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EFFECT OF COMPOSITE INGREDIENTS ON VIBRATION FREQUENCIES OF PLATES MADE OF FUNCTIONALLY GRADED MATERIAL

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The subject of this paper are thin plates with characteristic material structure: periodic in selected direction and smoothly varying along another. The aim of the contribution is to formulate and apply averaged model describing the free vibrations of these plates. Modelling procedure is based on the tolerance averaging technique (TAT). We analyze the plate in the rectangular as well as in the cylindrical coordinate systems, respectively. We are to obtain numerical solutions of this problem, using finite difference method, and to analyze the interrelation between the ingredients distribution and the first frequency of free vibrations of these plates. The presented general results are illustrated by the analysis of natural frequencies for two cases of plates: a plate band and an annular plate.

Keywords: functionally graded materials, thin plates, composites, the tolerance averaging technique, free vibrations.

1. INTRODUCTION

1.1. The subject of the consideration

The subject of this paper are thin plates with characteristic material structure: periodic in the selected direction and smoothly varying along another: the l-periodic structure along x_1 coordinate, but smoothly graded apparent (averaged) properties in the perpendicular direction of the x_1, along x_2 axis. (Figs. 1 and 2).

1.2. The aim of contribution

We would like to derive and apply a deterministic macroscopic model of the elastodynamics of considered plates. We will consider two special cases: the band plate and the annular plate. (Figs. 1 and 2).

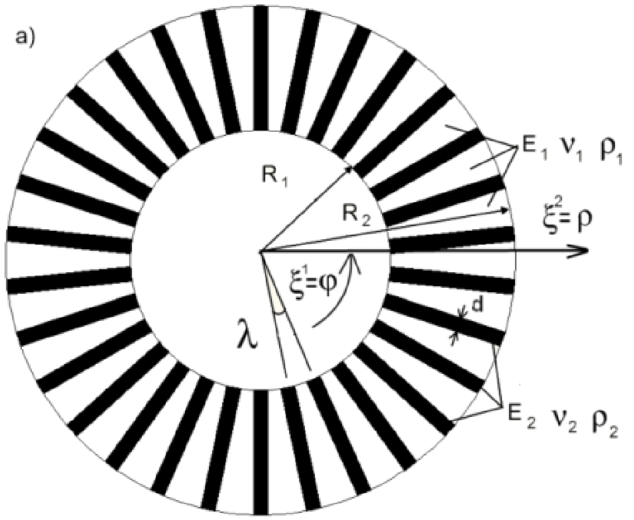

Fig. 1. Example of considered microheterogeneous plate in polar coordinates

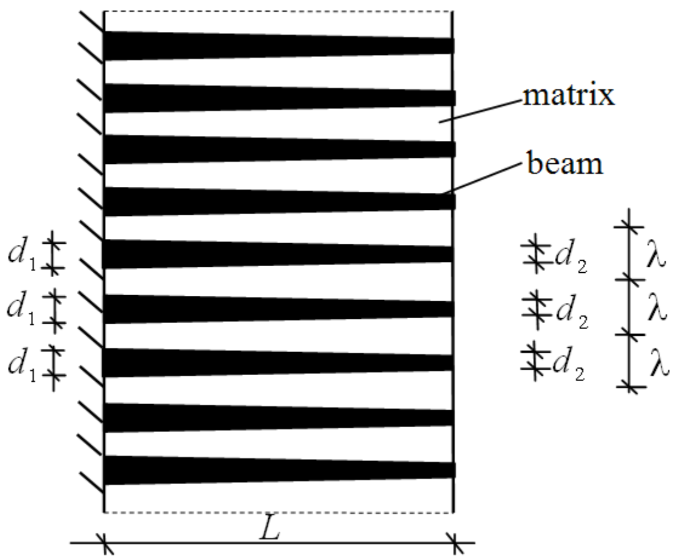

Fig. 2. Example of considered microheterogeneous plate in rectangular coordinates

1.3. The assumptions

We assume that the generalized period l is sufficiency small when compared to the characteristic length dimension of argument x^1.

2. THE ELASTODYNAMICS OF FGM PLATE

2.1. Introduction

The tolerance averaging technique allows the derivation of the model equations in a general way, for any coordinate system. This allows the combination of two seemingly independent problems: vibrations of the annular plate and the plate band within a single work.

2.2. The direct description

The bases of modeling procedure of FGM plate are [5]:

- strain-displacements relations:

$$\kappa_{\alpha\beta} = -w_{|\alpha\beta}, \tag{1}$$

where: k_{ab} is the curvature of the plate, *w* is displacement field.

- constitutive equations:

$$m^{\alpha\beta} = BH^{\alpha\beta\gamma\delta}\kappa_{\gamma\delta}, \tag{2}$$

where:

$$H^{\alpha\beta\gamma\delta} = \frac{1}{2}\left\{ \begin{array}{l} g^{\alpha\mu} g^{\beta\gamma} + g^{\alpha\gamma} g^{\beta\mu} - \\ + \upsilon\left(\in^{\alpha\gamma}\in^{\beta\mu} + \in^{\alpha\mu}\in^{\beta\gamma}\right) \end{array} \right\}, \tag{3}$$

$$B = \frac{E\delta^3}{12\left(1-\upsilon^2\right)}, \tag{4}$$

where:

E - Young module, δ - thickness of plate, ν - Poisson number, $\in^{ij}$ – component of Ricci tensor, g^{ij} - component of contravariant metric tensor.

These equations have highly oscillating coefficients, so they are difficult to solve.

2.3. The averaged description

The density of elastic and kinetic energy we write as functional in the form:

$$L_{\Pi} = -\frac{1}{2} BH^{\alpha\beta\gamma\mu} w_{|\alpha\beta} w_{|\gamma\mu} + \frac{1}{2} \mu \ddot{w} \ddot{w} \quad (5)$$

For further modeling we will use the TAT, as described by Cz. Wozniak and others in [10]. After micro-macro decomposition in the form:

$$w(\xi^1, \xi^2, t) = u(\xi^1, \xi^2, t) + q^A(\xi^1) V_A(\xi^1, \xi^2, t) \quad (6)$$

where

$u(\xi^1, \xi^2, t)$, , $V_A(\xi^1, \xi^2, t)$ are slovly-varying functions in x^2-direction: [10]

$$u(\xi^1, \xi^2, t) \in SV_{\Delta}(T),\ V_A(\xi^1, \xi^2, t) \in SV_{\Delta}(T),$$

and using averaging operator

$$\langle f \rangle (\xi^2) \equiv \frac{1}{\lambda} \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} f(\xi^1, \xi^2) d\xi^1 \quad (7)$$

we obtain averaging density of elastic and kinetic energy as follows:

$$\begin{aligned} \langle L_{\Pi} \rangle = &-\frac{1}{2} \langle BH^{\alpha\beta\gamma\mu} \rangle u_{|\alpha\beta} u_{|\gamma\mu} - \langle BH^{11\gamma\mu} q^A_{|11} \rangle V_A u_{|\gamma\mu} - 2 \langle BH^{12\gamma\mu} q^A_{|1} \rangle V_{A|2} u_{|\gamma\mu} + \\ &- \langle BH^{22\gamma\mu} q^A \rangle V_{A|22} u_{|\gamma\mu} - \frac{1}{2} \langle BH^{1111} q^A_{|11} q^B_{|11} \rangle V_A V_B - \langle BH^{1122} q^A_{|11} q^B \rangle V_A V_{B|22} + \\ &- 2 \langle BH^{1212} q^A_{|1} q^B_{|1} \rangle V_{A|2} V_{B|2} - \frac{1}{2} \langle BH^{2222} q^A q^B \rangle V_{A|22} V_{B|22} + \\ &+ \frac{1}{2} \langle \mu \rangle \ddot{u} \ddot{u} + \langle \mu q^A \rangle \ddot{V}_A \ddot{u} + \frac{1}{2} \langle \mu q^A q^A \rangle \ddot{V}_A \ddot{V}_B \end{aligned} \quad (8)$$

Next we use the tolerance averaging procedure and we obtain the following system of equations:

$$\begin{aligned} &\langle BH^{\alpha\beta\gamma\delta} w^0_{|\gamma\delta} \rangle_{}\alpha\beta} + \left(\langle BH^{\alpha\beta 11} q^A_{|11} \rangle V_A \right)_{|\alpha\beta} + \left(\langle BH^{\alpha\beta 22} q^A \rangle V_{A|22} \right)_{|\alpha\beta} + \langle \rho \ddot{w} \rangle = \langle p \rangle \\ &\langle q^A{}_{|11} BH^{11\gamma\delta} \rangle w^0_{|\gamma\delta} + \langle q^A_{|11} BH^{1111} q^B_{|11} \rangle V_B + \langle q^A_{|11} BH^{1122} q^B \rangle V_{B|22} + \\ &+ \left(\langle q^A{}_{|11} BH^{22\gamma\delta} \rangle w^0_{|\gamma\delta} \right)_{|22} + \left(\langle q^A BH^{2211} q^B_{|11} \rangle V_B \right)_{|22} + \left(\langle q^A BH^{2222} q^B \rangle V_{B|22} \right)_{|22} + \\ &+ \langle q^A \rho q^B \rangle \ddot{V}_B = \langle q^A p \rangle \end{aligned} \quad (9ab)$$

The above system consists of *N+1* differential equations with continuous and slowly-varying functions of argument ξ^2 as coefficients.

3. MODEL EQUATIONS IN RECTANGULAR AND POLAR COORDINATES.

3.1. Model equations for a plate band

The previous considerations have been independent of the coordinate system. Further considerations we will perform for specific coordinate systems: for a plate band in rectangular coordinates and for an annular plate in polar coordinates. After simple manipulation we obtain from equations (9ab) the following system of N+1 differential equations describing dynamic behavior of the plate band

$$\partial_{22}(< B^{2222} > \partial_{22} w^0 + < B^{2211} q_{|11} > V + < B^{2222} q > \partial_{22} V) + < \mu > \ddot{w}^0 = 0,$$

$$\partial_{22}(< B^{2222} q > \partial_{22} w^0 + < B^{1122} q\, \partial_{11} q > V + < B^{2222} qq > \partial_{22} V) +$$
$$- 4\partial_2(< B^{1212} \partial_1 q \partial_1 q > \partial_2 V) + < B^{1122} \partial_{11} q > \partial_{22} w^0 + < B^{1122} \partial_{11} qq > \partial_{22} V + .$$
$$+ < B^{1111} \partial_{11} q \partial_{11} q > V + < \mu qq > \ddot{V} = \langle qp \rangle \tag{10}$$

Equations (10) represent a system of two partial differential equations for the averaged deflection $w^0(\cdot, t)$ and fluctuation amplitude $V(\cdot, t)$.

$$w^0(x_2, t) = \tilde{w}^0(x_2) e^{i\omega t} \qquad V(x_2, t) = \tilde{V}(x_2) e^{i\omega t}. \tag{11}$$

Substituting (11) into (10) we obtain equations for $\tilde{w}^0(x_2)$ and $\tilde{V}(x_2)$

$$\partial_{22}(< B^{2222} > \partial_{22} w^0 + < B^{2211} q_{|11} > V + \underline{\langle B^{2222} q \rangle\, \partial_{22} V}) + < \mu > \omega^2\, \tilde{w}^0 = 0,$$

$$\partial_{22}(\underline{< B^{2222} q > \partial_{22} w^0} + \underline{< B^{1122} q\, \partial_{11} q > V} + \underline{< B^{2222} q\, q > \partial_{22} V}) +$$
$$- 4\partial_2(< B^{1212} \partial_1 q \partial_1 q > \partial_2 V) + < B^{1122} \partial_{11} q > \partial_{22} w^0 + \underline{< B^{1122} \partial_{11} qq > \partial_{22} V} + .$$
$$+ < B^{1111} \partial_{11} q \partial_{11} q > V + < \mu qq > \omega^2\, \tilde{V} = 0 \tag{12}$$

Since $q(\cdot) \in O(\lambda^2)$, the inertial module $< \mu qq >$ and the underlined terms depend on the microstructure length parameter λ, hence aforementioned equations describe the microstructure length-scale effect on the natural frequencies of the plate under consideration.

3.2. Model equations for an annular plate

Let us consider the following polar coordinates: one circular coordinate ξ_1 in angle measure and another radial coordinate ξ_2 in linear measure. Model equations in these coordinates are more complicated than those written in

Cartesian coordinates. Mathematical derivation of the following equations can be found in [6]:

$$
\begin{aligned}
& w^0{}_{,112}\left(2\left\langle q^A BH^{2211}\right\rangle_{,2}\right) + w^0{}_{,1122}\left(\left\langle q^A BH^{2211}\right\rangle\right) + w^0{}_{,11}\begin{pmatrix}\left\langle q^A{}_{|11} BH^{1111}\right\rangle \\ +\left\langle q^A{}_{|11} BH^{2211}\right\rangle_{,22}\end{pmatrix} + \\
& + w^0{}_{,22}\begin{pmatrix}\left\langle q^A{}_{|11} BH^{1122}\right\rangle + 2\rho\left\langle q^A BH^{2211}\right\rangle_{,2} + \\ 2\left\langle q^A BH^{2211}\right\rangle + \left\langle q^A BH^{2222}\right\rangle_{,22}\end{pmatrix} + w^0{}_{,2222}\left(\left\langle q^A BH^{2222}\right\rangle_{,2222}\right) + \\
& w^0{}_{,222}\begin{pmatrix}\rho\left\langle q^A BH^{2211}\right\rangle + \\ 2\left\langle q^A BH^{2222}\right\rangle_{,2}\end{pmatrix} + w^0{}_{,2}\left(\rho\left\langle q^A{}_{|11} BH^{1111}\right\rangle + \rho\left\langle q^A BH^{2211}\right\rangle_{,22} + 2\left\langle q^A BH^{2211}\right\rangle_{,2}\right) + \\
& + V_B\left(\left\langle q^A{}_{|11} BH^{1111} q^B{}_{|11}\right\rangle + \left\langle q^A BH^{2211} q^B{}_{|11}\right\rangle_{,22}\right) + V_{B,2}\left(2\left\langle q^A BH^{2211} q^B{}_{|11}\right\rangle_{,2}\right) + \\
& + V_{B,22}\left(\left\langle q^A{}_{|11} BH^{1122} q^B\right\rangle + \left\langle q^A BH^{2211} q^B{}_{|11}\right\rangle + \left\langle q^A BH^{2222} q^B\right\rangle_{,22}\right) + \\
& + V_{B,222}\left(2\left\langle q^A BH^{2222} q^B\right\rangle_{,2}\right) + V_{B,2222}\left(\left\langle q^A BH^{2222} q^B\right\rangle\right) + \left\langle q^A \rho q^B\right\rangle \ddot{V}_B = \left\langle q^A p\right\rangle \\
& w^0{}_{,1111}\left(\left\langle BH^{1111}\right\rangle\right) + w^0{}_{,112}\left(\frac{4}{\rho}\left\langle BH^{1212}\right\rangle + \frac{2}{\rho}\left\langle BH^{2211}\right\rangle + 4\left\langle BH^{1212}\right\rangle_{,2} + 2\left\langle BH^{2211}\right\rangle_{,2}\right) + \\
& + w^0{}_{,1122}\left(2\left\langle BH^{2211}\right\rangle + 4\left\langle BH^{1212}\right\rangle\right) + w^0{}_{,11}\begin{pmatrix}-\frac{4}{\rho^2}\left\langle BH^{1212}\right\rangle - 2\left\langle BH^{1111}\right\rangle - \frac{4}{\rho}\left\langle BH^{1212}\right\rangle_{,2} + \\ +\left\langle BH^{2211}\right\rangle_{,22} + \frac{2}{\rho}\left\langle BH^{1122}\right\rangle_{,2} - \rho\left\langle BH^{1111}\right\rangle_{,2}\end{pmatrix} + \\
& + w^0{}_{,22}\left(2\left\langle BH^{2211}\right\rangle - \rho^2\left\langle BH^{1111}\right\rangle + \rho\left\langle BH^{2211}\right\rangle_{,2} + \left\langle BH^{2222}\right\rangle_{,22} + \frac{1}{\rho}\left\langle BH^{2222}\right\rangle_{,2}\right) + \\
& + w^0{}_{,2222}\left(\left\langle BH^{2222}\right\rangle\right) + w^0{}_{,222}\left(\frac{2}{\rho}\left\langle BH^{2222}\right\rangle + 2\left\langle BH^{2222}\right\rangle_{,2}\right) + \\
& + w^0{}_{,2}\left(-3\rho\left\langle BH^{1111}\right\rangle + \frac{2}{\rho}\left\langle BH^{2211}\right\rangle + \rho\left\langle BH^{2211}\right\rangle_{,22} + 4\left\langle BH^{2211}\right\rangle_{,2} - \rho^2\left\langle BH^{1111}\right\rangle_{,2} + \frac{1}{\rho}\left\langle BH^{2222}\right\rangle_{,2}\right) + \\
& + V_{A,11}\left(\left\langle BH^{1111} q^A{}_{|11}\right\rangle\right) + V_A\left(\left\langle BH^{2211} q^A{}_{|11}\right\rangle_{,22} + \frac{2}{\rho}\left\langle BH^{2211} q^A{}_{|11}\right\rangle_{,2} - \rho\left\langle BH^{1111} q^A{}_{|11}\right\rangle_{,2} - 2\left\langle BH^{1111} q^A{}_{|11}\right\rangle\right) + \\
& + V_{A,2}\left(2\left\langle BH^{2211} q^A{}_{|11}\right\rangle_{,2} + \frac{2}{\rho}\left\langle BH^{2211} q^A{}_{|11}\right\rangle - \rho\left\langle BH^{1111} q^A{}_{|11}\right\rangle\right) + \\
& + V_{A,22}\left(\left\langle BH^{2211} q^A{}_{|11}\right\rangle + \left\langle BH^{2222} q^A\right\rangle_{,22} - \frac{1}{\rho^2}\left\langle BH^{2222} q^A\right\rangle + \frac{2}{\rho}\left\langle BH^{2222} q^A\right\rangle_{,2} - \rho\left\langle BH^{1122} q^A\right\rangle_{,2} - 2\left\langle BH^{1122} q^A\right\rangle\right) + \\
& + V_{A,1122}\left(\left\langle BH^{2211} q^A\right\rangle\right) + V_{A,222}\left(2\left\langle BH^{2222} q^A\right\rangle_{,2} + \frac{2}{\rho}\left\langle BH^{2222} q^A\right\rangle - \rho\left\langle BH^{1122} q^A\right\rangle\right) + V_{A,2222}\left(\left\langle BH^{2222} q^A\right\rangle\right) + \langle\rho\rangle\ddot{w} = \langle p\rangle
\end{aligned}
\tag{13}
$$

where:

$$H^{1111} = \frac{1}{(\xi_2)^4},\ H^{2222} = 1,\ H^{1122} = H^{2211} = \frac{\upsilon}{(\xi_2)^2},\ H^{1212} = H^{2121} = H^{2112} = H^{1221} = \frac{1-\upsilon}{2(\xi_2)^2}$$

4. APLICATIONS AND NUMERICAL RESULTS

4.1. Introduction

The following assumptions will be introduced now:

– displacement field disjoined:

$$w = w^0 + qV \,, \tag{14}$$

where:

$$w^0 \in SV_\Delta(T),\; V \in SV_\Delta(T),$$

– no external loading: *p=0*

– harmonic vibration:

$$w^0(\xi^\alpha, t) = \overline{w}(\xi^\alpha)\cos(\omega t), \quad V(\xi^\alpha, t) = \overline{V}(\xi^\alpha)\cos(\omega t), \tag{15}$$

– shape function:

$$q(\cdot) = \lambda^2\left(\cos\left(\frac{2\pi\xi^1}{\lambda}\right) + C\right), \tag{16}$$

where constant C we receive from equation:

$$\langle q\rho \rangle = 0 \,, \tag{17}$$

as:

$$C = \frac{\lambda\xi^2\left(-\rho_1 + \rho_1\cos\left(\frac{2\pi d}{\xi^2\lambda}\right) - \rho_2 - \rho_2\cos\left(\frac{2\pi d}{\xi^2\lambda}\right)\right)}{2\pi\left(\rho_1 d + \rho_2\lambda\xi^2 - \rho_2 d\right)} \,. \tag{18}$$

In both examples (4.2 and 4.3) we used the same following materials:

– matrix: E_1 = 69GPa, ν_1 = 0.3, ρ_1 = 2720 kg/m^3

– walls: E_2 = 210GPa, ν_2 = 0.3, ρ_2 = 7800 kg/m^3

4.2. Numerical results for band plate

We consider the following example: free vibrations of thin plate band. This plate is shown in Figure 2 in the rectangular coordinate system. We must formulate boundary conditions. Boundary conditions will be written in the form for left-hand side clamped and right-hand side freely below:

$$w|_{\xi^2=0}=0,\ \left(\frac{\partial w}{\partial \xi^2}\right)|_{\xi^2=0}=0,\ \left(\frac{\partial w^2}{\left(\partial \xi^2\right)^2}\right)|_{\xi^2=L}=0,\ \left(\frac{\partial^3 w}{\left(\partial \xi^2\right)^3}\right)|_{\xi^2=L}=0, \tag{19}$$

because (14):

$$w^0\,|_{\xi^2=0}=0,\ \left(\frac{\partial w^0}{\partial \xi^2}\right)|_{\xi^2=0}=0,\ V\,|_{\xi^2=0_i}=0\quad \left(\frac{\partial V}{\partial \xi^2}\right)|_{\xi^2=0}=0,$$

$$\left(\frac{\partial^2 w^0}{\left(\partial \xi^2\right)^2}\right)|_{\xi^2=L}=0,\left(\frac{\partial^3 w^0}{\left(\partial \xi^2\right)^3}\right)|_{\xi^2=L}=0,\ \left(\frac{\partial^2 V}{\left(\partial \xi^2\right)^2}\right)|_{\xi^2=L}=0,\left(\frac{\partial^2 V}{\left(\partial \xi^2\right)^3}\right)|_{\xi^2=L}=0 \tag{20}$$

and for both sides simply supported:

$$w|_{\xi^2=0}=0,\ \left(\frac{\partial^2 w}{\left(\partial \xi^2\right)^2}\right)|_{\xi^2=0}=0,\ w|_{\xi^2=L}=0\ ,\ \left(\frac{\partial^2 w}{\left(\partial \xi^2\right)^2}\right)|_{\xi^2=L}=0, \tag{21}$$

because (14):

$$w^0\,|_{\xi^2=0}=0,\ \left(\frac{\partial w^0}{\left(\partial \xi^2\right)^2}\right)|_{\xi^2=0}=0,\ V\,|_{\xi^2=0}=0\ ,\ \left(\frac{\partial V}{\left(\partial \xi^2\right)^2}\right)|_{\xi^2=0}=0,$$

$$w^0\,|_{\xi^2=L}=0,\ \left(\frac{\partial w^0}{\left(\partial \xi^2\right)^2}\right)|_{\xi^2=L}=0,\ V\,|_{\xi^2=L}=0,\ \left(\frac{\partial V}{\left(\partial \xi^2\right)^2}\right)|_{\xi^2=L}=0. \tag{22}$$

We use finite difference method to obtain numerical solutions. We write the own computer program in MS Visual C++ for solving this problem. We could change any geometrical and material parameters of plate and hence we can obtain first frequency of free vibrations. Hence, we shall analyze the influence of material proportion and microstructure parameter onto the frequency of free vibrations.

The following geometrical data will be applied:

– microstructure parameter λ = 0.3 m,
– the thickness of plate h = 3cm,

– band plate span L = 3m.

Below, some numerical results will be presented:

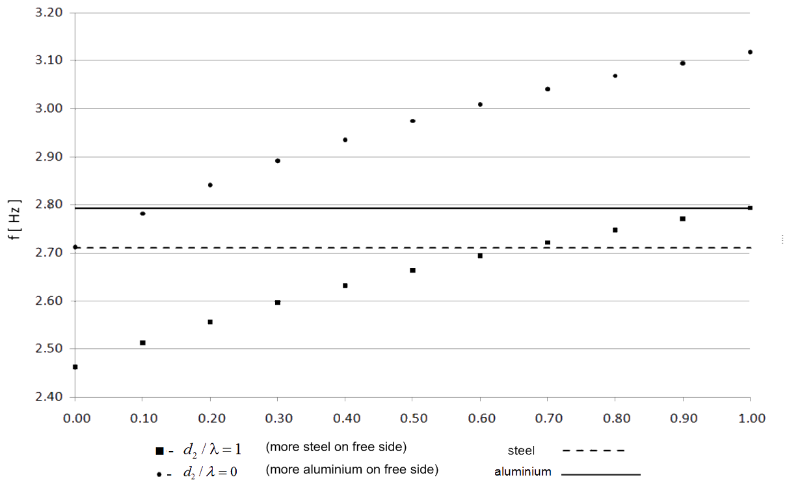

Fig. 3. Dependency the first frequency of free vibrations on share of composite ingredients for the plate band (left- hand side clamped, right-hand side free)

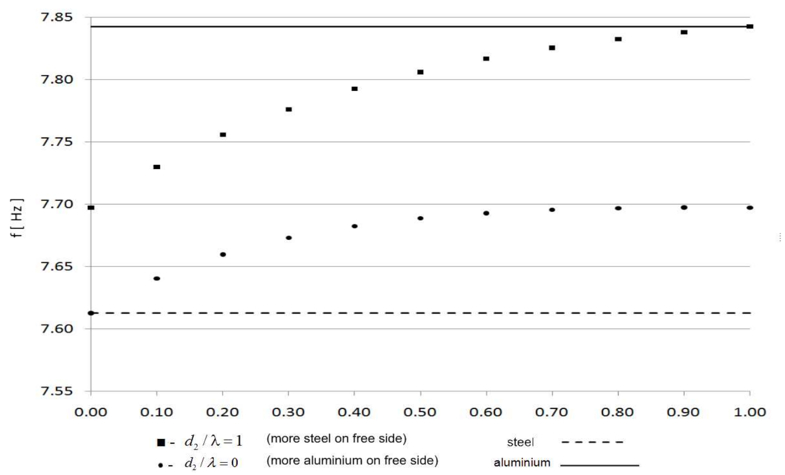

Fig. 4. Dependency the first frequency of free vibrations on share of composite ingredients for the plate band (both sides simply support)

4.3. Numerical results for the annular plate

Next we consider an example: free vibrations of thin annular plate. This plate is shown in Figure 1 in polar coordinates. We write boundary conditions for both side clamped below:

$$w|_{\xi^2=r_i}=0 \quad \text{and} \quad \left(\frac{\partial w}{\partial \xi^2}\right)|_{\xi^2=r_i}=0\,, \tag{23}$$

because (14):

$$w^0\,|_{\xi^2=r_i}=0\,,\ \left(\frac{\partial w^0}{\partial \xi^2}\right)|_{\xi^2=r_i}=0\,,\ V\,|_{\xi^2=r_i}=0\,,\quad \left(\frac{\partial V}{\partial \xi^2}\right)|_{\xi^2=r_i}=0\,, \tag{24}$$

and for freely supported both sides:

$$w|_{\xi^2=r_i}=0 \quad \text{and} \quad \left(\frac{\partial^2 w}{(\partial \xi^2)^2}+\frac{1}{\xi^2}\frac{\partial w}{\partial \xi^2}\right)|_{\xi^2=r_i}=0\,, \tag{25}$$

because (14):

$$w^0\,|_{\xi^2=r_i}=0\,,\ \left(\frac{\partial^2 w^0}{(\partial \xi^2)^2}+\frac{1}{\xi^2}\frac{\partial w^0}{\partial \xi^2}\right)|_{\xi^2=r_i}=0\,,\ \ V\,|_{\xi^2=r_i}=0$$

$$\left(\frac{\partial^2 V}{(\partial \xi^2)^2}+\frac{1}{\xi^2}\frac{\partial V}{\partial \xi^2}\right)|_{\xi^2=r_i}=0\,. \tag{26}$$

The above equations system is more difficult to solve than the equations system for the plate band [6]. We use finite difference method to find numerical solution of this equations system. We write the own computer program in MS Visual C++ for solving this problem. Similarly as in the previous example we could change any geometrical and material parameters of plate and we received first and higher frequencies of free vibrations and shapes of displacement field corresponding to it. Hence, we shall analyze the influence of material share and microstructure parameter on frequencies of free vibrations.

For example, the following data materials will be applied:

- matrix: E_1 = 20 GPa, ν_1= 0.3, ρ_1= 2800 kg/m^3,
- walls: E_2 = 220 GPa, ν_2= 0.3, ρ_2= 7800 kg/m^3,

and geometrical data:

- angle of periodic cell λ = 0.032416rad,
- thickness of plate h = 3cm,
- ring width L = 3m,
- internal radius R_1 = 4m,
- external radius R_1 = 7m.

Below, some numerical results will be presented:

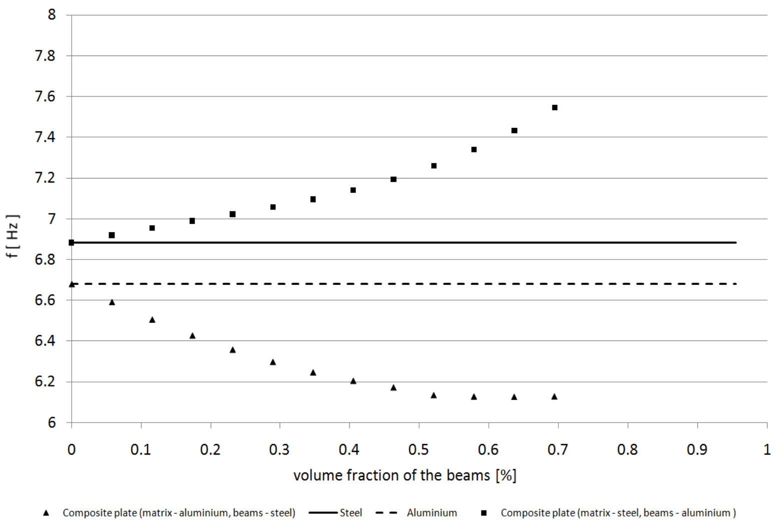

Fig. 5. Dependency of the first frequency of free vibrations on share of composite ingredients for the annular plate (free support on internal and external circuit)

4.4. The discusion of the numerical results

Particularly noteworthy is the case in which the first frequency of free vibrations of the composite do not fall between the first frequencies of free vibrations of the plates made of homogeneous materials: the wall and the matrix respectively. This situation occurs for the bracket plate band and the free support on both borders annular plate with certain geometric shares. It is connected with the asymmetry of boundary conditions and different values of the elasticity modules and density of individual materials. In the case of the annular plate we have the asymmetry of boundary conditions, due to a difference of internal and external radius: when both radiuses approach infinity the annular plate seeks to a plate band and phenomenon disappeares: the first frequency of free vibrations of the composite is placed between the first frequencies of free vibrations for homogeneous plates. For the plate band bracket similar results can be found in [1].

5. CONCLUSIONS

After modeling and analysis of the obtained results, some conclusions could be made:

- the tolerance averaging technique can be successfully applied to formulate averaging model of dynamic behavior of composite plates made from functionally graded material,

- the obtained model is described by equations with functional but smooth coefficients in contrast to direct description (equations with non-continuous and highly oscillating coefficients),
- the first frequency of free vibrations for both side freely supported composite plate band is between frequencies of homogeneous plates made respectively of material of beams and material of matrix,
- the first frequency of free vibrations for the bracket composite band plate is not placed between the first frequencies of free vibrations of respectively homogeneous plates.

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WPŁYW SKŁADNIKÓW KOMPOZYTU NA CZĘSTOŚCI DRGAŃ PŁYT ZBUDOWANYCH Z MATERIAŁÓW O FUNKCYJNEJ GRADACJI WŁASNOŚCI

S t r e s z c z e n i e

Przedmiotem niniejszej pracy są płyty cienkie posiadające charakterystyczną geometrię: periodyczną w jednym kierunku i zmieniającą się w sposób płynny w drugim. Celem rozważań jest zbudowanie modelu uśrednionego opisującego dynamiczne zachowanie tego typu płyty. Procedura modelowania jest oparta na technice tolerancyjnego uśredniania zaprezentowanej w pracy Woźniaka i Wierzbickiego [10]. Wyprowadzone równania modelu płyty są zapisane w układzie biegunowym dla płyty pierścieniowej oraz w układzie kartezjańskim dla przypadku pasma płytowego. Następnie zostało zaprezentowane rozwiązanie numeryczne za pomocą metody różnic skończonych oraz przeanalizowano wpływ udziału składników kompozytu na pierwszą częstość drgań własnych płyty.