

## STATIONARY HEAT CONDUCTION IN LAMINATES WITH NON-UNIFORM DISTRIBUTION OF LAMINAS

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In this paper it is analysed a stationary heat conduction in laminates made of two materials non-periodically distributed as microlaminas along one direction. It is assumed that this laminate has a functionally (transversally) graded macrostructure along this direction. Some effects of the microstructure in a distribution of a temperature are investigated using the tolerance modelling, cf. the book edited by Cz. Woźniak et al. [6].

Keywords: laminates, transversally graded structure, nonhomogeneous distribution of laminas, heat conduction

### 1. INTRODUCTION

In this contribution there are considered laminates, which are made of two materials. It is assumed that these materials are distributed non-periodically along a direction normal to laminas. Every lamina consists of two sub-laminas and has thickness  $\lambda$ . On the macroscopic level these composites have averaged (macroscopic) properties continuously varying along the direction normal to laminas, cf. Fig. 1a. On the microlevel, their microstructure is defined by a non-uniform distribution function –  $\lambda=\lambda(x)$ , cf. Fig. 1b. These laminates are treated as made of *functionally graded materials (FGM)*, cf. [10]. Hence, they can be called *transversally graded laminates with non-uniform distribution of laminas*.

FGM-type laminates are usually analysed by using methods proposed for macroscopically homogeneous structures, e.g. periodic laminates, cf. [10]. Between these methods there can be mentioned those based on *the asymptotic homogenization*, cf. [5]. *Models with microlocal parameters* are also used to analyse heat conduction in periodic laminates, cf. [7]. Unfortunately, equations of these models neglect usually the effect of the microstructure size.

On the other hand, this effect can be described in the framework of the *tolerance modelling*, proposed to the modelling non-stationary problems for periodic composites in the book [12]. This method was adopted to investigate various problems of FGM-type structures in a series of papers, e.g. for a heat conduction in transversally graded laminates in [1-4] and in longitudinally graded composites in [8, 9]. Certain summarisons of applications of this technique for composites and structures of this kind can be found in [11, 6].

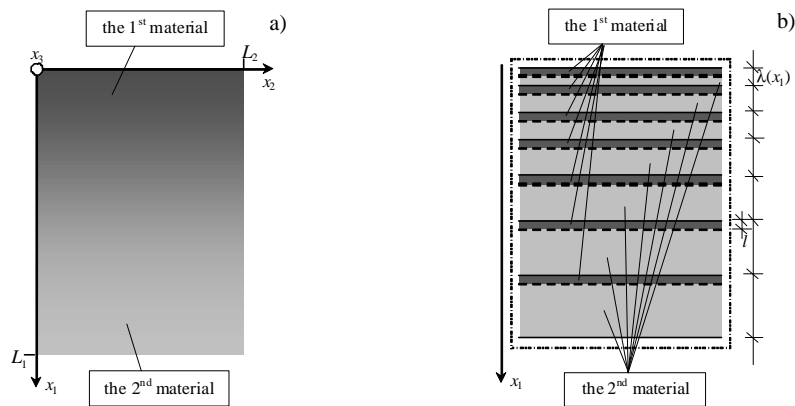


Fig. 1. A cross section of a transversally graded laminate: a) on the macroscopic level, b) on the microscopic level – a non-uniform distribution of laminas

The main aim of this note is to use the *tolerance model equations of heat conduction for transversally graded laminates with non-uniform distribution of laminas (TGL)*, cf. Fig. 1b, to investigate a stationary heat conduction along the direction normal to laminas. Here, there are shown distributions of the total temperature in the laminate. Some effects of the cell distribution and material properties on the temperature are presented.

## 2. MODELLING FOUNDATIONS

It is assumed that subscripts  $i, j, \dots$ , are related to the coordinate system  $Ox_1x_2$ , and run over 1, 2. Let us denote:  $x \equiv x_1$ ; by  $t$  the time coordinate; by  $\partial_i$  derivatives of  $x_i$ ; and also  $\partial \equiv \partial_1$ . The layer thickness along the  $x$ -axis is equal  $H$  and the length dimension along the  $x_2$ -axis is equal  $L$ . The region on the plane  $Ox_1x_2$  occupied by this layer is denoted by  $\Omega \times \Xi$ , where  $\Omega \equiv (0, H)$ ,  $\Xi \equiv (0, L)$ . The laminate is made of two materials distributed in  $m$  laminas having the varied thickness  $\lambda$ . These materials have heat conduction tensors with components  $k'_{ij}, k''_{ij}$ ,  $i, j=1, 2$ . Let the 1<sup>st</sup> material have the constant thickness  $l$  (cf. Fig. 1b), such that  $l \ll H$ . This thickness is called *the microstructure parameter*. Every  $n^{\text{th}}$

lamina has not constant thickness  $\lambda_n$ . Hence, the  $n^{\text{th}}$  lamina consists of two homogeneous sub-laminas having thicknesses  $\lambda'_n = l = \text{const}$  and  $\lambda''_n = \lambda_n - l$ , cf. Fig. 1b. Let us introduce material volume fractions in the  $n^{\text{th}}$  lamina defined as  $v'_n \equiv l/\lambda_n$ ,  $v''_n \equiv \lambda''_n/\lambda_n$ . Sequence  $\{v'_n\}$ ,  $n=1, \dots, m$ , is monotone and satisfies condition  $|v'_{n+1} - v'_n| \ll 1$ , for  $n=1, \dots, m-1$ . Since  $v'_n + v''_n = 1$  sequence  $\{v''_n\}$  satisfies similar conditions. We can approximate sequences  $\{v'_n\}$ ,  $\{v''_n\}$ ,  $n=1, \dots, m$ , by continuous functions  $v'(\cdot)$ ,  $v''(\cdot)$ , which describe the gradation of material properties along the  $x$ -axis. Similarly, we can approximate sequence  $\{\lambda_n\}$  of laminas thicknesses by function  $\lambda(x)$ . The functions  $v'(\cdot)$ ,  $v''(\cdot)$  are called *the fraction ratios of materials*, cf. [4]. Let us also introduce *the non-homogeneity ratio*  $v$  defined by  $v(\cdot) \equiv [v'(\cdot)v''(\cdot)]^{1/2}$ . Although for these laminates a basic cell in  $\Omega$  cannot be determined, we can define a certain non-uniform cell distribution in  $\Omega$  as  $\Omega(x) \equiv [x - \lambda(x)/2, x + \lambda(x)/2]$ , where  $\lambda(x)$  is called *the cell distribution function*. All these functions ( $v'(\cdot)$ ,  $v''(\cdot)$ ,  $\lambda(\cdot)$ ) are assumed to be slowly-varying, cf. [11]. Hence, the layer under consideration is called *the transversally graded laminated layer (the TGL layer)*.

Let  $\theta$  denote the unknown temperature field. Moreover, the stationary heat conduction problem in the TGL layer is analysed within the Fourier's model, i.e. it is described by the following equation (without heat sources):

$$\partial_i(k_{ij}\partial_j\theta) = 0, \quad (2.1)$$

where coefficients  $k_{ij} = k_{ij}(x)$  are highly-oscillating, tolerance-periodic, non-continuous functions in  $x$ . Since equation (2.1) is not a good tool to investigate heat conduction problems it can be replaced by differential equations having slowly-varying coefficients using *the tolerance modelling*, cf. [6].

### 3. BASIC CONCEPTS

In the modelling some introductory concepts defined in [11, 6] are used, e.g. an averaging operator, a tolerance-periodic function, a slowly-varying function, a highly oscillating function. Here, some of them are mentioned.

The *averaging operator* for an arbitrary integrable function  $f$  (which can also depend on  $x_2$ ), defined in  $\overline{\Omega}$ , has the form

$$\langle f \rangle(x) = \lambda(x)^{-1} \int_{x-\lambda(x)/2}^{x+\lambda(x)/2} f(\xi) d\xi, \quad x \in [\lambda(x)/2, H - \lambda(x)/2]. \quad (3.1)$$

It can be observed that for tolerance-periodic function  $f$  of  $x$ , its averaged value calculated from (3.1) is a slowly-varying function in  $x$ , cf. [11, 6].

Denote by  $\partial^k f$  the  $k$ -th gradient of function  $f=f(x)$ ,  $x \in \Omega$ ,  $k=0,1$ ,  $\partial^0 f \equiv f$ ; by  $\tilde{f}^{(k)}(\cdot, \cdot)$  – a function defined in  $\overline{\Omega} \times R^m$ ; and by  $\delta$  – the tolerance parameter.

Function  $f \in H^1(\Omega)$  is the *tolerance-periodic function*,  $f \in TP_\delta^1(\Omega, \Omega)$ , if for  $k=0,1$  the following conditions hold:

- (i)  $(\forall x \in \Omega) (\exists \tilde{f}^{(k)}(x, \cdot) \in H^0(\Omega)) [ \|\partial^k f|_{\Omega, \cdot} - \tilde{f}^{(k)}(x, \cdot)\|_{H^0(\Omega, \cdot)} \leq \delta ]$ ,
- (ii)  $\int_{\Omega(\cdot)} \tilde{f}^{(k)}(\cdot, z) dz \in C^0(\overline{\Omega})$ .

where the *periodic approximation of  $\partial^k f$  in  $\Omega(x)$* ,  $x \in \Omega$ , is denoted by  $\tilde{f}^{(k)}(x, \cdot)$ ,  $k=0,1$ .

Function  $F \in H^1(\Omega)$  is the *slowly-varying function*,  $F \in SV_\delta^1(\Omega, \Omega)$ , if

- (i)  $F \in TP_\delta^1(\Omega, \Omega)$ ,
- (ii)  $(\forall x \in \Omega) [\tilde{F}^{(k)}(x, \cdot)|_{\Omega(x)} = \partial^k F(x), k=0,1]$ .

Periodic approximation  $\tilde{F}^{(k)}$  of  $\partial^k F(\cdot)$  in  $\Omega(x)$  is a constant function for every  $x \in \Omega$ .

Function  $\phi \in H^1(\Omega)$  is the *highly oscillating function*,  $\phi \in HO_\delta^1(\Omega, \Omega)$ , if

- (i)  $\phi \in TP_\delta^1(\Omega, \Omega)$ ,
- (ii)  $(\forall x \in \Omega) [\tilde{\phi}^{(k)}(x, \cdot)|_{\Omega(x)} = \partial^k \phi(x), k=0,1]$ ,
- (iii)  $\forall F \in SV_\delta^\alpha(\Omega, \Omega) \exists \tilde{f} \equiv \phi F \in TP_\delta^\alpha(\Omega, \Omega)$   
 $\tilde{f}^{(k)}(x, \cdot)|_{\Omega(x)} = F(x) \partial^k \tilde{\phi}(x)|_{\Omega(x)}, k=1$ .

If  $\alpha=0$  then we denote  $\tilde{f} \equiv \tilde{f}^{(0)}$ .

Let  $h(\cdot)$  be a highly oscillating, continuous function,  $h \in HO_\delta^1(\Omega, \Omega)$ , defined on  $\overline{\Omega}$ , with piecewise continuous and bounded gradient  $\partial^1 h$ . Function  $h(\cdot)$  is the *fluctuation shape function of the 1<sup>st</sup> kind*, if it depends on  $l$  as a parameter and satisfies conditions:

- (1°)  $\partial^k h \in O(l^{1-k})$  for  $k=0,1$ ,  $\partial^0 h \equiv h$ ,
- (2°)  $\langle h \rangle(x) \approx 0$  for every  $x \in \Omega$ .

A set of all fluctuation shape functions of the 1<sup>st</sup> kind is denoted by  $FS_\delta^1(\Omega, \Omega)$ .

#### 4. MODELLING ASSUMPTIONS

Following [11, 6] and using the introductory concepts two fundamental modelling assumptions can be formulated.

The *micro-macro decomposition* is the fundamental assumption, in which it is assumed that temperature  $\theta$  can be decomposed in the form

$$\theta(x, x_2) = W(x, x_2) + h(x)Q(x, x_2), \quad (4.1)$$

with basic unknowns  $W(\cdot, x_2), Q(\cdot, x_2) \in SV_\delta^1(\Omega, \Omega)$  and the known fluctuation shape function  $h(\cdot) \in FS_\delta^1(\Omega, \Omega)$ . Function  $W(\cdot, x_2)$  is called *the macrotemperature*, but the additional unknown  $Q(\cdot, x_2)$  is *the fluctuation amplitude*. It is assumed that  $h(\cdot)$  is continuous, linear across every sub-lamina thickness and of an order  $O(l)$  function, which satisfies conditions (1°)-(2°). It can be given by:

$$h(x) = \begin{cases} -l\sqrt{3} \frac{v(\bar{x})}{v''(\bar{x})} \left[ 2 \frac{x}{\lambda(\bar{x})} + v'(\bar{x}) \right] & \text{for } x \in \left( -\frac{1}{2}\lambda(\bar{x}), -\frac{1}{2}\lambda(\bar{x}) + \lambda(\bar{x})v''(\bar{x}) \right), \\ l\sqrt{3} \frac{v(\bar{x})}{v'(\bar{x})} \left[ 2 \frac{x}{\lambda(\bar{x})} - v''(\bar{x}) \right] & \text{for } x \in \left( \frac{1}{2}\lambda(\bar{x}) - \lambda(\bar{x})v'(\bar{x}), \frac{1}{2}\lambda(\bar{x}) \right), \end{cases} \quad (4.2)$$

with  $\bar{x}$  being a centre of  $\Omega(\cdot)$ , cf. [11].

The next modelling assumption is *the tolerance averaging approximation*, in which it is assumed that terms  $O(\delta)$  are negligibly small, e.g.:

$$\begin{aligned} \langle f \rangle(x) &= \langle \bar{f} \rangle(x) + O(\delta), & \langle fF \rangle(x) &= \langle f \rangle(x)F(x) + O(\delta) \\ \langle f\partial(hF) \rangle(x) &= \langle f\partial h \rangle(x)F(x) + O(\delta), \\ x \in \Omega; \quad 0 < \delta \ll 1; \quad f &\in TP_\delta^1(\Omega, \cdot), \quad F \in SV_\delta^1(\Omega, \cdot), \quad h \in FS_\delta^1(\Omega, \cdot). \end{aligned}$$

## 5. TOLERANCE MODELLING

Following [6] the modelling procedure is outlined here. In the first step, the action functional is formulated

$$A(\theta(\cdot)) = \int_{\Omega \Xi} \Lambda(z, \partial_i \theta(z, \xi, t), \theta(z, \xi, t)) d\xi dz, \quad (5.1)$$

with the lagrangean  $\Lambda(\cdot, \partial_i \theta, \theta) \in HO_\delta^0(\Omega, \Omega)$  given by

$$\Lambda = \frac{1}{2} \partial_i \theta k_{ij} \partial_j \theta. \quad (5.2)$$

Hence, the Euler-Lagrange equation takes the form

$$\partial_j \frac{\partial \Lambda}{\partial \partial_j \theta} - \frac{\partial \Lambda}{\partial \theta} = 0. \quad (5.3)$$

Using the principle of stationary action from equation (5.3) (combined with (5.2)) the fundamental equation of the Fourier's heat conduction (2.1) is derived. The second step is the application of the tolerance modelling to action

functional (5.1). Substituting micro-macro decomposition (4.1) to (5.1) and averaging by (3.1), we obtain the tolerance averaging of functional  $A(\theta(\cdot))$

$$A_h(W, Q) = \iint_{\Omega \Xi} \langle \Lambda_h \rangle (x, \partial_i W, \partial_2 Q, \vartheta, Q) d\xi dx,$$

with the averaged lagrangean  $\langle \Lambda_h \rangle$  in the form

$$\begin{aligned} \langle \Lambda_h \rangle = & \frac{1}{2}(\partial_i W \langle k_{ij} \rangle \partial_j W + Q \langle \partial h k_{11} \partial h \rangle + \partial_2 Q \langle k_{22} h^2 \rangle + \partial_2 Q + \\ & + \partial_i W \langle k_{i1} \partial h \rangle + Q \langle \partial h k_{j1} \rangle \partial_j W). \end{aligned} \quad (5.4)$$

Using the principle of stationary action to lagrangean  $A_h$  Euler-Lagrange equations take the form:

$$\begin{aligned} \partial_i \frac{\partial \langle \Lambda_h \rangle}{\partial (\partial_i W)} - \frac{\partial \langle \Lambda_h \rangle}{\partial W} &= 0, \\ \partial_2 \frac{\partial \langle \Lambda_h \rangle}{\partial \partial_2 Q} - \frac{\partial \langle \Lambda_h \rangle}{\partial Q} &= 0. \end{aligned} \quad (5.5)$$

The above equations have slowly-varying, functional coefficients.

## 6. TOLERANCE MODEL EQUATIONS

Substituting (5.4) to (5.5) we obtain the averaged heat conduction equations:

$$\begin{aligned} \partial(\langle k_{11} \rangle (x) \partial W) + \langle k_{22} \rangle (x) \partial_{22} W + \partial(\langle k_{11} \partial h \rangle (x) Q) &= 0, \\ \langle k_{11} \partial h \rangle (x) \partial W + \langle k_{11} \partial h \partial h \rangle (x) Q - l^2 (\nu(x))^2 \langle k_{22} \rangle (x) \partial_{22} Q &= 0, \end{aligned} \quad (6.1)$$

involving a term depending explicitly on the microstructure parameter  $l$ . The above equations have slowly-varying, functional coefficients, in contrast to equation (2.1) with functional, non-continuous, highly oscillating coefficients.

Heat conduction for transversally graded laminates is described in the framework of *the tolerance model* by equations (6.1) together with micro-macro decomposition (4.1). These equations make it possible to investigate the effect of the microstructure size on heat transfer for these composites. For the TGL layer we have to formulate boundary conditions for the macrotemperature  $W$  on the edges  $x=0, H, x_2=0, L$ , but for the fluctuation amplitude  $Q$  on the edges  $x_2=0, L$ . Unknowns  $W, Q$  have a physical sense under conditions  $W(\cdot, x_2) \in SV_\delta^1(\Omega, \Omega)$   $Q(\cdot, x_2) \in SV_\delta^1(\Omega, \Omega)$ , being *a posteriori* evaluation of tolerance parameter  $\delta$ .

## 7. APPLICATIONS – HEAT CONDUCTION ACROSS LAMINAS

### 7.1. Introduction

Let us assume that the TGL layer subjected to a thermal gradient in the direction parallel to the  $x$ -axis. Thus, temperature  $\theta$  is a function of  $x$ ,  $\theta=\theta(x)$ , and then the tolerance model unknowns are also functions only of  $x$ ,  $W=W(x)$ ,  $Q=Q(x)$ . Let  $k \equiv k_{11}$ ,  $k' \equiv k'_{11}$ ,  $k'' \equiv k''_{11}$  be heat conduction coefficients in sub-laminas.

Denote

$$K(x) \equiv v'(x)k' + v''(x)k'', \quad \tilde{K}(x) \equiv 2\sqrt{3}v(x)(k' - k''), \quad \bar{K}(x) \equiv 12(v'(x)k'' + v''(x)k').$$

Equations (6.1) can be written as:

$$\begin{aligned} \partial[K(x)\partial W + \tilde{K}(x)Q] &= 0, \\ Q &= -\bar{K}(x)[\tilde{K}(x)]^{-1}\partial W. \end{aligned} \quad (7.1)$$

Substituting (7.1)<sub>2</sub> into (7.1)<sub>1</sub> and denoting

$$K^{eff}(x) \equiv K(x) - [\tilde{K}(x)]^2[\bar{K}(x)]^{-1} \quad (7.2)$$

we have only one equation

$$\partial[K^{eff}(x)\partial W] = 0. \quad (7.3)$$

Using the above equation, the fluctuation amplitude  $Q$ , cf. (7.1)<sub>2</sub>, and micro-macro decomposition (4.1) taking the form

$$\theta(x) = W(x) + h(x)Q(x), \quad (7.4)$$

the stationary heat conduction across laminas in the TGL layer can be described.

### 7.2. Exact solutions to the model equations

Since equation (7.3) has slowly-varying coefficients, defined by known functions, e.g. by  $v'$ ,  $v''$ ,  $v$ ,  $h$ , it can be integrated to find the solution. For the considered layer having constant coefficients  $k'$ ,  $k''$  and the fluctuation shape function given by (4.2) the effective heat conduction coefficient  $K^{eff}$  is equal

$$K^{eff}(x) = k''k''[k' + (k'' - k')v'(x)]^{-1}. \quad (7.5)$$

Denoting

$$p(x) = \int v'(x)dx,$$

assuming the following boundary conditions for macrotemperature  $W$ :

$$x=0: \quad W(0)=T; \quad x=H: \quad W(H)=0, \quad (7.6)$$

and bearing in mind formula (7.5) we obtain the macrotemperature in the form:

$$W(x) = C_1[x(k'')^{-1} + (k'' - k')(k'k'')^{-1}p(x)] + C_2, \quad (7.7)$$

where constants  $C_1, C_2$  are:

$$C_1 = T \frac{k'k''}{(k'' - k')[p(0) - p(H)] - k'H}, \quad C_2 = -T \frac{k'H + (k'' - k')p(H)}{(k'' - k')[p(0) - p(H)] - k'H}. \quad (7.8)$$

From (7.1)<sub>2</sub> the fluctuation amplitude  $Q$  can be calculated

$$Q(x) = \frac{\sqrt{3}}{6}(k'' - k')(k'k'')^{-1}v(x)C_1. \quad (7.9)$$

Substituting (7.7) and (7.9) to (7.4) we obtain the formula for temperature  $\theta$ ,

$$\theta(x) = \{x(k'')^{-1} + (k'' - k')(k'k'')^{-1}[p(x) + \frac{\sqrt{3}}{6}h(x)v(x)]\}C_1 + C_2, \quad (7.10)$$

which with constants  $C_1, C_2$ , (7.8), and the fluctuation shape function  $h$ , (4.2), determines the “exact” distribution of the temperature in the TGL layer in the framework of the tolerance model.

### 7.3. Computational results

Let the layer thickness  $H$  be coupled with the microstructure parameter  $l$  by the relation  $H=2(m-1)l$ , where  $m$  is the number of laminas. We consider three cell distribution functions:

1) the linear function (denoted by  $\alpha=1$ )

$$\lambda(\bar{x}) = \bar{x} \frac{2(H-ml)}{H(m-1)} + l; \quad (7.11)$$

2) the square function (denoted by  $\alpha=2$ )

$$\lambda(\bar{x}) = \bar{x}^2 \frac{6m(H-ml)}{(m-1)(2m-1)H^2} + l; \quad (7.12)$$

3) the cubic function (denoted by  $\alpha=3$ )

$$\lambda(\bar{x}) = \bar{x}^3 \frac{4m(H-ml)}{(m-1)^2H^3} + l; \quad (7.13)$$

with  $\bar{x}$  as the centre of “cell”. Calculations are also made for a periodic laminate with the constant function  $\lambda(x)=2l$  (denoted by  $\alpha=0$ ). The fraction ratios of materials are defined in the form:

$$v'(\bar{x}) = l(\lambda(\bar{x}))^{-1}, \quad v''(\bar{x}) = 1 - v'(\bar{x}).$$

Selected calculational results are shown in Fig. 2. Calculations are made for  $m=20$ , thus the ratio  $l/H=0.026$ . Fig. 2 shows curves of temperature  $\theta$  given by formula (7.10) versus coordinate  $x \in [0, H]$ . Diagrams are made for ratios



$k''/k'=1/3$  (a) and  $k''/k'=1/5$  (b). Some results of macrotemperature  $W$  and the fluctuation amplitude  $Q$  can be found in [4].

Under the obtained results some remarks can be formulated:

1. Values of temperature  $\theta$  depend on the cell distribution functions, i.e.:
  - a) the biggest values are for the cubic function (7.13),
  - b) the smallest values are for the linear function (7.11).
2. Values of the temperature depend on ratio  $k''/k'$  of heat conduction coefficients  $k', k''$  of material properties, i.e.:
  - a) the temperature increase with the decreasing of ratio  $k''/k'$ ,
  - b) the temperature in the periodic layer (the case  $\alpha=0$ ) is independent of ratio  $k', k''$ .

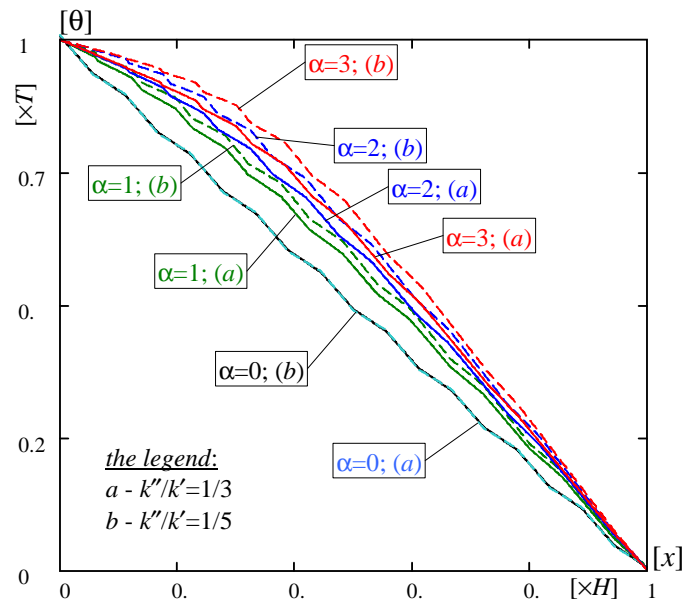


Fig. 2. Diagrams of temperature  $\theta$  versus  $x$ -coordinate (for cell distribution functions: linear ( $\alpha=1$ ), square ( $\alpha=2$ ), cubic ( $\alpha=3$ ); and the periodic distribution ( $\alpha=0$ ))

## 8. REMARKS

The tolerance modelling, cf. the book edited by Cz. Woźniak et al. [6], leads from the differential equation of heat conduction with highly-oscillating, non-continuous, tolerance-periodic coefficients to the system of differential equations with slowly-varying coefficients. The tolerance model equations take into account the effect of the microstructure size on the heat conduction.

Under the results of the example we can observe:

- Exact analytical solutions to the tolerance model equations can be obtained for the stationary heat conduction.
- Distributions of the temperature depend on:
  - the cell distribution functions in the TGL layer,
  - differences between heat conduction coefficients  $k'$ ,  $k''$ , i.e. ratio  $k''/k'$ .

Other applications of the tolerance model to various problems of heat conduction for the transversally graded laminates will be shown in forthcoming papers.

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#### STACJONARNE PRZEWODNICTWO CIAPŁA W LAMINATACH O NIERÓWNOMIERNYM ROZMIESZCZENIU WARSTW

##### Streszczenie

W pracy rozpatrywany jest stacjonarny problem przewodnictwa ciepła w laminatach charakteryzujących się „wolną” zmianą własności makroskopowych (uśrednionych) w kierunku prostopadłym do warstw. Przyjęto, że rozmieszczenie warstw jest nierównomierne. Wpływ mikrostruktury na rozkład temperatury całkowitej zbadano wykorzystując modelowanie tolerancyjne, por. Cz. Woźniak et al. [6].