Appl. Math. and Comp. Sci., 1998, Vol.8, No.1, 145–158

A ROBUST TRACKING CONTROL SCHEME FOR A CLASS OF NONLINEAR SYSTEMS WITH FUZZY NOMINAL MODEL

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A novel robust tracking control scheme is proposed for a class of nonlinear systems. It is shown that a nonlinear system is first approximated by a fuzzy nominal model by aggregating a set of linearized local subsystems and then a conventional linear feedback controller is designed to stabilise the nominal system. In order to obtain good tracking performance for the controlled nonlinear system, a variable structure compensator is introduced to eliminate the effects of the approximation error and uncertainties between the fuzzy nominal model and the nonlinear system. A simulation example using a one-link rigid robotic manipulator is given in support of the proposed control scheme.

1. Introduction

In the design of modern and classical control systems, the first step is to establish a suitable mathematical model to describe the behaviour of the controlled plant. However, in practical situations, such a requirement is not feasible because the controlled systems have high nonlinearities and uncertain dynamics, and simple linear or nonlinear differential equations cannot sufficiently represent the corresponding practical systems, and therefore, the designed controller based on such a model cannot guarantee the good performance such as stability and robustness. During the last few years, fuzzy logic control has been suggested as an alternative way to conventional control techniques for complex nonlinear systems due to the fact that fuzzy logic combines human heuristic reasoning and expert experience to approximate a certain desired behaviour function (see e.g. Takagi and Sugeno, 1985; Cao *et al.*, 1996; Wang *et al.*, 1996). However, the asymptotic error convergence and stability of the closed loop system may not be obtained due to the approximation error and uncertainties of the fuzzy model.

This paper presents a novel robust tracking control methodology for a class of nonlinear systems by combining the merits of fuzzy logic and variable structure control. Here, fuzzy logic is used to formulate a nominal system model by aggregating a

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set of linearized local subsystems which identify the nonlinear system approximately, and a fuzzy nominal feedback controller is designed to guarantee that the output tracking error of the nominal system with respect to a desired trajectory converges to zero. Then, a variable structure compensator is designed to eliminate the effects of the approximation error and uncertainties between the nonlinear system and fuzzy nominal model.

The organisation of the rest of the paper is as follows. In Section 2, the fuzzy nominal model for a class of nonlinear systems is introduced. In Section 3, a robust tracking control scheme is presented, and the stability and robustness of the closed loop system are discussed in detail. In Section 4, a simulation example using a one-link rigid robotic manipulator is given in support of the proposed control scheme. Section 5 gives concluding remarks.

2. Fuzzy Nominal Model

2.1. Linearization of the System

Consider the following single-input and single-output dynamic system:

$$x^{(n)}(t) = f(\boldsymbol{x}) + b(\boldsymbol{x})u \tag{1}$$

where the scalar x is the system output, the scalar u stands for the control input, $\boldsymbol{x} = [x, \dot{x}, \ldots, x^{(n-1)}]^T \equiv [x_1, x_2, \ldots, x_n]^T$ denotes the state vector, $f(\boldsymbol{x})$ is a linear or nonlinear function, and $b(\boldsymbol{x})$ is the (possibly state dependent) control gain. The control objective is to force the plant state vector $\boldsymbol{x} = [x, \dot{x}, \ldots, x^{(n-1)}]^T$ to follow a specified desired trajectory, $\boldsymbol{x}_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]$. Assuming that $f(\boldsymbol{x})$ and $b(\boldsymbol{x})$ are differentiable with respect to $\boldsymbol{x}$, we can linearize (1) at some point $(\boldsymbol{x}_i, u_i)$ by Taylor's method such that

$$\begin{aligned} x^{(n)} &= x_i^{(n)} + \left(\frac{\partial f}{\partial \boldsymbol{x}}\bigg|_{\boldsymbol{x}_i} + u_i \frac{\partial b}{\partial \boldsymbol{x}}\bigg|_{\boldsymbol{x}_i} \right) \tilde{\boldsymbol{x}} + b(\boldsymbol{x}_i)\tilde{u} \\ x_i^{(n)} &= f(\boldsymbol{x}_i) + b(\boldsymbol{x}_i)u_i \end{aligned} \tag{2}$$

where

$$\tilde{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x}_i, \qquad \tilde{u} = u - u_i$$

and u_i can be obtained from the following equilibrium condition (Palm and Rehfusess, 1997)

$$\dot{\boldsymbol{x}}_i = 0$$

From (2), we have the following local linearized error dynamic equation:

$$\dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}_i \tilde{\boldsymbol{x}} + \boldsymbol{B}_i \tilde{u} \tag{3}$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ a_0 & a_1 & \dots & \dots & a_{n-1} \end{bmatrix}, \qquad B_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b(x_i) \end{bmatrix} \tag{4}$$

$$a_0 = \frac{\partial f}{\partial x}\bigg|_{x_i} + u_i \frac{\partial b}{\partial x}\bigg|_{x_i}, \qquad a_1 = \frac{\partial f}{\partial \dot{x}}\bigg|_{x_i} + u_i \frac{\partial b}{\partial \dot{x}}\bigg|_{x_i}, \dots$$

Remark 1. Equation (3) is in a controllable form, and hence the feedback control law

$$\tilde{u} = -K_i \tilde{x} \tag{5}$$

can be designed by using conventional linear system theory (Ogata, 1990) so that the eigenvalues of $(A_i - B_i K_i)$ are the specified ones. The feedback gain K_i can be obtained by using Ackerman's formula (Ogata, 1990)

$$K_i = [0, \dots, 0, 1] Q_i^{-1} \alpha(A_i) \tag{6}$$

where

$$\alpha(s) = s^n + \alpha_n s^{n-1} + \dots + \alpha_2 s + \alpha_1$$

is a desired stable polynomial, and

$$Q_i = \begin{bmatrix} B_i & A_i B_i & A_i^2 B_i & \cdots & A_i^{n-1} B_i \end{bmatrix}$$

2.2. Fuzzy Nominal Model

Many physical systems are very complex so that it is very difficult to obtain their rigorous mathematical models. In recent years, fuzzy logic has been applied to the field of system modelling and control engineering (Takagi and Sugeno, 1985; Wang and Mendel, 1992; Feng *et al.*, 1997) by means of combining human heuristic reasoning and expert experience. In this paper, a fuzzy nominal model is established by the following fuzzy inference rules which include local linearized subsystems and feedback controllers:

$$R^i: \quad \text{IF} \quad x_1 \text{ is } F_1^i \quad \text{AND} \quad \dots \quad \text{AND} \quad x_n \text{ is } F_n^i$$

$$\text{THEN} \quad \dot{\tilde{x}} = A_i \tilde{x} + B_i \tilde{u}, \quad \tilde{u} = -K_i \tilde{x}, \quad i = 1, 2, \dots, l \tag{7}$$

where R^i denotes the i-th fuzzy inference rule, l stands for the number of inference rules, $F_j^i (j = 1, 2, \dots, n)$ are fuzzy sets, $\tilde{x} = x - x_d$ is the tracking error of the system with the desired trajectory x_d and $\tilde{u} = u - u_d$.

Let $\mu_i(\boldsymbol{x})$ be the normalized membership function of the inferred fuzzy set F^i where

$$F^i = \bigcap_{j=1}^{n} F_j^i \tag{8}$$

and

$$\sum_{i=1}^{l} \mu_i(\boldsymbol{x}) = 1 \tag{9}$$

By using a standard fuzzy inference method, i.e. a singleton fuzzifier, product fuzzy inference and centre-average defuzzifier, the following global tracking error fuzzy nominal model for the controlled nonlinear system can be obtained:

$$\dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}_0\tilde{\boldsymbol{x}} + \boldsymbol{B}_0\tilde{u} \tag{10}$$

$$\tilde{u} = -\boldsymbol{K}\tilde{\boldsymbol{x}} \tag{11}$$

where

$$\boldsymbol{A}_0 = \sum_{i=1}^{l} \mu_i \boldsymbol{A}_i, \qquad \boldsymbol{B}_0 = \sum_{i=1}^{l} \mu_i \boldsymbol{B}_i, \qquad \boldsymbol{K} = \sum_{i=1}^{l} \mu_i \boldsymbol{K}_i \tag{12}$$

Remark 2. In this paper, we assume that the fuzzy nominal model is globally controllable, i.e. $(\boldsymbol{A}_0, \boldsymbol{B}_0)$ is a controllable pair.

The nonlinear system (1) can then be expressed as the following error dynamic equation:

$$\dot{\tilde{\boldsymbol{x}}} = (\boldsymbol{A}_0 + \Delta\boldsymbol{A})\tilde{\boldsymbol{x}} + (\boldsymbol{B}_0 + \Delta\boldsymbol{B})\tilde{u} + \Delta\boldsymbol{f} \tag{13}$$

where $\Delta\boldsymbol{A}$, $\Delta\boldsymbol{B}$ represent the approximation error and system uncertainties, and $\Delta\boldsymbol{f}$ denotes all the residual errors which cannot be covered by $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{B}$, including the higher order terms in Taylor's expansions and possibly other disturbances.

For a further analysis, the following assumption is used in what follows (Man and Palaniswami, 1995).

Assumption 1. There exist a matrix $\boldsymbol{H} \in \mathbb{R}^{1\times n}$ and a scalar E such that

$$\begin{aligned}
&\Delta\boldsymbol{A} = \boldsymbol{B}_0\boldsymbol{H}\\
&\Delta\boldsymbol{B} = \boldsymbol{B}_0 E\\
&0 < \|\boldsymbol{H}\| < H_0\\
&0 < \|E\| < E_0\\
&0 < \|\Delta\boldsymbol{f}\| < f_0
\end{aligned} \tag{14}$$

Remark 3. The above assumption is known as *uncertain matching conditions* and have been used by several researchers (Man and Palaniswami, 1995; Shoureshi *et al.*, 1990; Tarn *et al.*, 1984). The upper bounds of the system uncertainties, namely H_0, E_0 and f_0, may be obtained by means of adaptive techniques in the Lyapunov sense (Man and Palaniswami, 1995).

The objective of this paper is to develop a robust tracking control scheme which ensures that the output tracking error $\tilde{x}$ converges asymptotically to zero.

3. Robust Tracking Control Scheme

The controller design of the nonlinear system is divided into two parts. First, a nominal feedback controller, as shown in (11), is designed based on the fuzzy nominal system model, which guarantees that the tracking error $\tilde{x}$ of the nominal system converges asymptotically to zero. Second, a variable structure compensator is designed based on an uncertain bound to eliminate the effects of the approximation error and uncertain dynamics so that the tracking error $\tilde{x}$ of the closed loop system with uncertain dynamics converges asymptotically to zero.

The nominal feedback controller for the fuzzy nominal model is derived directly from (11), i.e.

$$u_0 = -\boldsymbol{K}\tilde{\boldsymbol{x}} + u_d \tag{15}$$

where

$$u_d = \sum_{i=1}^{l} \mu_i u_i \tag{16}$$

Next, we consider the variable structure compensator design for the uncertain system (13). Let the control input in (1) have the following form:

$$u = u_0 + u_1 \tag{17}$$

where u_0 is the nominal feedback control given by (15), and u_1 is a compensator to deal with the effects of system uncertainties.

Using (14) and (17) in (13), we can write down the error dynamics of the closed loop system with uncertainties as

$$\dot{\tilde{\boldsymbol{x}}} = (\boldsymbol{A}_0 - \boldsymbol{B}_0\boldsymbol{K})\tilde{\boldsymbol{x}} + \boldsymbol{B}_0(\boldsymbol{H} - E\boldsymbol{K})\tilde{\boldsymbol{x}} + \boldsymbol{B}_0(1 + E)u_1 + \Delta f \tag{18}$$

In order to use variable structure theory to design the compensator u_1, we define the following switching hyperplane variable:

$$S = \boldsymbol{C}\tilde{\boldsymbol{x}} \tag{19}$$

where $\boldsymbol{C} = [c_1, c_2, \dots, c_n]$ is chosen such that the zeros of the polynomial $\boldsymbol{C}\tilde{\boldsymbol{x}} = 0$ are in the left half-plane and the matrix $\boldsymbol{C}\boldsymbol{B}_0$ is non-singular (Man and Palaniswami, 1995).

For the design of the compensator u_1 and the stability analysis of the tracking error dynamics (18), we give the following theorem.

Theorem 1. *If the nominal feedback control* u_0 *is given by (15) and the compensator* u_1 *is designed such that*

$$u_1 = \begin{cases} \dfrac{(SCB_0)}{\|SCB_0\|^2}\Big[-SC(A_0 - B_0K)\tilde{x} + \rho\Big] & \text{if } S \neq 0 \\ 0 & \text{if } S = 0 \end{cases} \tag{20}$$

where

$$\rho = \frac{1}{1 - E_0}\Big[-\|SC(A_0 - B_0K)\tilde{x}\| - \|SCB_0\|(H_0\|\tilde{x}\| + E_0\|K\tilde{x}\|) - \|S\|\,\|C\|f_0\Big] \tag{21}$$

then the tracking error $\tilde{x}$ *converges asymptotically to zero as time tends to infinity.*

Proof. Defining the Lyapunov function

$$v = \frac{1}{2}S^2 \tag{22}$$

Differentiating v with respect to time, we get

$$\begin{aligned} \dot{v} &= S\dot{S} \\ &= SC\big[(A_0 - B_0K)\tilde{x} + B_0(H - EK)\tilde{x} + B_0(1 + E)u_1 + \Delta f\big] \\ &= -ESC(A_0 - B_0K)\tilde{x} + SCB_0(H - EK)\tilde{x} + SC\Delta f + (1 - E)\rho \\ &\leq \|E\|\,\|SC(A_0 - B_0K)\tilde{x}\| + \|SCB_0\|\,(\|H\|\,\|\tilde{x}\| + \|E\|\,\|K\tilde{x}\|) \\ &\quad + \|S\|\,\|C\|\,\|\Delta f\| + \frac{1 - E}{1 - E_0}\Big[-\|SC(A_0 - B_0K)\tilde{x}\| \\ &\quad - \|SCB_0\|\,(H_0\|\tilde{x}\| + E_0\|K\tilde{x}\|) - \|S\|\,\|C\|f_0\Big] \\ &\leq -(1 - \|E\|)\|SC(A_0 - B_0K)\tilde{x}\| - \|S\|\,\|C\|(f_0 - \|\Delta f\|) \\ &\quad - \|SCB_0\|\Big[(H_0 - \|H\|)\|\tilde{x}\| + (E_0 - \|E\|)\|K\tilde{x}\|\Big] < 0 \text{ for } S \neq 0 \end{aligned} \tag{23}$$

Expression (23) constitutes the sufficient condition for the switching hyperplane variable S to reach the sliding mode

$$S = C\tilde{x} = 0 \tag{24}$$

On the sliding mode, the tracking error $\tilde{x}$ converges asymptotically to zero.

Remark 4. The robustness property of the proposed control scheme is obvious. First, the effects of the approximation error and uncertain dynamics can be eliminated by using the variable structure compensator. Second, the closed loop system is completely insensitive to uncertainties after the system error dynamics reaches the sliding mode.

Remark 5. To eliminate the chattering in the control input, the following boundary layer compensator (Man and Palaniswami, 1995) can be used in lieu of (20):

$$u_1 = \begin{cases} \dfrac{(SC\boldsymbol{B}_0)}{\|SC\boldsymbol{B}_0\|^2}\Big[-SC(\boldsymbol{A}_0 - \boldsymbol{B}_0\boldsymbol{K})\tilde{\boldsymbol{x}} + \rho\Big] & \text{if } \|SC\boldsymbol{B}_0\| \geq \delta_1 \\ \dfrac{(SC\boldsymbol{B}_0)}{\delta_1^2}\Big[-SC(\boldsymbol{A}_0 - \boldsymbol{B}_0\boldsymbol{K})\tilde{\boldsymbol{x}} + \rho\Big] & \text{if } \|SC\boldsymbol{B}_0\| < \delta_1 \end{cases} \tag{25}$$

where $\delta_1 > 0$.

The above boundary layer compensator can force the switching plane variable to move towards the sliding mode surface and then the control signal can be smoothed inside a boundary layer. This will achieve an optimal trade-off between the control bandwidth and tracking precision. Therefore the chattering and sensitivity of the controller to system uncertainties can be eliminated (Man and Palaniswami, 1995).

4. Simulation Example

In order to illustrate the proposed robust tracking control scheme, a simulation example is carried out for a one-link robotic manipulator. The dynamic equation of the manipulator is given by

$$ml^2\ddot{\Theta} + d\dot{\Theta} + mgl\cos(\Theta) = u \tag{26}$$

where $m = 1\,\text{kg}$ is the payload, $l = 1\,\text{m}$ is the length of the link, $g = 9.81\,\text{m/s}^2$ denotes the gravitational constant, $d = 1\,\text{kgm}^2/\text{s}$ stands for the damping factor, u is the control variable (kgm^2/s^2).

Assuming that we are interested in the dynamics of the system in the range $[-90^\circ, 90^\circ]$, the fuzzy nominal model can be obtained by linearizing the nonlinear equation (26) at the points 0°, $\pm 45^\circ$, $\pm 90^\circ$. The following fuzzy nominal model has been obtained:

$$\begin{aligned} R^1 :\ & \text{IF} \quad x_1 \text{ is about } 0^\circ \\ & \text{THEN } \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}_1\tilde{\boldsymbol{x}} + \boldsymbol{B}_1\tilde{\boldsymbol{u}} \\ R^2 :\ & \text{IF} \quad x_1 \text{ is about } -45^\circ \\ & \text{THEN } \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}_2\tilde{\boldsymbol{x}} + \boldsymbol{B}_2\tilde{\boldsymbol{u}} \\ R^3 :\ & \text{IF} \quad x_1 \text{ is about } +45^\circ \\ & \text{THEN } \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}_3\tilde{\boldsymbol{x}} + \boldsymbol{B}_3\tilde{\boldsymbol{u}} \end{aligned}$$

$$R^4: \text{IF} \quad x_1 \text{ is about } -90^o$$

$$\text{THEN } \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}_4\tilde{\boldsymbol{x}} + \boldsymbol{B}_4\tilde{u}$$

$$R^5: \text{IF} \quad x_1 \text{ is about } +90^o$$

$$\text{THEN } \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}_5\tilde{\boldsymbol{x}} + \boldsymbol{B}_5\tilde{u}$$

where

$$x_1 = \Theta, \;\; x_2 = \dot{\Theta}, \;\; \tilde{x} = [\tilde{x}_1, \tilde{x}_2]^T, \; u_i = mgl\cos(\Theta_i)$$

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \qquad \boldsymbol{B}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{A}_2 = \begin{bmatrix} 0 & 1 \\ -6.94 & -1 \end{bmatrix}, \qquad \boldsymbol{B}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{A}_3 = \begin{bmatrix} 0 & 1 \\ 6.94 & -1 \end{bmatrix}, \qquad \boldsymbol{B}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{A}_4 = \begin{bmatrix} 0 & 1 \\ -9.81 & -1 \end{bmatrix}, \qquad \boldsymbol{B}_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{A}_5 = \begin{bmatrix} 0 & 1 \\ 9.81 & -1 \end{bmatrix}, \qquad \boldsymbol{B}_5 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The fuzzy sets for x_1 are chosen as shown in Fig. 1.

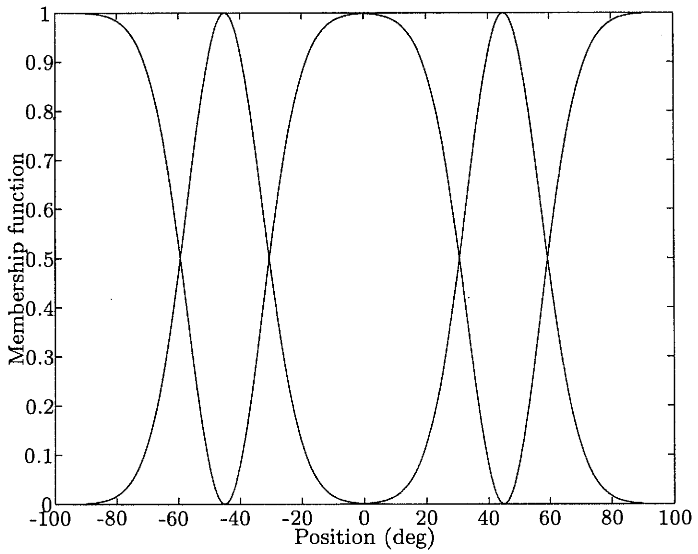


Fig. 1. Fuzzy sets of state x_1.

The desired closed loop poles for each local model are chosen as $[-4, -3]$. Thus the following feedback control gains are found by using the pole placement method:

$$\boldsymbol{K}_1 = [12\ \ 6], \quad \boldsymbol{K}_2 = [5.1\ \ 6], \quad \boldsymbol{K}_3 = [18.9\ \ 6], \quad \boldsymbol{K}_4 = [2.2\ \ 6], \quad \boldsymbol{K}_5 = [21.8\ \ 6]$$

The control objective in this simulation is to force the manipulator to follow the desired trajectory which is generated by the following reference model:

$$\begin{bmatrix} \dot{x}_d \\ \ddot{x}_d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -25 & -10 \end{bmatrix} \begin{bmatrix} x_d \\ \dot{x}_d \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_s \tag{27}$$

where u_s is chosen as in Fig. 2. The initial values of $\boldsymbol{x}$ and $\boldsymbol{x}_d$ are selected as

$$\big[x_1(0), \dot{x}_1(0)\big] = \big[17.2^\circ, 0\big], \qquad \big[x_d(0), \dot{x}_d(0)\big] = \big[0, 0\big]$$

and the sliding mode is prescribed as

$$S = [10, 1] \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$$

Moreover, the uncertain bounds are selected as

$$E_0 = 0.1, \qquad H_0 = 2, \qquad f_0 = 1$$

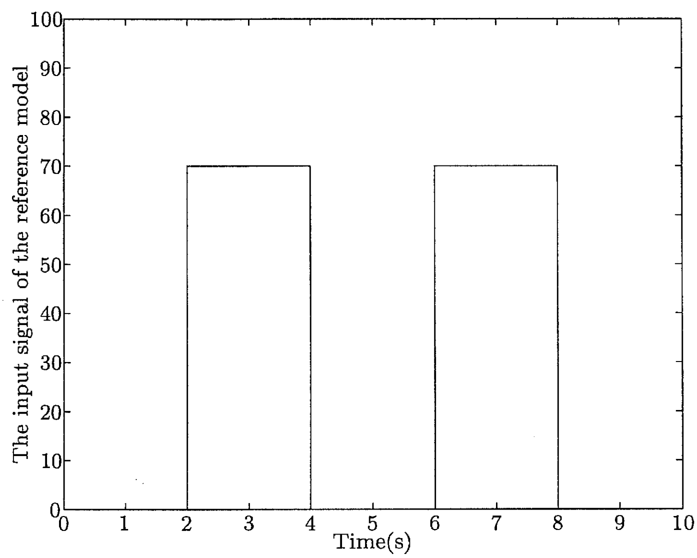


Fig. 2. Input signal u_s of the reference model.

Figure 3 shows the output tracking using only a fuzzy nominal feedback controller. It can be seen that due to large system uncertainties, the output cannot track the desired trajectory closely. Figure 4 shows the output tracking and control input using a fuzzy nominal feedback controller with a variable structure compensator. Clearly, the effects of the system uncertainties are eliminated and a good tracking performance is achieved. But there exists chattering in the control input. Figure 5 reveals a good performance of the closed loop system in which a fuzzy nominal feedback controller with a boundary layer compensator is used to eliminate the control chattering.

5. Conclusions

A robust tracking control scheme has been proposed for a class of nonlinear systems. The main contribution of this scheme is that a nominal system model for a nonlinear system is established by fuzzy synthesis of a set of linearized local subsystems, where the conventional linear feedback control technique is used to design a feedback controller for the fuzzy nominal system. A variable structure compensator is then designed to eliminate the effects of the approximation error and system uncertainties. Strong robustness with respect to large system uncertainties and asymptotic convergence of the output tracking error are obtained. A simulation example has also been given to illustrate the effectiveness of the proposed control scheme.

References

Cao S.G., Rees N.W. and Feng G. (1996): *Stability analysis and design for a class of continuous-time fuzzy control systems.* — Int. J. Control, Vol.64, No.6, pp.1069–1087.

Feng G., Cao S.G., Rees N.W. and Chak C.K. (1997): *Design of fuzzy control systems with guaranteed stability.* — Fuzzy Sets and Syst., Vol.85, No.1, pp.1–10.

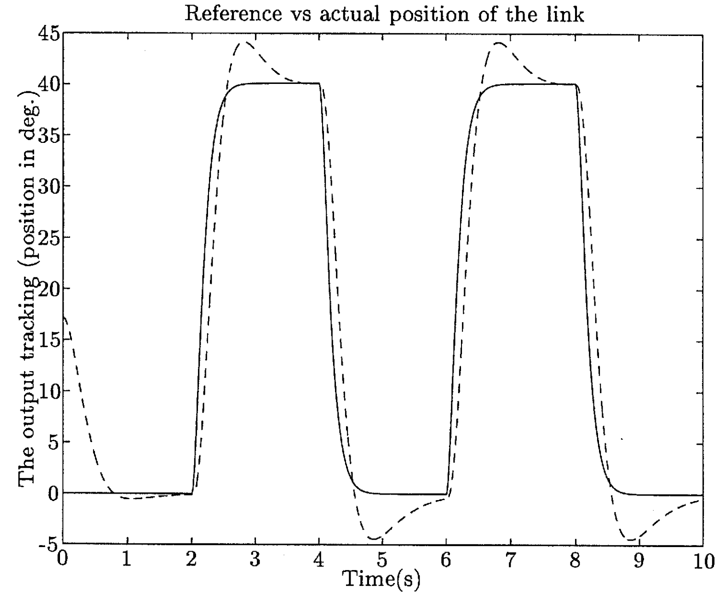
Man Z.H. and Palaniswami M. (1995): *A robust tracking control scheme for rigid robotic manipulators with uncertainty dynamics.* — Computers Elect. Eng., Vol.21, No.3, pp.211–220.

Ogata K. (1990): *Modern Control Engineering.* — Englewood Cliffs, New Jersey: Prentice-Hall.

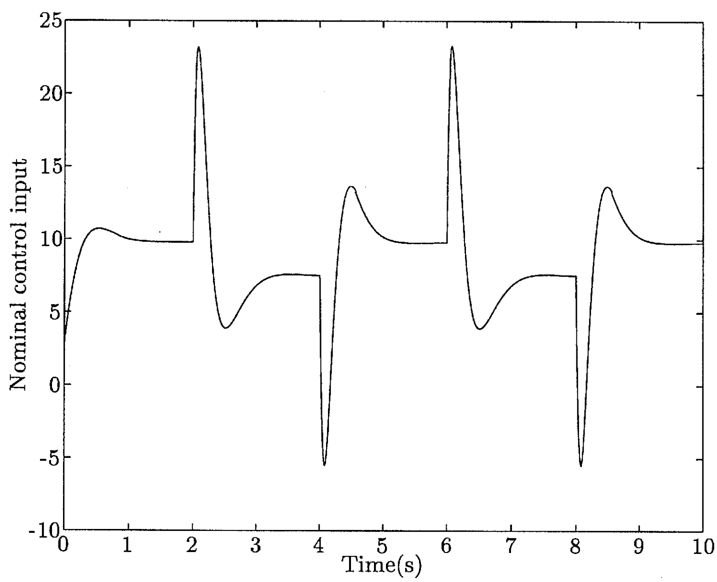
Palm R. and Rehfuess U. (1997): *Fuzzy controllers as gain scheduling approximators.* — Fuzzy Sets and Syst., Vol.85, No.2, pp.233–246.

Shoureshi R., Momot M.E. and Roesler M.D. (1990): *Robust control for manipulators with uncertainties.* — Automatica, Vol.26, No.2, pp.353–359.

Takagi T. and Sugeno M. (1985): *Fuzzy identification of systems and its application to modelling and control.* — IEEE Trans. Syst. Man Cybern., Vol.15, No.1, pp.116–132.

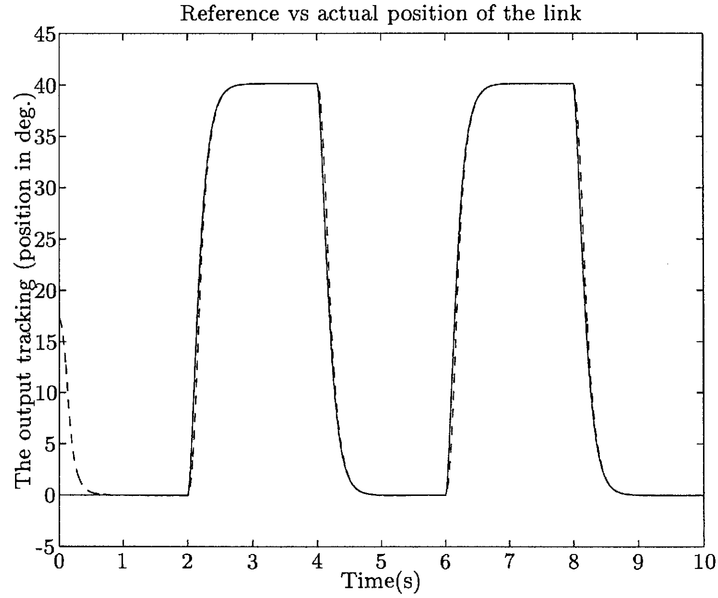


(a)

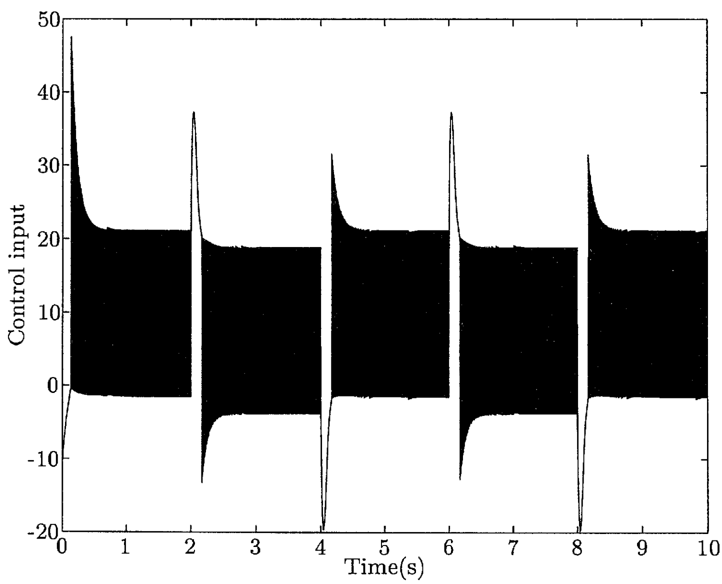


(b)

Fig. 3. Output tracking of the link using a fuzzy nominal feedback controller (a), where the solid line represents the desired trajectory, and nominal feedback control input (b).

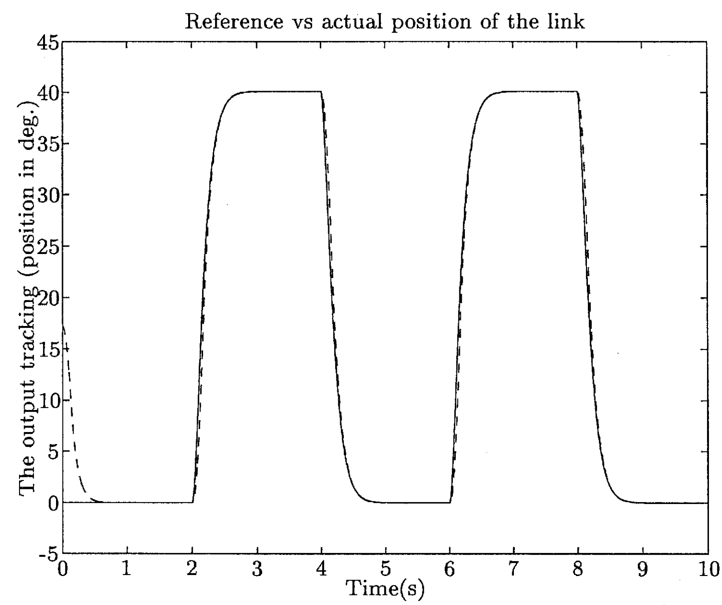


(a)

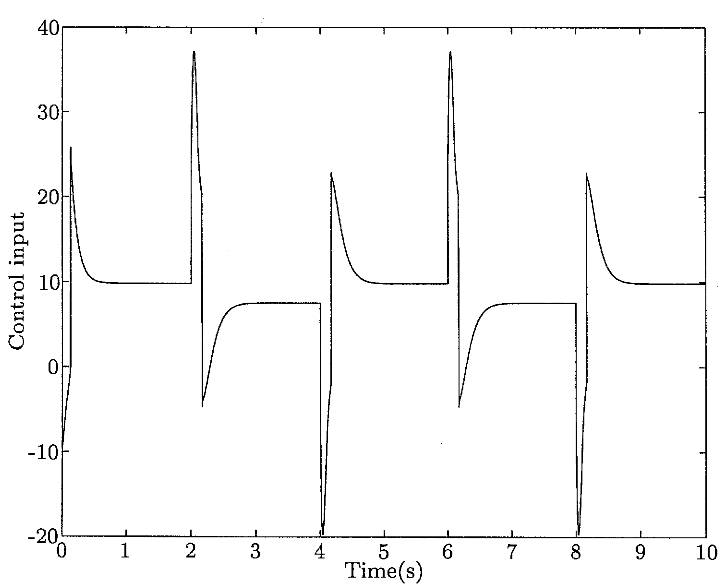


(b)

Fig. 4. Output tracking with a variable structure compensator (a) and control input (b).



(a)



(b)

Fig. 5. Output tracking using a boundary layer compensator for $\delta_1 = 0.04$ (a) and control input (b).

Tarn T.J., Bejczy A.K., Isidori A. and Chen Y. (1984): *Nonlinear feedback in robotic arm control.* — Proc. IEEE Conf. *Dec. Contr.*, Las Vegas, Nevada, pp.1569–1573.

Wang H.O., Tanaka K. and Griffin M.F. (1996): *An approach to fuzzy control of nonlinear systems: stability and design issues.* — IEEE Trans. Fuzzy Syst., Vol.4, No.1, pp.14–23.

Wang L.X and Mendel J.M. (1992): *Fuzzy basis function, universal approximation, and orthogonal least-squares learning.* — IEEE Trans. Neur. Netw., Vol.3, No.5, pp.807–814.