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A NEW SLIDING MODE APPROACH TO ASYMPTOTIC FEEDBACK LINEARISATION WITH APPLICATION TO THE CONTROL OF NON-FLAT SYSTEMS

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A dynamic sliding mode controller design method is proposed. The method uses a novel choice of the sliding surface to effect asymptotic linearisation of nonlinear differential input output systems and a class of state space systems. The stability of the overall system, i.e. a canonical state space form with a dynamic feedback, is analysed. This method is shown to be able to control a fairly general class of systems, including some which are not linearizable by dynamic feedback, using a chatter free control. The theoretical results are applied to the control of a particular single input system which is not dynamic feedback linearizable.

1. Introduction

The development of techniques for the linearisation of nonlinear systems is a topic of considerable interest to the control engineer. There are currently three broad approaches. Firstly, there are approximate techniques such as Jacobian linearisation. This method may only be applied to slowly varying systems which do not have a high degree of nonlinearity and has the inherent disadvantage that local controllability and observability may be effected by the linearisation process. Alternative approximate linearisation approaches seek to approximate a class of nonlinear systems with feedback linearisable nonlinear systems (Hauser *et al.*, 1992) and approximate slightly non-minimum phase systems with linearisable minimum phase systems (Hauser *et al.*, 1989). However, the range of problems to which these approximate methods may be applied is limited. The second main approach considers exact linearisation using static or dynamic feedback and coordinate transformation. Some conditions for such linearisation are given in (Charlet *et al.*, 1989; Isidori, 1989; Marino, 1988; Nijmeijer and van der Schaft, 1990). Again the range of problems which may be effectively solved using this method appears limited because the linearisability conditions are rather restrictive. Dynamic feedback linearisable systems are also called *flat* systems (Fliess *et al.*, 1993). The final approach involves an asymptotic feedback linearisation and is applicable for the control of both flat and non-flat systems. Sliding mode control methods have already been employed for effective asymptotic linearisation (Lu and Spurgeon, 1995; Sira-Ramirez, 1993a; 1993b).

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In this paper a new approach to asymptotic linearisation of nonlinear systems is developed using sliding mode control concepts. Traditional sliding mode methods as in (Sira-Ramirez, 1993a; 1993b) select a control independent sliding surface and as such, only produce dynamic sliding control policies when the differential input-output (I-O) system has control derivatives present. Such dynamic policies are desirable as they effectively reduce the chattering of the control signal which is an inherent disadvantage of many sliding mode schemes. This work uses a control dependent sliding surface which produces a broader class of dynamic controllers and thus presents a useful and more global approach to chatter-free sliding mode control. It will also be seen to produce a method for controller construction which is more widely applicable; essentially the method circumvents the usual restriction including the need for the system of interest to be expressible in 'regular form.'

The method will be employed to control a particular non-flat system. This system is thus not directly linearisable using static or dynamic feedback and provides a challenging problem for the design method. The asymptotic sliding mode control method under consideration here is shown to provide an alternative method to the high frequency control methods which have been previously employed to control such systems (Fliess *et al.*, 1995b).

The paper is structured as follows. Section 2 presents the necessary background regarding sliding mode control, develops the class of systems which are to be considered and contains the statement of a stability result which will be used for later proofs. The new sliding mode control approach is developed in Section 3. This includes a full stability analysis. Section 4 contains a detailed comparison of the proposed approach and traditional sliding mode methods. In Section Five, the method will be illustrated by considering asymptotic linearisation of a non-flat system.

The following notation will be used throughout the paper:

$$N_\delta(x_0) = \left\{ x \in \mathbb{R}^n \middle| \; \|x - x_0\| < \delta \right\}$$

where $\|\cdot\|$ is the Euclidian norm.

2. Background

2.1. Class of Systems Considered

For a given SISO system in state-space form which is locally observable,

$$\dot{x} = f(x,u) \tag{1}$$

$$y = h(x,u) \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$ and $f(x,u)$, $h(x,u)$ are sufficiently continuously differentiable, the following locally equivalent differential I-O system exists (van der Schaft, 1989):

$$y^{(n)} = \varphi(\widehat{y}, \hat{u}, t) \tag{3}$$

where $\hat{u} = (u, \ldots, u^{(\beta)})$, $\widehat{y} = (y, \ldots, y^{(n-1)})$.

Assumption 1.

(1) $\varphi(\cdot,\cdot,\cdot)$ is a C^1-function;

(2) (Regularity Condition)

$$\frac{\partial \varphi}{\partial u^{(\beta)}} \neq 0 \tag{4}$$

is satisfied with $\widehat{y} \in N_\delta(0)$ for all $t \geq 0$, some $\delta > 0$ and generically for $\hat{u}$.

Remark 1. Note that a large class of nonlinear systems, especially mechanical systems, are naturally in the form (3). Additionally, they may be in a combination of form (3) and a dynamic compensator which in the simplest case is a series of integrators of the control.

The design method considered in this paper is based on I-O systems of the form (3) which satisfy Assumption 1.

The system (3) has the GCCF (i.e. Generalized Controller Canonical Form) realisation (Fliess, 1990)

$$\begin{aligned} \dot{\zeta}_1 &= \zeta_2 \\ &\vdots \\ \dot{\zeta}_{n-1} &= \zeta_n \\ \dot{\zeta}_n &= \varphi(\zeta, \hat{u}, t) \end{aligned} \tag{5}$$

where $\zeta = (\zeta_1, \ldots, \zeta_n)$. The associated *zero dynamics* is defined as

$$\varphi(0, \hat{u}, t) = 0 \tag{6}$$

Here (3) is called *minimum phase* if there exist $\delta > 0$ and $\widetilde{u}_0 \in \mathbb{R}^\beta$ such that (6) is uniformly asymptotically (exponentially) stable for any initial condition $\widetilde{u}(0) \in N_\delta(\widetilde{u}_0)$, where $\widetilde{u} = (u, \ldots, u^{(\beta-1)})$. Otherwise, it is *non-minimum phase* (Fliess, 1990).

2.2. Flatness

Flatness is a notion of dynamic feedback linearisability proposed by Fliess *et al.* (1994; 1995a; 1995b). It is generally recognised that the problem of linearisation via dynamic state feedback and coordinate transformation is equivalent to that of linearisation with dynamic output feedback and associated properly chosen fictitious outputs. These outputs are called *linearizing* or *flat* outputs. More formally, consider nonlinear systems of the form

$$\dot{x} = f(x, \hat{u}) \tag{7}$$

where $x = (x_1, \dots, x_n)$, $\hat{u} = (u, \dot{u}, \dots, u^{(\beta)})$, $f = (f_1, \dots, f_n)$ are differential polynomials of x and $\hat{u}$. It is clear that the feedback involved is dynamic rather than static. The generalised nonlinear control system (7) is called *flat* if some fictitious output

$$y = y(x, \hat{u})$$

exists such that the state x and control variable u can be expressed, without integrating any differential equation, in terms of the flat output and its associated finite order of derivatives.

The works (Fliess *et al.*, 1994; 1995a; 1995b) demonstrate that some systems of practical significance are non-flat and hence non-trivial to control. Fliess and co-workers introduce the use of a particular high frequency control method to control such systems (Fliess *et al.*, 1995b). Fliess and Sira-Ramirez (1993) suggested a link between flatness and nonlinear sliding modes. Such non-flat systems thus provide an appropriate design case study for nonlinear sliding mode schemes.

2.3. A Brief Review of Sliding Mode Control

Much of the published work in sliding mode control employs a 'static' feedback approach (DeCarlo *et al.*, 1988; Utkin, 1992). The resulting controllers can be either continuous or discontinuous. It is generally recognised that the sliding mode control design strategy may be divided into two independent procedures which are concerned with the choice of the sliding surface and the choice of a reachability condition to ensure the sliding surface is reached.

Two particular choices of the sliding surface are given below:

(a) Choosing $s = s(x)$. This may be considered as a single geometric manifold which may be determined by a set of geometric equations (DeCarlo *et al.*, 1988; Utkin, 1992). This is termed a *static sliding surface.*

(b) Choosing the sliding surface which is a set of differential equations (Lu and Spurgeon, 1995; Sira-Ramirez, 1993a; 1993b; Slotine and Coetsee, 1986). This is, in fact, a bundle of geometric manifolds and is termed a *dynamic sliding surface.* Consider, for example, the work of Sira-Ramirez which will be used to draw a comparison with the work presented here (Sira-Ramirez, 1993a; 1993b). The sliding surface is chosen as

$$s = \sum_{i=1}^{n} a_i \zeta_i \tag{8}$$

where $(a_1, \dots, a_{n-1}, a_n)$ are the coefficients of the Hurwitz polynomial $\sum_{i=1}^{n} a_i \lambda^{i-1}$ of degree $n-1$. In order to distinguish this method from the proposed sliding mode method, this approach will be called the *direct sliding mode* method.

Having selected an appropriate sliding surface, a reachability condition must be employed to ensure that the sliding mode is reached. Perhaps the most popular reachability condition is to select a control such that $s\dot{s} < 0$ if $s \neq 0$. There are many possible reachability conditions which may be broadly defined as follows:

Definition 1. A *general sliding reachability condition* is defined as

$$\dot{s} = -\gamma(\kappa, s, t) \tag{9}$$

where $\kappa = [\kappa_1, \dots, \kappa_l]$ is a set of constant parameters such that for some fixed κ the following conditions are satisfied:

(1) $\gamma(\kappa, s, t)$ is continuous and bounded with respect to s if $s \neq 0$;

(2) $\gamma(\kappa, 0, t) = 0, \ t \geq 0$;

(3) equation (9) is globally uniformly asymptotically stable or $s \to 0$ in a finite time.

For most sliding mode design approaches, the following specific sliding reachability condition will be appropriate.

Definition 2. The *sliding reachability condition* is defined as

$$\dot{s} = -\gamma(\kappa, s) \tag{10}$$

where $\kappa = [\kappa_1, \dots, \kappa_l]$ is a set of constant parameters such that for some fixed κ

(1) $\gamma(\kappa, s)$ is a C^1-function of s if $s \neq 0$;

(2) $\gamma(\kappa, s)$ is a bounded for $s \in N_\delta(0)$;

(3) $\gamma(\kappa, 0) = 0$;

(4) $s\gamma(\kappa, s) > 0$ if $s \neq 0$.

For a sliding mode controller design using static feedback, it is necessary that the system assume a *regular form* and that the control variables appear linearly in the system in order to recover the control parameters from the reachability condition (Utkin, 1992). Thus far there is no global method which is practically implementable for nonlinear systems with nonlinear controls.

In addition, the sliding reachability condition may provide a discontinuous control signal which is often undesirable for a practical implementation. Further work has considered methods to reduce the chattering caused by the high frequency switching of the control signal. There are several effective ways to accomplish this task:

(a) Introduce a layer of thickness $0 < \epsilon \ll 1$ around the sliding surface such that, when $||s|| > \epsilon$, the controller developed from the sliding mode reachability condition is employed, and when $||s|| \leq \epsilon$, an alternative continuous control policy is employed (Slotine and Coetsee, 1986; Utkin, 1992).

(b) Adopt a dynamic sliding mode feedback where the effective filtering of the control reduces chattering naturally (Levant, 1993; Lu and Spurgeon, 1995; Sira-Ramirez, 1993a; 1993b; Slotine and Coetsee, 1986).

(c) Adopt a continuous reachability condition (Lu and Spurgeon, 1995).

Method (b) is natural only when a dynamic sliding mode approach is adopted. However, relatively little work has been done in this new and exciting area. The next section will show how to develop a dynamic sliding mode controller, with all the attendant advantages, which will also perform asymptotic linearisation of a broad class of nonlinear systems by employing a particular choice of the sliding surface.

It may be concluded that (15) and (21) are equivalent in stability if the matrix

$$\begin{bmatrix} A & b \\ 0 & k \end{bmatrix} \tag{22}$$

is non-singular. This is the case because $k > 0$ (Corollary 1) and A is Hurwitz. Now setting $c = 0$ and replacing b with $-b$ in (16) leads to (19). ∎

Theorem 1. *(Overall stability of the indirect sliding mode) Consider the system (5). Let the following conditions be satisfied:*

(1) System (3) fulfils Assumption 1;

(2) The highest order derivative of control

$$u^{(\beta+1)} = p(\zeta, \hat{u}, t)$$

is solved out from

$$\dot{s} = -\gamma(\kappa, ks)$$

where $\gamma(\kappa, \cdot)$ is as in Definition 2;

(3) k is chosen such that

$$k > (Bb)^T C^{-1} (Bb) \tag{23}$$

where B satisfies eqn. (17), A is the companion matrix of the Hurwitz polynomial determined by (14) and

$$b = [0, \dots, 0, 1]^T$$

(4) The zero dynamics (6) is locally uniformly asymptotically stable.

Then the closed-loop system (13) is locally uniformly asymptotically stable.

Proof. Let $z = (z_1, \dots, z_\beta)$. Under the regularity condition in Assumption 1, and the coordinate transformation $(\zeta, \widehat{z}) \to (\zeta, s, z)$, (13) is equivalent to

$$\begin{aligned} \dot{\zeta}_1 &= \zeta_2 \\ \dot{\zeta}_2 &= \zeta_3 \\ &\vdots \\ \dot{\zeta}_n &= -\sum_{i=1}^{n} a_i \zeta_i + s \\ \dot{s} &= -\gamma(\kappa, ks) \end{aligned} \tag{24}$$

together with

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_\beta &= q(\zeta, s, z, t), \end{aligned} \tag{25}$$

where

$$u^{(\beta)} = \dot{z}_\beta = q(\zeta, s, z, t)$$

is obtained from (11).

Lemmas 1 and 2 guarantee the local uniform asymptotic stability of (24). Condition (4) guarantees the local uniform asymptotic stability of (25) when setting $\zeta = 0$, $s = 0$. On the other hand, $q(\zeta, s, z, t)$ is a C^1–function because φ and s are C^1-functions. Thus conditions (A1) and (A2) in Theorem 1 (Vidyasagar, 1980) are satisfied and the assertion is then deduced from Theorem 1 about the stability of triangular systems by letting $w_1 = (\zeta, s)$, $w_2 = z$. ■

Now it will be shown that under slightly more restrictive conditions, global results can be obtained.

Corollary 2. *Suppose that*

(1) no derivatives of the control appear in (3);

(2) the Regularity Condition in Assumption 1 holds globally for $\widehat{y}$ and u, and $t \geq 0$;

(3) for any $r_1 > 0$ there exists an $r_2 > 0$ such that if $\|\widehat{y}\| \leq r_1$ and $\varphi(\widehat{y}, u, t) - y^{(n)} = 0$ then $\|u\| \leq r_2$;

(4) $\gamma(\kappa, \cdot)$ is chosen as in Definition 2;

(5) $\int_0^s \gamma(\kappa, z)\, \mathrm{d}z$ is radially unbounded for s.

Then there exists at least one dynamic sliding mode controller such that the closed loop system (13), where the dynamic feedback is of first order, is globally uniformly asymptotically stable.

Proof. By the results in (Sanderberg, 1981), conditions (2) and (3) guarantee the existence of at least one global solution (12) in Step 3. $(\zeta, u) \rightarrow (\zeta, s)$ is a global coordinate transformation. The result is obvious from Theorem 1 because there is no zero dynamics in this case. ■

Remark 2. Even if the derivatives of the control do not appear in (5), the indirect sliding mode method still produces a dynamic feedback of dimension 1. This low pass filter will effectively eliminate the high frequency chattering caused by a discontinuous sliding reachability condition and/or disturbances.

3.3. Proper Choice of Initial Conditions

The zero dynamics of dimension β in (6) are used for a theoretical analysis only. For a practical application, the dynamic feedback in (13) resulting from the design method is of dimension $\beta+1$, and the properties of these higher order dynamics must be explored.

Definition 3. The *zero dynamics associated with the design method* is defined as

$$\left[\dot{s} + \gamma(\kappa, ks) = 0\right]_{\zeta=0} = 0 \tag{26}$$

where k is as in Theorem 1.

Note that these zero dynamics are equivalent to (6) in stability if the regularity condition is satisfied and if $\gamma(\kappa, s)$ is as in Definition 2.

This zero dynamics results in a further step of the design procedure which is associated with the proper choice of initial conditions for the closed-loop system (13).

Step 4. Choose $\hat{u}_0 \in \mathbb{R}^{\beta+1}$ and a $\delta > 0$ such that, for the initial condition $\hat{u}(0) \in N_\delta(\hat{u}_0)$,

(1) the Regularity Condition is satisfied;

(2) the zero dynamics (26), or equivalently (13) when $\zeta \equiv 0$, are uniformly asymptotically stable;

(3) all the initial conditions for (13) are compatible.

4. Comparison with Traditional Sliding Mode Approaches

Traditional sliding mode approaches usually employ a control independent sliding surface. It follows that dynamic sliding schemes only result when I-O systems which contain control derivatives are considered as in (Sira-Ramirez, 1993b). It should be noted that such dynamic sliding mode policies are desirable as they filter possibly discontinuous signals resulting in an effective reduction of control chattering. Further, if the highest order derivative of the control appears nonlinearly in the I-O system, then it may be difficult to recover expressions for the control from the chosen reachability condition using traditional sliding mode approaches. By using the new approach presented in this paper, the sliding surface is control dependent and therefore a dynamic controller which provides the chattering reduction can result regardless of the particular I-O system representation. In addition, the highest order derivative of the control always appears linearly in the expression for $\dot{s}$, which facilitates the controller design. The proposed method restricts the class of reachability conditions which may be used and provides extra constraints upon the design parameter k in the auxiliary equations (15) and (20). In the traditional sliding mode approach the system becomes equivalent to an $n-1$ dimensional linear asymptotically stable system when sliding. In the new approach presented in this paper, the sliding mode technique has been used to asymptotically linearise the original nonlinear system; in the limit the sliding system thus becomes equivalent to an n-dimensional linear asymptotically stable system.

5. Control of the Gas Jet System

Angular velocity control of a Gas Jet Actuator with one control is modelled by the Euler equations (Example 6.9 of (Nijmeijer and van der Schaft, 1990)):

$$\begin{aligned} \dot{x}_1 &= \mathcal{A}x_2x_3 + \alpha u \\ \dot{x}_2 &= -\mathcal{A}x_1x_3 + \beta u \\ \dot{x}_3 &= \gamma u \end{aligned} \tag{27}$$

The reference trajectory is $(0,0,h)$, $h \neq 0$. It is proved in (Nijmeijer and van der Schaft, 1990) that in a neighbourhood of $x(0) = (0,0,h)$, $h \neq 0$, if $\gamma \neq 0$, $\mathcal{A} \neq 0$, $\alpha = 0$ and $\beta \neq 0$:

(a) (27) is locally strongly accessible (p.90 in (Nijmeijer and van der Schaft, 1990));

(b) (27) is not feedback linearisable by a static state feedback (p.189 of (Nijmeijer and van der Schaft, 1990)). Thus according to (Charlet *et al.*, 1989), (27) is not feedback linearisable by dynamic feedback. It is therefore non-flat.

Choose $y = x_1$ as the artificial output, and $x_3 = v$ as the control. Then

$$\begin{aligned} \dot{y} &= \mathcal{A}x_2 v \\ \ddot{y} &= \mathcal{A}v(-\mathcal{A}x_1 v + \beta\,\dot{v}) + \mathcal{A}x_2\dot{v} \end{aligned}$$

Thus an I-O system is obtained as

$$\ddot{y} = \varphi = \mathcal{A}v(-\mathcal{A}yv + \beta\dot{v}) + \dot{y}\dot{v}/v \tag{28}$$

The function φ in (28) is smooth in a neighbourhood of $(y, \dot{y}, v, \dot{v}) = (0, 0, h, \dot{v}\ (0))$. Thus the conditions (A1) and (A2) of Theorem 1 in Vidyasagar (1980) are satisfied. Moreover,

$$\left[\frac{\partial \varphi}{\partial \dot{v}}\right]_{(y,\dot{y},v,\dot{v})=(0,0,v(0),\dot{v}(0))} = \mathcal{A}\beta h \neq 0$$

so the regularity condition is satisfied. Thus (28) is a proper I-O system.

The sliding reachability condition $\dot{s} = -ks$ (where $\kappa = 1$) will be used for constructing the controller.

Consider the sliding surface

$$s = a_1 y + a_2\dot{y} + \varphi$$

When $s = 0$ with $(y, \dot{y}) = (x_1, x_2) = (0, 0)$, the zero dynamics is

$$\mathcal{A}\beta v\dot{v} = 0 \tag{29}$$

which is trivial, and so asymptotically stable for any $v(0)$.

To determine the initial conditions for the dynamic compensator, $[\dot{s} + ks]_{x=0} = 0$ yields the following zero dynamics:

$$v\ddot{v} + 2\dot{v}^2 = -(k + a_2)v\dot{v}$$

Dividing both sides by $v\dot{v}$ and integrating with respect to t gives

$$\ln \left|\dot{v}v^2\right| = -(k + a_2)t + \ln c_1, \qquad c_1 > 0$$

or equivalently

$$\begin{aligned} \dot{v}v^2 &= c_1 e^{-(k+a_2)t} \\ \dot{v}(0)v^2(0) &= c_1 \end{aligned}$$

Integrating again, we obtain

$$\frac{1}{3}v^3 = -(k + a_2)^{-1} c_1 e^{-(k+a_2)t} + c_2$$

This leads to

$$v(t) \to [3c_2]^{1/3} \quad (t \to \infty)$$

Note that $x_3 = v$. If (x_1, x_2) tends rapidly to zero, the dynamic behaviour of x_3 is dominated by the zero dynamics. Thus to force $x_3 \to h$, it is necessary and sufficient that $[3c_2]^{1/3} = h$, which determines c_2. Then

$$\begin{aligned} \dot{v}(0)v^2(0) &= c_1 > 0 \\ \left[h^3 - 3c_1(k + a_2)^{-1}\right]^{1/3} &= v(0) \end{aligned}$$

determine $v(0)$ and $\dot{v}(0)$ if h and c_1 are given.

All the conditions of Theorem 1 are thus satisfied. Note that

$$\frac{\partial \hat{y}}{\partial(x_1, x_2)} = \begin{bmatrix} 1 & 0 \\ 0 & \mathcal{A}v \end{bmatrix}$$

which is control dependent. It follows that in a neighbourhood of $(y, \dot{y}, v) = (0, 0, h)$, the transformation $(y, \dot{y}, x_3) \leftrightarrow (x_1, x_2, x_3)$ is non-singular, i.e. regulation of $(y, \dot{y})$ implies that of (x_1, x_2), and finally $(x_1, x_2, x_3) \to (0, 0, h)$ as $t \to \infty$. The design parameters are chosen as $k = 8$, $\beta = 2.5$, $\mathcal{A} = 0.5$, $\gamma = 1.25$, $h = 0.6$, $c_1 = 5$, $(x_1(0), x_2(0)) = (-1.13, 1.52)$. It is determined from the above that $v(0) = 0.6704$, $\dot{v}(0) = 11.1265$. Let $(a_1, a_2, 1) = (110, 21, 1)$, and $C = I_{2\times 2}$. Then, according to (23),

$$k > (Bb)^T C^{-1} (Bb) = 7.2347$$

where B is obtained from (17) and $b = [0, 1]^{\mathsf{T}}$. Simulation results are shown in Fig. 1.

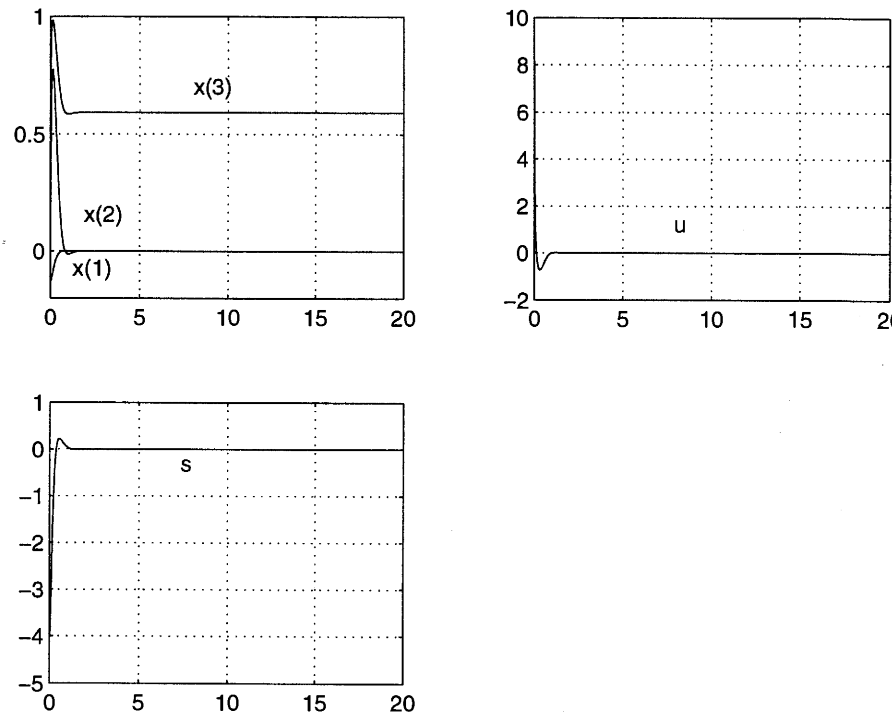

Fig. 1. Gas Jet with one control for the decoupled sliding reachability condition.

Remark 3. (i) The importance of the choice of initial conditions for the closed-loop system is seen when dynamic feedback is used to control nonlinear systems.

(ii) In this design method, x_3 is not observed by y and $\dot{y}$. Thus the effect of the control signal on x_3 is rather weak. The simulation results are reasonably good because the initial conditions $(x_1(0), x_2(0))$ are very small and $(x_1, x_2) \to (0,0)$ sufficiently quickly for the dynamic behaviour of x_3 to be dominated by the zero dynamics.

(iii) The application of this method to general nonlinear systems in state space form needs further considerations.

6. Conclusions

A large class of nonlinear systems can be modelled by differential input-output equations. Such models can then be used to develop dynamic controllers. This paper has addressed the application of sliding mode methods to this dynamic controller design problem. A particular choice of sliding surface is shown to provide asymptotic linearisation of the resulting closed-loop system. Note that this may be achieved with a chatter-free control signal. The stability analysis has also been performed. The method has the advantage that the highest order derivative of the control always appears linearly in the sliding mode reachability conditions which greatly facilitates controller construction; this is not the case in many previous methods for nonlinear sliding mode controller design. The application of sliding mode methods to the control of a single input non-flat Gas Jet system with one control has been used to illustrate the theoretical results. Note that this system is not linearisable using conventional dynamic feedback and thus provides a pertinent example.

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