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ON THE MODELLING OF DYNAMIC PROBLEMS FOR BIPERIODICALLY STIFFENED CYLINDRICAL SHELLS

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Thin linear-elastic cylindrical shells having a micro-periodic structure along two directions tangent to the shell midsurface (*biperiodic shells*) are objects of considerations. The aim of this contribution is to formulate a new mathematical non-asymptotic model for the analysis of dynamic problems for such shells. The model is derived by applying *the combined modelling procedure* presented in [11]. The combined modelling includes both *the asymptotic* as well as *the non-asymptotic (tolerance) modelling techniques*. The resulting *combined model* has constant coefficients and takes into account *the length-scale effect*. An important advantage of the proposed model is that it makes it possible to separate the macroscopic description of special dynamic problems from their microscopic description. Application of the resulting model equations to the analysis of a certain micro-vibration problem is presented.

Keywords: biperiodic cylindrical shells, dynamics, mathematical modelling, averaging of integral functionals, length-scale effect.

1. INTRODUCTION

Thin linear-elastic Kirchhoff-Love-type cylindrical shells with a periodically inhomogeneous structure along two directions tangent to the shell midsurface are analysed. By periodic inhomogeneity we shall mean periodically variable shell thickness and/or periodically variable inertial and elastic properties of the shell material. Shells of this kind are termed *biperiodic*. As an example we can mention cylindrical shells with periodically spaced families of thin stiffeners as shown in Fig. 1. The period of inhomogeneity is assumed to be very large compared with the maximum shell thickness and very small as compared to the midsurface curvature radius as well as the smallest characteristic length dimension of the shell midsurface.

Because properties of such shells are described by highly oscillating and non-continuous periodic functions, the exact equations of the shell theory are too complicated to apply to investigations of engineering problems. That is why a lot of different approximate modelling methods for shells of this kind have been proposed. Periodic cylindrical shells (plates) are usually described using *homogenized models* derived by means of *asymptotic methods*, cf. [3, 5, 10]. Unfortunately, in models of this kind *the effect of a cell size* (called *the length-scale effect*) on the overall shell behaviour is neglected.

The periodically densely stiffened shells are also modelled as homogeneous orthotropic structures, cf. [2, 6]. The orthotropic model equations with coefficients independent of the period length cannot be used to the analysis of phenomena related to the existence of microstructure length-scale effect (e.g. the dispersion of waves, the occurrence of additional higher-order free vibration frequencies and higher-order critical forces).

In order to analyse the length-scale effect in dynamic or/and stability problems, the new averaged non-asymptotic models of thin cylindrical shells with a periodic micro-heterogeneity either along two directions tangent to the shell midsurface (*biperiodic structure*) or along one direction (*uniperiodic structure*) have been proposed by Tomczyk in a series of papers, e.g. [14, 15, 16, 17, 18, 19, 22, 25], and also in the books [20, 21, 23, 24]. These, so called, *the tolerance models* have been obtained by applying *the non-asymptotic tolerance modelling technique*, proposed and discussed in the monographs [1, 11, 26, 28], to the known governing equations of Kirchhoff-Love theory of thin elastic shells (partial differential equations with functional highly oscillating non-continuous periodic coefficients). Contrary to starting equations, governing equations of the tolerance models have coefficients which are constant or slowly-varying and depend on the period length of inhomogeneity. Hence, these models make it possible to investigate the effect of a cell size on the global shell dynamics and stability. This effect is described by means of certain extra unknowns called *fluctuation amplitudes* and by known *fluctuation shape functions* which represent oscillations inside the periodicity cell. Moreover, it was shown that the tolerance models of uniperiodically and densely stiffened shells describe selected problems of the micro-dynamics of such shells, cf. [22, 23, 24]. It means that contrary to equations derived by using the asymptotic homogenised methods, the tolerance model equations make it possible to investigate the micro-dynamics of periodic shells independently of their macro-dynamics. In the papers and books, mentioned above, the applications of the proposed models to analysis of special problems dealing with dynamics as well as stationary and dynamical stability of periodically densely stiffened cylindrical shells have been presented. It was shown that the length-scale effect plays an important role in these problems and cannot be neglected.

It has to be emphasized that the non-asymptotic tolerance models of shells with uni- and biperiodic structure have to be led out independently, because they are based on different modelling assumptions. The governing equations for uniperiodic shells are more complicated. It means that *contrary to the asymptotic approach, the uniperiodic shell is not a special case of biperiodic shell*.

The application of the tolerance averaging technique to the investigations of selected dynamic problems for periodic plates can be found in many papers, e.g. in [4] and [7, 8], where dynamics of Hencky-Bolle-type plates and of Kirchhoff-type plates is analysed, respectively, in [12] and [13], where dynamics of wavy-type plates and of densely stiffened Kirchhoff-type plates is investigated, respectively. For review of application of the tolerance approach to the modelling of different periodic and also non-periodic structures the reader is referred to [1, 11, 26, 28].

The main aim of this contribution is to formulate a new mathematical non-asymptotic model for the analysis of special dynamic problems for biperiodic shells under consideration. The model is derived by applying *the combined modelling procedure*, presented in [11], to the known Euler-Lagrange equations which explicit form coincides with the governing equations of the simplified Kirchhoff-Love shell theory. The combined modelling technique is realized in two steps. In the first step *the macroscopic model equations*, being independent of the microstructure size, are derived by means of *the consistent asymptotic procedure*. Assuming that in the framework of the macroscopic model the solution to the problem under consideration is known, we can pass to the second step, which is based on *the tolerance (non-asymptotic) modelling*. The Euler-Lagrange equations derived in the second step depend on the cell size and hence, they are referred to as *the superimposed microscopic model equations*. Coefficients of the resulting equations are constant. *The main advantage of the combined model is that it makes it possible to separate the macroscopic description of some special problems from their microscopic description*.

The second aim of this contribution is to apply the obtained model to determine *the new additional higher order free micro-vibration frequencies*, occurring in periodic shells and depending on the cell length dimensions, independently of the lower (classical) free macro-vibration frequencies being independent of the period lengths.

Note, that *the combined model* for analysis of dynamic and/or stability problems for *uniperiodic cylindrical shells* has been proposed and discussed in

[24]. However, this model cannot be used to analysis of dynamic problems of *biperiodic shells*, being object of considerations in this paper.

It should be mentioned that the periodic cylindrical shells investigated here are widely applied in civil engineering, most often as roof girders and bridge girders. They are also widely used as housings of reactors and tanks. Periodic shells having small length dimensions are elements of air-planes, ships and machines.

In the subsequent section the basic denotations, preliminary concepts and starting equations will be presented.

2. FORMULATION OF THE PROBLEM

In this paper we investigate linear-elastic thin circular cylindrical shells. The shells are reinforced by families of ribs, which are periodically and densely distributed in circumferential and axial directions. Shells of this kind are termed *biperiodic*. Example of such shell is shown in Fig. 1.

In order to describe the shell geometry define $\Omega = (0, L_1) \times (0, L_2)$ as a set of points $\mathbf{x} \equiv (x^1, x^2)$ in R^2; x^1, x^2 being the Cartesian orthogonal coordinates parametrizing region $\Omega \subset R^2$. Let $O\bar{x}^1\bar{x}^2\bar{x}^3$ stand for a Cartesian orthogonal coordinate system in the physical space E^3. Points of E^3 will be denoted by $\bar{\mathbf{x}} \equiv (\bar{x}^1, \bar{x}^2, \bar{x}^3)$. A cylindrical shell midsurface *M* is given by its parametric representation $M \equiv \{\bar{\mathbf{x}} \in E^3 : \bar{\mathbf{x}} = \bar{\mathbf{r}}(x^1, x^2), (x^1, x^2) \in \Omega\}$, where $\bar{\mathbf{r}}(\cdot)$ is the smooth function such that $\partial\bar{\mathbf{r}}/\partial x^1 \cdot \partial\bar{\mathbf{r}}/\partial x^2 = 0$, $\partial\bar{\mathbf{r}}/\partial x^1 \cdot \partial\bar{\mathbf{r}}/\partial x^1 = 1$, $\partial\bar{\mathbf{r}}/\partial x^2 \cdot \partial\bar{\mathbf{r}}/\partial x^2 = 1$. It means that on *M* we have introduced the orthonormal parametrization and hence L_1, L_2 are length dimensions of *M*. It is assumed that x^1 and x^2 are coordinates parametrizing the shell midsurface along the lines of its principal curvature and along its generatrix, respectively, cf. Fig. 1.

Subsequently, sub- and superscripts $\alpha, \beta, \ldots$ run over sequence *1*, *2* and are related to midsurface parameters x^1, x^2; summation convention holds. The partial differentiation related to x^α is represented by ∂_α. Moreover, it is denoted $\partial_{\alpha\ldots\delta} \equiv \partial_\alpha \ldots \partial_\delta$. Differentiation with respect to time coordinate $t \in [t_0, t_1]$ is represented by the overdot. Denote by $a_{\alpha\beta}$ and $a^{\alpha\beta}$ the covariant and contravariant midsurface first metric tensors; respectively. For the introduced parametrization $a_{\alpha\beta} = a^{\alpha\beta} = \delta^{\alpha\beta}$ are the unit tensors.

Let $d(\mathbf{x})$ and r stand for the shell thickness and the constant midsurface curvature radius, respectively.

Denote by $b_{\alpha\beta}$ the covariant midsurface second metric tensor. For the introduced parametrization $b_{22} = b_{12} = b_{21} = 0$ and $b_{11} = -r^{-1}$.

Let λ_1 and λ_2 be the period lengths of the stiffened shell structure respectively in x^1- and x^2-directions, cf. Fig. 1. Define *the basic cell* Δ and *the cell distribution* (Ω, Δ) *assigned to* $\Omega = (0, L_1) \times (0, L_2) \subset R^2$ by means of:
$\Delta \equiv [-\lambda_1/2, \lambda_1/2] \times [-\lambda_2/2, \lambda_2/2]$,
$(\Omega, \Delta) \equiv \{\Delta(x^1, x^2) \equiv (x^1, x^2) + \Delta, (x^1, x^2) \in \overline{\Omega}\}$,
where point (x^1, x^2) is a centre of a cell $\Delta(x^1, x^2)$ and $\overline{\Omega}$ is a closure of Ω.

The diameter $\lambda \equiv \sqrt{(\lambda_1)^2 + (\lambda_2)^2}$ of Δ is assumed to satisfy conditions: $\lambda / d_{\max} >> 1$, $\lambda / r << 1$ and $\lambda / \min(L_1, L_2) << 1$. Hence, the diameter will be called *the microstructure length parameter*. In every cell $\Delta(\mathbf{x})$ we introduce local coordinates z^1, z^2 along the x^1- and x^2-directions, respectively, with the 0-point at the centre of the cell. It means that the cell Δ has two symmetry axes: for $z^1 = 0$ and $z^2 = 0$. Hence, inside the cell, the geometrical, elastic and inertial properties of the stiffened shell are described by symmetric (i.e. even) functions of $\mathbf{z} \equiv (z^1, z^2) \in [-\lambda_1/2, \lambda_1/2] \times [-\lambda_2/2, \lambda_2/2]$.

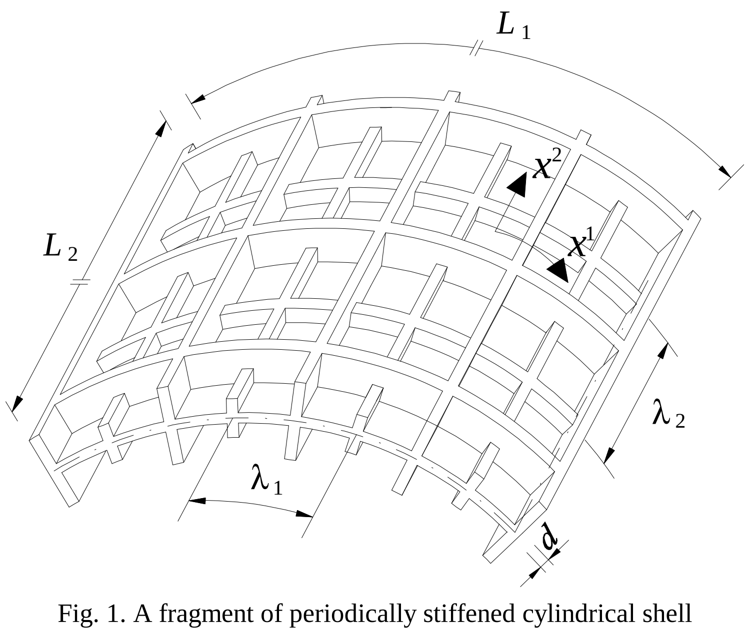

Fig. 1. A fragment of periodically stiffened cylindrical shell

A function $f(\mathbf{x})$ defined on Ω will be called Δ-periodic if for arbitrary points $(x^1, x^2), (x^1 \pm \lambda_1, x^2), (x^1, x^2 \pm \lambda_2), (x^1 \pm \lambda_1, x^2 \pm \lambda_2)$ it satisfies condition:
$f(x^1, x^2) = f(x^1 \pm \lambda_1, x^2) = f(x^1, x^2 \pm \lambda_2) = f(x^1 \pm \lambda_1, x^2 \pm \lambda_2)$ in the whole domain of its definition and it is not constant.

Denote by $u_\alpha = u_\alpha(\mathbf{x}, t)$, $w = w(\mathbf{x}, t)$, $\mathbf{x} \in \Omega$, $t \in (t_0, t_1)$, the midsurface shell displacements in directions tangent and normal to M, respectively. Elastic properties of the shell are described by shell stiffness tensors $D^{\alpha\beta\gamma\delta}(\mathbf{x})$, $B^{\alpha\beta\gamma\delta}(\mathbf{x})$. Let $\mu(\mathbf{x})$ stand for a shell mass density per midsurface unit area. In the problem considered here the external forces will be neglected.

Functions $\mu(\mathbf{x})$, $D^{\alpha\beta\gamma\delta}(\mathbf{x})$, $B^{\alpha\beta\gamma\delta}(\mathbf{x})$ and $d(\mathbf{x})$, $\mathbf{x} \in \Omega$, are assumed to be Δ-periodic with respect to arguments x^1, x^2.

It is assumed that the behaviour of the stiffened shell under consideration is described by the action functional

$$A(u_\alpha, w) = \int_0^{L_1}\int_0^{L_2}\int_{t_0}^{t_1} L(\mathbf{x}, \partial_\beta u_\alpha, \dot{u}_\alpha, \partial_{\alpha\beta} w, w, \dot{w})\, dt\, dx^2 dx^1 , \tag{2.1}$$

where lagrangian $L(\mathbf{x}, \partial_\beta u_\alpha, \dot{u}_\alpha, \partial_{\alpha\beta} w, w, \dot{w})$ is highly oscillating function with respect to $\mathbf{x}$ and has the well-known form, cf. [2, 27]

$$\begin{aligned} L = \frac{1}{2}(&D^{\alpha\beta\gamma\delta}\partial_\beta u_\alpha \partial_\delta u_\gamma + 2r^{-1} D^{\alpha\beta 11} w \partial_\beta u_\alpha + r^{-2} D^{1111} ww + \\ &+ B^{\alpha\beta\gamma\delta}\partial_{\alpha\beta} w \partial_{\gamma\delta} w - \mu a^{\alpha\beta}\dot{u}_\alpha \dot{u}_\beta - \mu \dot{w}^2)\, . \end{aligned} \tag{2.2}$$

Obviously, in the above formula it has been taken into account that $b_{11} = -r^{-1}$. Moreover, we recall that under the orthonormal parametrization introduced on the shell midsurface, the contravariant midsurface first metric tensor $a^{\alpha\beta}$ takes the following values: $a^{\alpha\beta} = 0$ for $\alpha \neq \beta$ and $a^{\alpha\beta} = 1$ for $\alpha = \beta$.

The principle of stationary action applied to A leads to the following system of Euler-Lagrange equations

$$\begin{aligned} &\partial_\beta \frac{\partial L}{\partial(\partial_\beta u_\alpha)} + \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{u}_\alpha} = 0\, , \\ &-\partial_{\alpha\beta}\frac{\partial L}{\partial(\partial_{\alpha\beta} w)} - \frac{\partial L}{\partial w} + \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{w}} = 0\, . \end{aligned} \tag{2.3}$$

After combining (2.3) with (2.2) the above system can be written in the form

$$\begin{aligned}&\partial_\beta(D^{\alpha\beta\gamma\delta}\partial_\delta u_\gamma)+r^{-1}\partial_\beta(D^{\alpha\beta11}w)=\mu a^{\alpha\beta}\ddot{u}_\beta\ ,\\ &r^{-1}D^{\alpha\beta11}\partial_\beta u_\alpha+\partial_{\alpha\beta}(B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta}w)+\rho^{-2}D^{1111}w=-\mu\ddot{w}\,.\end{aligned} \tag{2.4}$$

It can be observed that equations (2.4) coincide with the well-known governing equations of simplified Kirchhoff-Love theory of thin elastic shells, cf. [27]. In the above equations the displacements $u_\alpha = u_\alpha(\mathbf{x},t)$, $w = w(\mathbf{x},t)$ are the basic unknowns. For periodic shells coefficients of lagrangian L and hence also of equations (2.4) are highly oscillating non-continuous functions depending on $\mathbf{x}$ with a period λ. That is why equations (2.3) (or their explicit form (2.4)) cannot be directly applied to investigations of engineering problems. Our aim is to "replace" these equations by equations with constant coefficients depending on the microstructure size. To this end *the combined modelling technique* given in [11] will be applied. To make the subsequent analysis more clear, in the next Section we shall outline the basic concepts and the main assumptions of this approach, following the book [11] together with some results presented in [26].

3. MODELLING CONCEPTS AND ASSUMPTIONS

The combined modelling technique is based on two modelling procedures. The first of them is called *the consistent asymptotic modelling*. The second one is termed *the tolerance modelling*.

3.1. Basic concepts

The fundamental concepts of the tolerance modelling are those of tolerance determined by tolerance parameter, cell distribution, tolerance periodic function and its two special cases: slowly-varying and highly-oscillating functions. The tolerance approach is based on the notion of the averaging of tolerance periodic function.

The main statement of the modelling procedure is that every measurement as well as numerical calculation can be realized in practice only within a certain accuracy defined by *tolerance parameter* δ being a positive constant.

The concept of *cell distribution* (Ω,Δ) assigned to $\Omega=(0,L_1)\times(0,L_2)$ has been introduced in the previous Section.

A bounded integrable function $f(\cdot)$ defined on $\overline{\Omega}=[0,L_1]\times[0,L_2]$ (which can also depend on t as a parameter) is called *tolerance periodic* with respect to cell Δ and tolerance parameter δ, if roughly speaking, its values in an arbitrary cell $\Delta(\mathbf{x})$ can be approximated, with sufficient accuracy, by the corresponding values of a certain Δ-periodic function $f_{\mathbf{x}}(\mathbf{z}), \mathbf{z} \in \Delta(\mathbf{x}), \mathbf{x} \in \overline{\Omega}$. Function $f_{\mathbf{x}}$ is a Δ*-periodic approximation* of f in $\Delta(\mathbf{x})$. This condition has to be fulfilled by all derivatives of f up to the R-th order, i.e. by *all its derivatives which occur in the problem under consideration*; in the problem analysed here R is equal either 1 or 2. In this case we shall write $f \in TP_{\delta}^{R}(\Omega,\Delta)$. It has to be emphasized that for periodic structures being object of considerations in this paper function $f_{\mathbf{x}}(\mathbf{z}), \mathbf{z} \in \Delta(\mathbf{x}), \mathbf{x} \in \overline{\Omega}$ has the same analytical form in every cell $\Delta(\mathbf{x})$, $\mathbf{x} \in \overline{\Omega}$. Hence, $f_{\mathbf{x}}(\cdot)$ is independent of $\mathbf{x}$. In the general case, i.e. for tolerance periodic structures (i.e. structures which in small neighbourhoods of $\Delta(\mathbf{x})$ can be approximately regarded as periodic), $f_{\mathbf{x}} = f_{\mathbf{x}}(\mathbf{x},\mathbf{z}), \mathbf{z} \in \Delta(\mathbf{x}), \mathbf{x} \in \overline{\Omega}$.

Subsequently we will denote by $\partial \equiv (\partial_1, \partial_2)$ the gradient operator in Ω and by $\partial^k f(\cdot)$, $k = 0,1,..,R$, the k-th gradient of function $f(\cdot)$ defined in Ω, where $\partial^0 f(\cdot) \equiv f(\cdot)$. Let $f_{\mathbf{x}}^{(k)}(\mathbf{z})$, $\mathbf{z} \in \Delta(\mathbf{x})$ be a periodic approximation of $\partial^k f \in TP_{\delta}^{R}(\Omega,\Delta)$ in cell $\Delta(\mathbf{x})$, $\mathbf{x} \in \overline{\Omega}$, $k = 0,1,..,R$, $f_{\mathbf{x}}^{0}(\cdot) \equiv f_{\mathbf{x}}(\cdot)$.

A continuous bounded differentiable function $v(\mathbf{x})$ defined on $\overline{\Omega} = [0, L_1] \times [0, L_2]$ (which can also depend on t as a parameter) is called *slowly-varying* with respect to cell Δ and tolerance parameter δ, if

$$
\begin{aligned}
& v(\mathbf{x}) \in TP_{\delta}^{R}(\Omega,\Delta), \\
& v_{\mathbf{x}}^{(k)}(\mathbf{z}) = \partial^k v(\mathbf{x}), \quad k = 0,1,...,R, \quad \textit{for every } \ \mathbf{z} \in \Delta(\mathbf{x}), \mathbf{x} \in \overline{\Omega},
\end{aligned}
\tag{3.1}
$$

It means that periodic approximation $v_{\mathbf{x}}^{(k)}$ of $\partial^k v(\cdot)$ in $\Delta(\mathbf{x})$ is *a constant function* for every $\mathbf{x} \in \overline{\Omega}$. Under the above conditions we shall write $v \in SV_{\delta}^{R}(\Omega,\Delta)$.

Function $h(\mathbf{x})$ defined in $\overline{\Omega} = [0, L_1] \times [0, L_2]$ is called *the highly oscillating function* with respect to cell Δ and tolerance parameter δ, $h \in HO_{\delta}^{R}(\Omega,\Delta)$, if

$$
\begin{aligned}
& h(\mathbf{x}) \in TP_{\delta}^{R}(\Omega,\Delta), \\
& (\forall v(\mathbf{x}) \in SV_{\delta}^{R}(\Omega,\Delta)) \, (f = hv \in TP_{\delta}^{R}(\Omega,\Delta)), \\
& h_{\mathbf{x}}^{(k)}(\mathbf{z}) = \partial^k h_{\mathbf{x}}(\mathbf{z}), \\
& f_{\mathbf{x}}^{(k)}(\mathbf{z}) = \partial^k h_{\mathbf{x}}(\mathbf{z}) \, v(\mathbf{x}), \\
& \textit{for } k = 0,1,...,R \quad \textit{and for every } \ \mathbf{z} \in \Delta(\mathbf{x}), \mathbf{x} \in \overline{\Omega}.
\end{aligned}
\tag{3.2}
$$

In the problem considered here we also deal with *the highly-oscillating functions* which are Δ*-periodic*, i.e. they are special cases of *the highly-oscillating tolerance* Δ*-periodic functions*, defined above. Let $h(\mathbf{x})\in HO_\delta^R(\Omega,\Delta)$ be a λ-periodic function defined in $\overline{\Omega}$ which is continuous together with its gradients $\partial^k h, k=1,...,R-1,$ and has either continuous or a piecewise continuous bounded gradient $\partial^R h$. Function $h(\cdot)$ will be called *the fluctuation shape function*, if it depends on λ as a parameter and satisfies conditions $(3.2)_2$ and $(3.2)_4$, (in $(3.2)_4$ $\partial^k h_{\mathbf{x}}(\mathbf{z})$ is replaced by $\partial^k h(\mathbf{z})$), together with conditions:

$$\begin{gathered}\partial^k h\in O(\lambda^{R-k}), \quad k=0,1,...,R, \quad \partial^0 h\equiv h, \\ \int\limits_{\Delta(\mathbf{x})}\mu(\mathbf{z})h(\mathbf{z})d\mathbf{z}=0, \quad \mathbf{z}\in\Delta(\mathbf{x}), \\ \int\limits_{\Delta(\mathbf{x})}\partial^k h(\mathbf{z})d\mathbf{z}=0, \quad \mathbf{z}\in\Delta(\mathbf{x}), \quad \mathbf{x}\in\Omega, \quad k=1,2,...,R,\end{gathered} \tag{3.3}$$

where μ is a certain positive valued λ-periodic function defined in Ω.

Let $f(\cdot)\in TP_\delta^R(\Omega,\Delta)$. By *the averaging of tolerance periodic function* $f\equiv\partial^0 f$ and its derivatives $\partial^k f, k=1,2,...,R$, we shall mean function $<\partial^k f>(\mathbf{x})$, $\mathbf{x}\in\overline{\Omega}$, defined by

$$<\partial^k f>(\mathbf{x})\equiv\frac{1}{|\Delta|}\int\limits_{\Delta(\mathbf{x})} f_{\mathbf{x}}^{(k)}(\mathbf{x},\mathbf{z})d\mathbf{z}, \quad k=0,1,...,R, \quad \mathbf{z}\in\Delta(x), \quad \mathbf{x}\in\overline{\Omega}. \tag{3.4}$$

For periodic media periodic approximation $f_{\mathbf{x}}^{(k)}$ of $\partial^k f$ in $\Delta(\mathbf{x})$ is independent of argument $\mathbf{x}$ and $<\partial^k f>$ is constant. For tolerance periodic media $<\partial^k f>$ is a smooth slowly-varying function of $\mathbf{x}$.

Let $f(\mathbf{x},\partial^k g(\mathbf{x})), \quad k=0,1,...,R$ be a composite function defined in Ω such that $f(\mathbf{x},\partial^k g(\mathbf{x}))\in HO_\delta^0(\Omega,\Delta), \quad g(\mathbf{x})\in TP_\delta^R(\Omega,\Delta)$. The tolerance averaging of this function is defined by

$$< f(\mathbf{z},\partial^k g(\mathbf{z})>(\mathbf{x})\equiv\frac{1}{|\Delta|}\int\limits_{\Delta(\mathbf{x})} f_{\mathbf{x}}(\mathbf{x},\mathbf{z},g_{\mathbf{x}}^{(k)}(\mathbf{x},\mathbf{z})d\mathbf{z}, \quad \mathbf{z}\in\Delta(x), \quad \mathbf{x}\in\overline{\Omega}. \tag{3.5}$$

For periodically microheterogeneous shells under consideration function $f_{\mathbf{x}}$ is independent of $\mathbf{x}$ and $< f(\cdot, \partial^k g(\cdot) >$ is constant. It can be seen, that definition (3.4) is a special case of definition (3.5).

In the tolerance modelling of dynamic problems for periodic shells we also deal with *mean* (*constant*) *value* $< f >$ *of* Δ*-periodic integrable function* $f(\cdot)$ defined by

$$< f(\mathbf{z}) > \equiv \frac{1}{|\Delta|} \int_{\Delta(\mathbf{x})} f(\mathbf{z}) d\mathbf{z}, \quad \mathbf{z} \in \Delta(x), \quad \mathbf{x} \in \overline{\Omega}. \tag{3.6}$$

On passing from *tolerance averaging* to *the asymptotic averaging* we retain only the concept of *highly-oscillating function.* In the asymptotic approach we deal with *mean* (*constant*) *value* $< f >$ *of* Δ*-periodic function* $f(\cdot)$ defined by (3.6).

More general definitions of these concepts are given in [11, 26] and also in [1].

3.2. Modelling assumptions

The fundamental assumption imposed on the lagrangian under consideration in the framework of *the tolerance averaging approach* is called *the micro-macro decomposition.* It states that the displacement fields occurring in this lagrangian have to be *the tolerance periodic functions* in $\mathbf{x}$. Hence, they can be decomposed into *unknown averaged displacements* being *slowly-varying functions* in $\mathbf{x}$ and fluctuations represented by *known highly-oscillating functions* called *fluctuation shape functions* and by *unknown fluctuation amplitudes* being *slowly-varying* in $\mathbf{x}$.

The fundamental assumption imposed on the lagrangian under consideration in the framework of *the consistent asymptotic averaging approach* is called *the consistent asymptotic decomposition.* It states that the displacement fields occurring in this lagrangian have to be replaced by families of fields defined in an arbitrary cell and depend on small parameter $\varepsilon = 1/n, n = 1,2,...$. These families of displacements are decomposed into part described by unknown functions being continuously bounded in $\overline{\Omega}$ and highly-oscillating part depending on ε and represented by known fluctuation shape functions and by unknown functions being continuously bounded in $\overline{\Omega}$.

For details the reader is referred to [11, 26].

4. COMBINED MODELLING

A new mathematical model for the analysis of dynamic problems for biperiodically stiffened cylindrical shells under considerations will be formulated. In order to derive this model the new so-called *the combined modelling procedure*, proposed in [11], will be applied

The combined modelling includes both the asymptotic as well as the non-asymptotic modelling procedures.

The combined modelling technique is realized in two steps. The first step is based on *the consistent asymptotic procedure* which leads from starting equations (2.3) to the Euler-Lagrange equations *with constant coefficients* being independent of the microstructure cell size. Hence the model obtained in the first step is referred to as *the macroscopic model*. Assuming that in the framework of the macroscopic model the solution to the problem under consideration is known, we can pass to the second step, which is based on *the tolerance (non-asymptotic) modelling*. The Euler-Lagrange equations derived in the second step *have constant coefficients which depend on the cell size*. Hence, the model obtained in the second step is referred to as *the superimposed microscopic model*.

4.1. Step 1. Consistent asymptotic modelling

We start with *the consistent asymptotic averaging of lagrangian* L occurring in (2.1). To this end let us introduce two systems of the linear independent highly-oscillating periodic *fluctuation shape functions*, $h^a(\cdot)\in HO^1_\delta(\Omega,\Delta)$, $a=1,..,n$ and $g^A(\cdot)\in HO^2_\delta(\Omega,\Delta)$, $A=1,..,N$. These functions are assumed to be postulated *a priori* in every problem under consideration. They can be obtained by a certain periodic discretization of the cell. Now, we have to introduce *the consistent asymptotic decomposition* of displacements $u_\alpha = u_\alpha(\mathbf{z},t)$, $w = w(\mathbf{z},t)$, $\mathbf{z}\equiv(z^1,z^2)\in\Delta(\mathbf{x})$, $t\in(t_0,t_1)$, in an arbitrary cell $\Delta(\mathbf{x})$, $\mathbf{x}\in\Omega$

$$
\begin{aligned}
&u_{\varepsilon\alpha}(\mathbf{z},t)\equiv u_\alpha(\mathbf{z}/\varepsilon,t)=\overline{u}_\alpha(\mathbf{z},t)+\varepsilon h^a_\varepsilon(\mathbf{z})U^a_\alpha(\mathbf{z},t), \quad a=1,..,n,\\
&w_\varepsilon(\mathbf{z},t)\equiv w(\mathbf{z}/\varepsilon,t)=\overline{w}(\mathbf{z},t)+\varepsilon^2 g^A_\varepsilon(\mathbf{z})W^A(\mathbf{z},t), \quad A=1,..,N, \\
&\mathbf{z}\in\Delta_\varepsilon(\mathbf{x}),\ t\in(t_0,t_1),
\end{aligned}
\tag{4.1}
$$

where summation convention over a and A holds, and $\varepsilon=1/m,\quad m=1,2,...,$ $\Delta_\varepsilon\equiv(-\varepsilon\lambda/2,\varepsilon\lambda/2)$, $\Delta_\varepsilon(\mathbf{x})\equiv\mathbf{x}+\Delta_\varepsilon$, $\mathbf{x}\in\overline{\Omega}$, $h^a_\varepsilon(\mathbf{z})\equiv h^a(\mathbf{z}/\varepsilon)$, $g^A_\varepsilon(\mathbf{z})\equiv g^A(\mathbf{z}/\varepsilon)$. Unknown functions $\overline{u}_\alpha, U^a_\alpha$ in (4.1) are assumed to be continuous and bounded together with their first derivatives. Unknown functions $\overline{w}, W^A$ in (4.1) are assumed to be continuous and bounded together with their derivatives up to the second order.

Moreover $\overline{u}_\alpha, U^a_\alpha, \overline{w}, W^A$ are assumed to be independent of ε. This is the main difference between the asymptotic approach under consideration and approach which is used in the homogenisation theory, cf. [5, 9].

Due to the fact that lagrangian L defined by (2.2) is highly oscillating with respect to $\mathbf{x}$ there exists for every $\mathbf{x} \in \overline{\Omega}$, lagrangian $L_\mathbf{x}(\mathbf{z}, \partial_\beta u_\alpha, \dot{u}_\alpha, \partial_{\alpha\beta} w, w, \dot{w})$ which constitutes a Δ-periodic approximation of lagrangian L in $\Delta(\mathbf{x})$, $\mathbf{z} \in \Delta(\mathbf{x})$. Let $L_{\mathbf{x}\varepsilon}$ be a family of functions given by

$$
\begin{aligned}
L_{\mathbf{x}\varepsilon} &= L_\mathbf{x}(\mathbf{z}/\varepsilon, \partial_\beta u_{\varepsilon\alpha}, \dot{u}_{\varepsilon\alpha}, \partial_{\alpha\beta} w_\varepsilon, w_\varepsilon, \dot{w}_\varepsilon) = \\
&= \frac{1}{2}[D^{\alpha\beta\gamma\delta}(\mathbf{z}/\varepsilon)\partial_\beta u_{\varepsilon\alpha}\partial_\delta u_{\varepsilon\gamma} + 2r^{-1}D^{\alpha\beta 11}(\mathbf{z}/\varepsilon) w_\varepsilon \partial_\beta u_{\varepsilon\alpha} + \\
&\quad + r^{-2}D^{1111}(\mathbf{z}/\varepsilon) w_\varepsilon w_\varepsilon + B^{\alpha\beta\gamma\delta}(\mathbf{z}/\varepsilon)\partial_{\alpha\beta} w_\varepsilon \partial_{\gamma\delta} w_\varepsilon + \\
&\quad - \mu a^{\alpha\beta}\dot{u}_{\varepsilon\alpha}\dot{u}_{\varepsilon\beta} - \mu(\dot{w}_\varepsilon)^2].
\end{aligned}
\tag{4.2}
$$

Substituting the right-hand sides of (4.1) into (4.2) and taking into account that if $\varepsilon \to 0$ then every continuous and bounded function $f(\mathbf{z}, t)$, $\mathbf{z} \in \Delta_\varepsilon(\mathbf{x})$, $t \in (t_0, t_1)$, tends to function $f(\mathbf{x}, t)$, $\mathbf{x} \in \overline{\Omega}$, as well as after neglecting terms $O(\varepsilon)$, $O(\varepsilon^2)$ we arrive at

$$
\begin{aligned}
L_{\mathbf{x}\varepsilon} = L_\mathbf{x}(\mathbf{z}/\varepsilon, &\partial_\beta \overline{u}_\alpha(\mathbf{x},t) + \partial_\beta h^a(\mathbf{z}/\varepsilon) U^a_\alpha(\mathbf{x},t), \dot{\overline{u}}_\alpha(\mathbf{x},t), \\
&\partial_{\alpha\beta}\overline{w}(\mathbf{x},t) + \partial_{\alpha\beta} g^A(\mathbf{z}/\varepsilon) W^A(\mathbf{x},t), \overline{w}(\mathbf{x},t), \dot{\overline{w}}(\mathbf{x},t))
\end{aligned}
$$

Moreover, if $\varepsilon \to 0$ then, by means of a property of the mean value, cf. [9], the obtained result tends weakly to $L_0(\partial_\beta \overline{u}_\alpha, U^a_\alpha, \dot{\overline{u}}_\alpha, \partial_{\alpha\beta}\overline{w}, \overline{w}, W^A, \dot{\overline{w}})$, where $L_0 = \frac{1}{|\Delta|}\int_{\Delta(\mathbf{x})} L_\mathbf{x}(\mathbf{z}, \partial_\beta \overline{u}_\alpha, U^a_\alpha, \dot{\overline{u}}_\alpha, \partial_{\alpha\beta}\overline{w}, \overline{w}, W^A, \dot{\overline{w}})d\mathbf{z}$, $\mathbf{z} \in \Delta(\mathbf{x})$, $\mathbf{x} \in \overline{\Omega}$. It follows that

$$
\begin{aligned}
&L_0(\partial_\beta \overline{u}_\alpha, U^a_\alpha, \dot{\overline{u}}_\alpha, \partial_{\alpha\beta}\overline{w}, \overline{w}, W^A, \dot{\overline{w}}) = \\
&= \frac{1}{2}[< D^{\alpha\beta\gamma\delta}(\mathbf{z}) > \partial_\beta \overline{u}_\alpha \partial_\delta \overline{u}_\gamma + 2 < D^{\alpha\beta\gamma\delta}(\mathbf{z})\partial_\delta h^a(z) > \partial_\beta \overline{u}_\alpha U^a_\gamma + \\
&+ < D^{\alpha\beta\gamma\delta}(\mathbf{z})\partial_\beta h^a(\mathbf{z})\partial_\delta h^b(\mathbf{z}) > U^a_\gamma U^b_\alpha + 2r^{-1}(< D^{\alpha\beta 11}(\mathbf{z}) > \partial_\beta \overline{u}_\alpha \overline{w} + \\
&+ < D^{\alpha\beta 11}\partial_\beta h^a > \overline{w} U^a_\alpha) + r^{-2} < D^{1111}(\mathbf{z}) > (\overline{w})^2 + \\
&+ < B^{\alpha\beta\gamma\delta}(\mathbf{z}) > \partial_{\alpha\beta}\overline{w}\partial_{\gamma\delta}\overline{w} + 2 < B^{\alpha\beta\gamma\delta}(\mathbf{z})\partial_{\gamma\delta} g^A(\mathbf{z}) > \partial_{\alpha\beta}\overline{w} W^A + \\
&+ < B^{\alpha\beta\gamma\delta}(\mathbf{z})\partial_{\alpha\beta} g^A(z)\partial_{\gamma\delta} g^B(\mathbf{z}) > W^A W^B + \\
&- < \mu > a^{\alpha\beta}\dot{\overline{u}}_\alpha \dot{\overline{u}}_\beta - < \mu > (\dot{\overline{w}})^2], \qquad \mathbf{z} \in \Delta(\mathbf{x}),\ \mathbf{x} \in \overline{\Omega},
\end{aligned}
\tag{4.3}
$$

where denotation (3.6) has been used.

Function L_0, given above, is *the averaged form of lagrangian* L *defined by* (2.2) *under consistent asymptotic averaging*.

In the framework of consistent asymptotic modelling we introduce *the consistent asymptotic action functional* defined by

$$
A^0_{hg}(\overline{u}_\alpha, U^a_\alpha, \overline{w}, W^A) = \int_0^{L_1}\int_0^{L_2}\int_{t_0}^{t_1} L_0 \, dt\, dx^2 dx^1
$$

where L_0 is given by (4.3).

Under assumption that $\partial L_0 / \partial(\partial_\beta \overline{u}_\alpha), \partial L_0 / \partial(\partial_{\alpha\beta}\overline{w})$ are continuous, from the principle of stationary action for A^0_{hg}, we obtain

$$
\begin{aligned}
&\partial_\beta \frac{\partial L_0}{\partial(\partial_\beta \overline{u}_\alpha)} + \frac{\partial}{\partial t}\frac{\partial L_0}{\partial \dot{\overline{u}}_\alpha} = 0, \\
&-\partial_{\alpha\beta}\frac{\partial L_0}{\partial(\partial_{\alpha\beta}\overline{w})} - \frac{\partial L_0}{\partial \overline{w}} + \frac{\partial}{\partial t}\frac{\partial L_0}{\partial \dot{\overline{w}}} = 0, \\
&\frac{\partial L_0}{\partial U^a_\alpha} = 0, \quad a = 1,2,\ldots,n, \\
&\frac{\partial L_0}{\partial W^A} = 0, \quad A = 1,2,\ldots,N.
\end{aligned}
\tag{4.4}
$$

Combining (4.4) with (4.3) we arrive at the explicit form of *the consistent asymptotic model equations* for $\overline{u}_\alpha, \overline{w}, U^a_\alpha, W^A$

$$
\begin{aligned}
&< D^{\alpha\beta\gamma\delta} > \partial_{\beta\delta}\bar{u}_\gamma + r^{-1} < D^{\alpha\beta 11} > \partial_\beta \bar{w} + < D^{\alpha\beta\gamma\delta}\partial_\delta h^b > \partial_\beta U^b_\gamma + \\
&- < \mu > a^{\alpha\beta}\ddot{\bar{u}}_\beta = 0\,, \\
&< B^{\alpha\beta\gamma\delta} > \partial_{\alpha\beta\gamma\delta}\bar{w} + < B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} g^B > \partial_{\alpha\beta} W^B + r^{-1} < D^{11\gamma\delta} > \partial_\delta \bar{u}_\gamma + \\
&+ r^{-2} < D^{1111} > \bar{w} + r^{-1} < D^{11\gamma\delta}\partial_\gamma h^b > U^b_\delta - < \mu > \ddot{\bar{w}} = 0\,, \\
&< \partial_\beta h^a D^{\alpha\beta\gamma\delta}\partial_\delta h^b > U^b_\gamma = - < \partial_\beta h^a D^{\alpha\beta\gamma\delta} > \partial_\delta \bar{u}_\gamma + \\
&- r^{-1} < \partial_\beta h^a D^{\alpha\beta 11} > \bar{w}\,, \\
&< \partial_{\alpha\beta} g^A B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} g^B > W^A = - < \partial_{\alpha\beta} g^B B^{\alpha\beta\gamma\delta} > \partial_{\gamma\delta}\bar{w}\,.
\end{aligned} \tag{4.5}
$$

It can be shown that linear transformations $\mathbf{G}, \mathbf{E}$ given by $G^{ab}_{\alpha\gamma} = < \partial_\beta h^a D^{\alpha\beta\gamma\delta}\partial_\delta h^b >$, $E^{AB} = < \partial_{\alpha\beta} g^A B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} g^B >$, respectively, are invertible. Hence, solutions U^b_γ, W^A to $(4.5)_{3,4}$ can be written in the form

$$
\begin{aligned}
&U^b_\gamma = -(G^{-1})^{bc}_{\gamma\eta}\left[< \partial_\beta h^c D^{\beta\eta\mu\vartheta} > \partial_\vartheta \bar{u}_\mu + r^{-1} < \partial_\beta h^c D^{\beta\eta 11} > \bar{w} \right], \\
&W^A = -(E^{-1})^{AB} < \partial_{\alpha\beta} g^B B^{\alpha\beta\gamma\delta} > \partial_{\gamma\delta}\bar{w}\,,
\end{aligned} \tag{4.6}
$$

where $\mathbf{G}^{-1}$ and $\mathbf{E}^{-1}$ are the inverses of the linear transformations $\mathbf{G}, \mathbf{E}$, respectively. Substituting (4.6) into $(4.5)_{1,2}$ and setting

$$
\begin{aligned}
&D^{\alpha\beta\gamma\delta}_h \equiv < D^{\alpha\beta\gamma\delta} > - < D^{\alpha\beta\eta\chi}\,\partial_\chi h^a > (G^{-1})^{ab}_{\eta\zeta} < \partial_\chi h^b D^{\chi\zeta\gamma\delta} >\,, \\
&B^{\alpha\beta\gamma\delta}_h \equiv < B^{\alpha\beta\gamma\delta} > - < B^{\alpha\beta\mu\zeta}\partial_{\mu\zeta} g^A > (E^{-1})^{AB} < \partial_{\mu\zeta} g^B B^{\mu\zeta\gamma\delta} >,
\end{aligned} \tag{4.7}
$$

we arrive finally at the following form of Euler-Lagrange equations for $\bar{u}_\beta, \bar{w}$

$$
\begin{aligned}
&D^{\alpha\beta\gamma\delta}_h \partial_{\beta\delta}\bar{u}_\gamma + r^{-1} D^{\alpha\beta 11}_h \partial_\beta \bar{w} - < \mu > a^{\alpha\beta}\ddot{\bar{u}}_\beta = 0\,, \\
&B^{\alpha\beta\gamma\delta}_h \partial_{\alpha\beta\gamma\delta}\bar{w} + r^{-1} D^{11\gamma\delta}_h \partial_\delta \bar{u}_\gamma + r^{-2} D^{1111}_h \bar{w} + < \mu > \ddot{\bar{w}} = 0\,.
\end{aligned} \tag{4.8}
$$

Since functions $u_\alpha(\cdot,t)$, $w(\cdot,t)$ have to be uniquely defined in $\Omega \times (t_0, t_1)$, we conclude that $u_\alpha(\cdot,t)$, $w(\cdot,t)$ have to take the form

$$
\begin{aligned}
&u_\alpha(\mathbf{x},t) = \bar{u}_\alpha(\mathbf{x},t) + h^a(\mathbf{x}) U^a_\alpha(\mathbf{x},t)\,, \\
&w(\mathbf{x},\xi,t) = \bar{w}(\mathbf{x},t) + g^A(\mathbf{x}) W^A(\mathbf{x},t)\,, \quad \mathbf{x} \in \Omega,\ t \in (t_0, t_1)\,,
\end{aligned} \tag{4.9}
$$

with U^a_α, W^A given by (4.6).

Equations (4.8) together with formula (4.9) represent *the consistent asymptotic model* of Euler-Lagrange equations (2.4) derived from lagrangian (2.2). Coefficients in equations (4.8) are constant in contrast to coefficients in equations (2.4) which are discontinuous, highly oscillating and periodic. The above model is not able to describe the length-scale effect on the overall shell dynamics being independent of the microstructure cell size. That is why the model derived in the first step of combined modelling is referred to as *the macroscopic model* for the problem under consideration.

In the first step of combined modelling it is assumed that functions $\bar{u}_\alpha, \bar{w}$ obtained as solution to a certain boundary-initial value problem for consistent asymptotic equations (4.8) are known. Hence, there are also known functions

$$\begin{aligned} &u_{0\alpha}(\mathbf{x},t) = \bar{u}_\alpha(\mathbf{x},t) + h^a(\mathbf{x})U^a_\alpha(\mathbf{x},t)\,, \\ &w_0(\mathbf{x},\xi,t) = \bar{w}(\mathbf{x},t) + g^A(\mathbf{x})W^A(\mathbf{x},t)\,, \quad \mathbf{x}\in\Omega,\ t\in(t_0,t_1)\,, \end{aligned} \tag{4.10}$$

where U^a_α, W^A are given by means of (4.6).

4.2. Step 2. Superimposed modelling-tolerance approach

The second step of the combined modelling will be realized by means of *the tolerance procedure*, cf. [1, 11, 26, 28]. To this end we assume that $u_{0\alpha}$ and w_0 given by (4.10) are the known *tolerance periodic functions*, i.e. $u_{0\alpha}(\mathbf{x},t)\subset TP^1_\delta(\Omega,\Delta)$, $w_0(\mathbf{x},t)\subset TP^2_\delta(\Omega,\Delta)$, $\mathbf{x}\in\bar{\Omega}$, $t\in(t_0,t_1)$.

Let functions $c^k(\mathbf{x}), k=1,2,..,m$ and $b^K(\mathbf{x}), K=1,2,..,M$ be the new known λ-periodic in $\mathbf{x}$ *fluctuation shape functions*, $c^k(\cdot)\in HO^1_\delta(\Omega,\Delta)$, $b^K(\cdot)\in HO^2_\delta(\Omega,\Delta)$, such that $c^k\in O(\lambda),\ \lambda\partial_\alpha c^k\in O(\lambda)$, $b^K\in O(\lambda^2),\ \lambda\partial_\alpha b^K\in O(\lambda^2)$, $\lambda^2\partial_{\alpha\beta}b^K\in O(\lambda^2)$, $<\mu c^k>=<\mu b^K>=0$ and $<\mu c^k c^p>=<\mu b^K b^P>=0$ for $k\neq p, K\neq P$, where $\mu(\cdot)$ is the shell mass density being a λ-periodic function with respect to $\mathbf{x}$. In dynamic problems, the fluctuation shape functions c^k, b^K introduced in the second step of combined modelling represent either the principal modes of free periodic vibrations of the cell $\Delta(\mathbf{x})$ or physically reasonable approximation of these modes. Hence, they can be obtained as solutions to certain periodic eigenvalue problems describing free periodic vibrations of the cell, cf. [21]. Let functions $Q^k_\alpha(\mathbf{x},t)$, $k=1,2,..,m$ and $V^K(\mathbf{x},t)$, $K=1,2,..,M$ be the new unknowns called *fluctuation (microscopic) amplitudes* which are slowly-varying in $\mathbf{x}$, $Q^k_\alpha(\mathbf{x},t)\in SV^1_\delta(\Omega,\Delta)\subset TP^1_\delta(\Omega,\Delta)$, $V^K(\mathbf{x},t)\in SV^2_\delta(\Omega,\Delta)\subset TP^2_\delta(\Omega,\Delta)$.

We shall introduce *the extra decomposition superimposed on* $u_{0\alpha}, w_0$

$$u_{c\alpha}(\mathbf{x},t) = u_{0\alpha}(\mathbf{x},t) + c^k(\mathbf{x})Q_\alpha^k(\mathbf{x},t)\,,$$
$$w_b(\mathbf{x},t) = w_0(\mathbf{x},t) + b^K(\mathbf{x})V^K(\mathbf{x},t)\,, \quad \mathbf{x}\in\Omega,\ t\in(t_0,t_1)\,. \tag{4.11}$$

Summation convention over $k=1,2,..,m$ and $K=1,2,..,M$ holds. If $u_{0\alpha}$ and w_0 are known then the above formula will be referred to as *decomposition superimposed on the first step of combined modelling*.

Due to the fact that $u_{c\alpha}(\cdot,t)\in TP_\delta^1(\Omega,\Delta)$ and $w_b(\cdot,t)\in TP_\delta^2(\Omega,\Delta)$ there exist periodic approximations of these functions and of their pertinent derivatives in every $\Delta(\mathbf{x})$.

Bearing in mind properties of the slowly-varying and highly-oscillating functions, cf. (3.1), (3.2), the periodic approximations of $u_{c\alpha}(\mathbf{z},t)$, $\partial_\beta u_{c\alpha}(\mathbf{z},t)$ and $\dot{u}_{c\alpha}(\mathbf{z},t)$ in $\Delta(\mathbf{x})$, $\mathbf{z}\in\Delta(\mathbf{x})$, $\mathbf{x}\in\overline{\Omega}$, have the form

$$u_{c\alpha\mathbf{x}}(\mathbf{z},t) = u_{0\alpha}(\mathbf{x},t) + c^k(\mathbf{z})Q_\alpha^k(\mathbf{x},t)\,,$$
$$(\partial_\beta u_{c\alpha})_\mathbf{x}(\mathbf{z},t) = \partial_\beta u_{0\alpha}(\mathbf{x},t) + \partial_\beta c^k(\mathbf{z})Q_\alpha^k(\mathbf{x},t)\,, \tag{4.12}$$
$$\dot{u}_{c\alpha\mathbf{x}}(\mathbf{z},t) = \dot{u}_{0\alpha}(\mathbf{x},t) + c^k(\mathbf{z})\dot{Q}_\alpha^k(\mathbf{x},t)\,,$$

for every $\mathbf{x}\in\overline{\Omega}$, almost every $\mathbf{z}\in\Delta(\mathbf{x})$ and every $t\in(t_0,t_1)$.

The periodic approximations of $w_b(\mathbf{z},t)$, $\partial_{\alpha\beta}w_b(\mathbf{z},\xi,t)$ and $\dot{w}_b(\mathbf{z},t)$ in $\Delta(\mathbf{x})$, $\mathbf{z}\in\Delta(\mathbf{x})$, $\mathbf{x}\in\overline{\Omega}$, have the form

$$w_{b\mathbf{x}}(\mathbf{z},t) = w_0(\mathbf{x},t) + b^K(\mathbf{z})V^K(\mathbf{x},t)\,,$$
$$(\partial_{\alpha\beta}w_b)_\mathbf{x}(\mathbf{z},t) = \partial_{\alpha\beta}w_0(\mathbf{x},t) + \partial_{\alpha\beta}b^K(\mathbf{z})V^K(\mathbf{x},t)\,, \tag{4.13}$$
$$\dot{w}_{b\mathbf{x}}(\mathbf{z},t) = \dot{w}_0(\mathbf{x},t) + b^K(\mathbf{z})\dot{V}^K(\mathbf{x},t)\,,$$

for every $\mathbf{x}\in\overline{\Omega}$, almost every $\mathbf{z}\in\Delta(\mathbf{x})$ and every $t\in(t_0,t_1)$.

Setting $u_{c\alpha}\equiv u_\alpha,\ w_b\equiv w$, we obtain from (2.2) lagrangian $L_{cb}(\mathbf{x},\partial_\beta u_{c\alpha},\dot{u}_{c\alpha},\partial_{\alpha\beta}w_b,w_b,\dot{w}_b)\in HO_\delta^0(\Omega,\Delta)$, $\mathbf{x}\in\overline{\Omega}$. Since L_{cb} is highly oscillating with respect to $\mathbf{x}$ then there exists a periodic approximation $L_{cb\mathbf{x}}(\mathbf{z},(\partial_\beta u_{c\alpha})_\mathbf{x},\dot{u}_{c\alpha\mathbf{x}},(\partial_{\alpha\beta}w_b)_\mathbf{x},w_{b\mathbf{x}},\dot{w}_{b\mathbf{x}})$, $\mathbf{z}\in\Delta(\mathbf{x})$, of L_{cb} in every $\Delta(\mathbf{x})$, where functional arguments of $L_{cb\mathbf{x}}$ are given by means of (4.12), (4.13). Lagrangian $L_{cb\mathbf{x}}$ has the form of lagrangian (2.2) in which

$\partial_\beta u_{c\alpha} \equiv \partial_\beta u_\alpha, \dot{u}_{c\alpha} \equiv \dot{u}_\alpha, \partial_{\alpha\beta} w_b \equiv \partial_{\alpha\beta} w, w_b \equiv w, \dot{w}_b \equiv \dot{w}$ are replaced by $(\partial_\beta u_{c\alpha})_{\mathbf{x}}, \dot{u}_{c\alpha\mathbf{x}}, (\partial_{\alpha\beta} w_b)_{\mathbf{x}}, w_{b\mathbf{x}}, \dot{w}_{b\mathbf{x}}$, respectively. Substituting the right hand sides of approximations (4.12), (4.13) into lagrangian $L_{cb\mathbf{x}}$ and using tolerance averaging formula (3.5) we arrive at *the tolerance averaging of lagrangian* L_{cb} *in* $\Delta(\mathbf{x})$ *under superimposed decomposition* (4.11). Introducing the extra approximation $1+\lambda/r \approx 1$, the obtained result has the form

$$
\begin{aligned}
&< L_{cb} > (Q^k_\alpha, \dot{Q}^k_\alpha, V^K, \dot{V}^K) = \\
&= \frac{1}{2}[< D^{\alpha\beta\gamma\delta}\partial_\beta u_{0\alpha}\partial_\delta u_{0\gamma} > +2 < D^{\alpha\beta\gamma\delta}\partial_\delta c^k \partial_\beta u_{0\alpha} > Q^k_\gamma + \\
&+ < D^{\alpha\beta\gamma\delta}\partial_\beta c^k \partial_\delta c^l > Q^k_\gamma Q^l_\alpha + \\
&+ 2r^{-1}[< D^{\alpha\beta11}\partial_\beta u_{0\alpha} w_0 > + < D^{\alpha\beta11}\partial_\beta c^k w_0 > Q^k_\alpha] + \\
&+ r^{-2} < D^{1111} w_0 w_0 > + < B^{\alpha\beta\gamma\delta}\partial_{\alpha\beta} w_0 \partial_{\gamma\delta} w_0 > + \\
&+ 2 < B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} b^K \partial_{\alpha\beta} w_0 > V^K + < B^{\alpha\beta\gamma\delta}\partial_{\alpha\beta} b^K \partial_{\gamma\delta} b^L > V^K V^L + \\
&- < \mu a^{\alpha\beta}\dot{u}_{0\alpha}\dot{u}_{0\beta} > - < \mu(\dot{w}_0)^2 > + \\
&- \underline{< \mu c^k c^l >} a^{\alpha\beta}\dot{Q}^k_\alpha \dot{Q}^l_\beta - \underline{< \mu b^K b^L >} \dot{V}^K \dot{V}^L]\,.
\end{aligned}
\tag{4.14}
$$

Due to periodic structure of the shell averages $< \cdot >$ on the right-hand side of (4.14) are constant and calculated by means of (3.6).

Functional

$$
A_{cb}(Q^k_\alpha, V^K) = \int_0^{L_1}\int_0^{L_2}\int_{t_0}^{t_1} < L_{cb} > dt dx^2 dx^1 \,,
$$

where $< L_{cb} >$ is given by (4.14), is called *the tolerance averaging of functional* $A(u_\alpha, w)$ defined by (2.1) *under superimposed decomposition* (4.11). The underlined terms in (4.14) depend on microstructure length parameter λ.

The principle of stationary action applied to A_{cb} given above leads to the following system of equations for Q^k_α, V^K

$$
\begin{aligned}
&\frac{\partial}{\partial t}\frac{\partial < L_{cb} >}{\partial \dot{Q}^k_\alpha} - \frac{\partial < L_{cb} >}{\partial Q^k_\alpha} = 0\,, \\
&\frac{\partial}{\partial t}\frac{\partial < L_{cb} >}{\partial \dot{V}^K} - \frac{\partial < L_{cb} >}{\partial V^K} = 0\,.
\end{aligned}
\tag{4.15}
$$

Combining (4.15) with (4.14) we obtain finally the explicit form of the Euler-Lagrange equations

$$
\begin{aligned}
&-< D^{\alpha\beta\gamma\delta}\partial_\beta c^k \partial_\delta c^l > Q^l_\gamma - \underline{< \mu c^k c^l >} a^{\alpha\beta}\ddot{Q}^l_\beta = \\
&= r^{-1} < D^{\alpha\beta 11}\partial_\beta c^k w_0 > + < D^{\alpha\beta\gamma\delta}\partial_\delta c^k \partial_\beta u_{0\gamma} >, \quad k,l = 1,2,...,m,
\end{aligned}
\tag{4.16}
$$

$$
\begin{aligned}
&< B^{\alpha\beta\gamma\delta}\partial_{\alpha\beta} b^K \partial_{\gamma\delta} b^L > V^L \underline{+ < \mu b^K b^L >} \ddot{V}^L = - < B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} b^K \partial_{\alpha\beta} w_0 >, \\
&K,L = 1,2,...,M .
\end{aligned}
\tag{4.17}
$$

Let us observe that in the problem under consideration we have obtained system of governing equations which consists of two independent subsystems. The first from them is the system of $2m$ equations for fluctuation amplitudes Q^k_α, cf. (4.16), whereas the second one is the system of M equations for fluctuation amplitudes V^K, cf. (4.17). The right-hand sides of (4.16) and (4.17) are known under assumption that $u_{0\alpha}, w_0$ were determined in the first step of modelling.

Equations (4.16) and (4.17) have to be considered together with decomposition

$$
\begin{aligned}
&u_\alpha(\mathbf{x},t) = \overline{u}_\alpha(\mathbf{x},t) + h^a(\mathbf{x})U^a_\alpha(\mathbf{x},t) + c^k(\mathbf{x})Q^k_\alpha(\mathbf{x},t), \\
&w(\mathbf{x},t) = \overline{w}(\mathbf{x},t) + g^A(\mathbf{x})W^A(\mathbf{x},t) + b^K(\mathbf{x})V^K(\mathbf{x},t), \\
&\mathbf{x} \in \Omega, \ t \in (t_0,t_1), \quad a = 1,..,n, \quad k = 1,..,m, \quad A = 1,..,N, \ K = 1,..,M,
\end{aligned}
\tag{4.18}
$$

where functions $\overline{u}_\alpha, U^a_\alpha, \overline{w}, W^A$ have to be obtained in the first step of combined modelling, i.e. in the framework of *the consistent asymptotic modelling*. It follows that *the combined model* derived here is represented by

- *macroscopic model* defined by equations (4.8) for $\overline{u}_\alpha, \overline{w}$ with expressions (4.6) for U^a_α, W^A, obtained by means of *the consistent asymptotic modelling* and being independent of the microstructure length; it is assumed that in the framework of this model the solution (4.10) to the problem under consideration is known,
- *superimposed microscopic model equations* (4.16), (4.17) derived by means of *the tolerance (non-asymptotic) modelling*, some coefficients of these equations (underlined terms) depend on the microstructure length parameter λ,
- *decomposition* (4.18)

Coefficients of all equations derived in the framework of combined modelling are constant in contrast to coefficients in equations (2.4) which are discontinuous, highly oscillating and periodic.

The model proposed here can be applied to analyse the length-scale effect in selected problems of dynamics of biperiodically and densely stiffened cylindrical shells under consideration. Moreover, under special conditions it makes it possible to separate the macroscopic description of a certain problem from its microscopic description.

Applying *the tolerance modelling* directly to the decomposition (4.18) we also obtain the system of equations for $\overline{u}_\alpha, \overline{w}, U^a_\alpha, Q^k_\alpha, W^A, V^K$. However, this system is much more complicated then the system obtained in the framework of the combined modelling.

5. MICRO-DYNAMICS OF THE SHELL

Now, we are to show that the combined model, proposed here, makes it possible to study micro-dynamics of periodic shells under consideration independently of their macro-dynamics. To this end, instead of functions $c^k(\cdot), b^K(\cdot)$ in (4.16), (4.17) we introduce fluctuation shape functions $h^a(\cdot), a = 1,..,n,\quad g^A(\cdot), A = 1,..,N$, respectively, setting $n \equiv m, N \equiv M$. By means of the consistent asymptotic modelling we obtain

$$\begin{aligned}
&< D^{\alpha\beta\gamma\delta}\partial_\delta c^k \partial_\beta u_{0\gamma} > + r^{-1} < D^{\alpha\beta11}\partial_\beta h^a w_0 > = < D^{\alpha\beta\gamma\delta}\partial_\delta h^a > \partial_\beta \overline{u}_\gamma + \\
&+ < D^{\alpha\beta\gamma\delta}\partial_\beta h^a \partial_\delta h^b > U^a_\gamma + r^{-1} < D^{\alpha\beta11}\partial_\beta h^a > \overline{w} = 0\,, \quad a,b = 1,2,...,n\,, \\
&< B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} g^A \partial_{\alpha\beta} w_0 > = < B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} g^A > \partial_{\alpha\beta}\overline{w} + \\
&+ < B^{\alpha\beta\gamma\delta}\partial_{\alpha\beta} g^A \partial_{\gamma\delta} g^B > W^B = 0\,, \quad A,B = 1,2,...,N\,.
\end{aligned} \tag{5.1}$$

It means that the right-hand sides of equations (4.16) and (4.17) are equal to zero and the final result is given by equations

$$\begin{aligned}
&< D^{\alpha\beta\gamma\delta}\partial_\beta h^a \partial_\delta h^b > Q^b_\gamma + \underline{< \mu h^a h^b >} a^{\alpha\beta}\ddot{Q}^b_\beta = 0\,, \\
&a,b = 1,2,...,n\,,
\end{aligned} \tag{5.2}$$

$$\begin{aligned}
&< B^{\alpha\beta\gamma\delta}\partial_{\alpha\beta} g^A \partial_{\gamma\delta} g^B > V^B \underline{+ < \mu g^A g^B >} \ddot{V}^B = 0\,, \\
&A,B = 1,2,...,N\,.
\end{aligned} \tag{5.3}$$

Equations (5.2), (5.3) are independent of solutions $u_{0\alpha}$, w_0 obtained in the framework of *the macroscopic model* and hence describe selected problems of the shell micro-dynamics; e.g. the free micro-vibration problem. Moreover, the shell's micro-dynamics in the axial and circumferential directions can be analysed independently of its micro-dynamic behaviour in the direction normal to the shell midsurface.

It has to be emphasized that problems described by (5.2), (5.3) are related to unknown fields $u_\alpha = u_\alpha(\mathbf{x},t)$ and $w = w(\mathbf{x},t)$ by means of $u_\alpha = u_{0\alpha} + h^k Q_\alpha^k$, $w = w_0 + g^K V^K$, where $u_{0\alpha}, w_0$ are determined by the consistent asymptotic modelling.

At the end of this section, using equations (5.2), (5.3) we derive formulae for free micro-vibration frequencies of a certain closed biperiodically stiffened shell. The stiffened shell under consideration is treated as a shell with periodically varying thickness and periodically varying elastic and inertial properties. It is assumed that both the shell and stiffeners are made of homogeneous isotropic materials. We confine ourselves to the simplest form of the combined model in which $a = n = A = N = 1$. It assumed that the fluctuation shape functions are known in the problem under consideration. Let the investigated problem be rotationally symmetric with a period λ / r; hence $Q_1 \equiv Q_1^1$ in (5.2) is equal to zero and the remaining slowly-varying unknowns $Q_2 \equiv Q_2^1$, $V \equiv V^1$ of equations (5.2), (5.3) are independent of argument x^1. Obviously, the highly-oscillating fluctuation shape functions $h \equiv h^1$ and $g \equiv g^1$ are λ-periodic functions of both arguments x^1 and x^2. It is assumed that the edges $x^2 = 0, x^2 = L_2$ are simply supported, i.e. they are hinged with the support free, cf. [27].

Equations (5.2) and (5.3) reduce now to the form

$$(< D^{2112}(\partial_1 h)^2 > + < D^{2222}(\partial_2 h)^2 >)Q_2 + \underline{< \mu(h)^2 >} a^{22}\ddot{Q}_2 = 0\,, \qquad (5.4)$$

$$\begin{aligned}&(< B^{1111}(\partial_{11} g)^2 > + < B^{2222}(\partial_{22} g)^2 > + 2 < B^{1122}\partial_{11} g \partial_{22} g > + \\ &+ 4 < B^{1212}(\partial_{12} g)^2 >)V + \underline{< \mu(g)^2 >}\ddot{V} = 0\,.\end{aligned} \qquad (5.5)$$

Solutions to equations (5.4) and (5.5) will be taken in the form

$$\begin{aligned}Q_2(x^2,t) &= A_Q \cos(kx^2)\cos(\overline{\omega}_* t)\,,\\ V(x^2,t) &= A_V \sin(kx^2)\cos(\omega_* t)\,,\end{aligned} \qquad (5.6)$$

where $A_Q \neq 0, A_V \neq 0$ are micro-vibration amplitudes being arbitrary constants, $k = \pi / L_2$ is a wave number and $\overline{\omega}_*, \omega_*$ are frequencies of free micro-vibrations along the generating lines and in direction normal to the shell midsurface, respectively. It can be observed that solutions (5.6) are slowly-varying functions in argument x^2, because of, under assumption $\lambda / L_2 << 1$, the wave number k satisfies condition: $(k = \pi / L_2) << \pi / \lambda$.

Substituting $(5.6)_1$ and $(5.6)_2$ into (5.4) and (5.5), respectively, under extra denotations

$$\overline{D}^{22} \equiv < D^{2112}(\partial_1 h)^2 > + < D^{2222}(\partial_2 h)^2 >, \quad \mu_h \equiv \lambda^{-2} < \mu(h)^2 >,$$
$$\hat{B} \equiv < B^{1111}(\partial_{11} g)^2 > + < B^{2222}(\partial_{22} g)^2 > + 2 < B^{1122}\partial_{11} g \partial_{22} g > +$$
$$+ 4 < B^{1212}(\partial_{12} g)^2 >, \quad \mu_g \equiv \lambda^{-4} < \mu(g)^2 >,$$

we arrive at formulae for

- free micro-vibration frequency $\overline{\omega}_*$ in axial direction

$$\overline{\omega}_*^2 = \frac{\overline{D}^{22}}{\lambda^2 \mu_h}, \tag{5.7}$$

- free micro-vibration frequency ω_* in direction normal to the shell midsurface

$$\omega_*^2 = \frac{\hat{B}}{\lambda^4 \mu_g}. \tag{5.8}$$

The free micro-vibration frequencies derived above depend on microstructure length parameter λ. Hence, *they cannot be obtained in the framework of the commonly used asymptotic models of periodically stiffened shells*.

6. FINAL REMARKS

Thin linear-elastic Kirchhoff-Love-type circular cylindrical shells with a periodically inhomogeneous structure along the circumferential and axial directions are objects under consideration. Shells of this kind are termed *biperiodic*. As an example we can mention cylindrical shells with periodically spaced families of longitudinal and circular stiffeners as shown in Fig.1. Dynamic and stability behaviour of such shells are described by Euler-Lagrange equations (2.3) generated by the well known Lagrange function (2.2). The explicit form of (2.3), given by (2.4), coincides with the governing equations of the simplified Kirchhoff-Love theory for elastic shells. For periodic shells coefficients of these equations are highly oscillating non-continuous periodic functions. That is why the direct application of equations (2.4) to investigations of specific problems is non-effective even using computational methods.

In this contribution, *the new mathematical non-asymptotic model* for analysis of selected dynamic problems for periodic shells under consideration has been formulated by applying *the combined modelling procedure* given in [11]. Contrary to starting equations, *the resulting combined model equations have constant coefficients and take into account the length-scale effect*. The combined modelling technique is realized in two steps. The first step is based on *the consistent asymptotic averaging of lagrangian* (2.2) *under consistent asymptotic decomposition* (4.1) of the shell displacements. The resulting averaged form of lagrangian (2.2) is given by (4.3). Then, applying the principle of stationary action to *the consistent asymptotic action functional* defined by means of averaged lagrangian (4.3), we arrive at Euler-Lagrange equations (4.8) with constant coefficients which are independent of the microstructure cell size. Hence, the model obtained in the first step is referred to as *the macroscopic model*. Assuming that in the framework of the macroscopic model the solution (4.10) to the problem under consideration is known, we can pass to the second step. This step is based on *the tolerance averaging of lagrangian* (2.2) *under superimposed decomposition* (4.11). The resulting tolerance averaged form of lagrangian (2.2) is given by (4.14). Then, applying the principle of stationary action to *the tolerance averaged action functional* defined by means of averaged lagrangian (4.14), we arrive at Euler-Lagrange equations (4.16), (4.17) with constant coefficients which depend on the cell size (underlined terms). Hence, the model obtained in the second step is referred to as *the superimposed microscopic model*. Thus, *the new combined model*, proposed here, is represented by *macroscopic model equations* (4.8) together with expressions (4.6) and solution (4.10) and by *superimposed microscopic model equations* (4.16), (4.17) as well as by *decomposition* (4.18).

The important advantages of the new shell model proposed here are listed below.

- *The coefficients of the combined model equations are constant and some of them depend on the microstructure length parameter* λ. It means that the proposed model equations describe the effect of the cell size on the overall shell dynamics. Hence, they can be used to the analysis of many phenomena caused by the length-scale effect, e.g. for investigations of *the additional higher-order free vibration frequencies* occurring in the periodic shells.
- The resulting combined model equations are uniquely determined by the postulated *fluctuation shape functions,* which describe fluctuations of the shell displacements inside the cell from the qualitative point of view. The fluctuation shape functions introduced into macroscopic model by means of decomposition (4.1) can be obtained by a certain periodic discretization of

the cell while those introduced into superimposed microscopic model by means of decomposition (4.11) represent either the principal modes of the free periodic vibrations of the cell or physically reasonable approximations of these modes. In most problems the fluctuation shape functions specified in the first and second steps of combined modelling are different due to the different character of the macroscopic and the superimposed microscopic models. However, from the formal point of view the fluctuation shape functions of both the models can coincide.

- Under assumption that *the fluctuation shape functions* introduced in the first step of combined modelling coincide with those introduced in the second step, we have derived superimposed microscopic model equations (5.2), (5.3) which are independent of the solutions obtained in the framework of the macroscopic model. Taking into account this result we can conclude that *an important advantage of the combined model is that it makes it possible to separate the macroscopic description of some special problems from their microscopic description.* It means that in the framework of the combined model we can study micro-dynamics of periodic shells under consideration independently of their macro-dynamics.

Using superimposed microscopic model equations (5.2) and (5.3), the free micro-vibration frequencies caused by a periodic structure of a certain biperiodically stiffened shell have been derived independently of the macro-vibration frequencies. The results given by means of (5.7) and (5.8) depend on the microstructure length and *cannot be obtained in the framework of the commonly used asymptotic models for dynamic analysis of periodically stiffened shells*.

It is worth noting that the combined model for analysis of dynamic and stability problems for cylindrical shells with one-directional periodic structure (*uniperiodic shell*) was proposed and discussed in [24]. We recall that the tolerance models of uniperiodic shells are not the special cases of the tolerance models of biperiodic shells.

More detailed discussion of the combined model for dynamic analysis of biperiodic shells proposed in this contribution, will be presented separately.

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MODELOWANIE ZAGADNIEŃ DYNAMIKI W BIPERIODYCZNIE UŻEBROWANYCH POWŁOKACH WALCOWYCH

S t r e s z c z e n i e

W pracy wyprowadzono nowy nieasymptotyczny model służący do analizy dynamiki cienkich liniowo-sprężystych powłok walcowych typu Krchhoffa-Love'a, periodycznie i gęsto użebrowanych w dwóch kierunkach stycznych do powierzchni środkowej powłoki. Do wyprowadzania równań wykorzystano *technikę „combined modelling"* zaproponowaną w monografii [11]. Modelowanie jest dwuetapowe. W pierwszym etapie, stosując procedurę modelowania asymptotycznego, otrzymuje się *model makroskopowy* rozważanych powłok, mający stałe współczynniki, które nie zależą od długości okresu periodyczności mikrostruktury. Zakładając, że rozwiązanie danego problemu brzegowo-początkowego w ramach modelu makroskopowego jest znane, przechodzi się do etapu drugiego, w którym w oparciu o technikę modelowania tolerancyjnego wyprowadza się równania *modelu mikroskopowego nałożonego na model makroskopowy etapu pierwszego*. Równania modelu mikroskopowego mają stałe współczynniki zależne od wielkości komórki periodyczności. Proponowany *„combined model"* może być zastosowany do badania efektu skali w zagadnieniach dynamiki mikroperiodycznych powłok walcowych. *Zaletą modelu jest to, że umożliwia rozdzielenie makroskopowego opisu szczególnych zagadnień dynamiki powłok od ich opisu mikroskopowego*.