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VARIABLE- AND FIXED-STRUCTURE AUGMENTED INTERACTING MULTIPLE-MODEL ALGORITHMS FOR MANOEUVRING SHIP TRACKING BASED ON NEW SHIP MODELS[†]

EMIL SEMERDJIEV*, LUDMILA MIHAYLOVA*

Real-world tracking applications are related to a number of difficulties caused by the presence of different kinds of uncertainty, e.g. unknown or incompletely known system models and statistics of random processes or abrupt changes in the system modes of functioning. These problems are especially complicated in the marine navigation practice, where the commonly-used simple models of rectilinear or curvilinear target motions are not adequate for highly non-linear dynamics of the manoeuvring ship motion. A solution to these problems is to derive more suitable descriptions of real ship dynamics and to design adaptive estimation algorithms. After an analysis of basic hydrodynamic models, new ship models are derived in the paper. They are implemented in two versions of the Interacting Multiple Model (IMM) algorithm which has become very popular recently. The first one is a standard IMM version based on fixed model structures (FS's). They represent various modes of ship motion, distinguished by their rates of turns. The same rate of turn is additionally adjusted in the proposed new augmented versions of the IMM (AIMM) algorithm by using FS's and variable structures (VS's) of adaptive models estimating the current change in the system control parameters. Monte Carlo simulation experiments indicate that the VS AIMM algorithm outperforms the FS AIMM and FS IMM ones with respect to both accuracy and adaptability.

Keywords: Interacting Multiple Model (IMM) algorithm, model uncertainty, state and parameter estimation

1. Introduction

Tracking manoeuvring targets is a problem of great practical and theoretical interest. Real applications are related to a number of difficulties caused by the presence of different kinds of uncertainty due to the unknown or incompletely known system models and statistics of random processes, as well as because of their abrupt changes (Bar-Shalom, 1992; Bar-Shalom and Li, 1993; 1995; Best and Norton, 1997; Lerro

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* Bulgarian Academy of Sciences, Central Laboratory for Parallel Processing, 'Acad. G. Bonchev' Str., Bl. 25–A, 1113 Sofia, Bulgaria, e-mail: {signal, lsm}@bas.bg

and Bar-Shalom, 1993). These problems are especially complicated in the marine navigation practice, where the common models of rectilinear or curvilinear target motions are not adequate for the highly non-linear dynamics of a manoeuvring ship motion. A solution is to derive more suitable descriptions of the real ship dynamics and to design adaptive estimation algorithms. Such a solution is proposed in the paper. New ship models are derived in Section 2 after a brief analysis of the basic hydrodynamic models (Ermolaev, 1981; Ogawa and Kayama, 1977; Pershitz, 1973; Sobolev, 1976). These models are implemented in new versions of the Interacting Multiple Model (IMM) filter, one of the most cost-effective among the multiple model algorithms used for estimation of hybrid systems, i.e. systems with both continuous and discrete uncertainties (Bar-Shalom, 1992; Blom and Bar-Shalom, 1988; Li, 1996; Mazor *et al.*, 1998). A brief summary of the basic features of the Bayesian estimation algorithms and especially of the IMM filter is given in Section 3. Section 4 presents the proposed new IMM algorithms. They are based on an appropriate state vector augmentation, which includes the difference between the unknown control parameters and their values fixed in the IMM algorithm. Because of this model augmentation the resulting IMM algorithm is called here *augmented* (AIMM). Two versions of the AIMM algorithm are developed and evaluated. The first is a standard IMM version using a fixed set of models and is called the *fixed-structure* (FS) algorithm (Li, 1999). The models represent various modes of ship motion distinguished by their control parameter, i.e. the ship's rate of turn. The same rate of turn is additionally adjusted in the proposed new augmented versions of the IMM (AIMM) filter, respectively with fixed structure and *variable structure* (VS) (a variable set of models, with adaptive estimation of the current change of the system control parameters). The FS and VS AIMM algorithms are given in Section 4 and the results of a comparative performance evaluation of the algorithms are discussed in Section 5. Finally, conclusions and recommendations are summarized in Section 6.

2. Model Identification

In this section, some results of the research study described in (Semerdjiev and Bogdanova, 1995; Semerdjiev *et al.*, 1998; Semerdjiev and Mihaylova, 1998) are summarized. It should be noted that the high complexity of the hydrodynamic processes caused by the ship motion in deep and confined water and the wide variety of ship forms and sizes lead to various deterministic ship models. These models can be divided into two groups: precise models for particular ship forms and sizes (Sobolev's model (1976), the cubic model of Abkowitz (1964), the quadratic model of Norrbin (1981) and the MMG model (Ogawa and Kayama (1977)) and models of a greater generality but characterized by a lower accuracy (Pershitz, 1973; Nomoto, 1960). Here, the widely-used continuous-time (CT) Pershitz model is chosen as our basic model to assure a good trade-off between the model complexity and accuracy:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = K_V V_U \sin(\psi - \beta), \tag{1}$$

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = K_V V_U \cos(\psi - \beta), \tag{2}$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = K_V \omega, \tag{3}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\left(\frac{V_U}{L}\right)^2 (q_{31}\beta + s_{31}\delta) - \frac{V_U}{L} r_{31}\omega, \tag{4}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = -\frac{V_U}{L} (q_{21}\beta + h_1\beta\,|\beta| + s_{21}\delta) - r_{21}\omega, \tag{5}$$

$$V = V_U K_V, \tag{6}$$

$$K_V = \frac{V(\omega)}{V(0)} = \frac{V}{V_U} = \left(1 + 1.9\omega^2 L^2 V_U^{-2}\right)^{-1} \le 1,$$

where V_U is the *uniform* (rectilinear) ship velocity. The state vector of the model under consideration is $x = [X, Y, \psi, \omega, \beta, V]'$. It includes the ship coordinates, heading, rate of turn, drift angle and velocity δ being the control rudder angle deviation. The constant hydrodynamic coefficients q_{21}, r_{21}, s_{21}, h_1, q_{31}, r_{31} and s_{31} depend on the ship geometry, first of all on the ship length L (Voitkounski, 1985). Equations (3) and (6) illustrate the main feature of the dynamics, i.e. the *non-linear dependence between the ship's rate of turn and velocity.* This is the main difference between the above model and the other well-known simple models (Bar-Shalom, 1992; Best and Norton, 1997; Lerro and Bar-Shalom, 1993).

Very often (Pershitz, 1973; Voitkunski, 1985) the CT model (1)–(6) is simplified by substituting the factor

$$\beta_0 = \frac{-q + \sqrt{q^2 + 4h_1 r_{31} s\,|\delta|}}{2h_1 r_{31}}$$

which is computed off-line, for the factor $|\beta|$, where $q = q_{21}r_{31} - q_{31}r_{21}$ and $s = r_{21}s_{31} - r_{31}s_{21}$. The system of two first-order differential equations consisting of eqn. (4) and the modified equation (5) is further transformed into two independent second-order differential equations, omitting the negligible second derivatives:

$$2p\frac{\mathrm{d}\omega}{\mathrm{d}t}\frac{L^2}{V_U^2} + q\omega\frac{L}{V_U} + s_{31}\delta = 0, \tag{4'}$$

$$2p\frac{\mathrm{d}\beta}{\mathrm{d}t}\frac{L}{V_U} + q\beta + s_{21}\delta = 0, \tag{5'}$$

where $p = 0.5(q_{21}^* + r_{31})$, $q^* = q_{21}^* r_{31} - q_{31}r_{21}$, $q_{21}^* = q_{21} + h_1\beta_0$. The final CT model (1)–(3), (4′) and (6) is obtained by setting $\beta \equiv 0$.

The corresponding discrete-time (DT) model is as follows:

$$X_{k+1} = X_k + TV_k \sin\psi_k, \tag{7}$$

$$Y_{k+1} = Y_k + TV_k \cos\psi_k, \tag{8}$$

$$\psi_{k+1} = \psi_k + TV_k\left[\Omega_k + 0.5T\tau V_k\left(\Omega_k - \Omega_U\right)e^{TV_k\tau}\right], \tag{9}$$

$$\Omega_{k+1} = \Omega_k e^{TV_k\tau} + \Omega_U \left(1 - e^{TV_k\tau}\right), \tag{10}$$

$$V_k = V_U K_V = V_U \left(1 + 1.9\Omega_k^2 L^2\right)^{-1}, \tag{11}$$

where $k = 1, 2, \ldots$. Here T is the sampling interval and

$$\tau = \frac{-0.5p + \sqrt{0.25p^2 - q^*}}{L}\,[\mathrm{m}^{-1}], \quad \Omega_U = \frac{\omega}{V_U} = -\frac{[s_{31}\delta + \operatorname{sign}(\delta) q_{31}\beta_0]}{r_{31}L} \left[\frac{\mathrm{rad}}{\mathrm{m}}\right].$$

The agreement between the results obtained using the CT model (1)–(6) and those from the derived DT model (7)–(11) is demonstrated in (Semerdjiev *et al.*, 1998). That is why the DT model (7)–(11) is used for generation of the true data in the simulations to be presented in the sequel.

The final DT model, suitable for implementation in a Kalman filter, is obtained based on the following assumptions (Semerdjiev *et al.*, 1998; Semerdjiev and Mihaylova, 1998):

- The observed ship manoeuvres with a constant rate of turn:

  $$\Omega_{k+1} = \Omega_k, \quad \text{i.e. } \tau \equiv 0.$$

- The domain of unknown control parameters Ω_k can be 'covered' with a set of three control parameters corresponding to three basic kinds of ship motions: a uniform motion (Ω_U), as well as left and right turns (Ω_L and Ω_R, respectively):

  $$\Omega = [\Omega_U, \Omega_R, \Omega_L]' = [0, U, -U]',$$

  where U denotes a given constant rate of turn. The vector Ω accounts for all the ship manoeuvres and system noise in the band $[-U, U]$. A particular choice of U is made based on the marine practice and some important international navigation restrictions (Voitkounski, 1985).

- The attempt to introduce a vector of possible ship lengths has been recognised in (Semerdjiev *et al.*, 1998) as unsuccessful because of the small differences between the resulting models. The uncertainty concerning the ship geometry has been overcome by introducing a common constant average ship length l = const (Semerdjiev *et al.*, 1998).

Consequently, the final version of the sought ship model takes the following form:

$$X_{i,k+1} = X_{i,k} + TV_{i,k+1} \sin\psi_{i,k}, \tag{12}$$

$$Y_{i,k+1} = Y_{i,k} + TV_{i,k+1} \cos\psi_{i,k}, \tag{13}$$

$$\psi_{i,k+1} = \psi_{i,k} + TV_{i,k+1}\Omega_i, \tag{14}$$

$$V_{i,k+1} = K_{V,i} V_{U,k}. \tag{15}$$

The new state vector is $x_{i,k} = [X_{i,k}, Y_{i,k}, \psi_{i,k}, V_{U,k}]'$, $K_{V,i} = (1 + 1.9\Omega_i^2 l^2)^{-1}$ and $\Omega = [\Omega_U, \Omega_R, \Omega_L]' = [0, U, -U]'$, $i = 1,2,3$.

Another version of our model, based on the augmented state vector $x_{i,k}^a = [X_{i,k}, Y_{i,k}, \psi_{i,k}, V_{U,k}, \Delta\Omega_{i,k}]'$, is suggested in (Semerdjiev and Mihaylova, 1998):

$$X_{i,k+1} = X_{i,k} + TV_{i,k+1} \sin \psi_{i,k}, \tag{16}$$

$$Y_{i,k+1} = Y_{i,k} + TV_{i,k+1} \cos \psi_{i,k}, \tag{17}$$

$$\psi_{i,k+1} = \psi_{i,k} + TV_{i,k+1} \left(\Omega_i + \Delta\Omega_{i,k}\right), \tag{18}$$

$$V_{i,k+1} = K_{V,i} V_{U,k}, \tag{19}$$

$$\Delta\Omega_{i,k+1} = \Delta\Omega_{i,k}, \tag{20}$$

where $i = 1,2,3$. This model takes into account possible differences $\Delta\Omega_{i,k}$ between the unknown true ship rate of turn Ω_k and its values Ω_i fixed in the IMM algorithm. The influence of $\Delta\Omega_{i,k}$ on the velocity is not taken into account because of its insignificance.

It should also be noted that the above models can be used to cover simultaneous heading and velocity manoeuvres. For that purpose, it is only necessary to introduce velocity noise in the rectilinear motion model.

3. Standard IMM Algorithm

It is known (Bar-Shalom and Li, 1993; 1995) that in order to estimate the system state within the framework of the Bayesian approach, the computational and storage requirements increase exponentially with time, which makes the estimator hard to implement in real time. To circumvent this problem, suboptimal estimators with certain hypotheses management, such as pruning and merging, have been used, leading to algorithms such as generalized pseudo-Bayesian (GPB) algorithms of first order (GPB1), of second order (GPB2) and, in general, of order r (GPBr). It was shown in (Li, 1996; Bar-Shalom and Li, 1993; 1995) that the IMM algorithm is one of the most cost-effective schemes for estimation of hybrid systems. It yields the performance of GPB2 with the lower requirements of GPB1.

The IMM algorithm is recursive (Blom and Bar-Shalom, 1988; Bar-Shalom and Li, 1993; 1995; Li, 1996). Each cycle of the algorithm consists of four major steps: interaction (mixing), filtering, mode update and combination. In each cycle, the initial condition of the filter designed for a certain mode is obtained by interacting (mixing) the state estimates of all filters at the previous time moment under the assumption that this particular mode is in effect at the current time instant. This is followed by the filtering (prediction and update) step, performed in parallel for each mode. Then the combination (a weighted sum) of the updated state estimates from all filters yields the state estimate.

The standard IMM filter is used here to develop its versions to be suitable for ship tracking taking into account particular features of the ship models.

4. Augmented IMM Algorithms for Tracking Manoeuvring Ships

4.1. Fixed-Structure Augmented IMM Algorithm for Ship Tracking

In a general state-space form, the ship model and the measurement equation can be written down as follows:

$$x_k = f\left(x_{k-1}, \Omega_{k-1}\right) + g\left(\Omega_{k-1}\right) v_{k-1}, \tag{21}$$

$$z_k = h_k\left(x_k\right) + w_k, \tag{22}$$

where the state vector $x_k \in \mathbb{R}^{n_x}$ is estimated based on the measurement vector $z_k \in \mathbb{R}^{n_z}$ in the presence of an unknown true control parameter $\Omega_k \in \mathbb{R}^{n_\Omega}$. The mutually independent additive system and measurement noise vectors $v_k \in \mathbb{R}^{n_\nu}$ and $w_k \in \mathbb{R}^{n_z}$ are white and Gaussian, i.e. $\nu_k \sim N(0, Q_k)$, $w_k \sim N(0, R_k)$. Functions f, g and h are known and remain unchanged during the estimation procedure.

To estimate the difference $\Delta\Omega_{i,k}$ between the current true control parameter Ω_k and its value Ω_i fixed in the i-th IMM model, the system state model is augmented by the equation

$$\Delta\Omega_{i,k} = \Delta\Omega_{i,k-1}, \tag{23}$$

where

$$\Delta\Omega_{i,k} = \Omega_k - \Omega_i. \tag{24}$$

The state and system noise vectors of the i-th augmented model ($i = \overline{1,N}$) can be written down in the form

$$x^a_{i,k} = \left[\begin{array}{cc} x'_{i,k} & \Delta\Omega'_{i,k} \end{array}\right]' \in \mathbb{R}^{n_x+n_\Omega}, \quad \nu^a_{i,k} = \left[\begin{array}{cc} \nu'_{i,k} & \nu'_{\Omega_{i,k}} \end{array}\right]' \in \mathbb{R}^{n_\nu+n_\Omega}.$$

In general, the new *augmented* model is non-linear:

$$x^a_{i,k} = f^a\left(x^a_{i,k-1}, \Omega_i + \Delta\Omega_{i,k-1}\right) + g^a\left(\Omega_i + \Delta\Omega_{i,k-1}\right) v^a_{i,k-1}, \tag{25}$$

$$z_k = h^a\left(x^a_{i,k}, \Omega_i + \Delta\Omega_{i,k}\right) + w_k. \tag{26}$$

The functions $f^a(\cdot)$, $g^a(\cdot)$ and $h^a(\cdot)$ are known and remain unchanged during the estimation procedure. The equations of the corresponding Extended Kalman Filter (EKF) are derived by linearization of models (25) and (26). The functions $f^a\left(x_{i,k-1}, \Omega_i + \Delta\Omega_{i,k-1}\right)$ and $g^a\left(x_{i,k-1}, \Omega_i + \Delta\Omega_{i,k-1}\right)$ are expanded in the Taylor series up to first order around the filtered estimate $\hat{x}^a_{i,k-1/k-1}$ and the function $h^a(x_{i,k}, \Omega_i + \Delta\Omega_{i,k})$ is expanded up to first order around the predicted estimate $\hat{x}^a_{i,k/k-1}$ (Bar-Shalom and Li, 1993). Accordingly, the i-th EKF equations take the form

$$\hat{x}^a_{i,k/k} = \hat{x}^a_{i,k/k-1} + K^a_{i,k}\gamma_{i,k}, \tag{27}$$

$$\hat{x}^a_{i,k/k-1} = f^a\left(\hat{x}^a_{i,k-1/k-1}, \Omega_i + \Delta\hat{\Omega}_{i,k-1/k-1}\right), \tag{28}$$

$$\gamma_{i,k} = z_k - h^a\left(\hat{x}^a_{i,k/k-1}, \Omega_i + \Delta\hat{\Omega}_{i,k/k-1}\right), \tag{29}$$

$$P^a_{i,k/k-1} = \phi_i f^a_{xi,k-1} P^a_{i,k-1/k-1}\left(f^a_{xi,k-1}\right)' + Q^a_{i,k-1}, \tag{30}$$

$$S_{i,k} = h^a_{xi,k} P^a_{i,k/k-1}\left(h^a_{xi,k}\right)' + R_k, \tag{31}$$

$$K^a_{i,k} = P^a_{i,k/k-1}\left(h^a_{xi,k}\right)' S^{-1}_{i,k}, \tag{32}$$

$$P^a_{i,k/k} = P^a_{i,k/k-1} - K^a_{i,k} S_{i,k}\left(K^a_{i,k}\right)', \tag{33}$$

where $K^a_{i,k}$ is the filter gain matrix, $P^a_{i,k/k}$ and $Q^a_{i,k}$ are respectively the estimation error and system noise covariance matrices, $\gamma_{i,k}$ and $S_{i,k}$ denote respectively the filter innovation and its covariance matrix. The system and measurement Jacobians are $f^a_{xi,k-1} = \partial f^a(\hat{x}^a_{i,k-1/k-1}, \Omega_i + \Delta\hat{\Omega}_{i,k-1/k-1})/\partial\hat{x}^a_{i,k-1/k-1}$ and $h^a_{xi,k} = \partial h^a(\hat{x}^a_{i,k/k-1})/\partial\hat{x}^a_{i,k/k-1}$, respectively. Here $\phi_i \geq 1$ is the EKF fudge factor. The restrictions $\Omega_i + \Delta\hat{\Omega}_{i,k-1/k-1} \in [\Omega_{i,\min}, \Omega_{i,\max}]$ are imposed to provide a minimal model separation.

After the expansion of the ship models (12)–(15) and (16)–(20) in a Taylor time-series, three IMM algorithm versions are derived. The IMM algorithm based on the model (12)–(15) is further denoted by FS IMM, while the proposed AIMM algorithm based on the model (16)–(20) is denoted by FS AIMM.

4.2. Variable-Structure Augmented IMM Algorithm for Ship Tracking

The FS AIMM algorithm can be transformed into a new VS AIMM algorithm by substituting the random vector of control parameters $\Omega_{i,k}$ for the constant vector of deterministic parameters Ω_i. At the beginning of each EKF (before the state prediction step) in the IMM algorithm, the last filtered displacement $\Delta\hat{\Omega}_{i,k-1/k-1}$ corrects the old vector of control parameters $\Omega_{i,k-1}$:

$$\Omega_{i,k} = \Omega_{i,k-1} + \Delta\hat{\Omega}_{i,k-1/k-1} \qquad (\Omega_{i,0} = \Omega_i), \tag{34}$$

The new control parameters must satisfy the constraints

$$\Omega_{i,k} \in [\Omega_{i,\min}, \Omega_{i,\max}] \quad \text{for all } i.$$

Then the model displacement $\Delta\hat{\Omega}_{i,k-1/k-1}$ is set to zero, i.e.

$$\Delta\hat{\Omega}_{i,k-1/k-1} = 0. \tag{35}$$

Otherwise, it will be taken into account twice in the EKF equations.

Finally, it should be noted that the VS AIMM algorithm proposed here is general and depends neither on the system to be implemented, nor on the measurement models. It is an adaptive VS IMM algorithm using a minimal number of models and self-adjusting their location in the continuous parameter domain.

4.3. Implementation of the AIMM Algorithms

As for as the implementation of the AIMM algorithms in sea track-while-scan radars is concerned, some particular features of these sensors are taken into account by using the next measurement equation, i.e.

$$z_k = Hx_k + w_k,$$

where H is the measurement matrix,

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

and w_k is white Gaussian measurement noise with covariance matrix R_k. The polar measurements 'range-bearing' $z_k = [r_k, \beta_k]'$ are transformed, for convenience, to the Cartesian coordinates:

$$X_k = r_k \sin\beta_k, \quad Y_k = r_k \cos\beta_k.$$

The measurement vector takes on the new form $z_k = [X_k, Y_k]'$. Furthermore, the covariance matrix of the measurement errors becomes (Farina and Studer 1986)

$$R_{i,k} = \begin{bmatrix} \sigma_r^2 \sin^2\beta_k + r_k^2\sigma_\beta^2 \cos^2\beta_k & \left(\sigma_r^2 - r_k^2\sigma_\beta^2\right)\sin\beta_k\cos\beta_k \\ \left(\sigma_r^2 - r_k^2\sigma_\beta^2\right)\sin\beta_k\cos\beta_k & \sigma_r^2\cos^2\beta_k + r_k^2\sigma_\beta^2\sin^2\beta_k \end{bmatrix},$$

where σ_r and σ_β are the range and bearing standard deviations, respectively.

The Jacobi matrix computed based upon the model (12)–(15) has the form

$$f_{x_i,k} = \begin{bmatrix} 1 & 0 & TK_{V,i}\hat{V}_{U,k/k}\cos\hat{\psi}_{i,k/k} & TK_{V,i}\sin\hat{\psi}_{i,k/k} \\ 0 & 1 & -TK_{V,i}\hat{V}_{U,k/k}\sin\hat{\psi}_{i,k/k} & TK_{V,i}\cos\hat{\psi}_{i,k/k} \\ 0 & 0 & 1 & TK_{V,i}\Omega_i \\ 0 & 0 & 0 & K_{V,i} \end{bmatrix}.$$

The respective one based on the model (16)–(20) is

$$f^a_{x_i,k} = \begin{bmatrix} 1 & 0 & TK_{V,i}\hat{V}_{U,k/k}\cos\hat{\psi}_{i,k/k} & TK_{V,i}\sin\hat{\psi}_{i,k/k} & 0 \\ 0 & 1 & -TK_{V,i}\hat{V}_{U,k/k}\sin\hat{\psi}_{i,k/k} & TK_{V,i}\cos\hat{\psi}_{i,k/k} & 0 \\ 0 & 0 & 1 & TK_{V,i}\left(\Omega_i + \Delta\hat{\Omega}_{i,k/k}\right) & TK_{V,i}\hat{V}_{U,k/k} \\ 0 & 0 & 0 & K_{V,i} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Hard logic is introduced in all IMM algorithms to avoid an undesirable combination of the estimates $\hat{V}_{U,k/k}$, $\hat{V}_{L,k/k}$ and $\hat{V}_{R,k/k}$ (Semerdjiev *et al.*, 1998):

$$\hat{V}_{i,k/k} = \hat{V}_{U,k/k} \quad (i = 2,3),$$

$$\hat{V}_{k/k} = \hat{V}_{U,k/k} \quad \text{if} \quad \mu_{U,k} > 0.5,$$

where $\mu_{i,k}$ is the probability of the event 'the i-th model is topical at time k' and $\hat{V}_{k/k}$ is the overall (final) estimate of the ship velocity.

5. Performance Evaluation

5.1. Measures of Performance

The performance of the three foregoing IMM algorithms is compared with the use of Monte Carlo simulations. The *mean error (ME)* and the *root mean-square error (RMSE)* of each state component have been chosen as the measures of performance (Bar-Shalom and Li, 1993). The ME and the RMSE of both the estimated coordinates have been combined accordingly. The results of 100 independent runs, each lasting 200 scans (600 s, $T = 3$ s) are given in what follows.

The simulation parameters of the true model (7)–(11) are standard (Voitkounski, 1985; Semerdjiev *et al.*, 1998): $q_{21} = 0.331$, $r_{21} = -0.629$, $s_{21} = -0.104$, $h_1 = 3.5$, $q_{31} = -4.64$, $r_{31} = 3.88$, $s_{31} = -1.019$, $L = 99$ m, $\delta_{\min} = 3°$, $\delta_{\max} = 30°$. The initial conditions are $X_0 = Y_0 = 10000$ m, $\psi_0 = 45°$, $V_U = 30$ m/s.

It is assumed that initially the ship moves rectilinearly. The true ship trajectory is presented in Fig. 1. The applied pulse-wise rudder angle control law is

$$\delta = \begin{cases} \delta_{\max}, & k \in [51, 67], \\ 0, & k \notin [51, 67]. \end{cases}$$

The control parameters of FS IMM and FS AIMM algorithms are fixed as follows: $\Omega = [0, U, -U]'$, where $U = 0.0066$ rad/m (which corresponds to the turn rate of 360°/min). The VS AIMM uses the same control parameters at its initialization. For the VS AIMM algorithm it is assumed that $|\Omega_{i,\min}| = 0.0011$ and $|\Omega_{i,\max}| = 0.0066$.

The three IMM algorithms use the constant ship length $l = 69$ m. The EKF's fudge factors are also set constant for all IMM: $\phi = 1.03$.

In the example considered below, the covariance matrix of the error in the measurement is calculated for $\sigma_r = 100$ m and $\sigma_\beta = 0.3°$. The initial error covariance matrices $P_{i,0}$, the initial mode probability vectors μ and the transition probability matrices Pr are chosen as follows:

$$P_{i,0}^{\text{FS IMM}} = P_{i,0}^{\text{FS AIMM}} = \text{diag}\left\{ \sigma_X^2 \quad \sigma_Y^2 \quad \sigma_\psi^2 \quad \sigma_V^2 \right\},$$

$$P_{i,0}^{\text{VS AIMM}} = \text{diag}\left\{ \sigma_X^2 \quad \sigma_Y^2 \quad \sigma_\psi^2 \quad \sigma_V^2 \quad \sigma_{\Delta\Omega}^2 \right\},$$

$$\mu^{\mathrm{FS\,IMM}} = \mu^{\mathrm{FS\,AIMM}} = \mu^{\mathrm{VS\,AIMM}} = \begin{bmatrix} 0.95 \\ 0.025 \\ 0.025 \end{bmatrix},$$

$$\mathrm{Pr}^{\mathrm{FS\,IMM}} = \mathrm{Pr}^{\mathrm{FS\,AIMM}} = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix},$$

$$\mathrm{Pr}^{\mathrm{VS\,AIMM}} = \begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix},$$

$$\sigma_X = \sigma_Y = \sigma_r, \quad \sigma_\psi = 0.1^\circ, \quad \sigma_V = 10\,\mathrm{m}, \quad \sigma_{\Delta\Omega} = 0.01\,\mathrm{rad/m}.$$

It is supposed that there is no system noise in the models, i.e. $Q_i^a \equiv Q_i \equiv 0$. The Monte Carlo simulation results are shown in Figs. 2–12.

Generally, the VS AIMM algorithm is characterized by the best accuracy, the lowest peak dynamic errors and the shortest response time. These conclusions are confirmed by the ME and RMSE plots presented in Figs. 2–4 and Figs. 5–7, respectively. The average mode probabilities are given in Figs. 8–10. The ship moves uniformly at the beginning and at the end of the observed period, whereas in the midst of the time horizon it makes a right turn that is reflected in the mode probabilities. The VS AIMM algorithm also provides the best and fastest model recognition. From Figs. 11 and 12 it is obvious that the above excellent VS AIMM algorithm performance is due to the self-adjustment mechanism for appropriately control parameter tuning.

The technique proposed here for multiple-model ship tracking with a variable set of models can also be used in other applications.

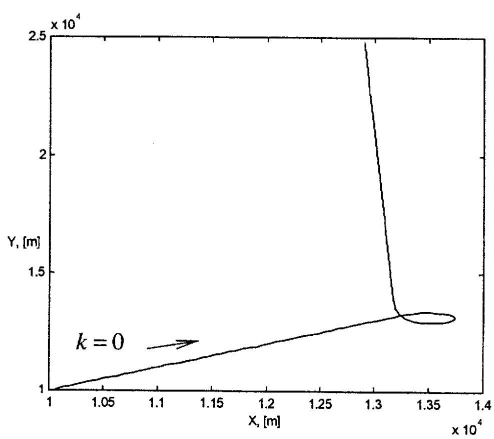

Fig. 1. True ship trajectory.

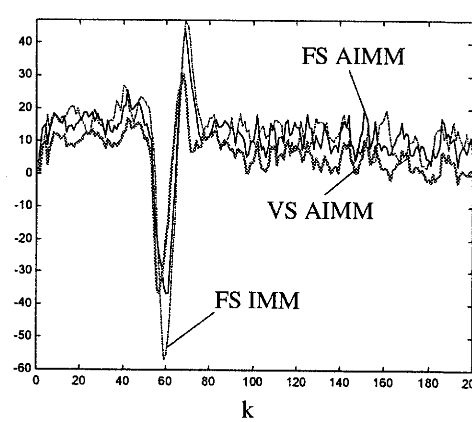

Fig. 2. ME of both estimated coordinates, [m].

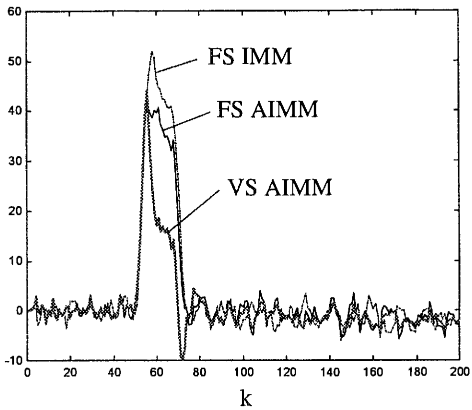

Fig. 3. Heading ME, [°].

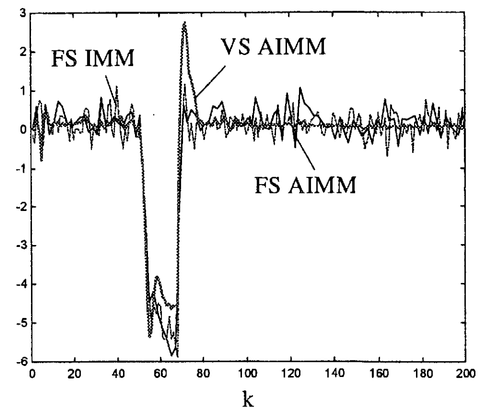

Fig. 4. Velocity ME, [m/s].

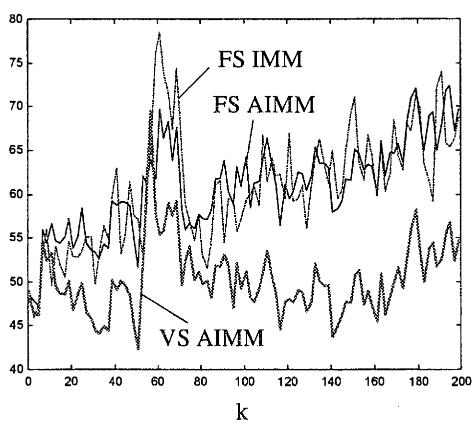

Fig. 5. RMSE of both estimated coordinates, [m].

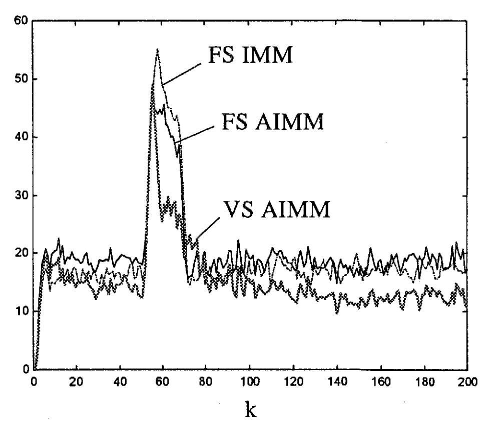

Fig. 6. Heading RMSE, [°].

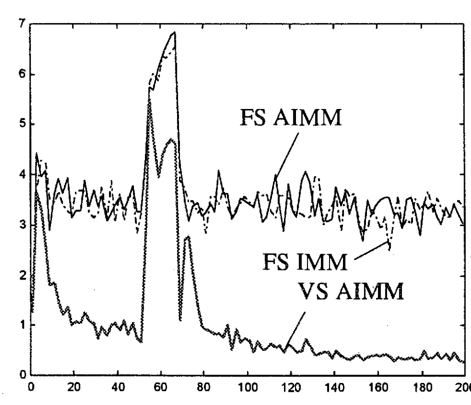

Fig. 7. Velocity RMSE, [m/s].

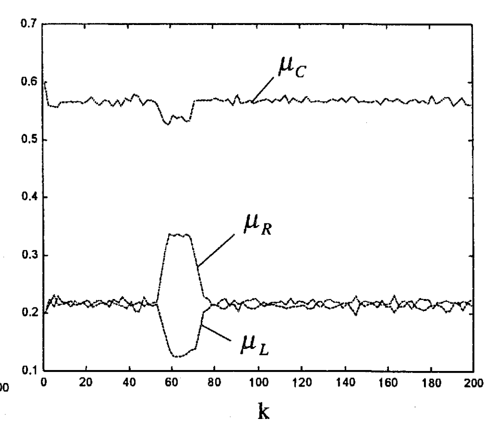

Fig. 8. Average mode probabilities of FS IMM.

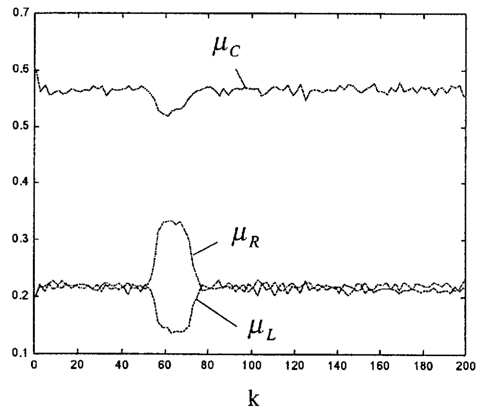

Fig. 9. Average mode probabilities of FS AIMM.

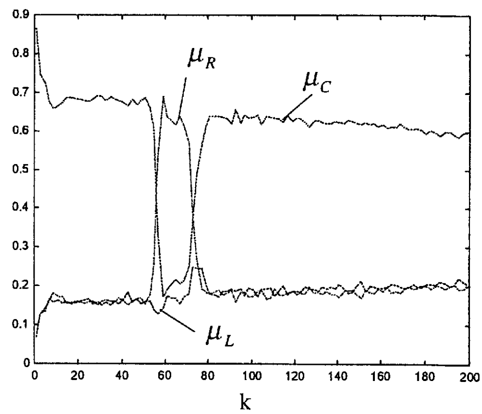

Fig. 10. Average mode probabilities of VS AIMM.

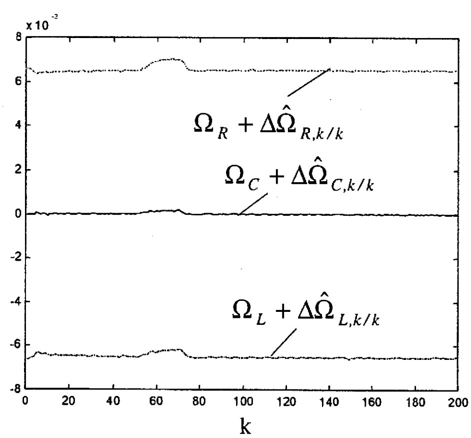

Fig. 11. Quantity $\Omega_i + \Delta\hat{\Omega}_{i,k/k}$ [rad/m] for FS AIMM.

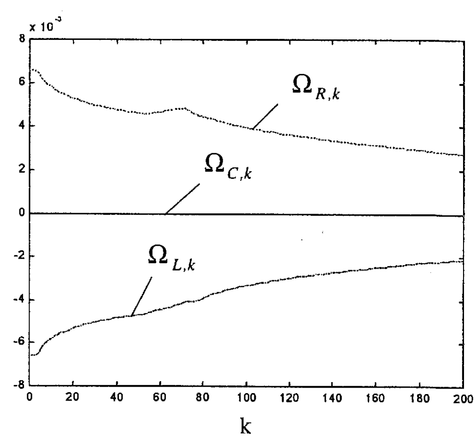

Fig. 12. Quantity $\Omega_{i,k}$ [rad/m] for FS AIMM.

6. Conclusions

Some new models which describe adequately the non-linear dynamics of the manoeuvring ship motion are derived in the paper for the purposes of manoeuvring ship tracking. A new variable-structure augmented IMM technique is also proposed. The designed ship models are implemented using a standard IMM algorithm and its two augmented IMM versions with fixed and variable model structures. The proposed new AIMM algorithms use augmented state vectors and models to compensate for the difference between the control parameters fixed in the IMM models and their current true values. Very good self-adjusting abilities are provided to the designed augmented IMM algorithms due to the estimated rate of turn. The accomplished extensive Monte

Carlo simulations show that the VS AIMM algorithm outperforms the FS AIMM and FS IMM ones with respect to the estimation accuracy and adaptability.

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