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# HEAT TRANSFER IN ROAD PAVEMENT STRUCTURE AND IDENTIFICATION OF ITS LAYERS THERMAL PARAMETERS

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A heat transfer problem for the road pavement system "layered plate – embankment – soil base" is formulated. There are suggested mathematical models and their realization with the use of finite element analysis and design software program by solving the layered plate on the ground foundation. Transient thermal state for this road structure is defined by discretization of the system described by different material properties. The theoretical results are compared with the experiment data for the real road pavement of Lubuski province road No 297 on the route Zagan-Kozuchow in Poland. Finally, two inverse problems are formulated, to identify respectively the boundary conditions and heat material parameters of road structure by smooth (least squares) and nonsmooth (minimax) optimization. It was shown that the criterion of least squares does not give trust results and only nonsmooth criterion of minimax lead to a trust result of the identification.

Keywords: pavement system, heat transfer problem, parameters and boundary conditions identification, least squares and minimax criteria

## 1. INTRODUCTION

The problems of heat transfer for the road structure are of great practical importance. They define the border of freezing of the soils which serves as a basis for the execution of a rain water drainage system [18]. The analysis of the problem of heat transfer for the road structure on the soil embankment and the soil base will qualify the pavement as an isolation layer.

The problems of heat transfer were early discussed as influences of the surroundings on the upper surface of the soils and pavements [16]. This paper examines the problem of heat transfer for the road pavement (pavement and base) as a layered isotropic plate on the soil embankment for the soil base. Direct and inverse mathematical models of heat transfer are suggested and the unsteady heat transfer is realized with the use of finite element analysis and design software COSMOS/M. The numerical results are compared with the experiment data for the real exploitation conditions on the Lubuski province road No 297 in Poland on the route Zagan-Kozuchow [1-3].

## 2. MATHEMATICAL MODEL AND NUMERICAL METHOD FOR THE HEAT TRANSFER PROBLEM

### 2.1. Model of heat transfer for the road pavement

A two dimensional direct problem of heat transfer is examined for the road pavement as a layered isotropic plate on the soil embankment and the soil base. The system of axial coordinates 0xy is used. The governing differential Fourier – Kirchhoff equation for heat transfer for the nonhomogeneous body is as follows:

$$\rho \frac{\partial (c_p T)}{\partial t} = \frac{\partial}{\partial x}\left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left( \lambda \frac{\partial T}{\partial y} \right) + q_{\backslash v}, \quad t > 0, \quad \mathbf{x} \in \Omega, \tag{2.1}$$

where $T(\cdot) := T(\boldsymbol{x}, t)$ is a temperature, $K$; $\boldsymbol{x} = (x, y)$ is a vector of the body point, $\boldsymbol{x} \in \Omega$; $\Omega$ is a region of two dimensional space for variables $\boldsymbol{x} \in \Omega \subseteq \boldsymbol{R}^2$; $t$ is time, s; ρ is density of the material, kg/m3; $c_p$ is specific heat for constant pressure, kJ/(kg·K); $\lambda$ is thermal conductivity, W/(m·K); $Q(\boldsymbol{x}, t)$ is the volumetric heat generation rate, kJ/(s·m$^3$); $\lambda$, $c_p$, and ρ are as the functions of coordinate $\boldsymbol{x}$, further constant for every layer of the body (Fig. 1).

In the general case, taking into account heat transfer and freezing for two-dimensional, nonhomogeneous ground medium, differential equation (2.1) we propose to write as follows (generalized equation, just as was done for 1D problem in [12]):

$$\rho \frac{\partial}{\partial t}\{d(i)T\} = \frac{\partial}{\partial x}\left( \lambda(i) \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left( \lambda(i) \frac{\partial T}{\partial y} \right) - K\rho W_0 \frac{\partial i(T)}{\partial t}, \quad t > 0, \quad \mathbf{x} \in \Omega \tag{2.2}$$

where $d(i)$ is also a "heat capacity" as a function of ice quantity $i$; $d(i)$ is analogy with specific heat $c_p$, $i$ is ice quantity for the fixed temperature $T$ [12]; $K$ is a heat of ice melting; $W_0$ is a soil moisture.

Further, the temperature of the body satisfies the initial as well as boundary conditions. The first condition

$$T(\mathbf{x},0) = \Phi_0(\mathbf{x}),\ \ \mathbf{x} \in \overline{\Omega}, \tag{2.3}$$

means that temperature $T(\cdot) = \Phi_0(\mathbf{x})$ for every body point $\mathbf{x}$ of region $\overline{\Omega}$ at the initial time $t = 0$ is known; the function $\Phi_0(\mathbf{x})$ is continuous for all points $\mathbf{x} \in \overline{\Omega}$, where $\overline{\Omega} = \Omega \cup \Gamma$; $\Gamma$ is the surface, $\Gamma := bd\ \Omega$; $bd$ is a symbol of the body $\Omega$ board. We assume conditions of type I as the boundary conditions on the external surface of the body (plate of pavement and soil massive) $\Gamma$ for every time $t$, which assign temperature distribution,

$$T(\mathbf{x},t) = \Phi(\mathbf{x},t),\ \ \mathbf{x} \in \Gamma,\quad t > 0; \tag{2.4}$$

$\Phi(\mathbf{x}, t)$ is a given a continuous function from $(\mathbf{x}, t)$ for all points of region $\Gamma$.

In this paper the border condition (2.3) can be analysed in different cases (Fig 1). For the layers of road pavement on the soil we consider it as a heterogeneous, unrestricted heat (semi – conductor) for a limited contour. Alternatively, in the first case of border, the temperature $T := \Phi_\infty$ of contour $\Gamma_-$ (on the infinity depth $y \to \infty$ for all points $x \in \Gamma_-$) is limited,

$$T(\mathbf{x},\infty,t) = \Phi_\infty < +\infty,\ \ \mathbf{x} \in \Gamma_-, y \to \infty,\quad t > 0. \tag{2.5}$$

In the second case on a definite depth of the contour $\Gamma_-$ border (on the depth $y = H$ for all points $\mathbf{x} \in \Gamma_-$) the temperature on $\Gamma_-$ should be definite as follows:

$$T(\mathbf{x},H,t) = \Phi_H < +\infty,\ \ \mathbf{x} \in \Gamma_-, y = H,\quad t > 0. \tag{2.6}$$

Furthermore, on the boundary between the road pavement and the surroundings $\Gamma_p$ (for the convection problem), we have to use boundary conditions of type III:

$$h\left[\Phi_s(\mathbf{x},t) - T_p(\mathbf{x},t)\right] = q,\ \ \mathbf{x} \in \Gamma_p,\quad t > 0, \tag{2.7}$$

where $h$ is a heat transfer (convection) coefficient, W/(m$^2$·K); $T_p$ is an ambient temperature; $q$ is a heat flux, W/m$^2$.

Finally, let us write the boundary conditions of mutual coupling for the pavement plate and the layers of the soil base (boundary conditions of type IV). We assume that on the internal surfaces $\Gamma_{i,j}$, between the bodies $i$ and $j$, there is an ideal heat contact. Then, for the temperature $T_i$ and $T_j$ as well as the bodies $i, j$ the following equations hold true

$$T_i(\mathbf{x},t) = T_j(\mathbf{x},t),\ \ \mathbf{x} \in \Gamma_{i,j},\quad t > 0; \tag{2.8}$$

$$\lambda_i\left[\partial T_i(x,t)/\partial_n\right] = \lambda_j\left[\partial T_j(x,t)/\partial_n\right],\ \ \mathbf{x} \in \Gamma_{i,j},\quad t > 0; \tag{2.9}$$

in a series for all the body numbers $(i, j) \in [1:5]$.

As a result, we have a two dimensional mixed boundary-value transient problem of the heat transfer (2.1), (2.3)-(2.9) lub (2.2)-(2.9) with boundary conditions of type I, II and IV.

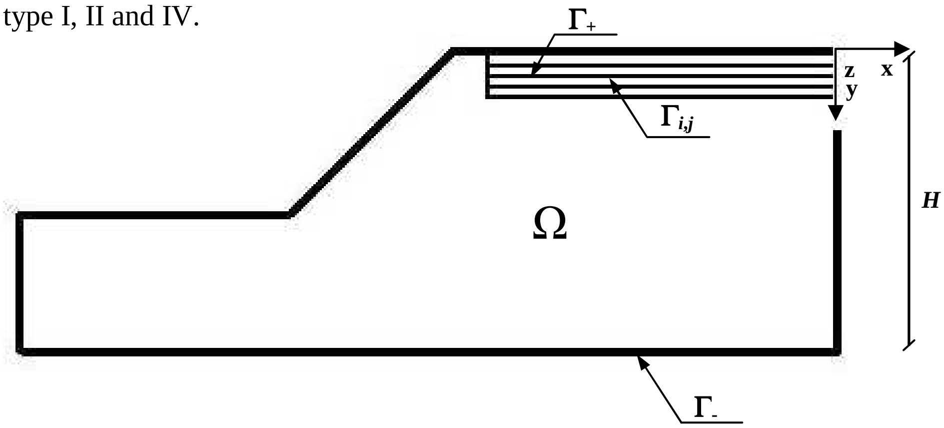


Fig 1. The model for the heat transfer problem

The stationary problem will be easier, due to $\frac{\partial T}{\partial t} = 0$ and $T(\cdot) := T(\boldsymbol{x})$ in the Eqn (2.1) lub (2.2), without initial condition (2.3); it includes Eqn (2.1) lub (2.2) and conditions (2.4)-(2.9).

### 2.2. Finite element method

Numerical solution of the given boundary-value problem will be performed by the finite element method. The equivalent variational formulation will be used for this aim. In this case the conditions (2.8)–(2.9) are fulfilled automatically for a heterogeneous (layered) body.

As a result of the diskretization of the transient heat transfer problem, the relations can be written in the form of the following matrix differential equation [28]

$$[K]\{T\}+[C]\frac{\partial}{\partial t}\{T\}=\{f\}, \tag{2.10}$$

where the global conductivity and specific heat matrices *K* and *C*, and the load vector *f* are obtained as a result of aggregation for separate finite elements; $\{T\}$ is vector of the temperature in the joints of finite elements.

The numerical solution of the problem (2.10) was realized by means of the COSMOS/M System [9]. Various boundary conditions were taken into account and particularly the arbitrary form of the initial conditions and the actual law of the temperature versus time variation of the air or of the external surface of the road cover.

It is also worth mentioning that the finite difference method is admissible for this problem [20].

## 3. DESCRIPTION OF THE ROAD STRUCTURE AND THE METHOD OF INVESTIGATION

Elements of the given embankment-type road structure are shown in Fig 2. The measurements were conducted on the Lubuski province road No 297 in Poland on the route Zagan-Kozuchow. Two experimental road sectors of 3 m width and 4 m length each were built up from the catalogue of standard structures of flexible and semi rigid pavements. The road structures were built up as embankment-type, formed from non-swelling soils; group G1 of soil bearing capacity – middle dimension of sandy particles, with the following cross-sections:

**Section 1**

1. SMA mixture-wearing course 0-12,8 grading – 5 cm thick.
2. Asphalt concrete binder course 0-20 grading – 6 cm thick.
3. Asphalt concrete base course 0-20 grading – 7 cm thick.
4. Crusher-run base course – 20 cm thick.
5. Natural base, sand with middle particles.

**Section 2**

1. SMA mixture-wearing course 0-12,8 grading – 5 cm thick.
2. Asphalt concrete binder course 0-20 grading – 7 cm thick.
3. Asphalt concrete base course 0-20 grading – 7 cm thick.
4. Cement stabilized granular aggregate base course 20 cm thick.
5. Natural base, sand with middle particles.

Characteristics of the materials are given in Table 1. Note that in the literature the data for the soil materials is visibly different; here their average values are taken.

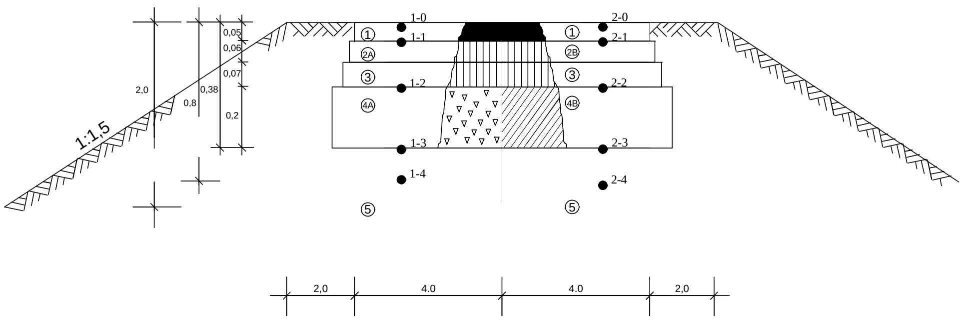


Fig 2. Elements of road structure

Table 1. Characteristics of the materials

| Materials | Thermal conductivity $\lambda$, W/(m·K) | Specific heat $c_p$, kJ/(kg·K) | Density $\rho$, kg/m$^3$ |
|---|---|---|---|
| SMA/asphalt concrete | 1.4 | 0.47 | 2530 |
| Crushed stone, stabilized mechanically | 0.4 | 0.35 | 1940 |
| Soil, stabilized by cement | 0.99 | 0.25 | 1880 |
| Sand with middle particles | 0.58 | 0.23 | 1600 |

The temperature sensors were set into the pavement. They were placed on the axis of a moving vehicle (between the wheels) for both sectors under the following pavement surface:

- 1 cm under the surface, in the wearing course ((1-0), (2-0), Fig. 2);
- 5 cm under the wearing course ((1-1), (2-1));
- 18 cm under the bituminous base course ((1-2), (2-2));
- 38/39 cm under the sub base ((1-3), (2-3));
- 80 cm, in the soil foundation ((1-4), (2-4)) - border of freezing for the Lubuski province.

Data which was measured in even temporary spaces was used to create a graph of schedule temperatures or unestablished conductivity of warmth.

## 4. NUMERICAL AND EXPERIMENTAL RESULTS

The numerical solution of the temperature distribution was obtained by means of the finite element system COSMOS/M [9] for the real material parameters of the road construction. The finite element TRIANG was used to discretize the domain of the road construction. The adopted mesh was shown in Fig. 3. The total number of elements was 9189, and the number of nodes, 4820, was the number of unknown nodal temperatures. The 10 hours time interval with the initial temperature conditions, which followed from the measurement data, was considered, and the time interval was constant, Dt=0,3 h. It means that the whole analysis was performed at 35 equal time intervals.

The numerical analysis was performed on the basis of temperature registrations. Measurements were executed for winter conditions as well as for summer conditions. They were done between 8 pm (20:00) on 01.02.2003 and 1 pm (13:00) on 02.02.2003 (for negative temperatures) as well as between 0.00 am and 11:30 pm on 17.05.2007 (for positive temperatures) for section 1 of the road No 297 in Poland.

The temperature distribution registered at 0:00 am on 17.05.2007 and at 11:30 pm on 17.05.2007 was treated as the initial conditions $\Phi(\mathbf{x})$ for the numerical analysis. The temperature $T_{mes}$ registered on the external layer surface ($y$ = 0 cm) of the road during the analysed 24 hours period was the main data $\Phi\,(\cdot)$ for the transient heat transfer problem. We assumed additionally that on the depth $y$ = - 80 cm the temperature is constant all the time and is equal to $\Phi(\cdot)=16{,}1^{o}$C for positive temperatures. As far as the material parameters are concerned, the data shown in Table 1 was adopted.

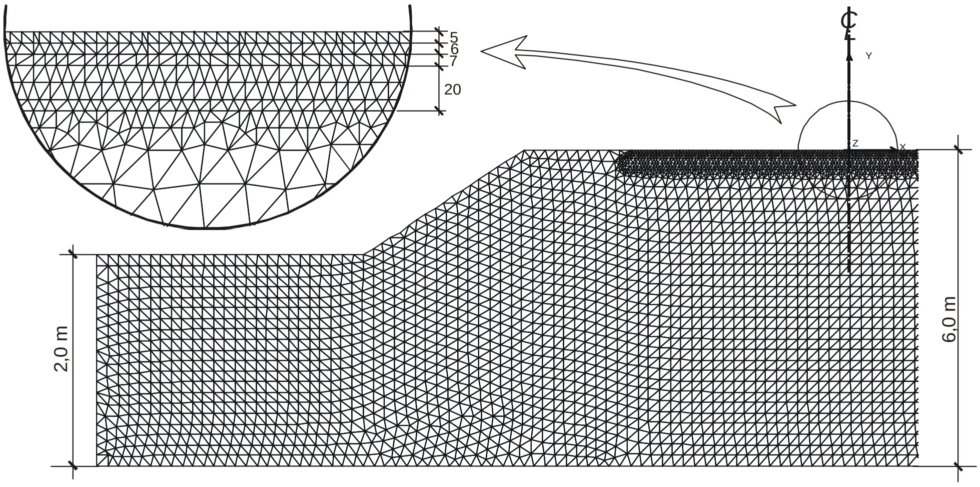


Fig 3. The FEM mesh used in calculations

The temperature distribution at the 24-th hour of the analysis (17.05.2007) was shown in Fig. 4.

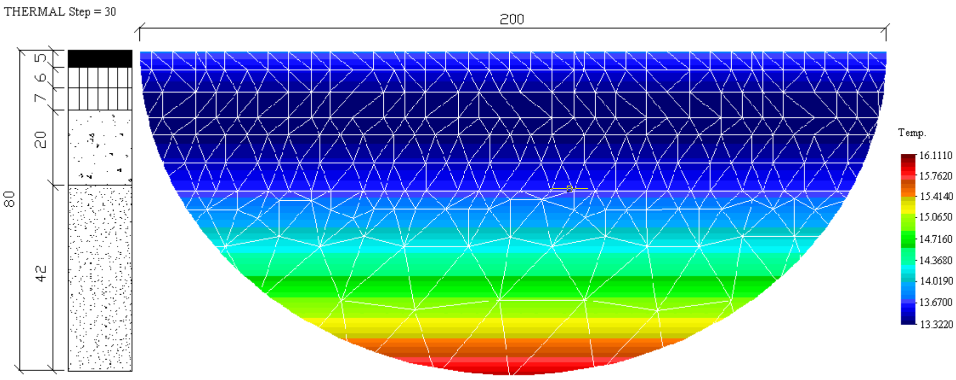


Fig 4. Temperature distribution at the 24-th hour of the analysis

## 5. INVERSE OPTIMIZATION PROBLEMS

The direct problems (2.1), (2.3)-(2.9) lub (2.2)-(2.9) are well-posed boundary-value problems. However, to improve the convergence of theoretical and experimental results, two inverse (ill-posed) problems are formulated, to identify respectively the boundary conditions and and heat material parameters of road structure. Note, that the problem of identification of mechanical systems is currently studied in extensive literature [5-7, 13-15, 17, 19, 22-26], including those devoted to problems of heat and mass transfer [4, 8, 9, 20, 21, 27].

### 5.1. Identification of boundary conditions by least squares method

A problem of transient heat transfer for the road pavement is analysed as an exact inverse formulation for the reconstruction of boundary conditions on the side and bottom surfaces $\Gamma_2$ of the analysed zone (Fig. 5).

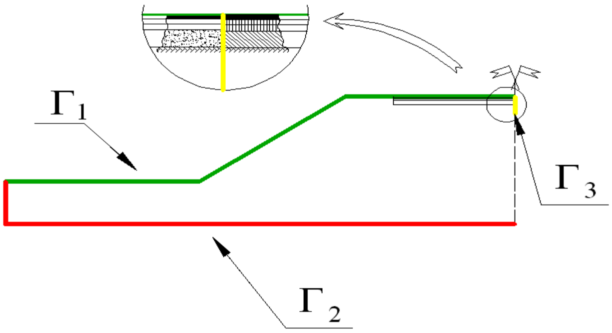


Fig 5. Measuring zone for the inverse optimization problem

At the same time, the boundary conditions on the upper surface $\Gamma_1$ are known and on the surface $\Gamma_3$ are measured. Thus, we have an inverse boundary problem, which can be presented as an optimization problem.

The governing differential equation (2.1) for heat transfer for the body is as follows (see Part 1)

$$\rho\frac{\partial(c_pT)}{\partial t}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+q_{\backslash v},\quad t>0,\quad \mathbf{x}\in\Omega. \tag{5.1}$$

Further, the temperature of the body satisfies the initial conditions (2.2) and the conditions (2.3) known on boundary $\Gamma_1$ have the following form:

$$T(\mathbf{x},0)=\Phi_0(\mathbf{x}),\quad \mathbf{x}\in\Omega, \tag{5.2}$$

$$T(\mathbf{x},t) = \Phi(\mathbf{x},t), \quad \mathbf{x} \in \Gamma_1, \quad t > 0; \tag{5.3}$$

The boundary conditions on $\Gamma_2$

$$T(\mathbf{x},t) := v(\mathbf{x},t), \quad (\mathbf{x},t) \in \Gamma_2, \tag{5.4}$$

are unknown and the function $v(\cdot)$ has to be found from the measuring of the temperature on $\Gamma_3$,

$$T(\mathbf{x},t) := \Psi(\mathbf{x},t), \quad (\mathbf{x},t) \in \Gamma_3. \tag{5.5}$$

Then inverse optimization problem can be formulated in such way:
find

$$J(w) = \min_{v} J(v), \tag{5.6}$$

where

$$J(v) = \int_{\Gamma_3} \left(T(v;\mathbf{x},t) - \Psi(\mathbf{x},t)\right)^2 ds, \tag{5.7}$$

subject to conditions (5.1) - (5.5).

The extremum problem (5.1) - (5.7) is a problem of optimal control. It contains the following unknowns: the field $T(\boldsymbol{x}, t)$, $\boldsymbol{x} \in \Omega$, of temperature in time $t$ as state variables, and the same one of the temperature field $v(\boldsymbol{x}, t)$ on $\Gamma_2$ as control variables. The optimal solution $w(\boldsymbol{x}, t)$ is the function $v$ of the temperature field on $\Gamma_2$ in time $t$.

This problem is ill-posed. Its solution is based on the technique of regularization of A.N. Tikhonov [20, 23, 24]. Assume the next smoothing form of functional (5.7)

$$J(v) = \int_{\Gamma_3} \left(\left(T(v;\mathbf{x},t) - \Psi(\mathbf{x},t)\right)^2 + \alpha v^2\right) ds, \tag{5.8}$$

where the Tikhonov factor $\alpha>0$ in the second term is a parameter of regularization, which depends on the difference in the first term of Eqn (5.8).

For the new well-posed problem (5.1) – (5.6), (5.8) we find the approximate solution using the method of iteration. The number of iteration in this method will be taken here as a parameter of regularization.

Analogously for the steady heat transfer, the problem (5.1) - (5.6.), (5.8) will be a simpler problem of optimal control, namely

$$J(v) \to \min_{v}, \tag{5.9}$$

$$J(v) = \int_{\Gamma_3} \left( \left( T(v;\mathbf{x}) - \Psi(\mathbf{x}) \right)^2 + \alpha v^2 \right) ds, \tag{5.10}$$

$$\frac{\partial}{\partial x}\left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left( \lambda \frac{\partial T}{\partial y} \right) = 0, \quad \boldsymbol{x} \in \Omega, \tag{5.11}$$

$$T(\mathbf{x}) = \Phi(\mathbf{x}), \quad \mathbf{x} \in \Gamma_1, \tag{5.12}$$

$$T(\mathbf{x}) := v(\mathbf{x}), \quad \mathbf{x} \in \Gamma_2, \tag{5.13}$$

$$T(\mathbf{x}) := \Psi(\mathbf{x}), \quad \mathbf{x} \in \Gamma_3. \tag{5.14}$$

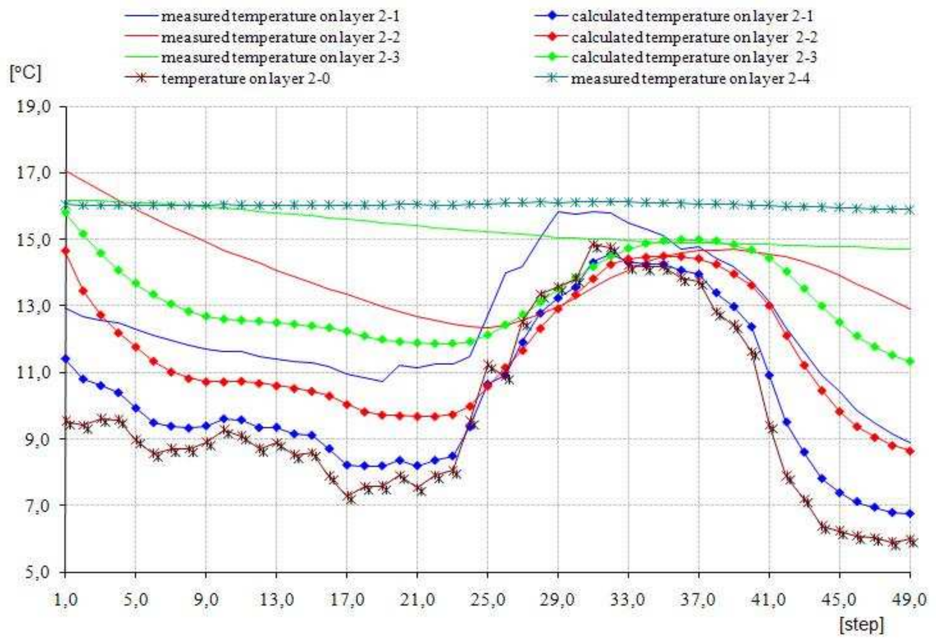


Fig 6. The temperature distribution registered on 17.05.2007 for the road pavement construction with crusher-run base course

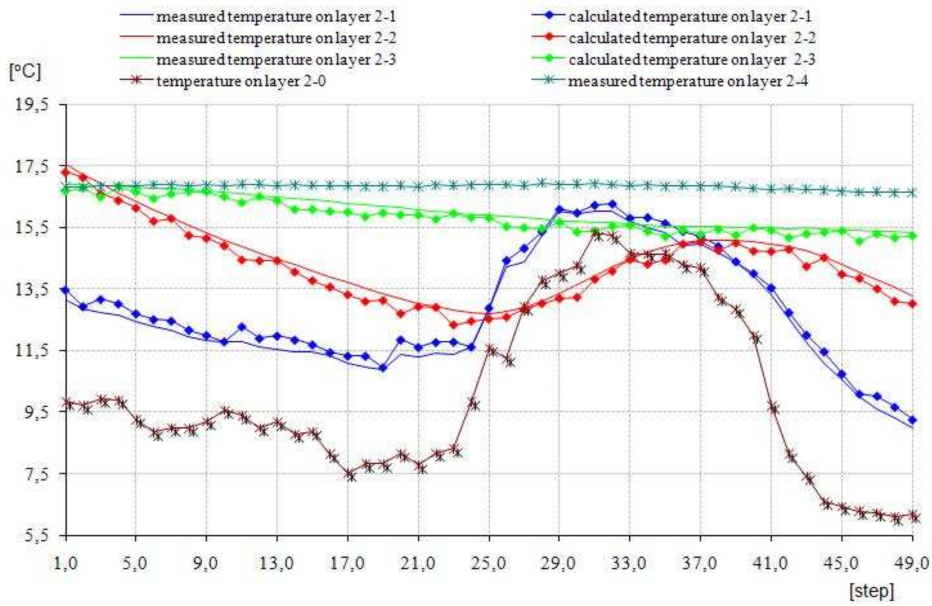


Fig 7. The temperature distribution registered on 17.05.2007 for the road pavement construction with crusher-run base course

The sensitivity of this problem is rather significant, and the results of direct and inverse solutions are different in the range of 10 %.

Another possible method might also be the finite time method of Beck [27].

### 5.2. Identification of heat parameters by least squares method

Identification of material parameters was carried out using the overdetermined inverse problem [21], where the number of parameters sought is not equal to the number of measurement points. To this end, results were analyzed temperature measurement read from the road surface for the different criteria for the objective function and the regularization parameters. The identification was performed by two criteria, namely the criterion of least squares and the criterion of minimax. The first, widely distributed method was early used and detally described in [3].

### 5.3. Identification of heat design parameters by minimax method

Another, less common method of solving inverse problems is to use a minimax method [10, 22]. This method consists of minimizing the maximum deviations (ie, differences in theoretical and experimental results). This method provides better results than the method of least squares, since it smooths out the resulting function, which becomes more adapted to the actual conditions of heat exchange.

According to the minimax criterion when the number of unknown parameters $n$ equals the number of measurement points $m$, $n = m$, the solution of the inverse problem will be the solution for $n$ equations of the form:

$$\sum_{j=1}^{m} \int_{0}^{t_0} \left| T_c(\mathbf{v}, t) - T_e(t) \right|_{ij} dt = 0, \quad i \in 1:n, \tag{5.15}$$

where:
$t_0$ is measurement time,
$T_e := (T_{ei}, i \in 1{:}m)$ is a vector of measured parameters,
$T_c := (T_{ci}, i \in 1{:}m)$ is a vector of parameters calculated in the formula (5.15),
$m$ is number of measured parameters,
$\boldsymbol{v} := (v_i, \mathrm{i} \in 1{:}n)$ is a vector of unknown parameters,
$n$ is number of unknown parameters.

Otherwise in a particular case when the number of unknown parameters $n$ is less than the number of measurement points $m$, $n \neq m$. Then the solution of the inverse problem will be the solution for the following functions:

$$\max_{t \in [0:t_0]} \left( \frac{2}{T_{e,\min} + T_{e,\max}} \left| T_c(\mathbf{v}, t) - T_e(t) \right| \right)_i = 0, \quad i \in 1:m, \tag{5.16}$$

where:
$T_{e,min,j}$, $T_{e,max,j}$ are the limits for the measured temperature $T_{e,j}$ in the $j$-th point.

Therefore, summary function φ of the normalized errors (deviations) as nonsmooth minimax criterion should be minimized:

$$\varphi(\mathbf{w}) = \min_{\mathbf{v}} \varphi(\mathbf{v}), \tag{5.17}$$

where:

$$\varphi(\mathbf{v}) = \max_{j \in [1:m]} \max_{t \in [0:t_0]} \left( \frac{2}{T_{e,\min} + T_{e,\max}} \left| T_c(\mathbf{v}, t) - T_e(t) \right| \right)_j \tag{5.18}$$

The solution of this ill-posed problem (5.17)-(5.18) is based on regularization technique of A.N. Tikhonov [19, 22]. The formula (5.18) has to replace by following smoothing one,

$$\varphi_\alpha(\mathbf{v}) = \max_{j\in[1:m]} \max_{t\in[0:t_0]} \left( \frac{2}{T_{e,\min} + T_{e,\max}} \left| T_c(\mathbf{v},t) - T_e(t) \right| \right)_j + \qquad (5.19)$$
$$+ \alpha \|\mathbf{v}\|^2,$$

where a Tikhonov factor α>0 in the second term is also a parameter of regularization,

$$\|\mathbf{v}\| = \sqrt{\sum_{i\in 1:n} (v_i^2)} \qquad (5.20)$$

For example, the problem of determining of thermal conductivities for the two first layers of road structures, namely for asphalt concrete and crushed stone, stabilized mechanically (see Part 3), was considered.

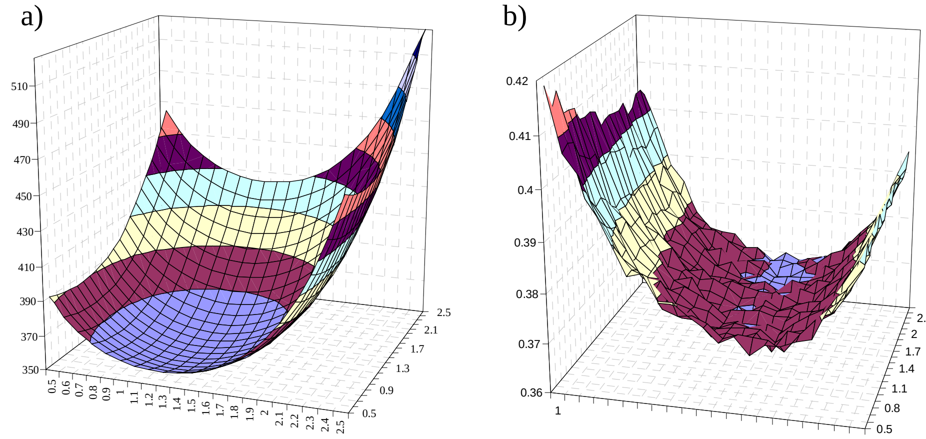


Fig 8. The criterion surfaces $\varphi(\mathbf{v})$ for the vector $\boldsymbol{v} = (\lambda_1, \lambda_2)$ of coefficients of thermal conductivity: a) the criterion of least squares, b) the criterion of minimax

As a result of numerical analysis for the identification based on least-squares method we can find the optimal values of thermal conductivity only for two from three test materials forming the layers of road construction (Fig 8.a). Therefore, this method was considered not very accurate and replaced it with identification based on the minimax criterion. Using the nonsmooth minimax optimization we can determine the optimal values of all test materials (Fig 8.b).

Finally we obtained the next optimal vector $\boldsymbol{v}^* = (\lambda_1^*, \lambda_2^*) := (1{,}70;\ 1{,}90)$, accordingly for layers of asphalt concrete and crushed stone, stabilized mechanically. This result may be considered as a valid one, since thermal conductivity $\lambda_1$* of asphalt concrete is close to the value of $\lambda_1$ = 1.84, obtained experimentally using a camera ISOMET 2104 [2]; the difference is 5.0%.

## 6. CONCLUSIONS

In a general case, it is advisable to realize the analysis of the 2D or 3D-problems for the heat transient regime of a real layered road structure, in the form of embankment or cavity with the use of the methods of identification for its mathematical models and a suitable computer software.

It is possible to find boundary conditions or physical and mechanical properties for inhomogeneous layered bodies using optimal control or overdetermined problem of multi-parameter identification respectivly. In these cases the criterion of least squares does not give trust results and only nonsmooth minimax criterion leads to a trust result of the identification of the boundary fields or material parameters sought.

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## WYMIANA CIEPŁA W KONSTRUKCJI NAWIERZCHNI DROGOWEJ I IDENTYFIKACJA PARAMETRÓW MATERIAŁOWYCH TWORZĄCYCH JEJ WARSTWY

S t r e s z c z e n i e

W pracy przedstawiono identyfikację parametów materiałowych oraz warunków granicznych ośrodka niejednorodnego, jaki stanowi konstrukcja nawierzchni drogowej. Problem ten rozwiązano poprzez analizę wymiany ciepła dla układu „płyta wielowarstwowa – nasyp – podłoże gruntowe”. Na podstawie wieloletnich badań doświadczalnych rozkładu temperatury sformułowane zostały proste i odwrotne zagadnienia nieustalonego przepływu ciepła, zrealizowane przy pomocy metody elementów skończonych. Dla identyfikacji stosowane były metody gładkiej i niegładkiej optymalizacji z kryterium najmniejszych kwadratów oraz kryterium minimaxu. Druga z metod okazała się skuteczną, ponieważ w odróżnieniu od pierwszej możliwe było wyznaczenie optymalnych wartości wszystkich badanych materiałów tworzących warstwy konstrukcji drogowej. Podane przykłady analizy numerycznej porównane zostały z wynikami badań doświadczalnych uzyskanych z poligonów zlokalizowanych na drodze wojewódzkiej nr 296 w miejscowości Kożuchów w Polsce.