CIVIL AND ENVIRONMENTAL ENGINEERING REPORTS

No. 10 2013

APPLICATION OF FUNDAMENTAL SOLUTIONS TO THE STATIC ANALYSIS OF THIN PLATES SUBJECTED TO TRANSVERSE AND IN-PLANE LOADING

Zdzisław PAWLAK*
Institute of Structural Engineering
Poznan University of Technology, Piotrowo 5, 61-138 Poznań

In this paper static analysis of Kirchhoff plates is considered. A transverse and in-plane loading is taken into consideration. The Finite Strip Method is used and the suitable fundamental solutions are applied. According to the finite strip method a continuous structure is divided into a set of identical elements simply supported on opposite edges. The unknowns are deflections and transverse slope variables along the nodal lines. The finite difference formulation is applied to express the equilibrium conditions of the discrete system. This reduces the number of degrees of freedom. The solution of a difference equation of equilibrium yields the fundamental function of the considered plate strip. The fundamental solution derived in this way, can be used to solve the static problem of a finite plate in the analogous way as the boundary element method is applied for continuous systems.

Keywords: Fundamental solutions, finite strip method, Kirchhoff plates, initial stability and static analysis

1. INTRODUCTION

The Finite Strip Method (FSM) was created as a numerical tool to solve specific engineering problems [8, 9]. This method is the alternative to the most popular Finite Element Method. Application of FSM does not require high number of degrees of freedom. The choice of the FSM to analyse structures requires finding and applying some types of functions called fundamental functions or fundamental solutions. A fundamental solution describes the behaviour of an infinite structure in the sense of generalized displacements and forces caused by a specific type of external loading.

* Corresponding author. E-mail: zdzislaw.pawlak@put.poznan.pl

The Boundary Element Method (BEM) which is often used in the thin and thick plates theory [2, 7, 11], was used to establish the critical forces. For the initial stability problem, the modified approach to the thin plate analysis with an assumed physical boundary condition was proposed by Guminiak and Sygulski [5], and also Guminiak [4]. Modelling of the plate bending problem with in-plane loading requires a modification of the governing boundary integral equation. It is necessary to introduce a set of internal collocation points in which the plate curvature should be found. The analysis of plates with a wide range of arbitrary shapes by BEM was discussed by Katsikadelis [6]. The author used the Analog Equation Method combined with BEM to establish distribution of in-plane forces, calculate critical forces and solve static problem with known in-plane forces. He presented the classic formulation of thin plate bending with corner concentrated forces and equivalent shear forces.

In this paper the critical forces were derived using the boundary element method and the procedure described by Guminiak [4, 5]. Moreover, the critical forces were derived analytically using the formula given in [12]. The static analysis based on the finite strip method (FSM) of an infinite plate strip with transverse and normal loading leads to the fundamental functions for the considered structure. A plate structure infinite in one direction, simply supported on its opposite edges is considered. The plate strips with such boundary conditions are commonly applied as bridge structures, as box or plate elements.

2. STATIC ANALYSIS OF A PLATE

According to the finite strip method [8] the continuous body is approximated by the regular mesh of identical finite strips of arbitrary width b and length L (see Fig. 1).

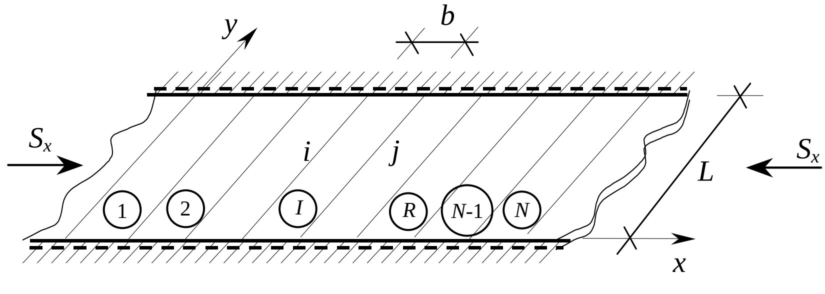

Fig. 1. An infinite plate strip discretization

The unknowns are deflections and transverse slope amplitudes along the nodal lines. Assuming a simply supported, four-degree-of-freedom finite strip for discretization (Fig. 2),

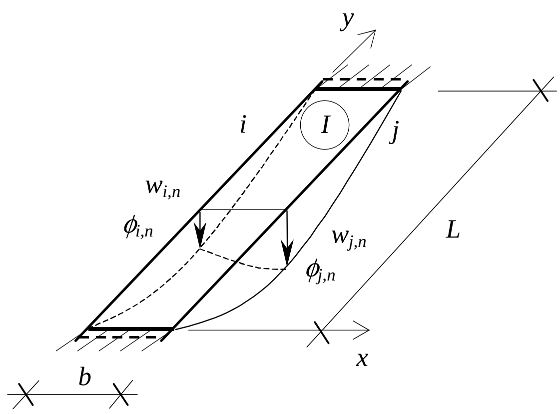

Fig. 2. A finite plate strip

the field of displacements for an arbitrary strip I is expressed in the combined form of harmonic series expansion:

$$w^I(x,y) = \sum_{n=1}^{\infty} \mathbf{N} \cdot \mathbf{q}_n^I \cdot \sin\frac{n\pi y}{L} \tag{1}$$

where: $\mathbf{q}_n^I = [w_{i,n} \quad \phi_{i,n} \quad w_{j,n} \quad \phi_{j,n}]^T$ is the vector of displacement amplitudes for n–th harmonic, $\mathbf{N}=[N_1 \;\; N_2 \;\; N_3 \;\; N_4]^T$ is the shape functions vector consisting of the well-known Hermite polynomials:

$$N_1 = 1 - \frac{3x^2}{b^2} + \frac{2x^3}{b^3}, \quad N_2 = x - \frac{2x^2}{b} + \frac{x^3}{b^2},$$
$$N_3 = \frac{3x^2}{b^2} - \frac{2x^3}{b^3}, \quad N_4 = -\frac{x^2}{b} + \frac{x^3}{b^2}. \tag{2}$$

For another boundary conditions of a finite strip one may use more complex trigonometrical functions in equation (1), i.e. for clamped edges $w^I(x,y) = \sum_{n=1}^{\infty} \mathbf{N} \cdot \mathbf{q}_n^I \cdot \frac{1}{2}\left(1 - \cos\frac{2n\pi y}{L}\right)$. The total displacements at the i-th nodal line may be derived as the sum of amplitudes obtained for an arbitrary n-th element of the harmonic series:

$$w_i = \sum_{n=1}^{\infty} w_{i,n} \cdot \sin\frac{n\pi y}{L} \ , \ \ \phi_i = \sum_{n=1}^{\infty} \phi_{i,n} \cdot \sin\frac{n\pi y}{L} \ , \ \ \phi_i = \frac{\partial w_i}{\partial x} . \tag{3}$$

Using the displacement functions (1) in the minimization procedure for the potential energy formula:

$$U_p^I = \frac{1}{2}\int_0^L\int_0^b \left(-M_x(w)\frac{\partial^2 w}{\partial x^2} - M_y(w)\frac{\partial^2 w}{\partial y^2} + 2M_{xy}(w)\frac{\partial^2 w}{\partial x \partial y} + S_x(w)\frac{\partial w}{\partial x} \right) dxdy \, , \tag{4}$$

we obtain the set of infinite number of linear equations:

$$\sum_{I=-\infty}^{\infty} \mathbf{K}^I \cdot \mathbf{q}^I + \sum_{I=-\infty}^{\infty} \mathbf{G}^I \cdot \mathbf{q}^I = \mathbf{P}^I \, , \tag{5}$$

where: $M_x(w)$, $M_y(w)$, $M_{xy}(w)$ are appropriate bending moments, $S_x(w)$ is the axial force, $\mathbf{q}^I = \begin{bmatrix} w_i & \phi_i & w_j & \phi_j \end{bmatrix}^T$ and $\mathbf{P}^I = \begin{bmatrix} T_i & m_i & T_j & m_j \end{bmatrix}^T$ are the displacement and force vectors for I-th strip, respectively, $\mathbf{K}^I$ is the stiffness matrix and $\mathbf{G}^I$ is the geometrical matrix of the finite strip element.

2.1. The element geometrical matrix

The geometrical matrix **G** for the finite strip of width b (see Fig. 2) can be derived from the expression:

$$\mathbf{G}^I = \int_0^b\int_0^L \mathbf{B}^T \cdot \mathbf{S} \cdot \mathbf{B} \, dy \cdot dx \, , \tag{6}$$

where:

$$\mathbf{B} = \begin{bmatrix} \mathbf{N}_{,x} \\ \mathbf{N}_{,y} \end{bmatrix}, \ \ \mathbf{S} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}, \tag{7}$$

σ_i are membrane stresses (constant across the plate thickness) produced by the in-plane forces acting at the finite strip borders. Using the equation (2) in the equation (6) leads to the geometrical matrix for any (I-th) finite element:

$$\mathbf{G}^I = \frac{S_x}{120hb} \begin{bmatrix} 72 & 6b & -72 & 6b \\ 6b & 8b^2 & -6b & -2b^2 \\ -72 & -6b & 72 & -6b \\ 6b & -2b^2 & -6b & 8b^2 \end{bmatrix}, \tag{8}$$

where h is the plate thickness.

2.2. The element stiffness matrix

The element stiffness matrix for a four-degree-of-freedom *I*-th strip can be derived from:

$$\mathbf{K}^I = \int_0^b \int_0^L \mathbf{B}^T \cdot \mathbf{D} \cdot \mathbf{B}\, dy \cdot dx\,, \tag{9}$$

where:

$$\mathbf{D} = \begin{bmatrix} D_x & D_1 & 0 \\ D_1 & D_y & 0 \\ 0 & 0 & D_{xy}/2 \end{bmatrix}.$$

The above mentioned flexural stiffness parameters for an orthotropic plate are:

$$D_x = \frac{E_x h^3}{12(1-\nu_p^2)}\,,\; D_y = \frac{E_y h^3}{12(1-\nu_p^2)}\,,$$
$$D_1 = \frac{\nu_p^2 \cdot (D_x + D_y)}{2}\,,\; D_{xy} = \frac{(1-\nu_p^2)\cdot(D_x + D_y)}{2}\,. \tag{10}$$

In the case of an isotropic plate these coefficients (10) have simpler form:

$$D = D_x = D_y = \frac{Eh^3}{12(1-\nu_p^2)}\,,\; D_1 = \nu_p \cdot D\,,\; D_{xy} = (1-\nu_p^2)\cdot D\,, \tag{11}$$

where: E is the Young's modulus and ν_p is the Poisson's ratio.

After some operations the stiffness matrix takes the form:

$$\mathbf{K_n} = \alpha_1 \cdot \mathbf{K_1} + \alpha_2 \cdot \mathbf{K}_2 + \alpha_3 \cdot \mathbf{K}_3 + \alpha_4 \cdot \mathbf{K}_4 \tag{12}$$

where $\mathbf{K}_i$ are number matrices:

$$\mathbf{K}_1 = \begin{bmatrix} 156 & 22b & 54 & -13b \\ 22b & 4b^2 & 13b & -3b^2 \\ 54 & 13b & 156 & -22b \\ -13b & -3b^2 & -22b & 4b^2 \end{bmatrix} \quad \mathbf{K}_2 = \begin{bmatrix} 6 & 3b & -6 & 3b \\ 3b & 2b^2 & -3b & b^2 \\ -6 & -3b & 6 & -3b \\ 3b & b^2 & -3b & 2b^2 \end{bmatrix} \tag{13a}$$

$$\mathbf{K}_3 = \begin{bmatrix} 36 & 18b & -36 & 3b \\ 18b & 4b^2 & -3b & -b^2 \\ -36 & -3b & 36 & -18b \\ 3b & -b^2 & -18b & 4b^2 \end{bmatrix} \quad \mathbf{K}_4 = \begin{bmatrix} 36 & 3b & -36 & 3b \\ 3b & 4b^2 & -3b & -b^2 \\ -36 & -3b & 36 & -3b \\ 3b & -b^2 & -3b & 4b^2 \end{bmatrix} \tag{13b}$$

α_i are coefficients depending on physical and geometrical parameters of the considered structure:

$$\alpha_1 = \frac{\alpha_n^4 LbD_x}{840},\ \alpha_2 = \frac{LD_y}{b^3},\ \alpha_3 = \frac{\alpha_n^2 LD_1}{30b},\ \alpha_4 = \frac{\alpha_n^2 LD_{xy}}{30b},\ \alpha_n = \frac{n\pi}{L}. \tag{14}$$

2.3. The equilibrium equations

The equilibrium equations are derived applying the finite element methodology. Having derived the element geometrical (8) and stiffness (12) matrices the equilibrium equations for the *n*-th harmonic element, after assembling two adjacent elements *R*-th and (*R*+1)-th (Fig. 3) are of the form:

$$\begin{aligned} T_{r,r-1} + T_{r,r+1} &= P_r \\ m_{r,r-1} + m_{r,r+1} &= m_r \end{aligned} \tag{15}$$

where $T_{i,j}$ and $m_{i,j}$ are forces derived for each element using equation (5).

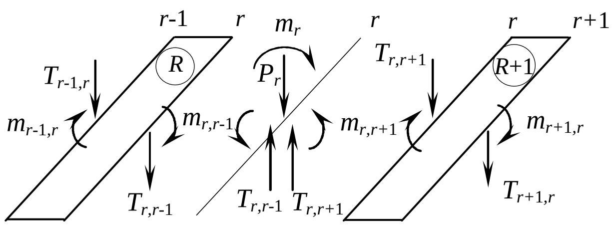

Fig. 3. Forces acting at a nodal line *r*

For a regular system the equilibrium conditions (15) can be written in the form of difference equations equivalent to the FEM matrix formulation [9]:

$$\begin{cases} \left[\beta_1\Delta^2 + \beta_0\right]\cdot w_r - \beta_2\left(E - E^{-1}\right)\cdot \phi_r = \beta_p \cdot P_r \\ \beta_2\left(E - E^{-1}\right)\cdot w_r + \left[\beta_3\Delta^2 + \beta_4\right]\cdot \phi_r = \beta_m \cdot m_r \end{cases} \tag{16}$$

where:

$$\beta_p = \sin(y_p, \alpha_n)\ ,\ \beta_m = \sin(y_m, \alpha_n)\ ,\ \beta_0 = 420\alpha_1\ ,\ \alpha_g = \frac{S_x}{120bhD_x}\ ,$$

$$\begin{aligned}
\beta_1 &= 6(9\alpha_1 - \alpha_2 - 6\alpha_3 - 6\alpha_4 - 12\alpha_g)\ ,\\
\beta_2 &= b(13\alpha_1 - 3\alpha_2 - 3\alpha_3 - 3\alpha_4 - 6\alpha_g)\ ,\\
\beta_3 &= b^2(-3\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4 - 2\alpha_g)\ ,\\
\beta_4 &= 2b^2(\alpha_1 + 3\alpha_2 + 3\alpha_3 + 3\alpha_4 + 6\alpha_g)\ ,
\end{aligned} \tag{17}$$

E^n is the shifting operator (see [1]):

$$\begin{aligned}
E^n(f_r) &= f_{r+n}\ ,\\
\Delta_r^2 = \Delta^2 &= (E + E^{-1} - 2)
\end{aligned} \tag{18}$$

is the second-order difference operator

$$\Delta_r^2 f_r = (E + E^{-1} - 2) f_r = f_{r+1} + f_{r-1} - 2f_r\ , \tag{19}$$

α_i are the functions of harmonic number *n* given by (14), P_r and m_r are the forces and moments acting at the nodal line *r* (with co-ordinates y_p and y_m, respectively). After elimination of the slope function ϕ_r, the equilibrium conditions are transformed into one fourth-order difference equation with one unknown w_r (nodal transverse displacement amplitude for the *n*-th harmonic):

$$\left[B_4\Delta^4 + B_2\Delta^2 + B_0\right] w_r = \beta_p\left[\beta_3\Delta^2 + \beta_4\right]\cdot P_r + \beta_2\beta_m\left(E - E^{-1}\right)\cdot m_r \tag{20}$$

where:

$$B_0 = \beta_0\beta_4\ ,\ B_2 = \beta_0\beta_3 + \beta_1\beta_4 + 4\beta_2^2\ ,\ B_4 = \beta_1\beta_3 + \beta_2^2 \tag{21}$$

For the regular infinite plate strip, equation (20) is equivalent to the set of infinite number of equilibrium conditions derived using the finite strip methodology (FSM). The solution of this equation enables one to determine the state of deformation of the entire considered structure.

3. THE FUNDAMENTAL SOLUTION

Solution of the finite difference equilibrium equation (20) yields the fundamental functions for the considered system. In order to solve the static problem of the structure loaded by the force $P_0 = P_r\delta_{r,0} = P\cdot\delta_{r,0}$ ($M_r = 0$, $\delta_{r,0}$ – Kronecker delta) we use the discrete Fourier transform in x direction [10]:

$$F\left[f_r\right] = \tilde{f}(\alpha) = \sum_{r=-\infty}^{\infty} f_r\ e^{ir\alpha}\ ,$$
$$F^{-1}\left[\tilde{f}(\alpha)\right] = f_r = \frac{1}{2\pi}\int_{-\pi}^{\pi}\tilde{f}(\alpha)\,e^{-ir\alpha}\cdot d\alpha\ . \tag{22}$$

Applying both transforms (22) to the equilibrium equation (20) yields the formula:

$$w_r = \frac{P}{\pi}\int_0^{\pi}\frac{\left(S_1\cos(\alpha)+S_2\right)\cos(r\alpha)}{\cos^2(\alpha)+B_m\cos(\alpha)+C_m}\cdot d\alpha\ , \tag{23}$$

where:

$$B_m = \left(B_2 - 4B_4\right)/\left(2B_4\right)\ ,\ C_m = \left(4B_4 - 2B_2 + B_0\right)/\left(4B_4\right),$$
$$S_1 = \beta_3\beta_p/\left(2B_4\right)\ ,\ S_2 = \left(\beta_4 - 2\beta_3\right)\beta_p/\left(4B_4\right). \tag{24}$$

The solution, i.e. the nodal displacement amplitude may be expressed in the form of the following recurrent relation:

$$w_r = \frac{P}{\pi}\left[S_1\cdot F_1(r) + S_2\cdot F_2(r)\right] \tag{25}$$

where

$$F_1(r) = 2^{r-1}\cdot C(r+1) - \binom{r}{1}2^{r-3}\cdot C(r-1) +$$
$$+\frac{r}{2}\binom{r-3}{1}2^{r-5}\cdot C(r-3) - \frac{r}{3}\binom{r-4}{2}2^{r-7}\cdot C(r-5) + \ldots \tag{26}$$

$$F_2(r) = 2^{r-1}\cdot C(r) - \binom{r}{1}2^{r-3}\cdot C(r-2) +$$
$$+\frac{r}{2}\binom{r-3}{1}2^{r-5}\cdot C(r-4) - \frac{r}{3}\binom{r-4}{2}2^{r-7}\cdot C(r-6) + \ldots \tag{27}$$

The integrals occurring in the above formula:

$$C(n)=\int_0^\pi \frac{\cos^n(\alpha)}{\cos^2(\alpha)+B_m\cos(\alpha)+C_m}\,d\alpha \tag{28}$$

can be easily solved in an analytical way. The formula (25) expresses the deflection amplitude along the nodal line *r* for an arbitrary *n*-th element of the harmonic series in a closed analytical form. From the equilibrium equations (16) the following relations for the transverse slope amplitudes are obtained:

$$\begin{aligned}\theta_1 &= -\frac{\beta_p P_r}{2\beta_2}+\frac{\beta_1}{\beta_2}w_1-\frac{2\beta_1-\beta_0}{2\beta_2}w_0,\\ \theta_{r+1} &= \frac{2\beta_3-\beta_4}{\beta_3}\theta_r-\theta_{r-1}-\frac{\beta_2}{\beta_3}(w_{r+1}-w_{r-1}).\end{aligned} \tag{29}$$

The functions of displacements at the nodal line *r* are in the form of the sums:

$$w(r,y)=\sum_{n=1}^{N}w_r(n)\cdot\sin\frac{n\pi y}{L},\quad \theta(r,y)=\sum_{n=1}^{N}\theta_r(n)\cdot\sin\frac{n\pi y}{L}, \tag{30}$$

where N is the number of harmonic elements, $w_r(n)$ and $\theta_r(n)$ are amplitudes obtained from (25) and (29), respectively.

The fundamental functions (30) for the infinite strip enable one to solve the static problem of a rectangular plate with finite dimensions, according to the indirect BEM.

4. NUMERICAL EXAMPLES

A problem of the initial stability of rectangular plates subjected to uniformly distributed loading q and compressive forces N_x is considered. All types of boundary conditions are introduced in the analysis.

The plate properties are as follows: Young's modulus E = 205 GPa, Poisson's ratio v = 0.3. The number of finite strips chosen for discretization was 6 and 12. Analytical solutions for the problem of initial stability of Kirchhoff plates were evaluated basing on the procedures given by Girkmann [3], Timoshenko and Woinowsky-Krieger [13] and Timoshenko and Gere [12].

4.1. The square simply-supported plate

The square plate, simply-supported on all edges and subjected to the uniformly distributed transverse loading and constant loading acting in plane is considered. The plate dimensions are $l = l_x = l_y$ = 1.0 m, the plate thickness h = 0.02 m, the uniformly distributed transverse loading p = 100 kN/m^2 and the constant loading acting in-plane N_x (Fig. 4).

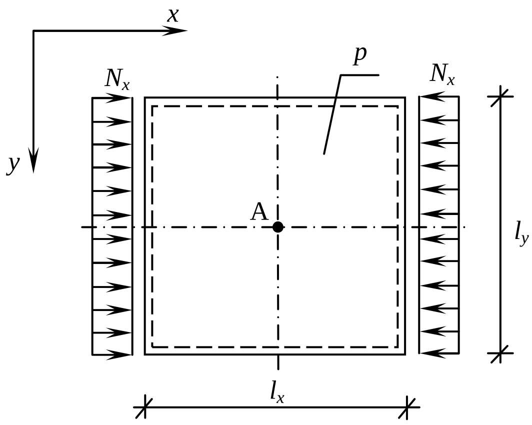

Fig. 4. Square, simply-supported plate subjected to the uniformly distributed loading *p* and constant loading in-plane N_x

The values of critical forces obtained using the analytical solution given in [12] and applying the BEM formulation presented in [5] are shown in Table 1. In the considered approach the plate boundary was divided into ten elements.

Table 1. The values of critical force

N_{cr} [kN/m]	Analytical solution	BEM solution
1	5928.993	5978.358
2	9264.052	9450.545
3	16.469.42	17102.466

The results obtained for the first critical force are presented in Table 2. The calculations were carried out for various values of in-plane loading. The constant loading N_x was assumed to be lower than the critical force.

Table 2. Deflection and bending moment at the point A

N_x	$w_A \cdot D/(pl^4)$	$M_x^A/(pl^2)$
0.0	0.004081	0.049700
$0.25 \cdot N_{cr}$	0.006903	0.070470
$0.50 \cdot N_{cr}$	0.012212	0.123610
$0.75 \cdot N_{cr}$	0.052933	0.533570

4.2. The square plate, simply-supported on two opposite edges with two clamped edges

In this example the square plate, simply-supported on two opposite edges with two clamped edges, subjected to the uniformly distributed transverse loading and constant loading acting in plane is considered. The plate dimensions are $l = l_x = l_y$ = 1.0 m, the plate thickness h = 0.02 m, the uniformly distributed transverse loading p = 100 kN/m^2. The calculations were carried out for a few values of constant load N_x , which acts in plane (Fig. 5).

The value of the critical force for the considered plate obtained analytically [12] and derived applying BEM procedure [5] equals $N_{cr} = 10090$ kN/m and $N_{cr} = 11635$ kN/m, respectively. The values of deflection and bending moment at the middle point of the plate are shown in Table 3.

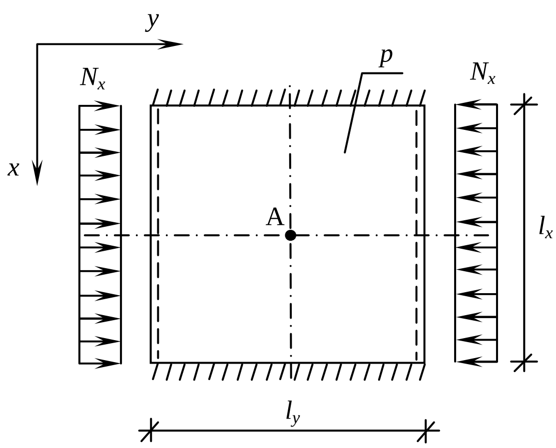

Fig. 5. Square plate, simply-supported on two opposite edges with two edges clamped, subjected to the uniformly distributed loading p and constant in plane loading N_x

Table 3. Deflection and bending moment at the point A

N_x	$w_A \cdot D/(pl^4)$	$M_x^A/(pl^2)$
0.0	0.002174	0.03430
$0.25 \cdot N_{cr}$	0.002792	0.04477
$0.50 \cdot N_{cr}$	0.003891	0.06335
$0.75 \cdot N_{cr}$	0.006397	0.10553

4.3. The rectangular plate, simply-supported on all edges

The rectangular plate, simply-supported on all edges subjected to the uniformly distributed transverse loading and constant loading acting in-plane is considered (Fig. 6).

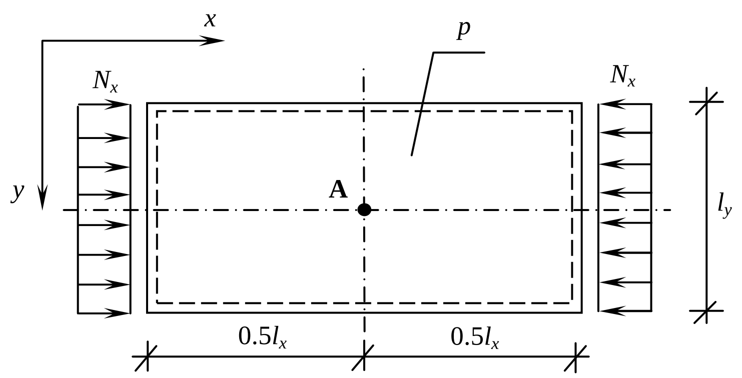

Fig. 6. Rectangular, simply-supported plate, subjected to the uniformly distributed loading p and constant in-plane loading N_x

The plate dimensions are l =0.5 l_x = l_y = 1.0 m, the plate thickness h = 0.02 m, the uniformly distributed transverse loading p = 100 kN/m^2. The value of constant loading N_x , which acts in-plane depends on the critical force (see Tab. 4).

Table 4. Deflection and bending moment at the point A

N_x	$w_A \cdot D/(pl^4)$	$M_x^A/(pl^2)$
0.0	0.01017	0.04434
$0.25 \cdot N_{cr}$	0.01358	0.05194
$0.50 \cdot N_{cr}$	0.01726	0.06208
$0.75 \cdot N_{cr}$	0.02340	0.07025

In this case the value of the critical force was derived applying BEM methodology [5]. For the considered plate the critical force equals N_{cr} = 5983 kN/m. The value of deflection and bending moment at the middle point of the plate are shown in Table 4.

4.4. The rectangular plate, simply-supported on two opposite edges and with two free edges

The rectangular plate, simply-supported on two opposite edges with two free edges is considered. Apart from the uniformly distributed transverse loading p = 100 kN/m^2 , the constant load N_x acts in-plane (Fig. 7). The plate dimensions are l_y = 2.0 m, l_x = 1.0 m, the plate thickness is h = 0.02 m.

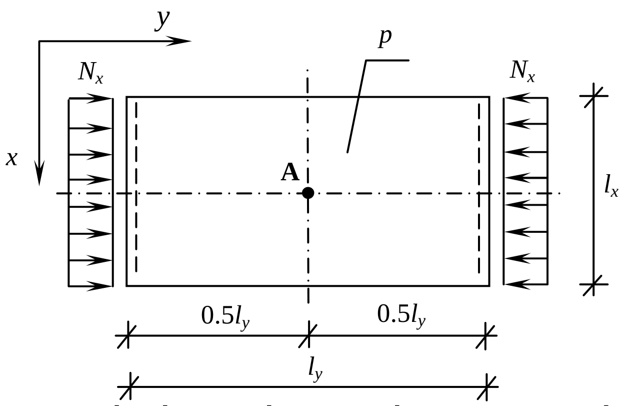

Fig. 7. Rectangular plate, simply-supported on two opposite edges and with two free edges, subjected to the uniformly distributed loading p and constant in-plane loading N_x

The results derived using the BEM procedure given in [5] yield the critical force value N_{cr} = 403 kN/m. The fundamental functions obtained using the FSM enable to derive the deflections and bending moments for the considered plate at the middle point A (Table 5).

Table 5. Deflection and bending moment at the point A

N_x	$w_A \cdot D/(pl^4)$	$M_x^A/(pl^2)$
0.0	0.19830	0.46548
$0.25 \cdot N_{cr}$	0.28101	0,70641
$0.50 \cdot N_{cr}$	0.48291	1,29549
$0.75 \cdot N_{cr}$	1.72861	4,93285

As it was expected the values of deflection and bending moment at the middle point for each example plate increase with the growth of the in-plane loading.

5. CONCLUDING REMARKS

In this paper the static analysis of thin plates with a transverse and in-plane loading was considered. The equilibrium conditions for an infinite strip were derived in the form of one difference equation. The solution of this equation, i.e. the fundamental function for an infinite plate strip, was derived basing on the finite strip method (FSM). This method is an important alternative to the most popular Finite Element Method, because it does not require a high number of degrees of freedom. The fundamental solution derived in this way, can be used to solve the static problem of a finite plate. Moreover, plates simply supported on their opposite edges and loaded in-plane are commonly applied as bridge structures, as box or plate elements. The numerical results demonstrate the effectiveness and efficiency of the proposed method.

REFERENCES

1. Babuška I., *The Fourier Transform in the Theory of Differences Equation and Its Applications*. Arch. Mech. Stos. **4**, 11, 1959.
2. Brebbia C.A., Telles J.C.F. and Wrobel L.C.: *Boundary Element Techniques, Theory and Applications in Engineering*. Springer-Verlag, Berlin Heidelberg, New York, Tokyo, 1984.
3. Girkmann K.: *Plane Girders* (in Polish). Warszawa, Arkady Publishing House, 1957.
4. Guminiak M.: *Thin plates analysis by the boundary element method using new formulation of a boundary condition* (in Polish). PhD Thesis, Poznan University of Technology, Poznań, Poland, 2004.
5. Guminiak M., Sygulski R.: *Initial stability of thin plates by the boundary element method* (in Polish). Proc. of Stability of Structures Xth Symposium, Zakopane, 8-12 September, Technical University of Lodz, Poland, 173-180, 2003.
6. Katsikadelis J.T.: *Boundary elements. The second volume: plate analysis* (in Greek). 2010.
7. Katsikadelis J.T., Sapountzakis E.J., Zorba E.G.: *A BEM approach to static and dynamic analysis of plates*. Computational Mechanics, **7** (1), (1990), 31-42.
8. Loo Y. C., Cusens A. R.: *The Finite Strip Method in Bridge Engineering*, New York, Viewpoint Publications, 1978.
9. Pawlak Z., Rakowski J.: *Singular solutions for two-dimensional discrete systems by Difference Equation Method*. Proc. of the Second International Conference on Difference Equations and Applications, Veszprem (Hungary), 7-11 August, 1995.
10. Rakowski J., *A critical analysis of quadratic beam finite elements*, International Journal for Numerical Methods in Engineering, **31**, (1991) 949-966.
11. Shi G.: *Flexural vibration and buckling analysis of orthotropic plates by the boundary element method*. International Journal of Solids Structures, **12** (26), (1990), 1351-1370.
12. Timoshenko S., Gere J. M.: *Theory of elastic stability*. McGraw-Hill Book Company, Inc., New York, Toronto, London, 1961.
13. Timoshenko S., Woinowsky-Krieger S.: *Theory of plate and shells*. McGraw-Hill Book Company, Inc., New York, Toronto, London, 1959.

ZASTOSOWANIE FUNKCJI FUNDAMENTALNYCH W ANALIZIE STATYCZNEJ PŁYT CIŃKICH OBCIĄŻONYCH POPRZECZNIE I W PŁASZCZYŹNIE

S t r e s z c z e n i e

W pracy przedstawiono analizę statyczną płyt cienkich, obciążonych zarówno poprzecznie jak i w płaszczyźnie, z wykorzystaniem metody pasm skończonych. Zgodnie z zasadami metody pasm skończonych, ciągły i nieograniczony układ aproksymowany jest nieskończoną liczbą identycznych elementów, którymi są pasma skończone swobodnie podparte na przeciwległych bokach. Niewiadomymi są tzw. amplitudy ugięć i kątów obrotu na liniach węzłowych, czyli na brzegach swobodnych pasma skończonego. Po określeniu macierzy sztywności i macierzy geometrycznej elementu skończonego wyprowadzone zostało różnicowe równanie równowagi, które obowiązuje dla każdej linii węzłowej pomiędzy elementami. Główną zaletą tej metody jest możliwość przedstawienia warunków równowagi dla całego rozważanego układu w postaci jednego równania rekurencyjnego. Rozwiązanie wpomnianego równania dla regularnego, dyskretnego pasma płytowego nazywane jest funkcją fundamentalną. Rozwiązanie fundamentalne otrzymane w ten sposób zostało wykorzystane do rozwiązania problemu statyki płyty o skończonych wymiarach, w sposób analogiczny jak metoda elementów brzegowych w statyce układów ciągłych. Podstawową korzyścią wynikającą ze stosowania metody elementów brzegowych (BEM) oraz metody pasm skończonych (FSM) jest mniejszy nakład obliczeniowy w porównaniu z innymi, podobnymi metodami.