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NONLINEAR ACTUATOR FAULT ESTIMATION OBSERVER: AN INVERSE SYSTEM APPROACH VIA A T–S FUZZY MODEL

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Based on a Takagi–Sugeno (T–S) fuzzy model and an inverse system method, this paper deals with the problem of actuator fault estimation for a class of nonlinear dynamic systems. Two different estimation strategies are developed. Firstly, T–S fuzzy models are used to describe nonlinear dynamic systems with an actuator fault. Then, a robust sliding mode observer is designed based on a T–S fuzzy model, and an inverse system method is used to estimate the actuator fault. Next, the second fault estimation strategy is developed. Compared with some existing techniques, such as adaptive and sliding mode methods, the one presented in this paper is easier to be implemented in practice. Finally, two numerical examples are given to demonstrate the efficiency of the proposed techniques.

Keywords: actuator fault estimation, Takagi–Sugeno fuzzy models, robust sliding mode observer, inverse system method.

1. Introduction

To improve system performance efficiency, maintainability and reliability can be achieved by designing Fault-Tolerant Control (FTC), which relies on early detection of fault, using Fault Detection and Isolation (FDI) procedures, and on fault accommodation or system reconfiguration strategies, to achieve the system goal in spite of the faults. The fault-tolerant design approach can be mainly classified into two types: passive and active (Staroswiecki and Gehin, 2001). In the passive approach, the same controller is used throughout the normal case as well as the fault case such that this passive fault-tolerant controller can be easily implemented (Gu *et al.*, 2010; Pang and Tang, 2010). An active FTC system compensates for the effect of the fault by synthesizing a new control strategy based on online accommodation (Xu *et al.*, 2011a; Guo *et al.*, 2010).

System reconfiguration is the strategy with which the goals are achieved by switching off the faulty part of the system and controlling only its healthy part (Staroswiecki

and Gehin, 2001). For system reconfiguration, FDI algorithms should only detect and isolate the faults (Shumsky, 2007). The design and analysis of such algorithms have received considerable attention during the past two decades. Fruitful results can be found in several excellent survey papers (Zhang and Jiang, 2008; Isermann, 2005) and books (Chen and Patton, 1999; Vachtsevanos *et al.*, 2006; Isermann, 2006).

Most research work on FDI has concentrated on linear systems and only limited results for nonlinear systems have been reported. Early work on fault diagnosis for a class of nonlinear systems was investigated by Seliger and Frank (1991) using the unknown input observer approach (Guan and Saif, 1991), while recently some results on FDI for nonlinear systems have been obtained, e.g., by Edwards *et al.* (2000), Yan and Edwards (2007), or Jiang *et al.* (2006) based on nonlinear observers, Staroswiecki and Gehin (2001) based on parity space approaches, and Christophe *et al.* (2002) exploiting the relationship between the two methods. More recently, Persis and Isidori

(2001) investigated the problem of fault detection and isolation for nonlinear systems using a differential geometric approach. Observer design was dealt with by Edwards *et al.* (2000) to maintain a sliding motion even in the presence of faults which are detected by analysing the so-called equivalent output injection.

Fault accommodation is the strategy with which the goals are achieved by controlling the fault system (Staroswiecki and Gehin, 2001), which means that only the controller is reconfigured. If a fault is detected and isolated, the fault also needs to be estimated so that its effect can be compensated by adapting (reconfiguring) the control algorithm. Compared with FDI only, fault estimation is not an easy task. However, some results for fault diagnosis/estimation have been obtained based on adaptive observers (Ding and Frank, 1993; Jiang *et al.*, 2001; Jiang *et al.*, 2010) unknown input observers (Fu *et al.*, 2004) and using a learning approach (Polycarpou, 2001). But uncertainty exists in the model of an actual plant, so fault estimation for uncertain systems has been studied. Jiang *et al.* (2006) applied robust/sliding-mode observers to estimate the faults for an affine nonlinear system with uncertainty. Yan and Edwards (2007) used sliding-mode observers to estimate faults, but the estimation error depends on the bounds on the uncertainty.

In recent years, there has been a growing interest in the Takagi–Sugeno (T–S) fuzzy modeling technique since it is a powerful solution that bridges the gap between linear and nonlinear control systems (Zhang and Jiang, 2010; Jiang *et al.*, 2010; Gao *et al.*, 2010; Nguang *et al.*, 2007; Wu *et al.*, 2011). The important advantage of a T–S fuzzy system is its universal approximation of any smooth nonlinear function by mixing some local linear system models. This greatly facilitates the analysis and synthesis of complex nonlinear systems. Many important results on the analysis and synthesis for a T–S fuzzy system have been reported (cf. Zhou *et al.*, 2007; Nguang and Shi, 2003; Lendek *et al.*, 2010a; Gao *et al.*, 2009; Takagi and Sugeno, 1985; Boukezzoula *et al.*, 2003). Because of the advantage of T–S fuzzy systems in approximating complex nonlinear systems, the fault diagnosis observer based on a T–S fuzzy system was investigated using adaptive observers (Jiang *et al.*, 2010; Gao *et al.*, 2010; Lendek *et al.*, 2010a; Lendek *et al.*, 2010b; Zhang and Jiang, 2010) or unknown input observers (Lendek *et al.*, 2010a; Lendek *et al.*, 2010b; Chen and Saif, 2010).

Based on the aforementioned works, this paper develops a methodology for comprehensive estimation algorithm of actuator faults in nonlinear systems. The T–S fuzzy modeling technique is firstly employed to approximate the nonlinear dynamic system, and an fault model is developed. Then, two different actuator fault estimation strategies are proposed. For the first strategy, a T–S fuzzy observer has been designed based on the T-S fuzzy model and a sliding mode technique, which is used to observe

the state for the nonlinear system. The designed filter estimates a high-order derivative of the output. Next, using the inverse system of the nonlinear system, the actuator fault can be estimated. The second strategy, combining a T–S fuzzy observer and the inverse system, is used to estimate and observe the actuator fault and state. It does not have to use adaptive and sliding mode techniques. Apart from that the paper not only outlines estimation of actuator faults, but also provides the state observer. The estimated actuator fault inputs could be potentially employed for the development of a fault tolerant control system. Finally, simulation results are presented to demonstrate the effectiveness of the proposed strategies.

2. Problem formulation and preliminaries

Consider the nonlinear invertible system

$$
\begin{aligned}
\dot{x}(t) &= f(x(t), u(t), d(t), f_a(t)), \\
y(t) &= h(x(t)),
\end{aligned}
\tag{1}
$$

where $x(t) \in \mathbb{R}^r$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^n$ is the output vector, $f_a(t) \in \mathbb{R}^m$ is the actuator fault signal, $d(t) \in \mathbb{R}^p$ is the unmeasurable disturbance, and $f(\cdot)$ and $h(\cdot)$ are nonlinear functions. The continuous actuator fault is modeled by a "fault pattern" as in Zhang and Jiang (2010), Yang (2004), as well as Patton *et al.* (2001). The "fault signal" $f_a(t) < f_{aM}$ represents the unexpected disturbance in the control channel and can be constant or time varying.

Assume the system is locally observable that and the disturbance $d(t)$ can be identified. According to input equivalent disturbance (LED) of Xie *et al.* (1999), the system (1) is re-written in the following form:

$$
\begin{aligned}
\dot{x}(t) &= f(x(t), (u(t) + \mu(t))), \\
y(t) &= h(x(t)).
\end{aligned}
\tag{2}
$$

This model lumps an actuator fault, the time-varying parameter, unmeasurable disturbance and unmodeled dynamics into $\mu(t)$.

Nonlinear systems can be approximated as locally linear systems is much the same way as nonlinear functions can be approximated as piecewise linear functions. Systems (2) can be represented by T–S fuzzy models of the following forms:

IF $z_1(t)$ is M_1^i, $z_2(t)$ is M_2^i and $\cdots\cdots$ $z_q(t)$ is M_q^i, THEN

$$
\dot{x}(t) = A_i x(t) + B_i u(t) + B_i \mu(t), \tag{3}
$$

$$
y(t) = C_i x(t), \qquad i = 1, \dots, L. \tag{4}
$$

This is referred to as a Takagi–Sugeno model. The quantities $z(t) = [z_1(t)\ z_2(t) \dots z_q(t)]$ are the premise variables, and $M_1^i, \dots, M_q^i$ are fuzzy sets. A_i, B_i and C_i

are known real constant matrices with appropriate dimensions. Each of the L local models of (3) and (4) is a linear time-invariant model. A fuzzy combination of these local models results in the global model

$$\dot{x}(t) = \sum_{i=1}^{L} h_i(z(t))[A_i x(t) + B_i u(t) + B_i \mu(t)], \quad (5)$$

$$y(t) = \sum_{i=1}^{L} h_i(z(t)) C_i x(t), \quad (6)$$

where the membership grades $h_i(z(t))$ are defined as

$$h_i(z(t)) = \frac{\nu_i(z(t))}{\sum_{i=1}^{L} \nu_i(z(t))},$$

$$\nu_i(z(t)) = \prod_{j=1}^{q} M_j^i(z(t)). \quad (7)$$

Hence, $h_i(z)$ satisfies the following conditions:

$$h_i(z(t)) \in [0,1], \qquad \sum_{i=1}^{L} h_i(z(t)) = 1. \quad (8)$$

From (5) and (6) we can derive

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + B(t)\mu(t), \quad (9)$$
$$y(t) = C(t)x(t), \quad (10)$$

where $A(t)$, $B(t)$ and $C(t)$ are given as

$$A(t) = \sum_{i=1}^{L} h_i(z(t)) A_i,$$

$$B(t) = \sum_{i=1}^{L} h_i(z(t)) B_i, \quad (11)$$

$$C(t) = \sum_{i=1}^{L} h_i(z(t)) C_i.$$

In other words, the global model, which is a fuzzy combination of L local linear time invariant models, can be represented as a time-varying model. If the premise variables $z(t)$ are functions of the state or control, then the model is nonlinear. However, if the premise variables are independent of the state or control, then the model is linear.

Now we define L continuous time signals $x_i(t)$ and L continuous time signals $y_i(t)$ as

$$x_i(t) = h_i(z(t))x(t), \quad y_i(t) = h_i(z(t))y(t). \quad (12)$$

From these definitions and (8), it can be seen that

$$x(t) = \sum_{i=1}^{L} x_i(t), \quad y(t) = \sum_{i=1}^{L} y_i(t). \quad (13)$$

The dynamic behavior of $x_i(k)$ and $y_i(k)$ is presented in the following lemma.

Lemma 1. *The T–S fuzzy model (5) and (6) can be described as follows:*

$$\dot{x}_i(t) = A_i x_i(t) + h_i(z(t)) B_i u(t)$$
$$+ h_i(z(t)) B_i \mu(t), \quad (14)$$
$$y_i(t) = C_i x_i(t), \quad i = 1, \dots, L. \quad (15)$$

Proof. From (12)–(14), we obtain

$$\dot{x}(t) = \sum_{i=1}^{L} \dot{x}_i(t)$$

$$= \sum_{i=1}^{L} [A_i x_i(t) + h_i(z(t)) B_i u(t) + h_i(z(t)) B_i \mu(t)]$$

$$= \sum_{i=1}^{L} A_i h_i(z(t)) x(t) + \sum_{i=1}^{L} h_i(z(t)) B_i u(t)$$

$$+ \sum_{i=1}^{L} h_i(z(t)) B_i \mu(t). \quad (16)$$

Now we can use (11) to obtain

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + B(t)\mu(t), \quad (17)$$

where $A(t)$, $B(t)$ and $C(t)$ are given in (11). This is exactly the dynamic behavior of the global system as described in (9), which shows that (9) does indeed describe the dynamic behavior of x_i. A similar method can be used to show that the premises of the lemma also result in

$$y(t) = C(t)x(t). \quad (18)$$

■

3. Estimation algorithm via a fuzzy robust observer

3.1. Design of a T–S robust sliding mode observer.
A robust Sliding Mode Observer (RSMO) is used to solve the state estimate problem for uncertain systems. In this section we modify the RSMO for the system given by (14) and (15), if the following assumption holds.

Assumption 1. The matrices $h_i(z(t))B_i$, C_i are full rank, (A_i, C_i) are observable, uncertain vector functions are $h_i(z(t))B_i\mu(t) = \Delta F_i$ and a scalar function $\alpha(t,y)$ is such that

$$\bar{F}_i = \mu(t), \quad \|\Delta \bar{F}_i\| \le r\|u(t)\| + \alpha(t, y_i), \quad (19)$$

where r is a known positive real coefficient.

There exists an exponentially convergent RSMO for the system described by (14) and (15), which is given by

$$\dot{\hat{x}}_i = A_i \hat{x}_i + h_i(z(t)) B_i u - K_i(\hat{y}_i - y_i)$$
$$+ h_i(z(t)) B_i \nu, \quad (20)$$
$$\hat{y}_i = C_i \hat{x}_i, \quad (21)$$

where $\dot{\hat{x}}_i$ is the i-th local state estimate K_i is the observer feedback gain matrix, ν is the control input.

Let $e_i(t) = \hat{x}_i(t) - x_i(t)$. From (14), (15), (20) and (21), the dynamics of the observation error are given by

$$\frac{\mathrm{d}}{\mathrm{d}t}e_i = (A_i - K_iC_i)e_i - h_i(z(t))B_i\Delta\bar{F}_i$$
$$+ h_i(z(t))B_i\nu \qquad (22)$$

Let $A_i^0 = (A_i - K_iC_i)$. At a finite time, if state variables of the deviation system (22) are asymptotically convergent to the origin, then the states can be estimated by (14) and (15).

In order to design the RSMO which is given by (20), first, design the sliding mode surface as

$$s_i = M_ie_i = F_iC_ie_i = F_i(C_i\hat{x}_i - y_i). \qquad (23)$$

Hence, the parameter matrix F_i is designed for the sliding mode surface.

In this paper, the following sliding mode strategy ν is used for RSMO (22):

$$\nu = \begin{cases} 0, & \text{if} \quad \|s_i^T M_i h_i B_i\| = 0, \\ -\dfrac{(s_i^T M_i h_i B_i)^T}{\|s_i^T M_i h_i B_i\|}(\rho_i\|s_i\|\|M_i h_i B_i\| + \Delta_i), \\ & \text{otherwise,} \end{cases} \qquad (24)$$

where

$$\Delta_i = \eta\Big(\frac{1}{2}\Big)^\beta\|s_i\|^{2\beta}$$

and the parameters $\beta > 0$, $0 < \eta < 1$, $\rho_i = r\|u(t)\| + \alpha(t, y_i)$.

In order to facilitate the proof, let $e_i = [e_i^1, e_i^2]^T$. The error system (22) can be written as follows:

$$\dot{e}_i^1(t) = A_i^{011}e_i^1(t) + A_i^{012}e_i^2(t), \qquad (25)$$
$$\dot{e}_i^2(t) = A_i^{021}e_i^1(t) + A_i^{022}e_i^2(t)$$
$$- h_i B_{i2}\bar{F}_i + h_i B_{i2}\nu, \qquad (26)$$

where $[A_{011}, A_{012}; A_{021}, A_{022}] = A_0$, $[0, B_{i2}]^T = B_i$. The sliding mode (22) can be written as follows:

$$s_i = M_{i1}e_i^1 + M_{i2}e_i^2, \qquad (27)$$

where $[M_{i1}, M_{i2}] = M_i$. Define the following matrices:

$$A_i^M = A_i^{011} - A_i^{012}M_{i2}^{-1}M_{i1}, \qquad (28)$$
$$A_i^s = \frac{1}{2}(M_i^T M_i A_i^0 + A_i^{0T}M_i^T M_i). \qquad (29)$$

The following theorem provides the design method of the robust sliding-mode observer (20). The designed observer is robust with respect to nonlinear uncertain parts, and can asymptotically estimate the state of the system (14).

Theorem 1. *For an RSMO, using the sliding mode (23) and the control input of the observer (24) to design the parameter matrix K_i and sliding mode parameter matrix M_i, respectively, make A_i^0 and A_i^M Hurwitz matrices. Moreover, $\lambda_{\max}(A_i^s) \le 0$, where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of matrix. Then RSMO is robust to nonlinear uncertain parts ΔF_i, and the states of the system (14) can be estimated asymptotically. The convergence speed of the error system (22) is determined by the eigenvalues of A_i^M.*

Proof. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}s_i^T s_i = \frac{1}{2}e_i^T M_i^T M_i e_i. \qquad (30)$$

Its time derivative with respect to (30) is

$$\begin{aligned} \dot{V}(t) &= s_i^T \dot{s}_i = e_i^T M_i^T M_i e_i \\ &= e_i^T M_i^T M_i(A_i^0 e_i - h_i B_i\Delta\bar{F}_i + h_i B_i\nu) \\ &= \frac{1}{2}e_i^T(M_i^T M_i A_i^0 + A_i^0 M_i^T M_i)e_i \\ &\quad - s_i^T M_i h_i B_i\Delta\bar{F}_i + s_i^T M_i h_i B_i\nu \\ &\le \lambda_{\max}(A_i^s)\|e_i\|^2 - s_i^T M_i h_i B_i\Delta\bar{F}_i + s_i^T M_i h_i B_i\nu \\ &\le \|s_i\|\|M_i h_i B_i\|\|\Delta\bar{F}_i\| \\ &\quad - (\rho_i\|s_i\|\|M_i h_i B_i\| + \eta(\frac{1}{2})^\beta\|s_i\|^{2\beta}). \end{aligned} \qquad (31)$$

By (19) and (24), design the parameter $\rho_i \ge \|\Delta\bar{F}_i\|$. Hence

$$\begin{aligned} \dot{V}(t) &\le -\Big(\frac{1}{2}\Big)^\beta\eta\|s_i\|^{2\beta} \le -\eta V^\beta, \quad \forall t \ge 0, \\ V(0) &\ge 0. \end{aligned} \qquad (32)$$

The error system (22) can be obtained on sliding the surface $s_i = 0$ in a limit time T_r, $T_r = [\eta(1-\beta)]^{-1}V^{1-\beta}(0)$. When the system (34) reaches the sliding surface $s_i = 0$, the dynamic performance of the system (22) is decided by the linear sliding mode (23). Through Eqn. (27), we can obtain $s_i = M_{i1}e_i^1 + M_{i2}e_i^2 = 0$, or $e_i^2 = M_{i2}^{-1}M_{i1}e_i^1$, substituting it into (33), and obtain the reduced order system equation

$$\dot{e}_i^1(t) = (A_i^{011} - A_i^{012}M_{i2}^{-1}M_{i1})e_i^1(t) = A_i^M e_i^1(t). \qquad (33)$$

The design parameters of the sliding matrix M_i make A_i^M a Hurwitz matrix, so that the system (33) is asymptotically stable. The convergence speed of the error system (22) is determined by the eigenvalues of A_i^M. Hence, the observer is robust to nonlinear uncertain parts and can be used to estimate the state of the system (14). ∎

We combine the state observer which is described by Eqns. (20) and (21) for the local systems given in (14) and (15) to obtain a state estimator for the T–S fuzzy model given in (3) and (4). Since we know from (13) that

$x(t) = \sum_{i=1}^{L} x_i(t)$, we can combine the local state estimates in (20) to estimate the state of the T–S fuzzy model (3) and (4)

$$\hat{x}(t) = \sum_{i=1}^{L} \hat{x}_i(t). \qquad (34)$$

Theorem 2. *The state estimation given by (20) and (34) is an unbiased estimate of the true state of the T–S fuzzy model given by (3) and (4).*

Proof. We can use (13) and (34) to derive the error in the state estimate as

$$\tilde{x}(t) = x(t) - \hat{x}(t) = \sum_{i=1}^{L} x_i(t) - \sum_{i=1}^{L} \hat{x}_i(t). \qquad (35)$$

Therefore, knowing from Theorem 1 that $\lim_{t \to \infty}(x_i(t) - \hat{x}_i(t)) = 0$, we obtain

$$\lim_{t \to \infty} \tilde{x}(t) = 0. \qquad (36)$$

∎

Remark 1. An important advantage of a T–S fuzzy system is its universal approximation of any smooth nonlinear function by blending some local linear system models. This greatly facilitates the analysis and synthesis of the complex nonlinear system. Considering the advantage of the T–S fuzzy system, in this paper, a robust sliding mode observer based on a T–S fuzzy system is to be designed for nonlinear systems.

3.2. Determination of the inverse system. Consider the nonlinear system (2). It is defined as the system S_0, and it is described by the following equation:

$$S_0 : \begin{cases} \dot{x} = f(x, u + \mu), \\ g_0(y, x) = 0, \end{cases} \qquad (37)$$

where $g_0(y, x) = y - h(x)$, and $x \in M_0 \subset \mathbb{R}^r, u \in L_0 \subset \mathbb{R}^m, y \in N_0 \subset \mathbb{R}^n$. Then, a system sequence S_1, $S_2, \ldots, S_k, \ldots$ is defined in the recursive way from S_0. Generally, S_k can be described by

$$S_k : \begin{cases} \dot{x} = f(x, u + \mu), \\ g_k(y, y', \ldots, y^{(k)}, x, u + \mu) = 0, \end{cases} \qquad (38)$$

where $x \in M_k, u \in L_k, [y, y', \ldots, y^{(k)}] \in N_k$.
  Set

$$\eta_k = \max_{\Omega_k} \left\{ \text{rank}[\frac{\partial}{\partial(u+\mu)} g_k(y, y', \ldots, y^k, x, u + \mu)] \right\},$$

where $\Omega_k = [L_k, M_k, N_k]$,

$$M_{k+1} = \left\{ x \mid x \in M_k, \text{rank}\left[\frac{\partial}{\partial(u+\mu)} h_k(\cdot)\right] = \eta_k \right\},$$

$$L_{k+1} = \left\{ u + \mu \mid u + \mu \in L_k, \right.$$
$$\left. \text{rank}\left[\frac{\partial}{\partial(u+\mu)} \qquad h_k(\cdot)\right] = \eta_k \right\},$$
$$\overline{N}_{k+1} = \left\{ y, \ldots, y^{(k)} \mid (y, \ldots, y^{(k)}) \in N_k, \right.$$
$$\left. \text{rank}\left[\frac{\partial}{\partial(u+\mu)} h_k(\cdot)\right] = \eta_k \right\}. \qquad (39)$$

From (38), it follows that the disturbance/fault is given by

$$u + \mu = g_k^{-1}(x, y, y', \ldots, y^k) = u_c, \qquad (40)$$

and u_c can be obtained by the above equation. However, the actual state must be observed while the state is unmeasurable. Consequently $\hat{u}_c \to u_c$ as $\hat{x} \to x$ by the following theorem.

Theorem 3. *Let the observer*

$$\hat{u}_c = g_k^{-1}(\hat{x}, y, y', \ldots, y^{(k)})$$

be used to estimate the input vector u_c. If the state estimate of $\hat{x} \to x$, then $\hat{u}_c \to u_c$.

Proof. Let $e = \hat{x} - x$. Then the observer u_c can be described as

$$\hat{u}_c = g_k^{-1}(x + e, y, y', \ldots, y^{(k)})$$
$$= g_k^{-1}(x, y, y', \ldots, y^{(k)}, e). \qquad (41)$$

Obviously, the function $g_k^{-1}(\cdot)$ has the properties

$$g_k^{-1}(x, y, y', \ldots, y^{(k)}, e)\mid_{e=0} = g_k^{-1}(x, y, y', \ldots, y^k). \qquad (42)$$

Hence, the function $g_k^{-1}(x, y, y', \cdots, y^{(k)}, e)$ can be expanded in a Taylor polynomial at $e = 0$. Thus

$$\hat{u}_c = g_k^{-1}(x, y, y', \cdots, y^k, e)\Big|_{e=0} + \frac{\partial(g_k^{-1})}{\partial x} e + o(e^2), \qquad (43)$$

where

$$\frac{\partial(g_k^{-1})}{\partial x} \triangleq \frac{\partial(g_k^{-1}(x, y, y', \ldots, y^{(k)}))}{\partial x}.$$

If $\partial(g_k^{-1})/\partial x$ is bounded, namely,

$$\left| \frac{\partial(g_k^{-1})}{\partial x} \right| \leq \sigma,$$

it is easy to see that if $\hat{x} \to x$, then $\hat{u}_c \to u_c$. ∎

Remark 2. In the actual plant, $|\partial(g_k^{-1})/\partial x|$ is bounded, and

$$\lim_{e \to 0} \frac{\partial(g_k^{-1})/\partial x}{e} = 0.$$

3.3. Actuator fault estimation . Because of the design of an observer for uncertain systems, the actual state can be estimated by an RSMO when a fault occurs. In this section, we propose a novel fault detection and estimation strategy based on an inverse system method. From (40), it follows that the actual fault is given by

$$\mu = g_k^{-1}(x, y, y', \ldots, y^k) - u(t). \qquad (44)$$

It is easy to see that the state, output and high-order derivative of the output are the input in (44). In real-world systems, the k-th derivative of the output y is unavailable, so we introduce the following:

$$
\begin{bmatrix} \dot{y}_e \\ \ddot{y}_e \\ y_e^{(3)} \\ \vdots \\ y_e^{(a)} \end{bmatrix} =
\begin{bmatrix}
-\frac{1}{\tau} & 0 & 0 & \cdots & 0 \\
-\frac{1}{\tau^2} & -\frac{1}{\tau} & 0 & \ddots & 0 \\
-\frac{1}{\tau^3} & -\frac{1}{\tau^2} & -\frac{1}{\tau} & \ddots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-\frac{1}{\tau^{a-1}} & -\frac{1}{\tau^{a-2}} & -\frac{1}{\tau^{a-3}} & \cdots & -\frac{1}{\tau}
\end{bmatrix}
$$
$$
\times
\begin{bmatrix} y_e \\ \dot{y}_e \\ \ddot{y}_e \\ \vdots \\ y_e^{(a-1)} \end{bmatrix} +
\begin{bmatrix} \frac{1}{\tau} \\ \frac{1}{\tau^2} \\ \frac{1}{\tau^3} \\ \vdots \\ \frac{1}{\tau^{a-1}} \end{bmatrix} y. \qquad (45)
$$

Note that (45) is obtained by repeated use of the filter $\tau\dot{\omega} + \omega = \nu$ to obtain $\dot{\omega}$ as a filtered derivative of ν. The successive derivatives of the output y shown in Eqn. (45) are obtained by the repeated use of such a filter. Hence, as $\tau \to 0$, $y_e^{(k)} \to y^{(k)}$, $k = 1, \ldots, a$. Thus the following equation can be obtained for the actuator fault:

$$\hat{\mu} = g_k^{-1}(\hat{x}, y, y_e', \ldots, y_e^k) - u(t). \qquad (46)$$

Therefore, according to Theorem 2 and (46), where $\hat{x} \to x$ and $y_e^{(k)} \to y^{(k)}$, we obtain $\hat{\mu} \to \mu$.

A flow chart in Fig. 1 gives a clear idea of the overall design procedure,
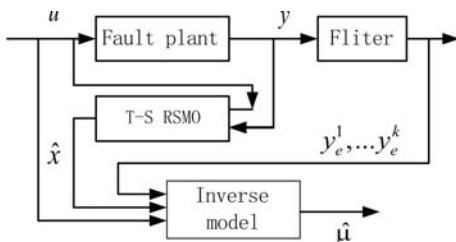

Fig. 1. Actuator fault estimate scheme via a T–S robust sliding mode observer.

Remark 3. LED is used to design the estimation algorithm for the system for an actuator fault. Compared with the results of Yan and Edwards (2007), the estimation algorithm does not need to know the bounds of the disturbance. The estimation algorithm can be applied to FTC for a nonlinear uncertain system, while the information of the estimate is used to accommodate the control command.

4. Estimation algorithm based on an integrated fuzzy observer and an inverse system

Fault estimation based on an adaptive observer has good accuracy and robustness for unknown parameters. At the same time, fault estimation based on an adaptive observer is based on a pure integral term. However, the speed and accuracy of fault estimation cannot be satisfactory. For some of the above problems, Zhang *et al.* (2009) proposed a fast adaptive fault estimation. However, this method is mainly focused on linear time invariant systems. Indeed, the established form of the actuator fault may be very time-varying, so estimation based on an adaptive observer may not be satisfactory to estimate rapid time-varying parameters. Sliding mode observer-based fault estimation has a proportional term at heart, which can quickly ensure the requirement for a time-varying fault. However, a steady-state error will exist for a constant fault.

In this section, the second observer algorithm is proposed based on a combined fuzzy observer and an inverse system without adaptive and sliding mode techniques.

4.1. Inverse system of the T–S fuzzy form. The fuzzy inverse model or the T–S inverse model is used to control nonlinear systems (e.g., Babuska, 1998; Boukezzoula *et al.*, 2003; 2007), mainly for input and output data identification. Essentially, fuzzy inverse control is a data-driven control method primarily for Single-Input Single-Output (SISO) system. For Multiple-Input Multiple-Output (MIMO) systems, in this section a form of a T–S fuzzy inverse system is given by a dynamic inverse method to design $K(t) = \sum_{i=1}^{L} K_i h_i$.

Consider the system in (17). Let the vector relative degree of the system from the output $y \in \mathbb{R}^n$ to the input $u \in \mathbb{R}^m$ be $[r_{u1} \ r_{u2}, \ldots, r_{un}]$. Hence

$$C^j A^k B = 0, \quad k = 0, 1, \ldots, r_{uj} - 2, \qquad (47)$$
$$C^j A^{r_{uj}-1} B \neq 0, \quad j = 0, 1, \ldots, n. \qquad (48)$$

It follows that

$$y_j^{r_{uj}} = (C^j A^{r_{uj}})x(t) + (C^j A^{r_{uj}-1}B)(u(t) + \mu(t)),$$
$$j = 0, 1, \ldots, n.$$
$$(49)$$

Hence

$$
\begin{bmatrix} y_1^{r_{u1}} \\ y_2^{r_{u2}} \\ \vdots \\ y_m^{r_{um}} \end{bmatrix} = \begin{bmatrix} C^1 A^{r_{u1}-1} B \\ C^2 A^{r_{u2}-1} B \\ \vdots \\ C^m A^{r_{um}-1} B \end{bmatrix} (u(t) + \hat{\mu}(t))
$$
$$
+ \begin{bmatrix} C^1 A^{r_{u1}} x \\ C^2 A^{r_{u2}} x \\ \vdots \\ C^m A^{r_{um}} x \end{bmatrix}, \tag{50}
$$

where y_j is the j-th output of y. From (50), it follows that the actuator fault is given by

$$
\mu(t) = \begin{bmatrix} C^1 A^{r_{u1}-1} B \\ C^2 A^{r_{u2}-1} B \\ \vdots \\ C^m A^{r_{um}-1} B \end{bmatrix}^{-1}
$$
$$
\times \left\{ \begin{bmatrix} y_1^{r_{u1}} \\ y_2^{r_{u2}} \\ \vdots \\ y_m^{r_{um}} \end{bmatrix} - \begin{bmatrix} C^1 A^{r_{u1}} x \\ C^2 A^{r_{u2}} x \\ \vdots \\ C^m A^{r_{um}} x \end{bmatrix} \right\} - u(t). \tag{51}
$$

4.2. Estimation algorithm via an inverse-based fuzzy observer.
In this section, an estimation algorithm via an inverse-based fuzzy observer is proposed in the following theorem. The same effect can be achieved as in the first estimation algorithm. However, no adaptive or sliding mode technologies are used in the design.

Theorem 4. *Consider the following observer and actuator fault estimator:*

$$
\dot{\hat{x}}(t) = \sum_{i=1}^{L} h_i(z)[A_i \hat{x}(t) + B_i u(t)
$$
$$
- K_i(\hat{y}(t) - y(t)) + B_i \hat{\mu}(t)], \tag{52}
$$

$$
\hat{\mu}(t) = \begin{bmatrix} C^1 A^{r_{u1}-1} B \\ C^2 A^{r_{u2}-1} B \\ \vdots \\ C^m A^{r_{um}-1} B \end{bmatrix}^{-1}
$$
$$
\times \left\{ \begin{bmatrix} y_{e1}^{r_{u1}} \\ y_{e2}^{r_{u2}} \\ \vdots \\ y_{em}^{r_{um}} \end{bmatrix} - \begin{bmatrix} C^1 A^{r_{u1}} \hat{x} \\ C^2 A^{r_{u2}} \hat{x} \\ \vdots \\ C^m A^{r_{um}} \hat{x} \end{bmatrix} \right\} - u(t). \tag{53}
$$

Suppose it is used to estimate the state vector $\hat{x}$ and the magnitude of the actuator fault $\hat{\mu}$. From the filter equation (45), let the observer gain matrix K_i be chosen such that

the system

$$
\dot{\tilde{x}} = \sum_{i=1}^{L} \sum_{j=1}^{L} h_i h_j \Big(A_i - K_i C_j - B_i
$$
$$
\times \begin{bmatrix} C_1 A^{r_{u1}-1} B \\ \vdots \\ C_m A^{r_{um}-1} B \end{bmatrix}^{-1} \begin{bmatrix} C^1 A^{r_{u1}} \\ \vdots \\ C^m A^{r_{um}} \end{bmatrix} \Big) \tilde{x} \tag{54}
$$

is stable. Then the filter constant $\tau \to 0$, $\hat{\mu} \to \mu$ and $\hat{x} \to x$ as $t \to \infty$.

Proof. Let $\tilde{x} = \hat{x} - x$ be the estimation error, while (51) and (53) are substituted into the equation for the estimation error. Then

$$
\dot{\tilde{x}} = \sum_{i=1}^{L} \sum_{j=1}^{L} h_i h_j [(A_i - K_i C_j) \tilde{x} + B_i(\hat{\mu} - \mu)]
$$
$$
= \sum_{i=1}^{L} \sum_{j=1}^{L} h_i h_j \Big(A_i - K_i C_j - B_i
$$
$$
\times \begin{bmatrix} C_1 A^{r_{u1}-1} B \\ \vdots \\ C_m A^{r_{um}-1} B \end{bmatrix}^{-1} \begin{bmatrix} C^1 A^{r_{u1}} \\ \vdots \\ C^m A^{r_{um}} \end{bmatrix} \Big) \tilde{x} \tag{55}
$$
$$
+ \begin{bmatrix} C_1 A^{r_{u1}-1} B \\ \vdots \\ C_m A^{r_{um}-1} B \end{bmatrix}^{-1} \begin{bmatrix} y_{e1}^{r_{u1}} - y_1^{r_{u1}} \\ \vdots \\ y_{em}^{r_{um}} - y_m^{r_{um}} \end{bmatrix}.
$$

Hence, as $\tau \to 0$, we get $y_e^{(r)} \to y^{(r)}$. Thus it follows that, as $t \to \infty$, $\hat{\mu} \to \mu$ and $\hat{x} \to x$. ∎

The estimation error equation (54) can be represented as

$$
\dot{\tilde{x}} = \sum_{i=1}^{L} \sum_{j=1}^{L} h_i h_j \left(\bar{A}_i - K_i C_j \right) \tilde{x}, \tag{56}
$$

where

$$
\bar{A}_i = A_i - B_i \begin{bmatrix} C_1 A^{r_{u1}-1} B \\ \vdots \\ C_m A^{r_{um}-1} B \end{bmatrix}^{-1} \begin{bmatrix} C^1 A^{r_{u1}} \\ \vdots \\ C^m A^{r_{um}} \end{bmatrix}.
$$

To design the parameters matrix K_i, make $\bar{A}_i - K_i C_i$ a Hurwitz matrix. The matrix K_i can guarantee the asymptotical stability of the estimation error (55) via a T–S fuzzy model in the case of actuator faults. At same time, the actuator fault value can be estimated.

To give a clear idea of the overall design procedure, we provide a flow chart in Fig. 2.

Remark 4. An important advantage of a T–S fuzzy system is its universal approximation of any smooth nonlinear function by "blending" some local linear system models. This greatly facilitates the analysis and synthesis of
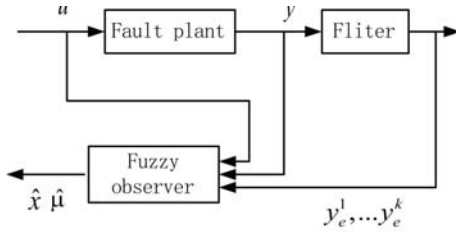
Fig. 2. Disturbance/fault estimate scheme via an integrated T–S fuzzy observer and inverse system.

complex nonlinear systems. Consequently, Theorem 3 in Section 4 is only used to design the observer time-varying feedback gain $K(t) = \sum_{i=1}^{L} h_i(z)K_i$. Actually, Eqns. (52) and (53) are replaced as follows:

$$\dot{\hat{x}}(t) = f(\hat{x}, (u + \hat{\mu})) - \sum_{i=1}^{L} h_i(z)[K_i(\hat{y} - y)],$$

$$\hat{\mu} = g_k^{-1}(\hat{x}, y, y'_e, \ldots, y_e^k) - u(t).$$

Remark 5. The estimation algorithm which is presented in this section does not require the system to be minimum phase, which not only provides information for fault detection but also enables estimation of an actuator fault. Compared with the first estimation method in Section 3, it does not need adaptive and sliding mode techniques, and parameter selection is easy, i.e., only the parameter matrix K_i have to be designed to make $\bar{A}_i - K_iC_i$ a Hurwitz matrix. Compared with the results of Yan and Edwards (2007) as well as Lendek *et al.* (2010a), this estimation method is much simpler in design and easier to realize.

Remark 6. Geometric theory is used to design the fault observer based on decoupling techniques in the works of Kabore and Wang (2001) as well as Kabore *et al.* (2000). This will make the design process complicated. Hence these methods are difficult to promote for engineering applications. Some differences between the approach of Kabore and Wang (2001) and Kabore *et al.* (2000) and ours concern two aspects:

- The fault diagnosis approaches of Kabore and Wang (2001) as well as Kabore *et al.* (2000) were presented for affine systems based on decoupling techniques, but our study is focused on general nonlinear systems and uses the T–S fuzzy approach.

- Our paper deals with actuator failures, while the above scientists studied system faults in their works. Hence, the results presented in our paper have wider application potentials, which cover more general practical systems.

5. Illustrative examples

5.1. Example 1. In this section, the first proposed estimation strategy will be demonstrated with an example,

which is a nonlinear continuous system taken from the work of Chang and Yeh (2006). It is described as follows under the actuator fault:

$$\dot{x}_1(t) = \{-21.96 + 21.96 \cdot \cos(x_1(t))\} \cdot \sin(x_1(t))$$
$$+ 6.1 - 5.55 \cos(x_1(t)) \cdot x_2(t) + \omega_1(t)$$
$$+ \frac{u(t) + \mu(t)}{1.54 - 0.54 \cdot \cos(x_1(t))},$$
$$\dot{x}_2(t) = 3x_1(t) + 0.11x_2(t) + \omega_2(t),$$
$$y(t) = 3x_1(t) + 2x_2(t).$$
$$(57)$$

We apply the technique called *local approximation in fuzzy partition space* and presented by Tanaka and Wang (2001) to construct the structure of a T–S fuzzy model. The spirit of this method is to approximate a nonlinear term by judiciously chosen linear terms. In this example, we assume that the state variable $x_1(t)$ is measurable and its range is of $x_1(t) \in [-\pi/2, \pi/2]$. Besides, the nonlinear terms $\sin(x_1(t))$ and $\cos(x_1(t))$ in nonlinear system equations (57) are approximated by the following two rules:

Rule 1: When $x_1(t) \to 0$, it is assumed that $\sin(x_1(t)) \to x_1(t)$ and $\cos(x_1(t)) \to 1$.

Rule 2: When $x_1(t) \to \pm\pi/2\,(|x_1(t)| < \pi/2)$, it is assumed that $\sin(x_1(t)) \to 2/\pi x_1(t)$ and $\cos(x_1(t)) \to \cos(88^o \times \pi/180^o)$.

Based on the above representation, one can obtain the following two-rule (i.e., $L = 2$) T–S fuzzy model.

Plant Rule 1: IF $x_1(t)$ is about 0, THEN

$$\dot{x}(t) = A_1x(t) + B_1u(t) + B_1\mu(t),$$
$$y(t) = C_1x(t).$$

Plant Rule 2: IF $x_1(t)$ is about $\pm\pi/2(|x_1(t)| < \pi/2)$, THEN

$$\dot{x}(t) = A_2x(t) + B_2u(t) + B_2\mu(t)$$
$$y(t) = C_2x(t),$$

where

$$A_1 = \begin{pmatrix} 0 & 0.55 \\ 3 & 0.11 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} -13.98 & 5.9063 \\ 3 & 0.11 \end{pmatrix},$$
$$B_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} 0.65 \\ 0 \end{pmatrix}.$$

We can choose the control input as

$$u(t) = \sum_{i=1}^{L} h_i(z)\mathcal{K}_i\hat{x}(t).$$

Let us use the feedback control gains $\mathcal{K}_i$ of Chang and Yeh (2006),

$$\mathcal{K}_1 = \begin{pmatrix} -3.8753 & -5.2832 \end{pmatrix},$$
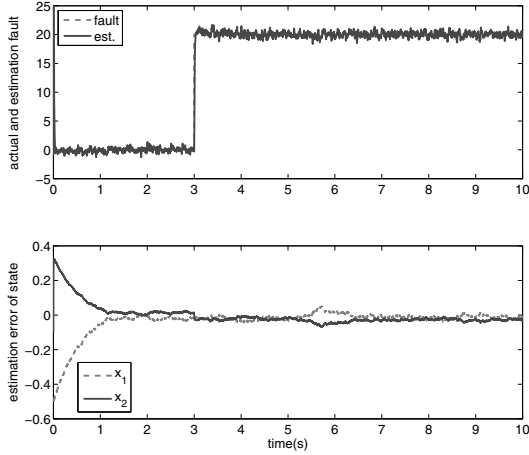$$\mathcal{K}_2 = \begin{pmatrix} 12.1769 & -11.7884 \end{pmatrix}.$$

Fig. 3. Unknown constant fault estimate and the observer state error.
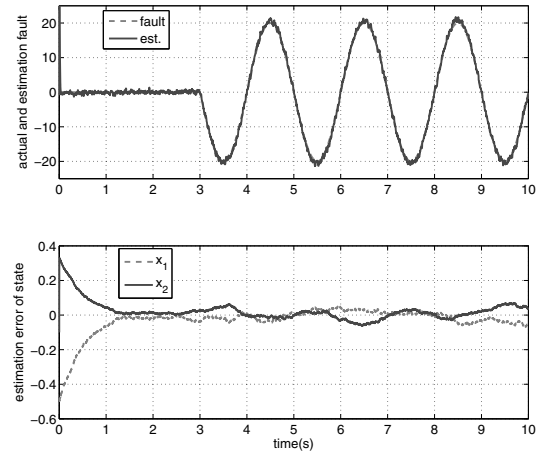

Fig. 4. Unknown time-varying fault estimate and the observer state error.

By solving the derived stability conditions of Theorem 1, the designed observer gains are

$$K_1 = \begin{pmatrix} 0.340 & 0.0854 \end{pmatrix}^T,$$
$$K_2 = \begin{pmatrix} 0.340 & 0.0854 \end{pmatrix}^T.$$

The parameter matrices F of the sliding mode surface which is defined by (35) can be obtained as $F_1 = 3.31$ and $F_2 = 4.46$. The parameters of the sliding mode surface are selected as $\beta = 10$, $\eta = 0.1$, $\rho_i = 5$ by Eqn. (36), and the filter parameters as $\tau = 0.005$. It is supposed that the unknown disturbance $\omega(t)$ is band-limited white noise with power 0.001. Using the method proposed in Section 3, we can calculate the inverse system of (57). We apply the above observed-state feedback control gains $\mathcal{K}_1$ and $\mathcal{K}_2$, the observer gains K_1 and K_2, and a constant actuator fault is assumed as

$$\mu(t) = \begin{cases} 0, & 0 \leq t < 3 \\ 20, & 3 \leq t \leq 10 \end{cases}$$

As seen in Fig. 3, the actuator fault and the states are estimated well with the first estimation approach. In simulation, the initial states are given as $[x_1(0), x_2(0)]^T = [0.1, 0.5]^T$.

Secondly, it is assumed that a time-varying actuator fault $\mu(t)$ appears as

$$\mu(t) = \begin{cases} 0, & 0 \leq t < 3, \\ 20\sin(\pi t), & 3 \leq t \leq 10. \end{cases}$$

The estimation of a time-varying actuator fault and the error of states are displayed in Fig. 4.   ◆

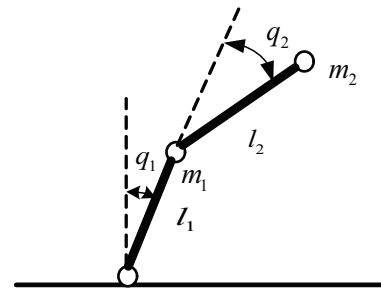**5.2. Example 2.** In this subsection, we test the proposed second estimation strategy of a two-link planar robot manipulator with an actuator fault, cf. Fig. 5. The dynamic equation of the two-link robot system is given as follows:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau, \qquad (58)$$

where

$$M(q) = \begin{bmatrix} (m_1 + m_2)l_1^2 & m_2 l_1 l_2(s_1 s_2 + c_1 c_2) \\ m_2 l_1 l_2(s_1 s_2 + c_1 c_2) & m_2 l_2^2 \end{bmatrix},$$

$$C(q, \dot{q}) = m_2 l_1 l_2(c_1 s_2 - s_1 c_2) \begin{bmatrix} 0 & \dot{q}_2 \\ -\dot{q}_1 & 0 \end{bmatrix},$$

$$G(q) = \begin{bmatrix} -(m_1 + m_2)l_1 g s_1 \\ -m_2 l_2 g s_2 & 0 \end{bmatrix},$$

$q = [q_1, q_2]$, q_1 and q_2 are generalized coordinates, $M(q)$ is the moment of inertia, $C(q, \dot{q})$ includes Coriolis and centripetal forces, and $G(q)$ is the gravitational force. Other quantities include link masses m_1, m_2, link lengths l_1, l_2, angular positions q_1, q_2, torques applied $\tau = [\tau_1, \tau_2]^T$, acceleration due to gravity $g = 9.8(m/s^2)$. For brevity, we use the notation $s_1 = \sin(q_1)$, $s_2 = \sin(q_2)$, $c_1 = \cos(q_1)$ and $c_2 = \cos(q_2)$. Let $x_1 = q_1$, $x_2 = q_2$, $x_3 = \dot{q}_1$ and $x_4 = \dot{q}_2$.

Fig. 5. Configuration of two-link robot systems.

For all simulations, the parameters are given in Table 1. The initial conditions are assumed to be $x(0) =$

Table 1. Parameters of the two link-robot.

Inertia parameter	Link 1	Link 2
m (kg)	1.0	1.0
l (m)	1.0	1.0

$[1, 1, 0, 0]^T$ and $\hat{x}(0) = [0, 0, 0, 0]^T$.

The T–S fuzzy model for the system in (58) is given by the following nine-rule fuzzy model:

Rule 1: IF $x_1(t)$ is about $-\pi/2$ and x_2 is about $-\pi/2$, THEN

$$\dot{x}(t) = A_1 x(t) + B_1 u(t) + \omega(t), \quad y(t) = C_1 x(t).$$

Rule 2: IF $x_1(t)$ is about $-\pi/2$ and x_2 is about 0, THEN

$$\dot{x}(t) = A_2 x(t) + B_2 u(t) + \omega(t), \quad y(t) = C_2 x(t).$$

Rule 3: IF $x_1(t)$ is about $-\pi/2$ and x_2 is about $\pi/2$, THEN

$$\dot{x}(t) = A_3 x(t) + B_3 u(t) + \omega(t), \quad y(t) = C_3 x(t).$$

Rule 4: IF $x_1(t)$ is about 0 and x_2 is about $-\pi/2$, THEN

$$\dot{x}(t) = A_4 x(t) + B_4 u(t) + \omega(t), \quad y(t) = C_4 x(t).$$

Rule 5: IF $x_1(t)$ is about 0 and x_2 is about 0, THEN

$$\dot{x}(t) = A_5 x(t) + B_5 u(t) + \omega(t), \quad y(t) = C_5 x(t).$$

Rule 6: IF $x_1(t)$ is about 0 and x_2 is about $\pi/2$, THEN

$$\dot{x}(t) = A_6 x(t) + B_6 u(t) + \omega(t), \quad y(t) = C_6 x(t).$$

Rule 7: IF $x_1(t)$ is about $\pi/2$ and x_2 is about $-\pi/2$, THEN

$$\dot{x}(t) = A_7 x(t) + B_7 u(t) + \omega(t), y(t) = C_7 x(t).$$

Rule 8: IF $x_1(t)$ is about $\pi/2$ and x_2 is about 0, THEN

$$\dot{x}(t) = A_8 x(t) + B_2 u(t) + \omega(t), \quad y(t) = C_8 x(t).$$

Rule 9: IF $x_1(t)$ is about $\pi/2$ and x_2 is about $\pi/2$, THEN

$$\dot{x}(t) = A_9 x(t) + B_9 u(t) + \omega(t), \quad y(t) = C_9 x(t),$$

where $x = [x_1, x_2, x_3, x_4]^T$, $u = [\tau_1, \tau_2]^T$. The parameter matrices A_i, B_i, C_i are given in Appendix. It

is supposed that the unknown disturbance $\omega(t)$ is band-limited white noise with power 0.001. It can be seen that $\text{rank}(CB) = 0$, so the proposed first estimation strategy cannot be designed in the robotic system.

The observer gains are given in Appendix by Theorem 3. We design the normal nonlinear controller based on backstepping control. The control objective is to force the system outputs q_1 and q_2 to track the sinusoidal desired trajectories $y_1^d = \sin(0.5t)$ and $y_2^d = \sin(0.5t)$. The parameters k_1 and k_2 of the controller can also be selected following Xu *et al.* (2011b).

We assumed that actuator faults are created as follows:

$$\mu_1(t) = \begin{cases} 0, & 0 \le t < 3 \\ 50\sin(2(t - 3)), & 3 \le t \le 10, \end{cases}$$

$$\mu_2(t) = \begin{cases} 0, & 0 \le t < 3 \\ 50(1 - e^{-4(t-3)}), & 3 \le t \le 10. \end{cases}$$

The system states are also estimated accurately as seen in Fig. 6. Figure 7 illustrates the result of actuator fault estimation. It can be seen that the proposed method is effective to estimate the actuator fault.
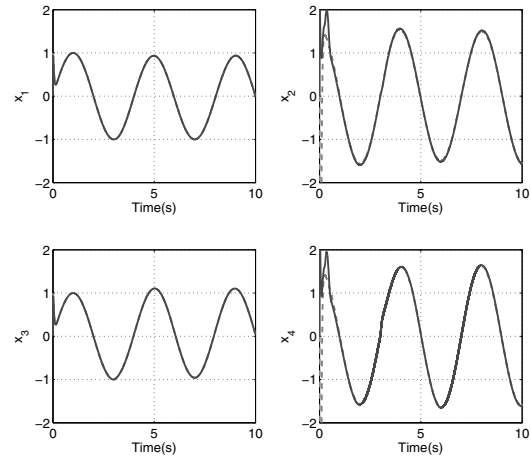

Fig. 6. Actual and estimated states (solid: actual, dotted: estimate).

The actuator fault and states can be estimated by the fault estimation observer without using adaptive and sliding mode technologies. The fault estimation observer can estimate a time-varying actuator fault quickly and the steady-state error for a constant actuator fault accurately. ♦

6. Conclusions

This paper developed a methodology for actuator fault diagnosis and quantitative estimation of actuator fault signals in nonlinear systems via a T–S fuzzy model. The T–S
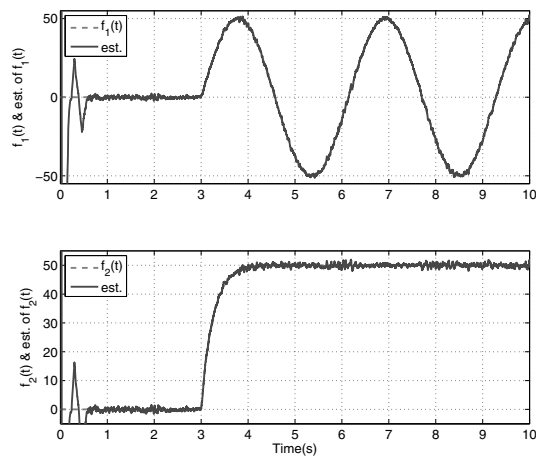
Fig. 7. Unknown fault input and its estimate.

fuzzy modeling approach is firstly employed to approximate the nonlinear dynamic system, and then the actuator fault model is presented. Two different actuator fault estimation strategies are developed. For the first strategy, a T–S fuzzy observer was designed based on a T–S fuzzy model and the sliding mode technique, which is used to observe the state of the nonlinear system. Next, using the inverse system of the nonlinear system, the actuator fault can be estimated. The second strategy, combining a T–S fuzzy observer with an inverse system, is to estimate and observe the actuator fault and state. Simulation results are used to show the effectiveness of the obtained results. Actuator fault estimation system design for T–S fuzzy systems with an approximation error and application to practical nonlinear systems are meaningful and challenging issues, which will be studied in our future work.

Acknowledgment

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Appendix

The parameter matrices A_i, B_i, C_i are given as

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 5.927 & -0.315 & -0.001 & -8.4 \times 10^{-6} \\ -6.859 & 3.155 & 0.002 & 6.2 \times 10^{-6} \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3.0482 & 0.1791 & -0.0011 & -0.0002 \\ 3.5436 & 2.5611 & 0.0313 & 1.14 \times 10^{-5} \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 6.2728 & 0.4339 & 0.0030 & -0.0001 \\ 9.1041 & -1.0574 & 0.0158 & -3.2 \times 10^{-5} \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 6.5435 & 1.2427 & 0.0017 & 0.0002 \\ -3.1873 & 5.1911 & -0.0306 & -1.8 \times 10^{-5} \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 11.1336 & -1.18145 & 0.0000 & 0.0000 \\ -9.0918 & 9.1638 & 0.0000 & 0.0000 \end{bmatrix},$$

$$A_6 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 6.1702 & 1.6870 & -0.000 & 0.0002 \\ -2.3559 & 4.5298 & 0.0314 & 1.1 \times 10^{-5} \end{bmatrix},$$

$$A_7 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 6.1206 & 0.6205 & -0.0041 & 0.0001 \\ 8.8794 & -1.0119 & -0.0193 & 4.4 \times 10^{-5} \end{bmatrix},$$

$$A_8 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3.6421 & 0.0721 & 0.0018 & 0.0002 \\ 2.4290 & 2.9832 & -0.0305 & -1.9 \times 10^{-5} \end{bmatrix},$$

$$A_9 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 6.2933 & -0.2188 & -0.0009 & -1.2 \times 10^{-5} \\ -7.4649 & 3.2693 & 0.0024 & 9.2 \times 10^{-6} \end{bmatrix}.$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \\ -1 & 2 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, \qquad B_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_5 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \\ -1 & 2 \end{bmatrix}, \qquad B_6 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_7 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, \qquad B_8 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_9 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \\ -1 & 2 \end{bmatrix}, \qquad C_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The observer gains of Example 2 are given as

$$L_1 = \begin{bmatrix} 4.5110 \times 10^2 & -4.4693 \times 10 \\ -7.5851 \times 10 & 2.5735 \times 10^2 \\ 1.2168 \times 10^3 & -1.7120 \times 10^2 \\ -2.4621 \times 10^2 & 7.7910 \times 10^2 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 4.6963 \times 10^2 & -8.1664 \times 10 \\ -8.0699 \times 10 & 2.6313 \times 10^2 \\ 1.2667 \times 10^3 & -2.7067 \times 10^2 \\ -2.6021 \times 10^2 & 7.9779 \times 10^2 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 4.0385 \times 10^2 & -1.1052 \times 10^2 \\ -6.4433 \times 10 & 2.7854 \times 10^2 \\ 1.0888 \times 10^3 & -3.5054 \times 10^2 \\ -2.0810 \times 10^2 & 8.4484 \times 10^2 \end{bmatrix},$$

$$L_4 = \begin{bmatrix} 4.3523 \times 10^2 & -5.5508 \times 10 \\ -8.8939 \times 10 & 2.5128 \times 10^2 \\ 1.1773 \times 10^3 & -1.9855 \times 10^2 \\ -2.8464 \times 10^2 & 7.6144 \times 10^2 \end{bmatrix},$$

$$L_5 = \begin{bmatrix} 3.7229 \times 10^2 & -4.1800 \times 10 \\ -5.7890 \times 10 & 2.4382 \times 10^2 \\ 1.0041 \times 10^3 & -1.6094 \times 10^2 \\ -1.8940 \times 10^2 & 7.3892 \times 10^2 \end{bmatrix},$$

$$L_6 = \begin{bmatrix} 4.3828 \times 10^2 & -5.9589 \times 10 \\ -9.2605 \times 10 & 2.5321 \times 10^2 \\ 1.1861 \times 10^3 & -2.0975 \times 10^2 \\ -2.9571 \times 10^2 & 7.6731 \times 10^2 \end{bmatrix},$$

$$L_7 = \begin{bmatrix} 4.0793 \times 10^2 & -1.0870 \times 10^2 \\ -6.6944 \times 10 & 2.7792 \times 10^2 \\ 1.1001 \times 10^3 & -3.4554 \times 10^2 \\ -2.1585 \times 10^2 & 8.4292 \times 10^2 \end{bmatrix},$$

$$L_8 = \begin{bmatrix} 4.6273 \times 10^2 & -7.6872 \times 10 \\ -7.9199 \times 10 & 2.6137 \times 10^2 \\ 1.2481 \times 10^3 & -2.5757 \times 10^2 \\ -2.5558 \times 10^2 & 7.9235 \times 10^2 \end{bmatrix},$$

$$L_9 = \begin{bmatrix} 4.4698 \times 10^2 & -4.2650 \times 10 \\ -7.6254 \times 10 & 2.5663 \times 10^2 \\ 1.2059 \times 10^3 & -1.6560 \times 10^2 \\ -2.4734 \times 10^2 & 7.7684 \times 10^2 \end{bmatrix}.$$