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NUMERICAL MODELLING OF SOIL ARCHING IN A SHALLOW BACKFILL LAYER

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Abstract

The paper presents the application of the finite element method into the modelling of soil arching. The phenomenon plays fundamental role in soil-shell flexible structures behaviour. To evaluate the influence of arching on a pressure reduction, a plain strain trapdoor under a shallow layer of backfill was simulated. The Coulomb-Mohr plasticity condition and the nonassociated flow rule were used for the soil model. The research examines the impact of the internal friction angle and the influence of the backfill layer thickness on the value of soil arching. The carried out analyses indicate that the reduction of pressures acting on a structure depends on the value of the internal friction angle, which confirms the earlier research. For a shallow backfill layer however, the reduction is only a local phenomenon and can influence only a part of the structure.

Keywords: arching, soil-shell structure, trapdoor problem, stress redistribution, FEM, Mohr-Coulomb model

1. INTRODUCTION

A rapid development of transport and community infrastructures makes soil-shell flexible structures more and more common. They are used to build small bridges, passages for animals or culverts. The basic element of such a system is a flexible shell structure (e.g. a thin steel shell), backfilled with compacted soil. The soil transfers the loads onto the shell and also provides a support for it. A characteristic feature of a soil-shell flexible structure is a reduction of pressure induced by soil acting on the structure, compared to the pressure acting

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on an analogous stiff structure. The reduction is a result of a phenomenon called arching. Unfortunately, due to the interaction of deformable elements of the system (the structure and the soil), arching is difficult to characterize by analytical models which impedes taking it into account in rational design.

Terzaghi [8] defined the arching phenomenon as a transfer of pressure induced by deforming soil masses onto adjacent stable soil. The idea of arching is explained in Fig. 1. The dashed line indicates the initial configuration of the system. The flexible structure, shown in Fig. 1a, experiences displacements caused by the weight of the backfill or/and by external vertical loads. The crown of the structure (point A) moves downward. Simultaneously, the shell's arms (points B and C) move outward compacting the surrounding soil and thus increasing mean stresses inside it. Shear stiffness of soil above the shell causes that its downward movement creates shear stresses which counteract these displacements. They are schematically shown on vertical planes D-E and F-G in Fig. 1b. As a result, stresses are transferred from the area to be lowered (block D-E-G-F) onto the neighbouring soil which constitutes outer zones of the backfill.

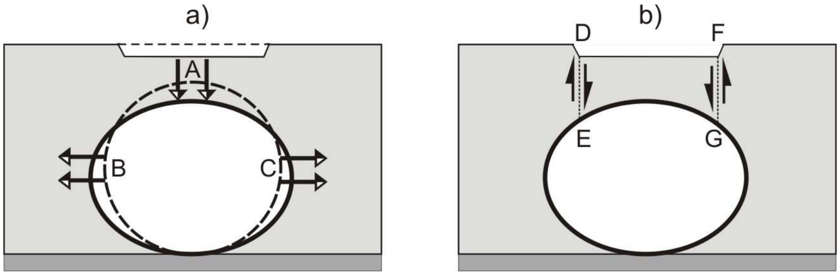

Fig. 1. A draft of an arching phenomenon: a) displacements of a flexible structure and the surrounding soil backfill, b) shear stresses induced by displacements

In soil-shell systems, alterations in mutual interactions between the structure and the soil play important roles. The alterations result from deformations and the changes in stiffness of the system components which arise both at the stage of their installation and during operation. Issues related to modelling of the installation process of a soil-shell structure, where subsequent sand backfill layers are placed and compacted, are discussed in [6].

The hereto paper presents the analysis of load redistribution caused by the system deformations, focusing on arching in a shallow backfill layer. To discuss the essence of soil pressure reduction and stress redistribution, a simplify geometry is investigated i.e. a flat structure moving downwards (a trapdoor) in plane strain conditions.

The influence of arching on the behaviour of soil-shell structures was extensively discussed in [1]. According to the authors of the book, the ratio between the recorded load P acting on a selected part of a flexible structure and the value of geostatic loading P_0 (resulting exclusively from the thickness and the unit weight of the volume of the soil overburden on the non-flexible structure) is a measure of the arching effect. As far as Polish-language publications are concerned, the extensive review of methods of considering loads acting on a soil-shell structure is presented in [4], whereas engineering methods of culvert designing are described in [9].

Soil arching was the subject of laboratory tests carried out by numerous authors. The results pioneering tests, published in 1936, are presented in Fig. 2 [7]. A draft of the experiment, consisting in lowering a trapdoor over a sand layer, is show in Fig. 2a. During the experiment, the values of vertical displacements and the loads acting on the trapdoor were recorded. Fig. 2b reveals that arching results in a considerable reduction of loads acting on the trapdoor (a small value of ratio P/P_0 indicates a considerable reduction of pressure, i.e. an essential arching effect). The scale of the reduction depends on the compactness of the examined soil and on the value of the trapdoor displacements. It can be expected that the thickness of the soil layer H in relation to the width of the trapdoor D will also be an influential factor.

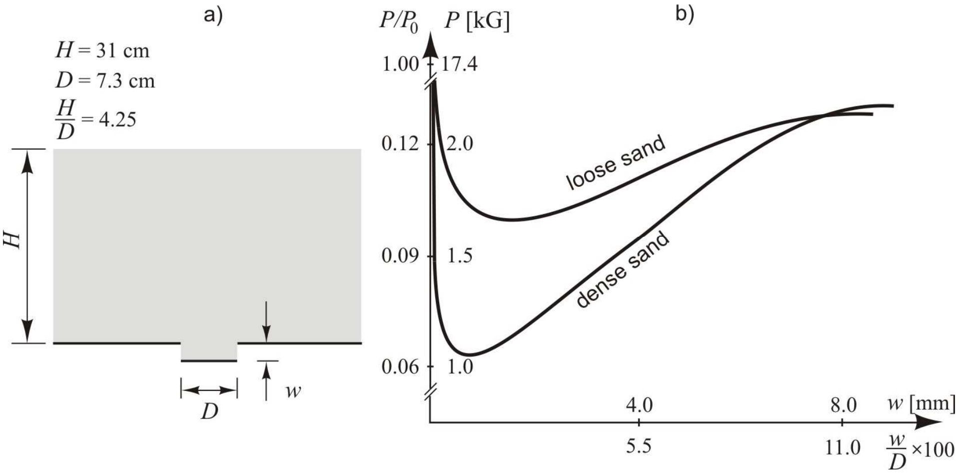

Fig. 2. The trapdoor test [7]: a) a layout of the test, b) the reduction of load P generated by soil on the trapdoor in the function of its displacements w

Arching phenomenon, in plane strain state conditions, was subject to analytical investigation in work [5]. Work [3] presents the analyses of arching in the flat strain state and in axial symmetry conditions performed with the use of the finite element method. Both papers applied the Mohr-Coulomb plasticity condition to model soil behaviour while loaded. The numerical results obtained in [3] allowed formation of an approximate analytical expression enabling the determination of the value of the reduction of soil pressure in relation to the thickness of a backfill and to the angle of internal friction.

The results of physical modelling tests of the failure mechanism of sandy soil over a deep trapdoor carried out in a centrifuge were discussed in [2]. The obtained results indicate an essential role of dilation in the process of soil deformations. Additionally, the index of density of soil (I_D), which is correlated with an internal friction angle, influenced significantly the magnitude of the surface displacements.

The discussed papers refer mainly to the investigations of an arching phenomenon in soil over a trapdoor with a thick layer of backfill. They do not refer to the pressure distribution along the trapdoor, instead they use resultant values. In practice, there are also instances of designs of structures with shallow backfill layers. This paper focuses on the study of arching in a shallow soil layer ($H/D \leq 1$) and the consequent distribution of pressure.

2. NUMERICAL MODEL

The assumed numerical model of the task includes: spatial discretization of the problem, elastic-plastic constitutive soil model, and the finite element method solution procedure. In order to simplify the task, a flat strain state was assumed. The task was solved with the use of Abaqus computer program.

Due to the practical importance of the investigated issue, the analyses were carried out for the commonly applied Mohr-Coulomb plasticity soil model, parameters of which are easily readable for engineers thanks to their clear physical interpretation. Additionally, their values can be determined in standard tests. For the main directions of the stress state, the failure envelope of the model is given in the following form:

$$\frac{\sigma_1+\sigma_3}{2}\sin\phi-\frac{\sigma_1-\sigma_3}{2}-c\cdot\cos\phi=0 \tag{2.1}$$

where ϕ is the angle of internal friction, and c is the soil cohesion (for engineering calculations, it is assumed that c=0 for sands, but in a damp media, the water meniscus generates the cohesion of an order of several kPa). To provide stabilization of the solutions in the discussed model, c=0.01 kPa was assumed.

Compacted sands, used for backfilling, show dilation phenomenon while shearing, i.e. the increase in its volume caused by the growth of shear deformations. Taking into account the results of research presented in [2], the hereto paper assumes dilation angle ψ different from zero, and the flow rule non-associated with the plasticity condition given by (2.1).

The geometry of the system similar to the one presented in Fig. 2a (a soil layer over the trapdoor) is analysed at various values of backfill thickness i.e. different values *H/D*. The solution takes advantage of the symmetry with respect to the vertical plane passing through the centre of the trapdoor (A-F – Fig. 3).

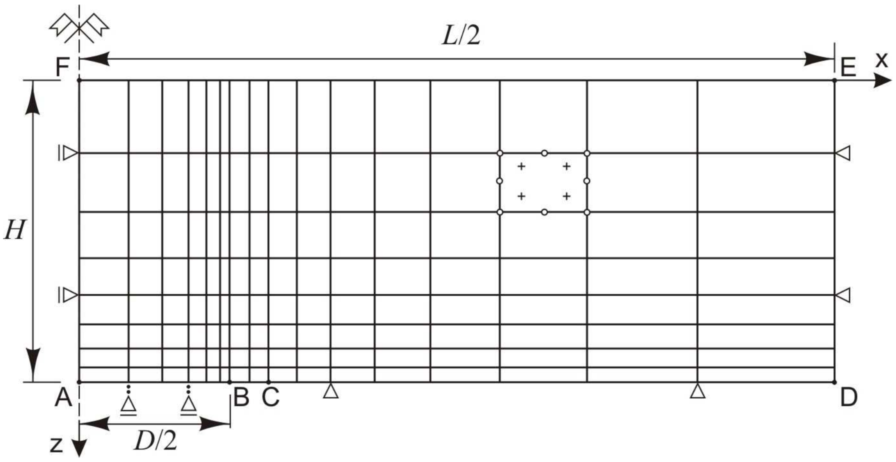

Fig. 3. The model of the task: a zone of induced displacements (a trapdoor) A-B, basic dimensions, FEM discretization and boundary conditions

Fig. 3 shows an exemplary finite element mesh for a system with the proportion *H/D* = 1. In the spatial discretization process, eight-node finite elements were used. Due to the adopted elastic-plastic model of soil, the elements are integrated in a reduced way. The layout of the integration points is shown on a selected finite element.

Soil deformations, caused by the structure's flexibility are modelled by induced displacements of the mesh nodes on the section A-B (Fig. 3). In order to avoid singularities in the stress distribution in the vicinity of point B, resulting from the discontinuity of displacements, a transition zone of linear variation of movements was applied along section B-C, as in [3]. Boundary conditions applied to the remaining edges are shown in Fig. 3.

In all the analysed examples, the load is exclusively induced by the soil dead weight, calculated for the unit density γ= 20 kN/m^3. The stresses generated by dead load in stationary conditions, where all elements are at rest, is called geostatic initial condition.

The model takes into account the non-linearity of the geometric equations. To solve the non-linear problem, a method with a constant increment of displacements was applied.

The paper presents the analysis of the influence of the thickness of the backfill layer as well as the influence of the values of parameters of the soil strength on the stress distribution in the basement of the layer. Two volumes of soil of dimensions H/D=1 and H/D=1/2 were examined. In the subsequent tests, a constant value of dimensions L=10 m and D=2 m were maintained, but the layer thickness H was changed.

In the following examples, the values of internal friction and dilation angles (ϕ /ψ) were assumed as follows: (30^0/ 15^0), (35^0/ 20^0), (40^0/ 25^0). Moreover, the following values of sand elasticity parameters were assumed: Young modulus E=100 MPa, the Poisson ratio v=0.3.

3. RESULTS AND DISCUSSION

Fig. 4a depicts the distribution of vertical components of stress tensor in the geostatic state. For homogeneous soil, a horizontal layout of contour lines is characteristic. Fig. 4b presents distribution of stresses induced by downward movement of the trapdoor (section A-B) by w=1 mm. The negative values of stresses visible in the figure refer to compression. The task was solved with the following data set: H/D=1, $\phi = 35^0$ and $\psi = 20^0$.

The reduction of stresses, shown in fig. 4a, over the section A-B is connected with their transfer towards stationary boundary conditions, which is revealed by a strong concentration of stresses on the right side of point B. An inconsiderable reduction of stresses is observed in the central part of the model.

The paper examines arching induced redistribution of vertical stresses acting on the trapdoor, and not only the load resultant, so the ratio of the current vertical stresses to the geostatic stresses $\sigma/(\gamma H)$ was calculated as a measure of arching (γ is the soil unit weight), instead of P/P_0. In order to examine the influence of the values of internal friction angle ϕ and the dilation angle ψ on arching, calculations of the problem were performed for the geometry as presented in Fig. 3, H/D=1 and three various sets of parameters ϕ and ψ. Values $\sigma/(\gamma H)$ were determined in each analyses in integration points located in the middle of the section A-B. The obtained results are presented in Fig. 5. The horizontal axes, upper and lower, indicate absolute values of induced vertical displacement of section A-B and relative displacements referred to the trapdoor length D, respectively. The vertical axis presents the values of the ratio of the current vertical load to the load on the trapdoor generated by geostatic stresses.

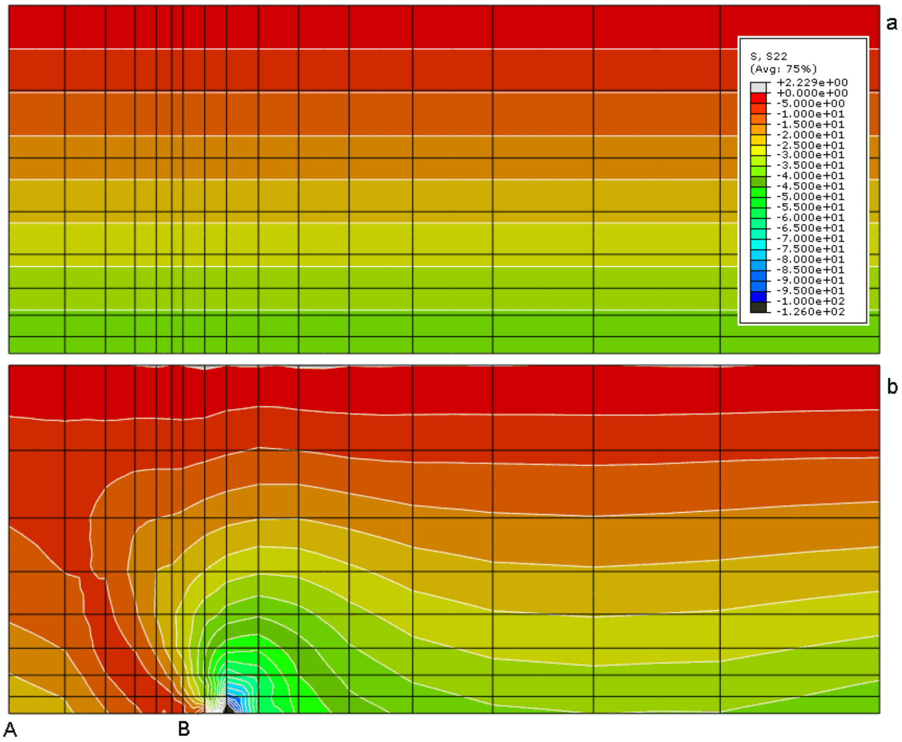

Fig. 4. Redistribution of vertical stresses in soil: a) initial geostatic state (ϕ=35^0, ψ=20^0), b) state after the trapdoor A-B is lowered by w=1 mm

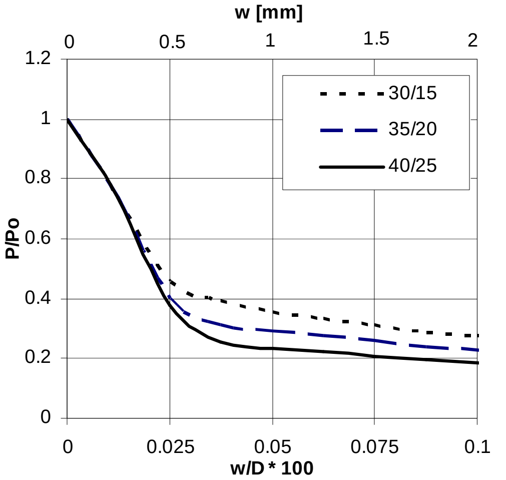

Fig. 5. Reduction of stresses over the trapdoor (H/D=1) in the function of vertical displacements w for various friction and dilation angles (ϕ / ψ)

The shapes of the curves presented in Fig. 5 show that at zero displacement of the trapdoor (w=0), there is no arching effect ($\sigma/(\gamma H)=1$). In the initial phase of inducing vertical displacements until reaching the value $w\approx0.3$ mm, the effect of stress reduction (arching) is independent of the soil conditions, i.e. of the internal friction angle. Above this value, the stress reduction is the greater the larger is internal friction angle of the sand.

Inducing displacements above the value $w\approx1$ mm results in the growth of arching, which is observed for all three curves in Fig. 5. At this stage, small areas of soil under tension begin to develop on the upper edge of the left part of the finite element mesh (Fig. 4a). It seems that since then the model does not describe the real behaviour of the material.

It is also interesting to study the distribution of stresses along the base of the model at different ratios *H/D*. Calculations were performed for *H/D*=1 and *H/D*=0.5. The results of calculations are presented in Fig. 6, where the dashed line refers to stresses induced by geostatic state. For both solutions, a relief of load is observed over the zone of induced displacements (over the trapdoor). Simultaneously, there is a transfer of stresses to the area of fixed boundary conditions. The stress distribution along the trapdoor is not uniform and depends on the ratio *H/D*. With a small thickness of backfill (*H/D*=0.5) there is no arching effect in the vicinity of the axis of symmetry of the trapdoor (x=0÷0.25 m).

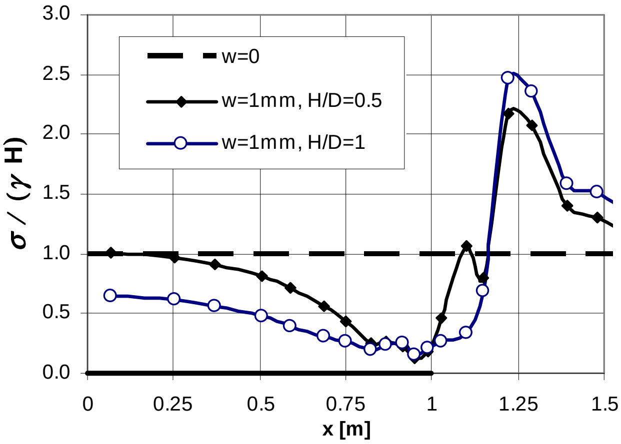

Fig. 6. Normalized vertical stress distribution along the trapdoor ($\phi=35^0$, $\psi=20^0$)

Fig. 6 shows that vertical stress concentration occurs in the stationary support zone ($x\approx1.25$ m). A comparison of the curves shows that greater arching effect along the trapdoor results in a greater stress concentration. In addition, there is a fluctuation of stresses in the transition zone (x≈1.1 m) caused by a considerable gradient of displacements and coarse finite element mesh in the area.

4. SUMMARY AND CONCLUSIONS

The article analyses soil arching over a soil-shell flexible structure or a trapdoor with shallow layer of backfill. In this kind of objects, a soil backfill surrounding the shell provides both a source of loading and a support for the structure. As a result of the shell flexibility and its deformation, rearrangement of mutual interactions is observed in the system. The measure of arching is the ratio of the current vertical stresses σ to the vertical geostatic stresses γH, wherein the low value of $\sigma/(\gamma H)$ refers to significant arching. The redistribution of interactions is possible thanks to the soil ability to convey shear stresses.

The paper investigates the influence of the internal friction angle and the dilation angle as well as the influence of the thickness of the backfill on the size of arching and the redistribution of pressures from the soil onto the structure. The research has been limited to the cases with shallow backfill layers ($H/D \leq 1$).

The carried out analyses lead to the following conclusions:

- An important factor influencing the size of arching is the soil strength. Since noncohesive soils are usually used as backfill, internal friction angle ϕ is a decisive parameter. The increase in the strength of the backfill (the increase in the sand compaction) results in the reduction of its pressure on the flexible structure. This conclusion confirms all former experiences. The influence of the soil dilation, as a factor independent from the friction angle should be additionally verified. The values of the dilation angles were arbitrarily assumed for particular values of angles ϕ.
- The thickness of the backfill layer, and consequently the proportion of dimensions H/D, is the second important factor influencing the value of the reduction of stresses over the flexible structure. The thicker the backfill, the larger the relative arching effect (at constant values of soil strength and stiffness). Obviously, the thicker backfill layer the greater its absolute weight.
- Reduction of stresses in one part of soil induced by the system flexibility, results in the increase in stresses in another part of soil, thus the phenomenon should be referred to as arching-related stress redistribution.
- The value of redistribution of interactions over the flexible structure varies along its length. Moreover, the reduction is of a local character and is limited to the soil adjacent to the one overtaking the loads from the unloaded

zone. In the case of a shallow backfill layer (inconsiderable value of *H/D* ratio) it may indicate that the pressure reduction zone does not include the desired part of the flexible structure.

- The coupling of the above factors influencing the value of arching (strength and stiffness parameters, variability of which were not tested in this article, and the analysed proportions of dimensions H/D), suggests that in order to design properly, the interaction between the structure and the adjacent soil should be taken into account. Treating the loadings as flexible and independent from the deformation state is a considerable simplification.

Further research should consider the influence of the density of the mesh of elements on the obtained results. In addition, the phenomenon of stress fluctuation in the zone of considerable displacement gradients should be examined. Another factor which should be verified, and which may influence the value of arching is the condition of the contact between the structure and the soil. Moreover, the influence of the constitutive model of soil on the aching should be tested.

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MODELOWANIE NUMERYCZNE PRZESKLEPIENIA GRUNTU W PŁYTKIEJ WARSTWIE ZASYPKI

Streszczenie

W artykule przedstawiono próbę modelowania numerycznego zjawiska przesklepienia gruntu w płytkiej warstwie zasypki. Zjawisko to ma podstawowe znaczenie w podatnych konstrukcjach gruntowo-powłokowych. W wyniku przesklepienia parcie gruntu na konstrukcję podatną jest znacznie mniejsze niż parcie na analogiczną konstrukcję sztywną. Redukcja parcia nad poddającą się obciążeniom wiotką konstrukcją wiąże się z redystrybucją naprężeń w gruncie na strefy zasypki, które nie podlegają deformacjom. W celu zbadania istoty zjawiska redystrybucji naprężeń analizowany jest układ o bardzo uproszczonej geometrii – tzw. problem zapadni. Zadanie rozwiązano z wykorzystaniem metody elementów skończonych. W rozwiązaniu przyjęto warunki płaskiego stanu odkształcenia. Założono sprężysto-plastyczny model zasypki gruntowej z warunkiem plastyczności Coulomba-Mohra i niestowarzyszonym prawem płynięcia. Jako miarę redukcji parcia wywołanego przesklepieniem przyjęto stosunek aktualnego parcia do parcia geostatycznego działającego na konstrukcję niepodatną. Badano wpływ kąta tarcia wewnętrznego i grubości warstwy zasypki na wartość redukcji parcia. Wzrost kąta tarcia skutkuje większą redukcją parcia gruntu, co potwierdza wcześniejsze wyniki badań. Przy małej grubości warstwy zasypki efekt redukcji parcia ma charakter lokalny i może być ograniczony jedynie do małych obszarów gruntu nie obejmując swym zasięgiem całej konstrukcji podatnej.

Słowa kluczowe: przesklepienie, konstrukcja gruntowo-powłokowa, problem zapadni, redystrybucja naprężeń, MES, model Coulomba-Mohra

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