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APPROXIMATION OF A MEAN TIME OF THE SLOT TRANSFER IN THE CAMBRIDGE RING

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In the paper, a popular version of the LAN called the Cambridge Ring is considered. Performance parameters of the Ring depend on its organization, size, number of users as well as on the Ring reliability. A mean time of the slot transfer, as measure of a performability (performance + reliability) of the Ring is proposed. The Stochastic Timed Inhibited Petri Net model of the Ring is used to derive an approximation of the mean time of the slot transfer.

1. Introduction

The Cambridge Ring which is a popular version of the Local Area Network (LAN), has been analysed for many years (Hopper *et. al.*, 1986; Tanenbaum, 1989). Recently new developments in VLSI technologies recall the interest of the LAN as a subject of research (Fulcher and McKerrow, 1990; Hopper *et. al.*, 1986; King and Mitrani, 1987; Liu and Rouse, 1984; Zamojski and Al–Aloosy, 1990). It is easier now to construct more complex rings (up to 255 stations) and it is also possible to increase the speed of the ring (up to 60 MHz (Fulcher and McKerrow, 1990)). Reliability parameters of network components such as computers and links have been improved so much that an occurence of *catastrophic* failures in a ring may be considered only as an academic problem. On the other hand *malfunctions* in the ring could not be completely eliminated from the ring operation yet. The rings suffered large time losses for renewal of operations destroyed by malfunctions. Repeated transfers of slots with detected malfunctions influence the real performance parameters which do not match the needs of ring users.

Performance parameters of the ring depend on its organization and size (number of slots, distance between stations, system response to failures etc.), number of users and user's needs for accessing the ring, as well as on the ring reliability. The measure that combines the performance and reliability is called *performability* (Bobbio, 1983; Zamojski and Al–Aloosy, 1990). In this paper we propose a *mean time of a slot transfer* as a performability measure of the Cambridge Ring.

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There is a lot of concurrent events (calls for empty slot, slots traffic, task execution in the stations, failures etc.) which may happen in different places of the ring at the same time. For these reasons the Petri net theory is proposed as a tool for modeling of LAN as systems with concurrent events (Al–Jaar and Desrochers, 1990; Bobbio, 1983; King and Mitrani, 1987; Li and Georganas, 1990; Zamojski, 1987; Zamojski and Al–Aloosy, 1990). The Stochastic Timed Inhibited Petri Net model of the Cambridge Ring given in (Zamojski and Al–Aloosy, 1990) is the basis for approximation of the mean time of a slot transfer in the Cambridge Ring. Special attention is paid to malfunctions (catastrophic failures are neglected) and repeated slot transfers caused by these malfunctions.

2. Cambridge Ring and its Petri Net Model

The idea of the Cambridge Ring is based on the principle of a circulating empty slot (Hopper *et. al.*, 1986; Tanenbaum, 1989). The ring links N stations (they are called nodes too) and a monitor station (Figure 1) which provides maintenance functions (slot generation, error detection etc.).

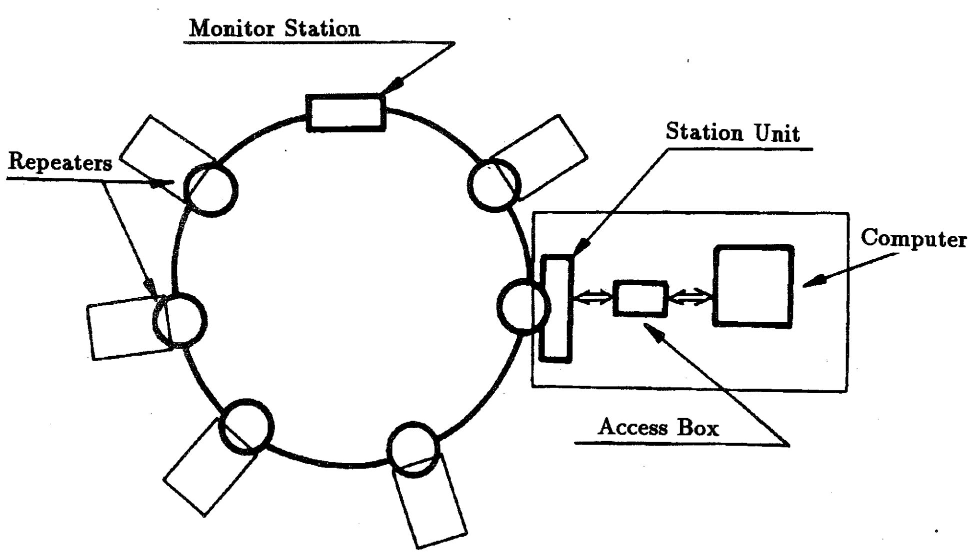

Fig. 1. The Cambridge Ring structure.

The slot contains 38 bits in a standard configuration. There is a field for transferred information (2 bytes), fields for source and destination addresses, and fields for other control–checking information.

A hardware protocol is given for data transmition. A station demanding transmition checks passing slots until it detects an empty one. The slot is then filled with two bytes of data, the sender and destination addresses. A receiving station also monitors the circulating slots. When the filled slot arrives to its destination, the station copies data into its registers and marks the response bits of the slot. The sending station waits for the returning slot and when it returns, the station checks the response of the destination (slot accepted or not, destination busy, etc.). The slot is then released (it is skipped to the next station) and the station waits for another empty slot transmit again.

Such a protocol ensures that a station does not monopolize a slot, so that all the stations get the same transmission possibility. Moreover, a station demanding data transmition should wait a finite time before it can send a slot.

There are N_A active stations and N_P passive $(N = N_A + N_P)$. A passive station may work only as repeator; its function is to regenerate passed signals (bits). The active station may be used as a source, or a destination or a repeator; that is it may realize all protocol functions and regenerate transferred bits too. There are malfunctions in the ring, detected by parity checking operation in each node. This checking operation is executed on each passing slot (full as well as empty). When a fault is detected in a slot, then the transfer is repeated after one full round route of the slot (details of these operations will be omitted here). When the repeated transfer is not finished successfully, then the ring will repeat it once more after 15 full round routes of the slot. The last unsuccessful transfer means that a failure exists in the ring. Note that time is lost in the ring to remove the faulty slot and to repeat its transfer.

The Cambridge Ring must realize the determined sequences of operations in accordance with the protocol sketched above. Some of these operations are realized randomly with probabilities depending on special conditions in the ring (for example: a reaction to request for an empty slot or a detected malfunction). Naturally a set and a sequence of realized operations depend on the node status (source or destination,active or passive repeater).

Let $F_i^{(k)}$ denote the i–th function realized in the k–th node and $O_i^{(k)}$ is a set of primitive operations $(o_{ji}^{(k)}, j = 1, ..., n^{(k)})$ of this node. The function $F_i^{(k)}$ may be defined as a n–tupple on the set $O_i^{(k)}$

$$F_i^{(k)} = \left\langle o_{1i}^{(k)}, o_{2i}^{(k)}, ..., o_{n^{(k)}i}^{(k)} \right\rangle \tag{1}$$

and, as the consequence of the latter ,there is a subset of primitive operations $O_i^{(k)}$ which must be executed for the purpose of the i–th function realization.

A route of a slot transfer may be divided into two parts (Figure 2);

i) source–destination; a full slot is transferred,

ii) destination–source; an ACK signal is transferred and an empty slot is skipped to the next node.

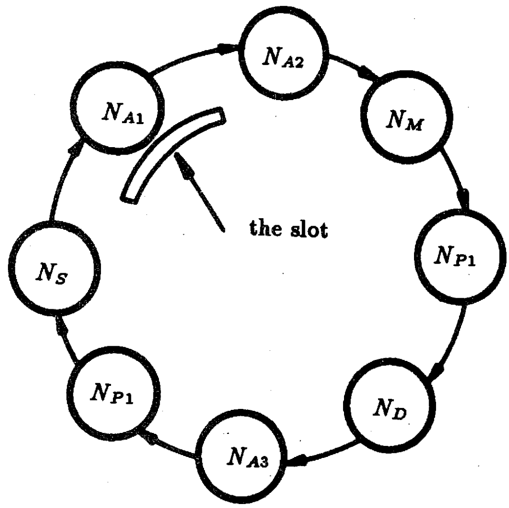

Fig. 2. The functional model of the Cambridge Ring,
N_A– active node, N_P– passive node, N_S– source node,
N_D– destination node, N_M– monitor.

Let N_{SD} denote a subset of nodes used during the first part of the route and N_{DS} during the second. In this way the function of the slot transfer, F_{ST} is

$$F_{ST} = \langle F_{SD}, F_{DS} \rangle \tag{2}$$

where F_{SD} and F_{DS} are functions realized during the first and second part of the route, respectively.

It is assumed that the index of the source station is 1, the index of destination station is D, and indices of consecutive stations increase to N in the direction of the slot motion. We have then

$$F_{SD} = \left\langle F_{SD1}^{(1)}, F_{SD1}^{(2)}, \ldots, F_{SD1}^{(D)} \right\rangle \tag{3}$$

and

$$F_{DS} = \left\langle F_{DS2}^{(D)}, F_{DS2}^{(D+1)}, \ldots, F_{DS2}^{(N)}, F_{DS2}^{(1)} \right\rangle \tag{4}$$

where $F_{SD1}^{(1)}$ is a function realized in the source node while waiting for an empty slot and sending a full one, $F_{SD1}^{(D)}$ is a function realized in the destination node while receiving the full slot, $F_{SD2}^{(D)}$ is a function realized in the destination node while sending the ACK signals, $F_{DS2}^{(1)}$ is a function realized in the source node while receiving the ACK signal and skipping the empty slot.

Each primitive operation $o_{ji}^{(k)}$ is realized during time $\tau_{ji}^{(k)}$ and the function $F_i^{(k)}$ is realized during time $T_i^{(k)}$, which is a sum of all primitive operation times needed for realization of this function and

$$T_i^{(k)} = \sum_{j=1}^{n(k)} Pr\{o_{ji}^{(k)}\}\tau_{ji}^{(k)} \tag{5}$$

where $Pr\{o_{ji}^{(k)}\}$ is a probability that the j–th operation of the k–th node is used during the i–th function realization.

The Stochastic Timed Inhibited Petri Nets (STIPN) are a very useful tool for modeling the behavior of the ring and its nodes (Al–Jaar and Desrochers, 1990; Bobbio, 1983; King and Mitrani, 1987; Li and Georganas, 1990; Zamojski, 1987). Consequently this model may be applied as a base for estimation of the function realization. Within the Petri net theory transitions model events (operations) and places describe the conditions needed for the realization of these events (more exactly: the condition realizations are modeled by the presence of tokens in input places of the transition).

The STIPN model of the Cambridge ring is proposed by Zamojski and Al–Aloosy (1990). The general integrated Petri Net model of the ring is given and it is shown how to decompose this model to the operations level. A token may use only one possible path from an input of the source node to an output of the destination node. This path creates a submodel of a function realization. The sum of the firing times of all transitions along this path is equal to the time realization of a considered operation.

It will be shown, how to estimate the realization time of function F_1: *receiving the data in the destination node.*

According to the STIPN model given in (Zamojski and Al–Aloosy, 1990), the submodel of this operation is shown in Figure 3. There are two main function paths:

i) upper–loading operation of the correct slot (without malfunctions),
ii) lower–creation of the maintenance packet (because a parity failure has been detected).

The token (i.e. slot) selects the lower path with the probability of a parity malfunction in the slot $(Pr\{M\})$ or the upper path with probability $1 - Pr\{M\}$.

Since the transitions model the operations then, the set of primitive operations is given by

$$O_i = \{o_{j1};\quad j = 1, 2, ..., 11\} \tag{6}$$

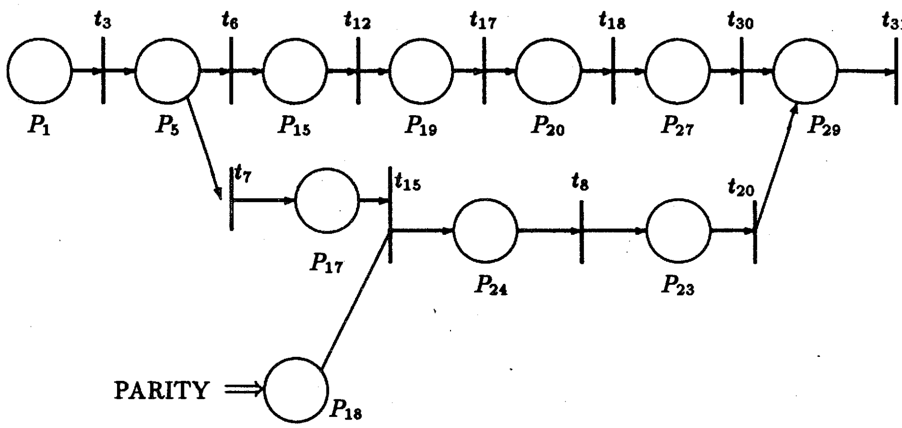

Fig. 3. The Petri Net submodel of the operation *Receiving data in the destination* (from: Zamojski and Al-Aloosy, 1990).

and function F_1 is realized as the tupple

$$\langle o_{11}, o_{21}, o_{31}, o_{41}, o_{51}, o_{61}, o_{71} \rangle \tag{7}$$

or

$$\langle o_{11}, o_{81}, o_{91}, o_{101}, o_{111}, o_{71} \rangle \tag{8}$$

In this way the realization time of function F_1 is

$$T_1 = \sum_{j=1}^{11} Pr\{o_{j1}^{(k)}\}\tau_{j1}^{(k)} = t_1 + Pr\{M\}(t_2 + t_3 + t_4 + t_5 + t_6)$$

$$+ \ (1 - Pr\{M\})(t_8 + t_9 + t_{10} + t_{11}) + t_7 \tag{9}$$

where $t_1, ..., t_{11}$ are firing duration times.

The Petri net submodels of functions realized in the source node, in the intermediate nodes (active and passive), and in the destination node may be analyzed in the same way. The times of functions realizations are estimated on the basis of these submodels.

As a few functions are realized in each node, then it is convenient to characterize the speed of a node by its mean transfer time

$$t_T^{(k)} = \sum_{i=1}^{I^{(k)}} Pr\{F_i^{(k)}\}T_i^{(k)} \tag{10}$$

where $t_T^{(k)}$ is the mean transfer time of the k-th node, $F_i^{(k)}$ is the i-th function realized in the k-th node, $T_i^{(k)}$ is the time of the realization of the i-th function in the k-th node (5), $I^{(k)}$ is a number of functions realized in the k-th node.

3. Mean Time of Slot Transfer of Cambridge Ring

Performance parameters of the ring depend on its organization and size, the number of users, volume of transported information, etc. and also on the ring reliability. The measure which combines the performance (conditions of a fault–free system performance) and reliability (probability of success) is called performability. The performability is defined as the probability that the system operates at an assigned accomplishment level with respect to faults, fault recovery, reconfiguration, degradation etc. The following performance indices may be used as performability mesuares of a system: response time, mean time of a task realization, mean number of realized tasks, etc. In this paper the ***mean time of a slot transfer*** (MTST) in the Cambridge Ring is defined as the performability measure of the ring. The MTST depends on the size of the ring (number of stations and distances between them, number of slots), condition of the ring operation (number of active and passive stations, users requests for accesses to ring resources), reliability of the ring (ratio of failures and malfunctions and ring reactions to them).

The Mean Time of Slot Transfer (MTST) is evaluated as a sum of the following main factors:

1. Mean Real Transfer time (MRT) of the full slot (together with the ACK signal receipt) in the real ring (some paths of the STIPN model are realized randomly with a given probability of malfunctions (10), and ring reactions on detected malfunctions are considered),
2. Mean Waiting Time (MWT) for an empty slot in the real ring (the waiting time depends on the number of active stations and their needs for access to the ring,and the number of the malfunctions, because the repeated transfers must be considered);

$$MTST = MRT + MWT \tag{11}$$

4. Estimation of MTST

4.1. Assumptions

There are N stations and some of them are in the active state (N_A) while the others are passive (N_P);

$$N = N_A + N_P \tag{12}$$

The ring is regular, that is:

i) each station is modeled as the Stochastic Timed Inhibited Petri Net,

ii) the transfer times between stations are equal to one another

$$\forall k : N_{Ak} = N_A, \quad N_{Pk} = N_P, \quad t_T^{(k)} = t_T \tag{13}$$

where $t_T^{(k)}$ denotes the transfer time from the k–th station to the $(k+1)$–th which is evaluated by (10),

iii) the active and passive stations delay all signals by 3 bits. The speed of the signal propagation in the ring is 2.3×10^8 *mps* and the bandwidth of the link is $10\,Mbits$. So the number of bits stored in the ring is estimated as

$$B_R \cong N\,3 + \frac{N\,L}{23} \tag{14}$$

where L is distance between the two consecutive stations,

iv) malfunctions of transfered bits are independent events with exponential distributions (with parameter k_m). The malfunctions of the slot create the Poisson process with rate $\lambda_M = 38k_m$,

v) the Cambridge Ring protocol states that after the first detected malfunction the ring waits through one free ring cycle and then tries to repeat the transfer operation. If it is not completed successfully the ring waits through 15 ring cycles and again attempts the same. The next fault is signaled as the ring breakdown (a failure).
Generally, it is noted that the time is lost in the ring while removing the faulty slot and repeating its transfer:

a) $T_{L1}-$ the lost time for successfully repeated first transfer

$$T_{L1} \cong MCT + T_M \tag{15}$$

b) $T_{L2}-$ the lost time for successfully repeated second transfer

$$T_{L2} = 15MCT + T_M \tag{16}$$

where MCT is the ring circlulating time, T_M is the time lost to remove the faulty slot and to create the maintenance packet. It is assumed that $T_M \cong k\,MCT$; $k = 3 - 4$,

vi) there are M slots in the ring. The gap is smaller than the slot and is neglected in further considerations,

vii) a user needs an access to the ring resources with probability P_A. It is assumed that for each user P_A is the same and is given by an exponential distribution with ratio λ.

4.2. Mean Real Transfer Time

The Mean Real Transfer (MRT) time of the full slot in the ring with $N = N_A + N_P$ stations is estimated as

$$MRT \cong MCT\,M_{M0} + M_{M1}(MCT + T_M) + M_{M1,2}(15\,MCT + T_M) \tag{17}$$

where MCT is the ring circle time evalueted for the full reliable ring i.e. without malfunctions during the system operation; $MCT \approx Nt_T$ and t_T is given by (10), M_{M0} is probability of the successful transfer of the full slot, M_{M1} is probability of malfunctions during the first transfer of the full slot, $M_{M1,2}$ is probability of malfunctions during the repeated transfer of the full slot.

The probability of the successful transfer of the full slot (38 bits) is approximated on the basis of assumption 4 as

$$M_{M0} = e^{-\lambda_M MCT} = e^{-38k_m MCT} \tag{18}$$

the full slot transfer is equal

$$M_{M1} = 1 - e^{-\lambda_M MCT} \tag{19}$$

Because the malfunctions are independent events then the probability that a malfunction occurs during the first and the second slot transfer is equal

$$M_{M1,2} = (M_{M1})^2 = \left(1 - e^{-\lambda_M MCT}\right)^2 \tag{20}$$

We substitute (18), (19) and (20) into (17) and obtain the formula for the Mean Real Transfer time:

$$\begin{aligned} MRT \cong\ & MCT\left\{e^{-\lambda_M MCT} + \left[1 - e^{-\lambda_M MCT}\right](1+k)\right. \\ & + \left.\left[1 + e^{-\lambda_M MCT}\right]^2 (15+k)\right\} \end{aligned} \tag{21}$$

where $k = 3 - 4$ (assumption 5).

4.3. Mean Waiting Time

The waiting time for an empty slot depends on its position in the ring at the moment when the source requests a transfer. The most optimistic case is that the slot is emptied in a station just before the source. The worst case is when the slot is emptied in the source and must be skipped to the next station, and each active node in a sequence fills the slot and waits for the ACK signal. Of course possible malfunctions will increase the waiting time.

Each active node needs an empty slot with probability P_A (assumption 7) and the transfer time to the next station is

$$P_A\,(t_{RT} + t_T) + (1 - P_A)\,t_T \tag{22}$$

where t_{RT} is real transfer time of the full slot.

There are M slots in the ring and N stations. Each slot serves $N_M = N/M$ stations. Each active station needs an empty slot with probability

$$P_A = 1 - e^{-\lambda t^*} \tag{23}$$

where t^* is a time of *thinking and waiting.*

When the slot is just emptied in the waiting station $(l = 1)$, it must be skipped to the consecutive node and the station waits for the next slot, then:

$l = 1 \quad t_T$

$l = 2 \quad P_A(MRT + t_T) + (1 - P_A)t_T = T_2$

$l = 3 \quad P_A(MRT + t_T) + (1 - P_A)t_T + T_2 = T_3$

...

$l \quad P_A(MRT + t_T) + (1 - P_A)t_T + T_{l-1} + T_{l-2} + ... + T_2 = T_l$

or

$$T_l = P_A MRT + t_T + T_{l-1} + T_{l_2} + ... + T_2.$$

The slot journey time from node $l = j$ to $l = 1$ is

$$T_j = (j - 1)(P_A MRT + t_T) \tag{24}$$

Because the slot may be at each node with a probability $Pr\{j\}$, then the Mean Waiting Time for an empty slot is

$$MWT = \sum_{j=2}^{N_M} Pr\{j\}T_j + t_T \tag{25}$$

The ring is regular, then $Pr\{j\} = M/N$ and

$$\begin{aligned} MWT &= \frac{M}{N}[(N_M - 1)(P_A MRT + t_T) + t_T] \\ &= \frac{M}{N}[(N_M - 1)P_A MRT + N_M t_T] \end{aligned} \tag{26}$$

where P_A is given by (23), MRT is given by (21).

There is a problem of an approximation of the *thinking and waiting time* $-t^*$, which may change from the smallest value t_T to the biggest $(N_M - 1)(MRT + t_T)$. This time depends on the slot position and the mean value of t^* is

$$Et^* = \frac{M}{N}(N_M - 1)MRT \tag{27}$$

and

$$P_A \cong 1 - e^{\left[-\lambda \frac{M}{N}(N_M - 1)MRT\right]} \tag{28}$$

4.4. Mean Time of the Slot Transfer

The MTST (11) may be approximated as the sum of terms given in (21) and (26). The approximates of the Mean Time of the Slot Transfer in the regular Cambridge ring with different numbers of stations are shown in Figure 4 and Figure 5.

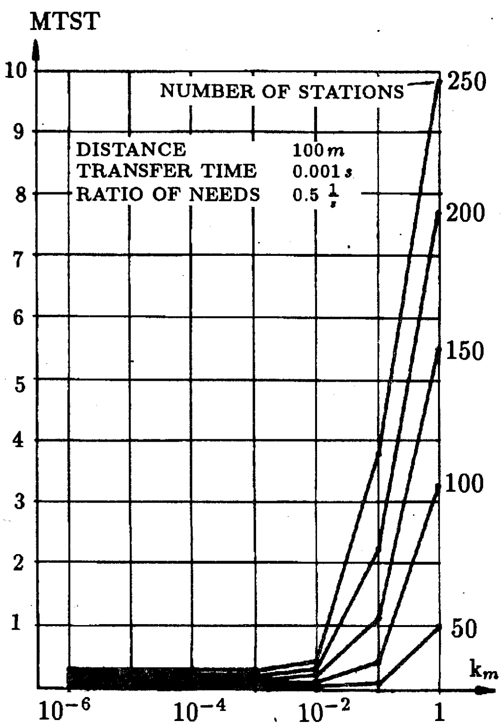

Fig. 4. The Mean Time of the Slot Transfer versus the probability of malfunctions in the Cambridge Ring (the transfer time: 0.001 s).

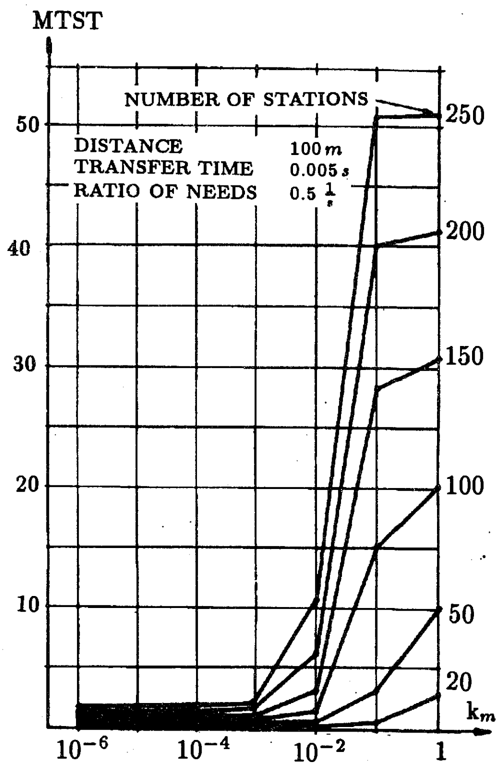

Fig. 5. The Mean Time of the Slot Transfer versus the probability of malfunctions in the Cambridge Ring (the transfer time: 0.005 s).

The reliability of each station is 1. Malfunctions occur in the ring with ratios given from the interval $[10^{-6}, 1]$. The distances between stations are equal to $100m$ and the transfer time t_T is equal $0.001s$ (Figure 4) and $0.005s$ (Figure 5). Number of stations in the ring is changing from 20 to 250 nodes.

The mean time of the full slot transfer versus ratios of malfunctions is shown on the figures. We note that the MTST grows rapidly for k_m bigger than 10^{-2}, and there is a *saturation state* for an *overload ring*.

5. Conclusions

The functional–reliability analysis of the Cambridge Ring is given above. A set of the faults of the rings is limited only to malfunctions and this assumption, as it was mentioned ring under consideration is in the Introduction, corresponds to rapid reduction of ratio of failures in computer systems of today. It is assumed that a malfunction produces faults among bits of a transferring slot. A parity checking operation ought to find this fault immediately in the following node and the whole transferring operation of this slot must be repeated together with the waiting operation for an empty slot.

Therefore, the mean time of slot transfer is considered as a performability measure of the ring. Organization and size of the ring (number of nodes and distance between them, bandwidth, number of slots), number of users and their needs for sending information and reliability (malfunctions) are considered. The fundamental conclusion of the analysis is: the Mean Time of Slot Transfer does not depend on realistic value of the malfunction ratio. Naturally, if malfunctions are very frequent then the MTST rapidly grows up – see figure 4 and 5.

The Petri net model of the Cambridge Ring is proposed. The model may be useful for a carefully analysis of the ring operation in real reliability conditions and for precise calculations of time relations between functions realized by the ring.

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References

Al–Jaar R.Y. and Desrochers A.A. (1990): *Performance evaluation of automated manufacturing systems using generalized stochastic Petri nets.*– IEEE Trans.on Robotics and Automation, v.6, No.6, pp.621–639.

Bobbio A. (1983): *A Petri net approach to the reliability and performability analysis of complex systems.*– Int. Conf. *Micro- Electronics and Teleinformatics*, Milano Fair Ground

Fulcher J. and McKerrow P. (1990): *Monitoring tool for the Cambridge Ring LAN.*– Computer Communications, v.13, No.10, pp.619–625.

Hopper A., Temple S. and Williamson R.. (1986): *Local Area Network Design.– London: Addison-Wesley Publishing Company.*

King P.J.B. and Mitrani I. (1987): *Modeling a slotted ring local area network.*– IEEE Trans. on Computers, v.C–36, No.5, pp.554–561.

Li M. and Georganas N.D. (1990): *Coloured generalized stochastic Petri nets for integrated systems protocol performance modeling.*– Computer Communications, v.13, No.7, pp.414–424.

Liu M.T. and Rouse D.M. (1984): *A study of ring network.*– In: Ring Technology Local Area Networks, (Eds. I.N. Dallas and E.B. Spratt), Berlin: Elsevier Science Publishers B.V. (North-Holland), IFIP, pp.1–39.

Tanenbaum A.S. (1989): *Computer Networks.*– New Jersey: Prentice Hall.

Zamojski W. (1987): *A reliability-functional computer system model based on Petri nets.*– IASED Symp. *Reliability and Quality Control*, Zurich: Acta Press, Anaheim, Calgary.

Zamojski W. and Al–Aloosy A.K.A. (1990): *A Petri net approach to the performability estimation of a ring transport system.*– Prep. nr 49/90, Inst. Engineering Cybernetics, Technical University of Wrocław, Poland.

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