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LIMIT ANALYSIS OF STEEL-CONCRETE COMPOSITE STRUCTURES WITH SLIP

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The paper proposes a modeling of the load carrying capacity of steel-concrete composite structures with slip. An analytical approach to the load carrying capacity calculation of multilayered composite elements and structures with slip is presented. The interaction of layers of composite beams and bars on the example of two- and three-layers, including thin-wall ones, was analyzed. The results are compared with numerical calculations using a program based on the finite element method. Then the problem of the limit analysis for the spatial skeleton structures is formulated as an optimization problem.

Keywords: limit analysis, steel-concrete composite structures, beams and bars, slip, FEM, optimization

1. INTRODUCTION

Steel-concrete composite structures mean connection all element parts of the construction in such a way that in the calculations could be treated as one system [9, 14, 21]. The tie of layers is made using different types of connectors. Due to the flexibility of the connection it is not always achieved full contact of layer of composite structures.

The problem of interaction of layers for composite structures has been the subject of many works, theoretical and experimental, for example [8, 10-12, 15-19, 22, 24], but it was not explained until the end.

The paper presents an analytical approach to calculate the limit load capacity of multilayered composite structural elements with slip. The problem of interaction layers was presented. The numerical calculations is performed on the base of program Abaqus/Standard [1] for beams with two-and three-layers,

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simply supported at ends and loaded with concentrated force in the middle of the span. Then, the bearing capacity of the complex skeleton structures has been formulated as a problem of optimization.

The analytical approach assumes perfectly plastic bearing layers material (reinforced concrete and steel) and continuous or discrete constraints in perfectly plastic contact of these layers.

In the numerical calculations of composite structures were included real constitutive relationships for material layers.

2. ANALYTICAL SOLUTIONS FOR COMPOSITE BEAM AND BARS WITH SLIP

2.1. Two-layer composite reinforced concrete-steel beam

First let consider a limit state of load carrying capacity for two-layer reinforced concrete-steel beam with slip loaded by the forces *F* (Figure 1). The cross-section of beam is constant. In this Figure „1" denotes a plastic hinge in cross-section of element with slip; "2" is the same, for element with a full connection, without slip.

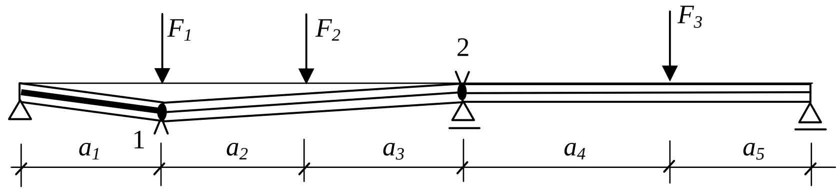

Fig.1. Two-layer reinforced concrete-steel beam in the limit state with slip

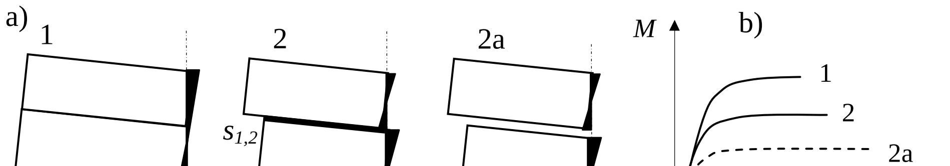

Fig. 2. Two-layer beam, a) destruction mechanisms 1…2a; b) relationship moment *M* vs angle φ for mechanisms 1…2a

For the typical finite element of beam there are possible the different mechanisms of element destruction (see Figure 2.a), including plastic hinge in cross-section with full connection of layers 1, the same for layers with slip 2 and next with brittle destruction of connectors 2a. The relationships between bending moment M and corresponding angle φ in cross-section for destruction mechanisms 1…2a are shown in Figure 2.b.

We analyse here a general case (Fig. 2.a,2) for typical element of the reinforced concrete-steel beam in the limit state with slip $s_{1,2}$ under the forces F and moments M_l, M_0. The normal σ_{0i} and tangent τ_{0i} stresses in the *i*-layers section are shown in Figure 3, i = [1:2]. Distributed forces $t_{01,2}$ in the connectors at the surface between layers 1,2 are assumed constant at length interval *a*.

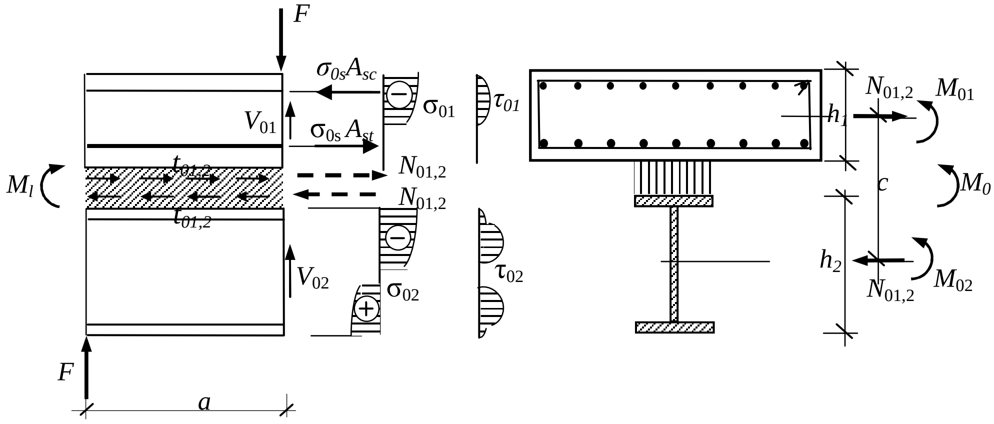

Fig.3. Forces and stresses in the reinforced concrete-steel beam in the limit state of load carrying capacity with slip

The contact forces for continuous or discrete constraints $N_{01,2}$ and the bending moment M_0 in cross-section may be written as follows:

$$N_{01,2} = at_{01,2} \quad \text{or} \quad N_{01,2} = \sum_{j\in J} a_j t_{01,2j}, \tag{2.1}$$

$$M_0 = M_{01} + M_{02} + N_{01,2}c, \tag{2.2}$$

where a_j, $t_{01,2j}$ are length and distributed force in the *j*-connector constraints; J is a set of *j*-connectors; c is the distance between the centers of weight (or stiffness) of the individual *i*-layers [3], M_{0i} is the plastic bending moment in *i*-layer, i = [1:2].

The value of c determines the possibility of layers slip in the limit state: when $c > (h_1 + h_2)/2$, then the maximum bending moment M_{0*} is formed in the cross-section with slip; when $c = (h_1 + h_2)/2$, then the plastic hinge is formed in the cross-section like for whole composite bar without slip (Fig. 2.a,1) with the bending moment M_{0*0} [24]. Finally for the scheme (Fig. 2.a,2a) in the Eqns (2.1) we have $t_{01,2}$ =0.

For calculating of the value c can be also used numerical approach. Very important will be to identify parameter c adopted in analytical and numerical model. The parameters of these models can be obtained in laboratory tests.

Bending moment M_0 can be moreover calculated for the whole element,

$$M_0 = M_l + Fa. \tag{2.3}$$

Internal forces in layers 1, 2 are limited by conditions of plasticity or strength

$$\varphi_1(N_{01,2}, V_{01}, M_{01}, K_1) \le 0, \tag{2.4}$$

$$\varphi_2(N_{01,2}, V_{02}, M_{02}, K_2) \le 0, \tag{2.5}$$

where K_1, K_2 are constant plastic/strength parameters. For the reinforced concrete layer, with rectangular cross-section and asymmetric reinforcing (Figure 3, $A_{st} > A_{sc}$), the function $\varphi(\cdot)$ is shown in Figure 4. This function can be calculated algorithmically [3, 21] using different computer programs, for example [4, 6, 23]. For the thin-walled steel layer function $\varphi(\cdot)$ can be calculated iteratively, taking into account the effective cross-section, for example, by PN-EN 1993-1-1 [13, 20]. For the I-section of beam in the case of loading by bending moment M with shear force V and longitudinal force N, function $\varphi(\cdot)$ for a limit surface M - N - V is shown in Figure 5.

In the limit state of element the load F will be maximum, max$F = F_{*0}$, or

$$F \to \max. \tag{2.6}$$

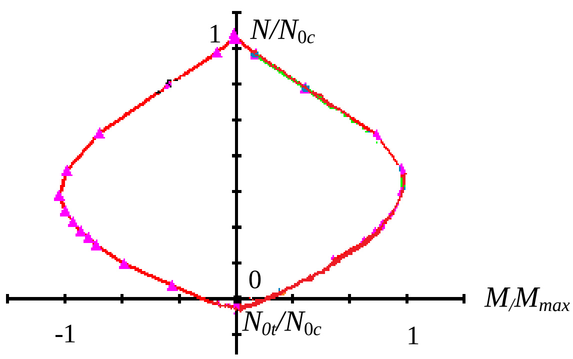

Fig. 4. Function of strength $\varphi(M, N)$ for the reinforced concrete layer

We can solve the optimization problem (2.1)-(2.6), replacing inequalities (2.4), (2.5) on equations, and from relationships (2.1)-(2.5) define the maximum load F_{*0} of element and bending moment M_{0*0} in cross-section.

Then we calculate the whole beam. For the beam as in Figure 1 the problem is not difficult to direct analysis. General case for complex spatial structures will be examined in 3rd Part.

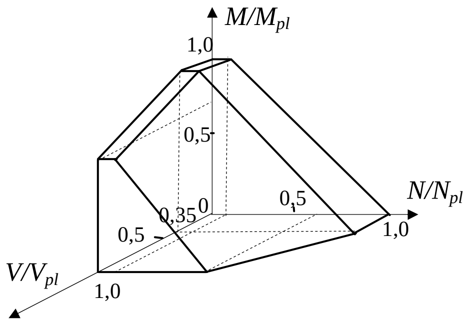

Fig. 5. Limit surface for *M* - *N* - *V* of steel I-beam cross section by PN-EN 1993-1-1

2.2. Three- and multilayer composite reinforced concrete-steel beam

For the calculation of three-layer reinforced concrete and steel beam was distinguished a typical element loaded by the forces and moments as shown in Figure 6.

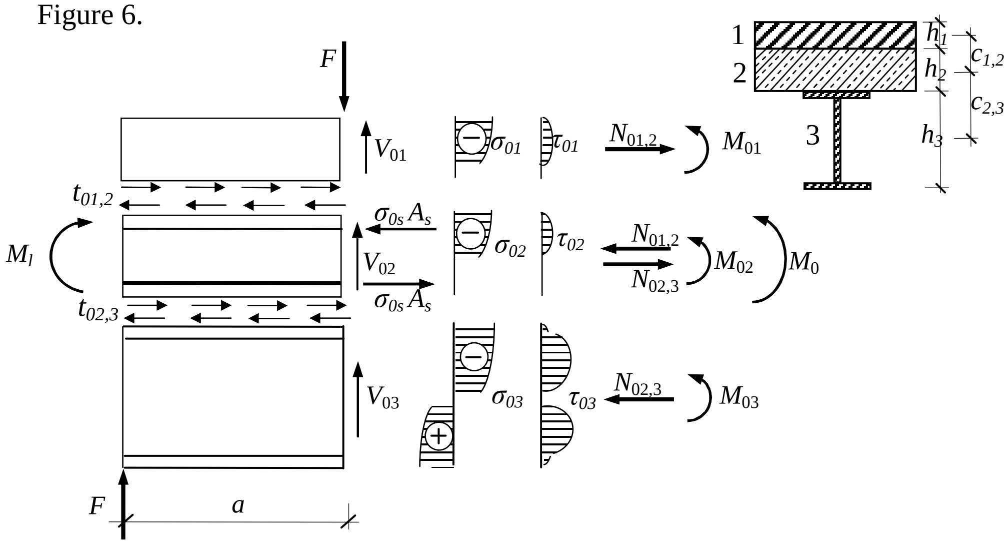

Fig. 6. Forces and stresses in the three-layer beam in the state of load carrying capacity with slip

The mechanisms of element destruction, including full connection 1 and different cases for yuielding of connectors 2…4 are presented in Figure 7.a. The relationships between bending moment M and corresponding angle φ in cross-section for these destruction mechanisms 1…4 are shown in Figure 7.b.

We assume first that slip occurs as a result of yuielding of connectors 1, 2 and 2, 3 for layers 1...3 (Fig.7.a,4).

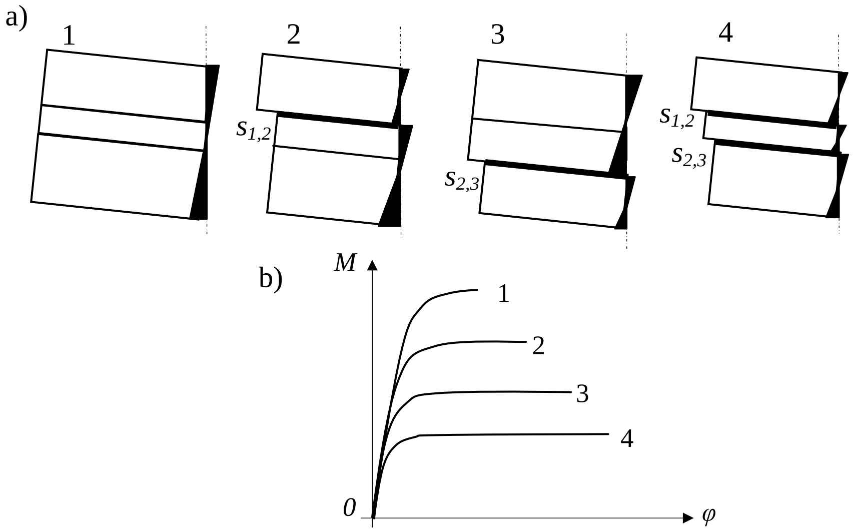

Fig. 7. Three-layer beam, a) destruction mechanisms 1…4; b) relationship moment M vs angle φ for mechanisms 1…4

The contact forces at the surfaces of layers and the bending moment in cross-section can be written as:

$$N_{01,2} = at_{01,2}, \qquad N_{02,3} = at_{02,3}, \tag{2.7}$$

$$M_0 = M_{01} + M_{02} + M_{03} + N_{01,2}c_{1,2} + N_{02,3}c_{2,3}, \tag{2.8}$$

where $t_{01,2}$ i $t_{02,3}$ are distributed forces in the connectors, respectively 1,2 and 2,3 for layers 1...3. Bending moment M_0 can be also written as (Figure 6):

$$M_0 = M_l + Fa. \tag{2.9}$$

Conditions of plasticity/strength in the layers 1...3 are defined as:

$$\varphi_1(N_{01,2}, V_{01}, M_{01}, K_1) \le 0, \tag{2.10}$$

$$\varphi_2(N_{01,2}, N_{02,3}, V_{02}, M_{02}, K_2) \le 0, \tag{2.11}$$

$$\varphi_3(N_{02,3}, V_{03}, M_{03}, K_3) \le 0, \qquad (2.12)$$

where the parameters K_1, ..., K_3 are similar to Part 2.1.

We solve the optimization problem (2.6)-(2.12), where inequalities (2.10)-(2.12) are replaced by the equations and from these relationships define the maximum load F_{*1} of element and bending moment M_{0*1} in cross-section.

Next, we assume that slip occurs as a result of yuielding for connectors 1,2 (Fig. 7.a,2) or connectors 2,3 in layers 1...3 (Fig. 7.a,3).

Like in Part 2.1, define the maximum loads F_{*2}, F_{*3} of composite beam and bendig moments M_{0*2}, M_{0*3} in cross-section with slip and force F_{*0} and bendig moment M_{0*0} in cross-section without slip (Fig. 7.a,1).

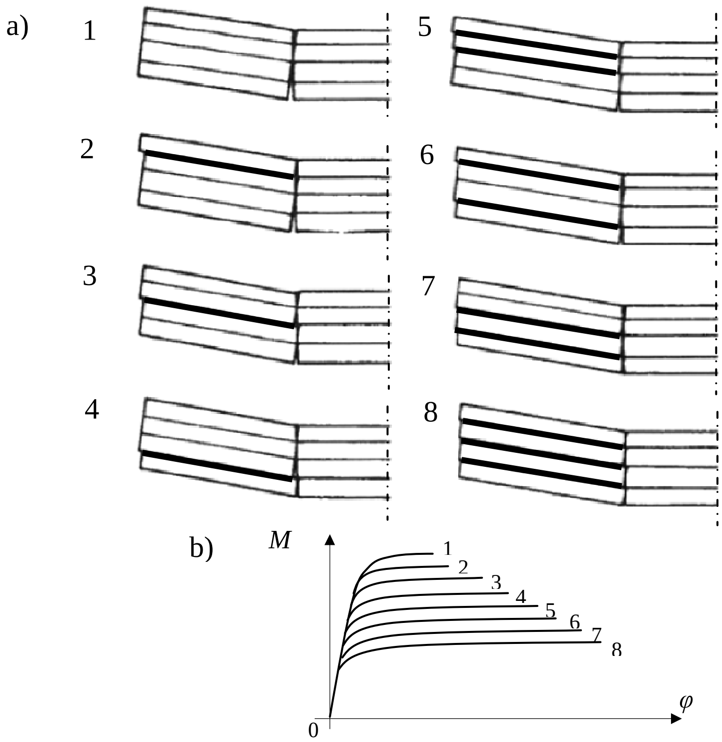

Fig. 8. Four-layer beam, a) destruction mechanisms 1…8; b) relationship moment M vs angle φ for mechanisms 1…8

Then the maximum load and bending moment will be as follows:

$$F = \max\{F_{*0}, F_{*1}, F_{*2}, F_{*3}\}, \quad M_{0*} = \min\{M_{0*0}, M_{0*1}, M_{0*2}, M_{0*3}\}. \qquad (2.13)$$

Finally we calculate the whole composite structures (see Part 3).

Similarly, the same formulae can be derived for multilayer elements and two or three-dimentional bars in skeleton sctructures with slip, where generalized forces (normal and shere forces, bending and twisting moments) and generalized plastic hinges are forming in the cross-section.

For the multilayer elements with m slip surfaces we have a set J of mechanisms of destruction, where $|J| = 2^m$. The case of four-layer beam, m =3 and $|J|$ = 8, is presented in Figure 8.

3. OPTIMIZATION PROBLEMS FOR CARRYING CAPACITY OF THE COMPOSITE STRUCTURES

In contrast to the optimization problem for the limit load analysis of usual structures with full-contact of layers [2, 5, 7, 25], we have to take into account here the possibility of slip for the composite ones (Figure 9). The hinges 1, 2 here were early defined in Figure 1.

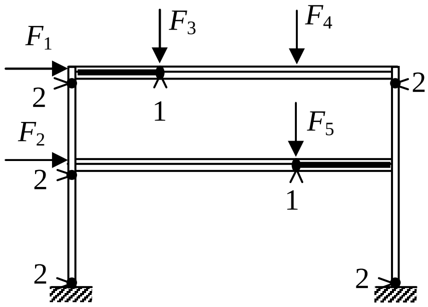

Fig. 9. Mechanism of the destruction for the composite frame, taking into account slip

Then the problem in the vector-matrix form (for the static principle) is formulated as follows.

We have to maximize the limit load multiplier μ for the vector load μ***F***,

$$\mu \to \max, \tag{3.1}$$

with restrictions:

- conditions of equilibrium of moments ***M*** in cross-sections and construction loads μ***F***,

$$\mathrm{A}\boldsymbol{M} = \mu \boldsymbol{F} \tag{3.2}$$

- conditions cross-sections yielding

$$\boldsymbol{M} - \min\{\boldsymbol{M}_{0*j}, j \in J\} \leq 0, \tag{3.3}$$

where $\boldsymbol{M}_{0*j},\ j \in J,$ is vector limit moments for j-mechanisms of destruction; J is a set of mechanisms of destruction, $|J| = 2^m$; m is a number of slip surfaces.

For the generalized forces in the spatial skeleton structures (normal N and shear V forces, bending M and twisting T moments) we apply the principle of generalized plastic joints; vector of moments $\boldsymbol{M}$ in equations (3.2) will be changed on a internal forces vector $\boldsymbol{S}$, $\boldsymbol{S} = (N, V_y, V_z, T, M_y, M_z)$,

$$\boldsymbol{AS} = \mu \boldsymbol{F} \tag{3.4}$$

and in conditions (3.3) of plasticity of cross-sections [2, 24],

$$\max_j \varphi_j(\boldsymbol{S}, \boldsymbol{K}_j, j \in J) \le \boldsymbol{0}, \quad j \in J, \tag{3.5}$$

where $\boldsymbol{K}_j,\ j \in J,$ denote the set of vectors with constant parameters, similar as in Parts 2.1 and 2.2.

The system of relations (3.1) - (3.3) is a linear programming problem while the system (3.1), (3.4), (3.5) belongs to non-linear programming problems for composite structures in the limit state of load carrying capacity with slip.

4. NUMERICAL ANALYSIS OF COMPOSITE BEAMS

The numerical solution of problem was found by the finite element method (FEM), using program Abaqus/Standard [1].

Calculations were carried out for the composite steel-concrete beams of following types: two-layer beam with height $h=h_2+h_3$ and three-layer one with an additional layer with height h_1 (Figure 10). Beams of length L = 5 m were simply supported at ends, and loaded by concentrated forces F at the center of span.

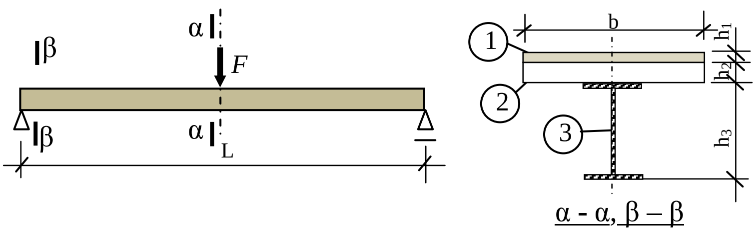

Fig. 10. Scheme with loading and cross-section of composite beam

The first beam has a concrete plate with a thickness h2 = 12 cm and width b = 80 cm connected with a steel I-beam (PN-300) with height h3 = 30 cm. The second beam has an additional concrete layer with a thickness h1 = 6 cm.

The material parameters for the components of steel beam and concrete middle layer were taken from paper [18]: for compressed concrete the modulus of elasticity E_c = 30745 MPa, compressive strength f_c =32,37 MPa and compressive strain, corresponding to the maximum compressive strength, ε_{c1} = 0,0022. For steel the yield stress were taken 273,0 MPa. And for additional concrete layer modulus of elasticity is equal E_c = 2000 MPa.

The numerical calculations were made for the following material models: a) plasticity with hardening for concrete; b) ideal plasticity for steel.

In the FEM analysis the concrete plates were modeled using eight-node solid elements (C3D8R), for the steel beam was used shell elements (S4R).

Were analyzed the following contact cases between composite beam layers:

1. Two-layer beam: a) full contact (without slip, Fig. 2.a,1); b) a contact, in which it is possible the slip between the upper surface of the steel beam and the lower plane of the plate (Fig. 2.a,2).
2. Three-layer beam: a) full contact (without slip, Fig. 7.a,1); b) slip between upper layer of concrete and upper surface of the middle concrete plate and full contact between upper surface of the steel beam and lower surface of the middle concrete plate (Fig. 7.a,2); c) slip between upper surface of the steel beam and lower surface of the middle concrete plate and full contact between upper layer of concrete and upper surface of the middle concrete plate (Fig. 7.a,3); d) slip in each plane of contact (Fig. 7.a,4).

Full contact of the individual layers of beams was carried out in Abaqus as a continuous contact of type „tie”. This way of connection of the layers to ensure the continuity of displacements was used, for instance, in the paper [12].

Flexibility of connection was modeled by definition of contact taking into account the slip between the layers of the beam. It was carried out by introducing the coefficient of friction μ between the layers: for concrete-steel contact μ = 0,5 and for the concrete-concrete contact μ = 0,6.

The aim of numerical calculation was to estimate limit load capacity of beams for different ways of connection modeling (full contact or contact with slip). The results of calculations are shown in Figures 11, 12.

Figure 11 presents the relationship between load F and the vertical displacement u_2 in the middle of the span of two-layer composite beam.The limit load value for the two-layer beam with full contact was estimated as 325 kN. If the beam model takes into account slip, this force is much smaller and amounts 200 kN.

For the three-layer beam with full contact the load limit value is about 500 kN. For individual contact cases is obtained lower values of the limit load. Decrease of beam carrying capacity is shown in Figure 12.

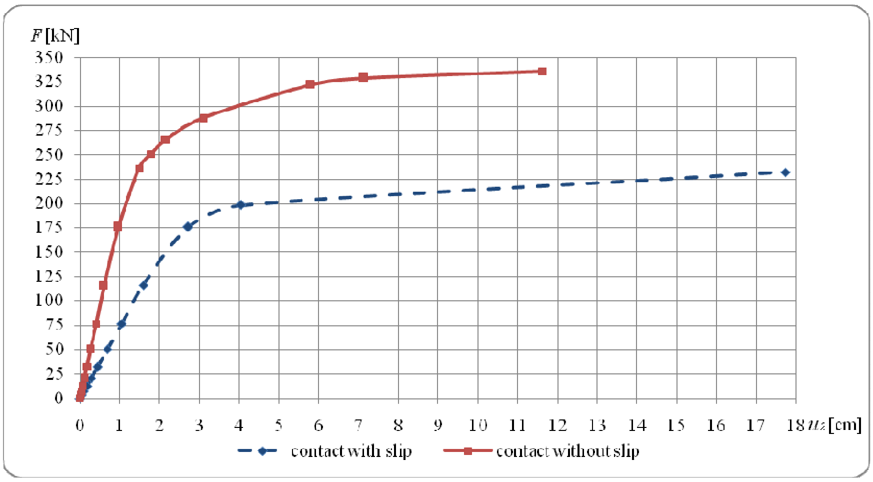

Fig. 11. The relationship of load F vs the vertical displacement u_2 in the middle of the span of two-layer composite beam

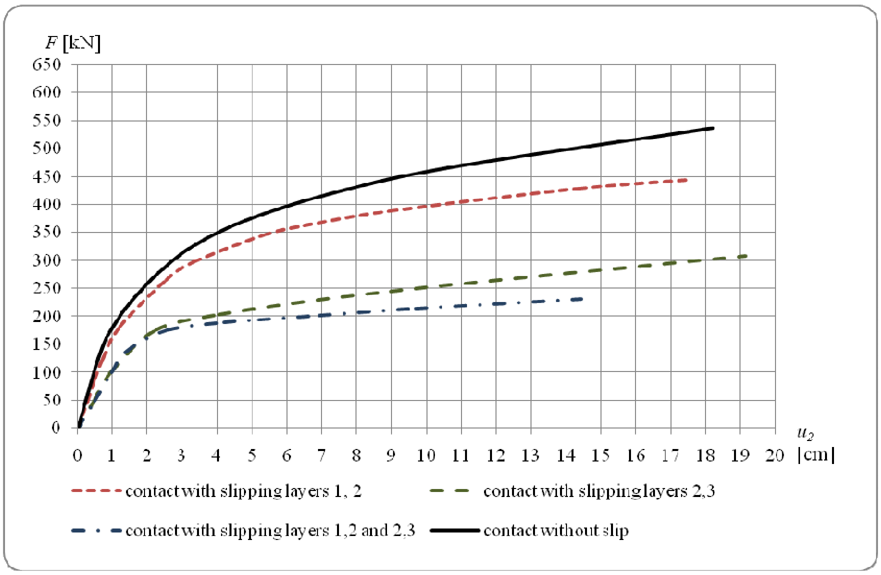

Fig. 12. The relationship of load F vs the vertical displacement u_2 in the middle of the span of three-layer composite beam

The displacement $s_{1,2}$ of concrete plate relatively to the steel beam for two-layer beams under vertical load F is shown in Figure 13.

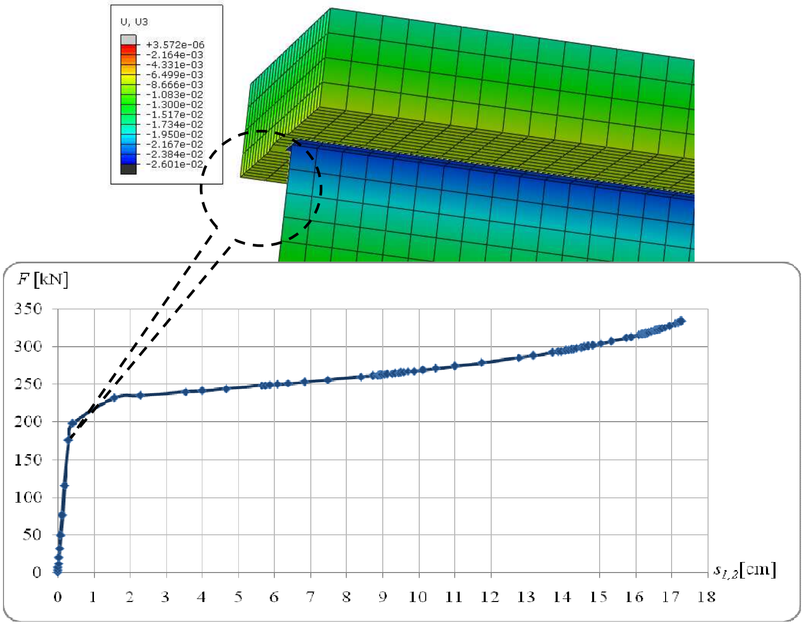

Fig. 13. The relationship of slip $s_{1,2}$ vs vertical force F in two-layer beam

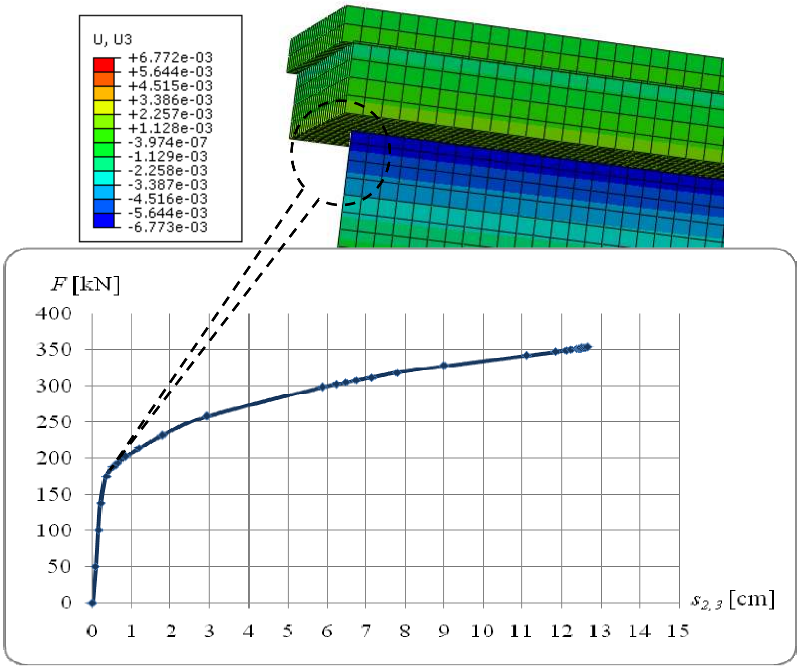

Fig. 14. The relationship of slip $s_{2,3}$ vs loading force F in three-layer beam

Relationship between slip $s_{1,2}$ and vertical force F is non-linear for the whole loading range. This applies to both the two- and three-layer beams taking into account the slip in each contact surface (Figure 14). The nonlinearity of relationship *F-s* is clearly greater beyond 80% of load limit.

Figure 15 shows the von Mises stress σ_{red} distribution in the middle section α-α for the limit state (at a load F = 325 kN) of the two-layer beam without slip.

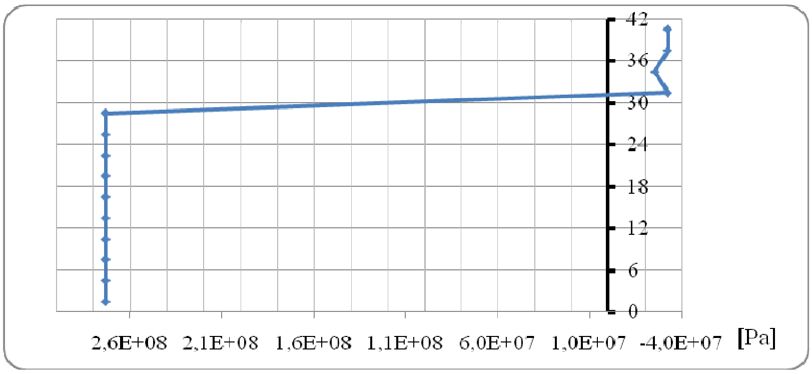

Fig. 15. Von Mises stresses σ_{red} in section α-α of two-layer beam without slip

The fields of stresses σ_{red} for concrete plate and steel I-beam without slip at a load F = 325 kN are shown in Figure 16.

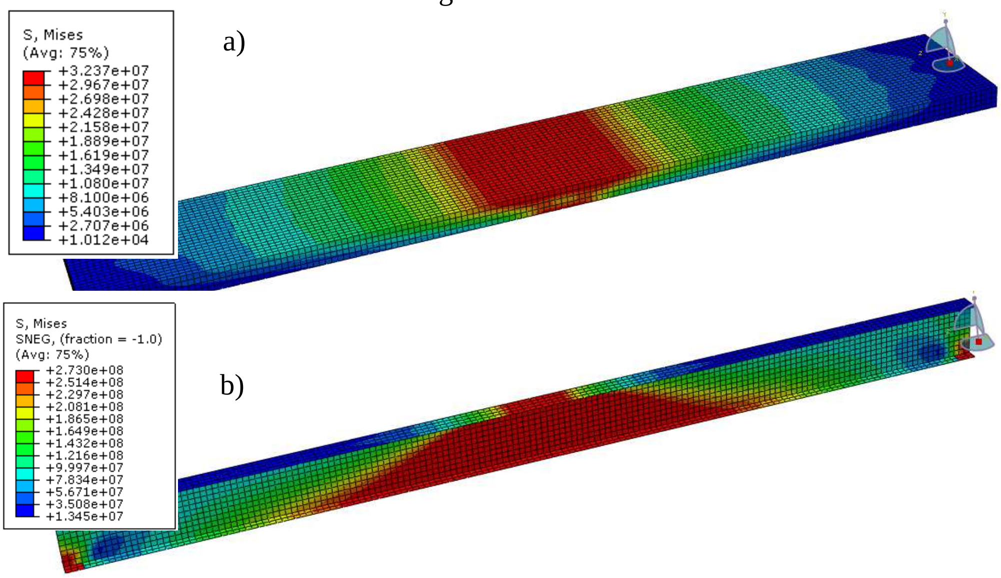

Fig. 16. The fields of stresses σ_{red} for two-layer beam without slip: a) in the plate, b) in the steel beam

The method of modeling of contact between layers have an influence on the strain distribution in the beam cross-section. In the case composite beams with slip at maximal loading F = 200 kN we can observe a discontinuity of strain in plane of contact (Figure 17) for the section β-β (see Figure 10) in contrast to beams with full contact of layers.

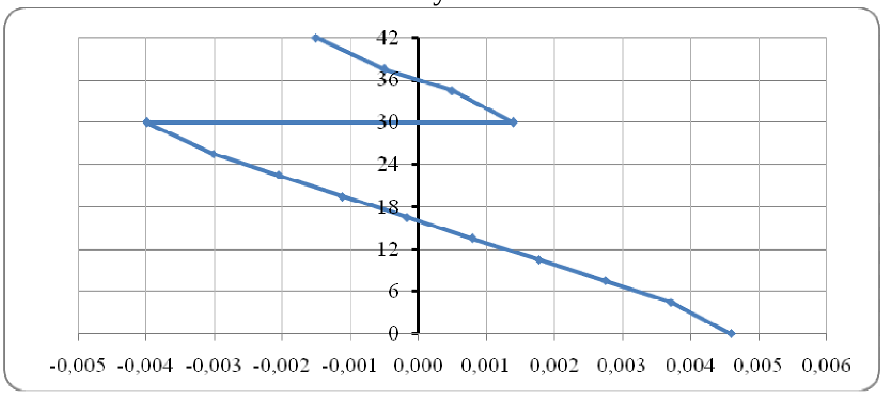

Fig. 17. Strains in section β-β of two-layer beam with slip

5. SUMMARY

The paper proposes a modeling of the load carrying capacity of steel-concrete multilayer composite structures with slip on the example of two-and three-layer beams. It presents an analitical approach for calculating the limit load capacity of composite elements and structures as an optimization problem. Analytical results are compared with numerical calculations. An approach given in the paper can be applied to composite structures, in which the layers are made from another materials. Further analysis can involve the limit load determining for beams and frames with more complex elements with different mechanisms of destructure.

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NOŚNOŚĆ GRANICZNA ZESPOLONYCH STALOWO-BETONOWYCH KONSTRUKCJI Z POŚLIZGIEM

S t r e s z c z e n i e

W referacie zaproponowano modelowanie nośności granicznej zespolonych stalowo-betonowych konstrukcji z poślizgiem. Przedstawiono analityczne podejście do obliczania nośności granicznej wielowarstwowych elementów i konstrukcji zespolonych z poślizgiem. Przeanalizowano współdziałanie warstw konstrukcji zespolonych, w tym cienkościennych, na przykładzie belek dwu- i trójwarstwowych. Wyniki zostały porównane z obliczeniami numerycznymi wykonanymi przy użyciu programu opartego o Metodę Elementów Skończonych. Następnie zagadnienie nośności granicznej przestrennych konstrukcji szkieletowych zostało określone jako problem optymalizacji.