscous dissipation across an unstable stretched surface inside a porous material and presents numerical solutions that account for thermal radiation and heat generation. Using the proper similarity transformation, the time-dependent PDE were converted to ODE. The time-dependent PDE were transformed into ODE using appropriate similarity transformation. The numerical solutions showed a high degree of consistency with previous findings (refer to Tab.1). It can be deduced from the study that

there is a reduction of the Nusselt number, temperature profile, skin friction coefficient, and dimensionless velocity profile as the unsteadiness parameter increases. The temperature profile and the decreased Nusselt number cause the Eckert number to decrease concurrently. While the heat generation parameter corresponds to a rise in temperature profile and reduced Nusselt number, the Prandtl number shows an increase with rising reduced Nusselt number and temperature profile. The porous parameter rises with a drop in velocity profile and skin friction coefficient. Furthermore, the radiation parameter rises as the temperature profile and Nusselt number decrease. Even though this research is quite thorough, more investigation in this area of using BVP4C is highly advised to improve our knowledge of boundary layer heat and mass transmission.

Nomenclature

- A – unsteadiness
 C_f – local skin friction coefficient
 C_p – specific heat due to constant pressure $[J \cdot K^{-1} \cdot kg^{-1}]$
 Ec – Eckert number
 $F(\zeta)$ – dimensionless stream function
 K – permeability $[N / A^2]$
 Nu – Nusselt number
 Pr – Prandtl number
 Q – heat source or sink
 Qr – radiation heat flux, $[kg / s^3]$
 R – thermal radiation parameter
 Re_L – local Reynolds number
 T – temperature of the fluid, $[K]$
 t – time, $[s]$
 T_W – surface temperature, $[K]$
 T_∞ – free stream temperature, $[K]$
 v_s – surface velocity, $[m / s]$
 $v(x)$ – fluid velocity in x -direction, $[m / s]$
 $v(y)$ – fluid velocity in y -direction, $[m / s]$
 x, y – cartesian coordinates along the surface and normal to it respectively, $[m]$

Greek letters

- α – mean absorption coefficient, $[l / cm]$
 γ – stretching rate, $[l / s]$
 δ – dimensionless heat source or sink
 η – similarity variable, (dimensionless space variable)
 θ – similarity temperature function,
 κ – thermal conductivity, $\left[\frac{kg \cdot m}{s^3 \cdot K} \right]$

λ – permeability parameter, $\left[\frac{N}{m^2} \right]$

μ – dynamic viscosity of the fluid, $\left[\frac{Ns}{m^2} \right]$

ν – kinematics viscosity, $\left[\frac{m^2}{s} \right]$

ρ – density of fluid, $\left[\frac{kg}{m^3} \right]$

σ – Stefan-Boltzmann constant, $\left[\frac{kg}{s^3 K^4} \right]$

ψ – stream function, $\left[\frac{m^2}{s} \right]$

Superscript

' – differentiation with respect to f

Subscripts

w – surface conditions

∞ – conditions far away from the surface

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