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Original Research Article

SHAKEDOWN ANALYSIS OF COMPOSITE STEEL-CONCRETE FRAME SYSTEMS WITH PLASTIC AND BRITTLE ELEMENTS UNDER SEISMIC ACTION

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Abstract

In this paper the earthquake analysis of composite steel-concrete frames is performed by finding solution of the optimization problem of shakedown analysis, which takes into account the nonlinear properties of materials. The constructions are equipped with systems bearing structures of various elastic-plastic and brittle elements absorbing energy of seismic actions. A mathematical model of this problem is presented on the base of limit analysis theory with partial redistribution of self-stressed internal forces. It is assumed that the load varies randomly within the specified limits. These limits are determined by the possible direction and magnitude of seismic loads. The illustrative example of such analysis of system is introduced. Some attention has been paid to the practical application of the proposed mathematical model.

Keywords: limit and shakedown analysis, composite steel-concrete frames, elastic-plastic and brittle elements, seismic-protected systems

1. INTRODUCTION

The earthquake analysis of buildings and structures is one of the most complicated problems in engineering practice.

Protection of buildings against seismic actions is achieved by equipping them with systems bearing structures of various elements absorbing energy of these

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actions [0]. As a result, the destruction of the basic structure is prevented. Some elements can be abruptly shut down (brittle elements), and some damaged as a result of plastic flow (elastic-plastic elements). By the way of illustration these may be ties, guys, fascicles, strands, and other elements, which are off in the process of earthquakes.

There are several methods for the earthquake analysis using for design of the buildings [2,3]. The nonlinear properties of materials may be taking into account using all of these methods. For example, the nonlinear (plastic) properties of the materials are taking into account by using behavior factor q for elastic analysis based on a response spectrum. Material nonlinearity is taken into account "directly" by inserting plastic hinges in the elements when using push-over analysis. More accurate solution, with taking into account physical and dynamic properties of the materials, can be obtained by using direct in-tegration method of earthquake accelerograms. Shear checks should be performed for the all elements during design by every of these methods, after that all elements will be modified to increase shear resisting and then recalculating of the system will be provided. In the case of a large-scale system with composite elements these checks may require many iterations and therefore calculating time will significantly increase.

The earthquake analysis of buildings and structures can be performed by finding solution of the optimization problem of shakedown analysis, which also takes into account the nonlinear properties of materials [4–15]. Such analysis has several advantages:

- External actions are introduced as the set of the load cases, that's why we can solve the problem for all direction of seismic load and for every scheme of live load at once;
- As part of the solution of the optimization problem we can take into account the elastic-plastic and brittle behavior of the elements [7,15,16].

This paper presents a mathematical model of the shakedown and limit analysis of the buildings with elastic-plastic and brittle elements. It is assumed that the load varies randomly within the specified limits. These limits are determined by the possible direction and magnitude of seismic loads, which can be found from elastic FEA analysis of the system using elastic response spectrum.

An illustrative example of plane composite steel-concrete frames system with the limited plastic redistribution of forces under seismic actions is shown.

2. A MATHEMATICAL MODEL OF STRUCTURES WITH SEISMIC-PROTECTED SYSTEMS

The proposed below mathematical model of the optimization problem of limit shakedown analysis will be used to the design of the high-rise building located in Minsk (see Figure 1), taking into account the construction stages.

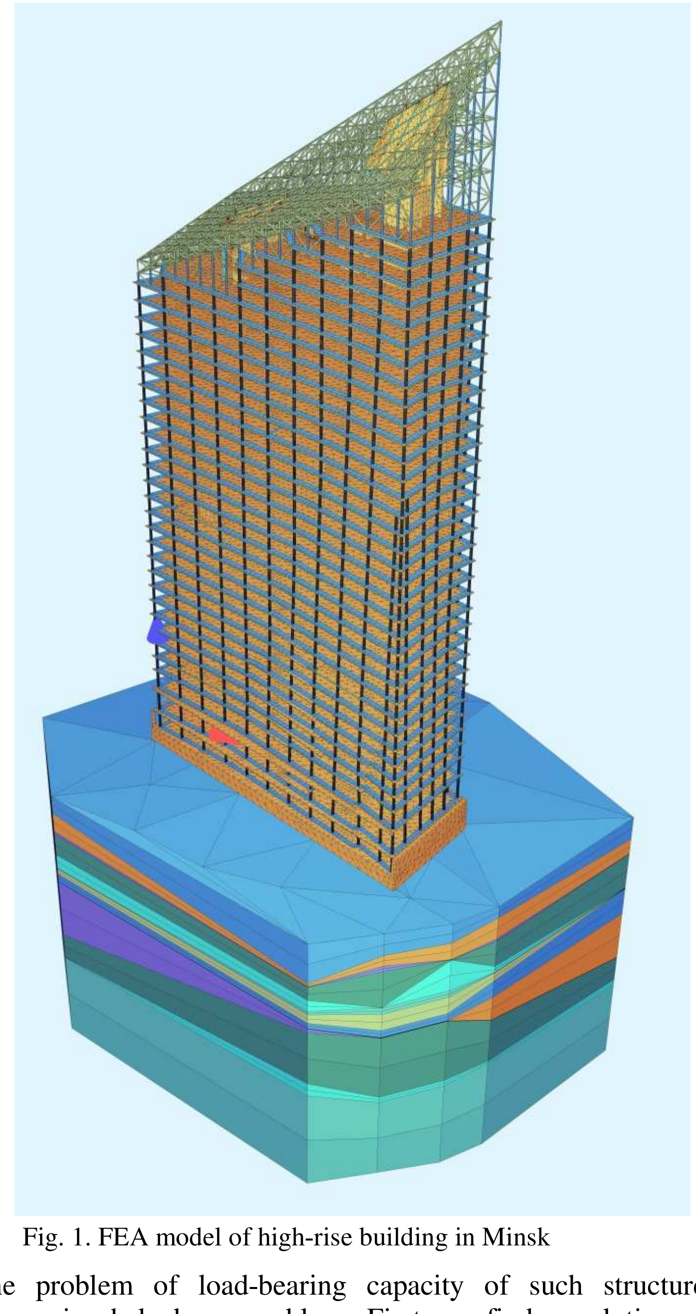


Fig. 1. FEA model of high-rise building in Minsk

We assume the problem of load-bearing capacity of such structures as a generalized dynamic shakedown problem. First we find a solution of the equation of motion for a damped discrete elastic system under load $\boldsymbol{F}(t)$ as

a function of time t. This vector belongs to the set $\Omega(\boldsymbol{F}(t), t)$, which generally is nonconvex [17, 18], but here it is approximated by the convex domain.
The “elastic” solution is used then as basis for analysis of inelastic system with the partial plastic redistribution of forces. Namely the problem of load-bearing capacity of structures made of perfectly elastic-plastic and elastic-brittle elements, under variable loads is formulated as follows. Find a parameter (safety factor) μ for load $\boldsymbol{F}$, as well as the vector of residual forces $S_p{}^r$ such, that

$$\mu \to \max , \tag{2.1}$$

$$S^e(t) = f(\mu F(t)) , \tag{2.2}$$

$$AS_p{}^r = 0 , \tag{2.3}$$

$$S_p{}^r = E_p S^r , \tag{2.4}$$

$$\varphi_{pl}(S^e(t) + S_p{}^r, S_{0,\,pl}) \le 0 , \tag{2.5}$$

$$\varphi_{br}(S^e(t), S_{0,\,pl})_i \le 0 , \quad i \in I_{br} , \tag{2.6}$$

$$F(t) \in \Omega(F_j(t), t) , \tag{2.7}$$

where $\boldsymbol{S}^e$ = a vector of internal forces; $\boldsymbol{S}_{0,pl}$, $\boldsymbol{S}_{0,br}$ = vectors of limit internal forces in the cross sections of elastic-plastic and elastic-brittle elements accordingly; $\boldsymbol{A}$ = matrix of static compatibility (equilibrium eguations) for residual internal forces $\boldsymbol{S}^r$ in the cross sections of elements; $\boldsymbol{S}^e$, $\boldsymbol{S}^r$ = vectors of elastic and residual internal forces in the cross sections of elements; $\boldsymbol{F}(t)$ = vector of load; I_{br} = set of i-th brittle elements; Ω(•) = set of loads $\boldsymbol{F}$, t = time.
The subscripts *pl* and *br* relate to the elastic-plastic and elastic-brittle elements, super-scripts *e* and *r* - to the elastic and residual internal forces.
The inequality constraints (2.6) in the case of brittle damage, for instance, due to normal or shear forces may be written consequently as $\mathrm{N}_{ed} \ge \mathrm{N}_{rd}(\mathrm{M}_{ed})$ or $\mathrm{V}_{ed} \ge \mathrm{V}_{rd}$; V_{ed} = shear force in the element; V_{rd} = shear resistance of the element; N_{ed} = normal force in the element; $\mathrm{N}_{rd}(\mathrm{M}_{ed})$ = normal force resistance of the element according to N-M interaction diagram.

A matrix $\boldsymbol{E}_p$ assigns a value (true or false) to the residual internal force in elastic-plastic or elastic-brittle elements accordingly (i.e. defines partial plastic redistribution of forces), its concept is as follows:

$$E_p = \mathrm{Diag}\begin{bmatrix} 1 \quad \text{if plastic element} \\ 0 \quad \text{if brittle element and } \varphi_{br}(\cdot)_i = 0, \quad i \in I_{br} \end{bmatrix}. \tag{2.8}$$

Equations (1)-(8) belong to a problem of nonlinear mathematical programming. The equation (3) with (4) can be formally eliminated here by the introduction in FEM programs corresponding linearly independent distortions $\boldsymbol{d}$ (or the initial displacements) in the construction:

$$\boldsymbol{Kq} + \boldsymbol{K}^d \boldsymbol{E}_p \boldsymbol{d} = 0, \tag{2.9}$$

where $\boldsymbol{K}$ - matrix of rigidity; $\boldsymbol{q}$ – vector of unknowns of FEM (usually as element displacements), $\boldsymbol{K}^d$ - matrix of distortions $\boldsymbol{d}$ influence on the reaction of finite elements.

Note, that we analyze here the geometrical linear systems (1st order theory); methods of analysis presented may also be applied to the geometrical nonlinear ones.

3. EXAMPLE OF SHAKEDOWN ANALYSES OF COMPOSITE FRAME

An example of shakedown analysis of plane composite steel-reinforced concrete braced frame with elastic-plastic and elastic-brittle elements (Figure 2) is given below.

Here the rods braces and all elements' cross-sections under shear and normal forces are assumed as brittle. On the contrary, elements' cross-sections under bending moments we presume as plastic.

There are several methods for the earthquake analysis of structures such as response spectrum analysis, direct integration time history analysis using accelerograms occurred earthquakes or artificial accelerograms and other [2]. All of these methods can be applied both in the linear and non-linear formulation. Nonlinear properties of the system under alternating seismic action may be taken into account in the optimization problem of elastic-brittle-plastic analysis.

The first step is to define the envelope of internal forces from arising seismic action in elastic stage of work. Seismic action is presented in the form of an elastic response spectrum. For this we use type 1 elastic response spectrum for ground type D, given in Eurocode 8. Peak ground acceleration is equal 10 m/s^2.

Response of structure to earthquake excitation we can compute as follows:

1. Define the structural properties
   - Determine the mass matrix m and the stiffness matrix k
   - Estimate the modal damping ratios ζ_n (was accepted at a rate of 5% in this example)
2. Determine the natural frequencies ω_n and natural modes φ_n of vibration

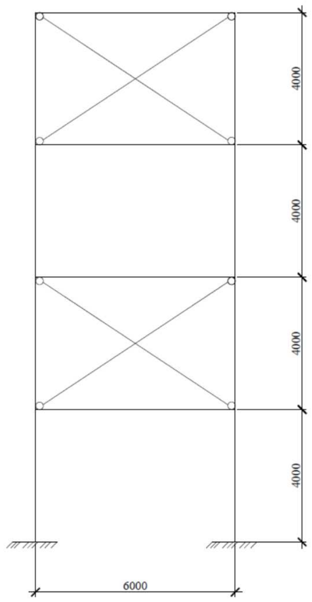


Fig. 2. Composite concrete frame

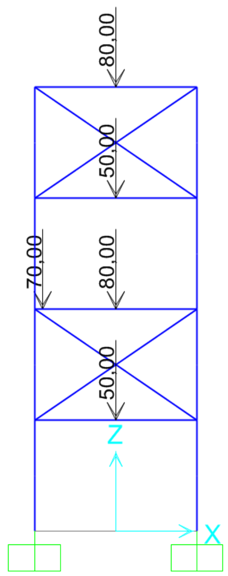


Fig. 3. Permanent loads acting on frame, *kN*

3. Compute the peak response in the n-th mode:
   - Determine A_n (acceleration ordinate) и D_n (displasment ordinate) from the response or design spectrum corresponding to natural period T_n and damping ratio ζ_n
   - Compute the displacements with $u_{jn} = \Gamma_n \phi_{jn} D_n$
   - Compute equivalent static forces f_n from $f_{jn} = \Gamma_n m_j \phi_{jn} A_n$
   - Compute the story forces, shear and overturning moment, and element forces, bending moments and shear, by static analysis of the structure subjected to lateral forces f_n
4. Determine an estimate for the peak value r of any response quantity by combining the peak modal values r_n according to square root of the sum of the squares (SRSS) or complete quadratic combination (CQC).

For each critical load cases, the design values of internal forces were determined by combing action for seismic design situation in according with Eurocode 0 [19]:

$$Ed = G_{kj,\sup}\left(G_{kj,\inf}\right) + A_{ed} + \psi_{2,i} Q_{k,i} \tag{3.1}$$

$G_{kj,sup}(G_{kj,inf})$ = unfavourable (favourable) characteristic permanent action (see Figure 3), A_{ed} = design seismic action, $\psi_{2,i}$ = factor by A1.2.2 [19], $Q_{k,i}$ = accompanying variable actions.

FEA software SAP2000 [20] is used for structure calculation in elastic stage. Design model is shown in Figure 2, the cross-sections of the frame elements are exposed in Table 1; strength class for concrete is C35/45.

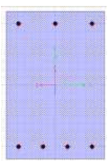
Design value of the element (section) resistance was determined according to Global Resistance Factor method described in Fib Model Code 2010 [21]:

$$R_d = R(f_{cR}, f_{yR}, f_{uR})/\gamma_R, \tag{3.2}$$

where γ_R=global safety coefficient is equal 1,3; $f_{cR} = 0,85 \cdot \alpha \cdot f_{ck}$; α=1; f_{yR} = 1,1; $\cdot f_{yk}$, $f_{tR} = 1,08 \cdot f_{yR}$.

The plastic moment capacity of all composite concrete members was calculated by moment-rotation (curvature) analyses according to [3,22]. The moment-rotation curve can be idealized with an elastic perfectly plastic response to estimate the plastic moment capacity of a member's cross-section [23]. The elastic portion of the idealized curved should pass through the point marking the first reinforcing bar yield. The idealized plastic moment capacity is obtained by balancing the areas between the actual and the idealized moment-rotation curves beyond the first reinforcing bar yield point. The idealized moment-curvature curves for members' cross-sections have shown in Table 2.

Table 1. Sections of frame members

№	Section	b(D), mm	H, mm	Longitudinal reinforcement, class		Steel section, class
				top	bottom	
1	 Beam	400	600	3Ø16, B500B	4Ø16, B500B	

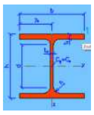
2	 Column	400		4Ø16, B500B		RO 377x6, S235
3	 rod brace					HE100A, S275

Table 2. The plastic moment capacity for members' cross-sections

Number of cross-section in Table 1 (axial force N, *kN*)	The plastic moment capacity M_p for positive moment, *kN·m*	The plastic moment capacity M_p for negative moment, *kN·m*
1	192	146.5
2 (-574)	284	284
2 (-197)	276	276

Transverse reinforcement of all concrete beams is made from bars Ø8 B500B at 200 mm (Figure 4). Resistance of the concrete beams to vertical shear designed in according with Eurocode 2 [24]. Design shear capacity $V_{Rd,s}$ is equal 226.7 *kN*. Resistance of the composite columns to vertical shear designed in according with Eurocode 4 [22]. The distribution of the total vertical shear V_{Ed} into the parts $V_{a,Ed}$ and $V_{c,Ed}$, acting on the steel section and the reinforced concrete core of the composite columns respectively assumed to be in the same ratio as the contributions of the steel section and the reinforced concrete core to the bending resistance $M_{pl,Rd}$ (see Table 3). Envelope diagram of shear forces is shown in Figure 5.

Table 3. Shear resistance of composite column

Part of section	Shear resistance, V_{Rd} , *kN*	Bending resistance $M_{pl,Rd}$, *kN·m*	Shear force (max) V_{Ed} , *kN*
Concrete core	296	102	30.2
Steel tube	630	202	60.4

Envelope diagram of elastic bending moments is shown in Figure 6.
To solve the optimization problem first we have to find the residual forces $\boldsymbol{S}^r$ in cross-sections of the elastic-plastic elements. This frame is 12 times statically indeterminate. Taking in account that brittle ties do not allow plastic redistribution, remain only 4 ties through moments in which the system can adapt to external actions remains.

Solving nonlinear optimization problem using sequence of linear programming tasks, we obtain the solution of optimization problem. Interaction between the moment capacity and the axial force (see Figure 7) was taken into account for the second iteration and the safety factor for load $\mu = 1{,}11$ was obtained for the vector of independent residual forces $\boldsymbol{X} = (-63{,}2; -22{,}95; 10{,}24; 0)$. Envelope diagram of elastic moments with the safety factor for load is shown in Figure 8. Result envelope diagram of moments considering redistribution of internal forces is shown in Figure 9.

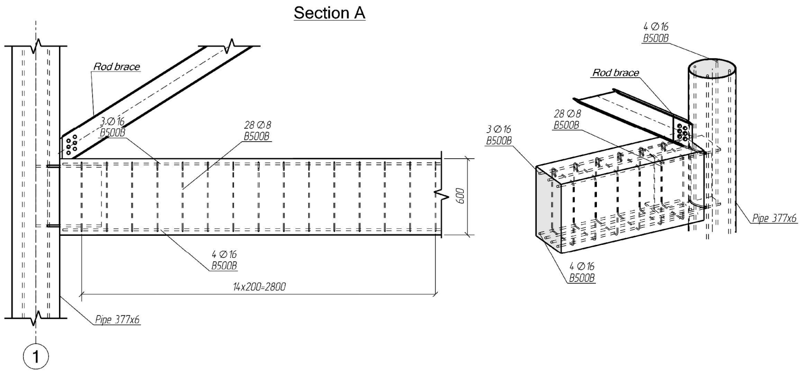


Fig. 4. Structural drawings of the frame , *kN*

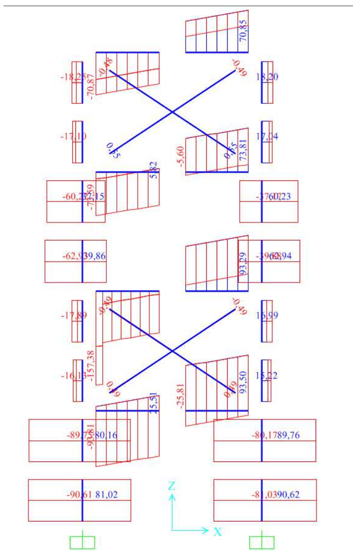


Fig. 5. Envelope diagram of shear forces, *kN*

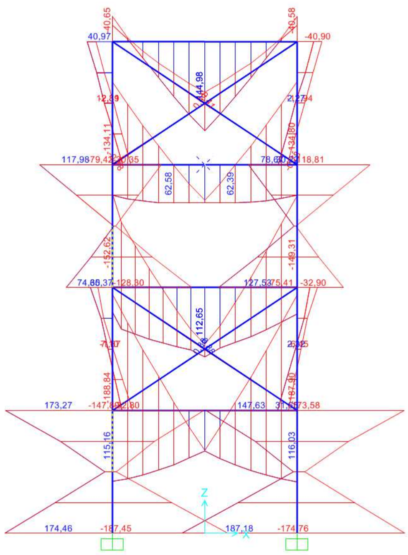


Fig. 6. Envelope diagram of “elastic” bending moments, *kN·m*

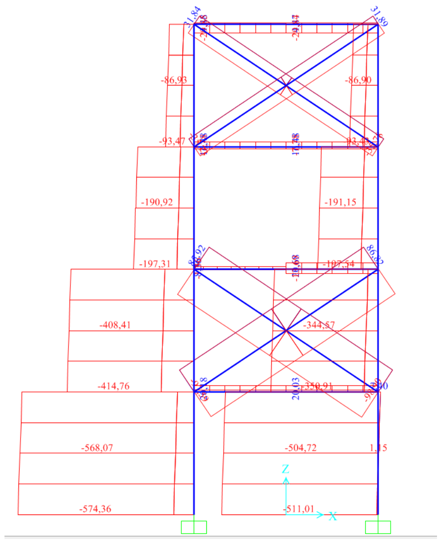


Fig. 7. Envelope diagram of axial forces, *kN*

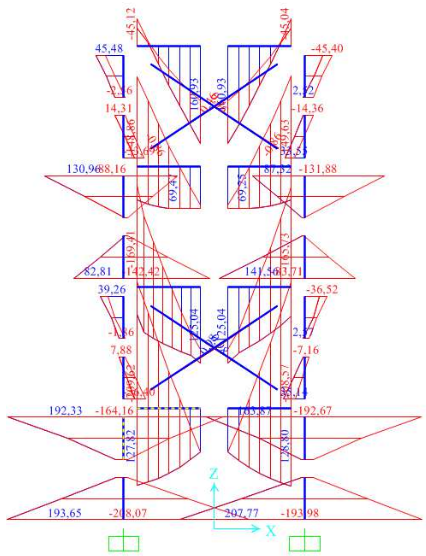


Fig. 8. Envelope elastic moments diagram (increased by the safety factor), *kN·m*

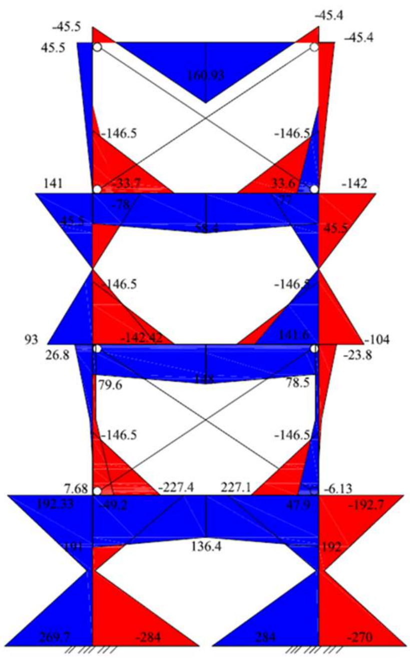


Fig. 9. Envelope diagram of shear forces, kN

4. CONCLUSIONS

In the paper a mathematical model of the optimization problem of shakedown analysis for seismic-protected systems is proposed. This analysis reveals additional reserve of bearing capacity of such structures, due to the ability to limited plastic redistribution of forces in elements, taking into account the plastic and brittle fracture of elements. Identification of reserves of bearing capacity is especially important to accidental design situations such as seismic action. The model can be used for the design of composite steel-concrete and reinforced concrete framed structures. Additionally, it allows both better dimensioning of sections and rebars.

An illustrative example of shakedown analysis for plane composite steel-reinforced concrete braced frame under seismic actions is presented. This mathematical model can be easily used for the analysis of spatial system.

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PRZYSTOSOWANIE ZESPOLONYCH STALOWO-BETONOWYCH UKŁADÓW RAMOWYCH ZAWIERAJĄCYCH PLASTYCZNE I KRUCHE ELEMENTY PRZY OBCIĄŻENIACH SEJSMICZNYCH

S t r e s z c z e n i e

W pracy przedstawiona analiza sejsmiczna stalowo-betonowych ram zespolonych za pomocom rozwiązywania problemu optymalizacji nośności granicznej i przystosowania, biorąc pod uwagę nieliniowe właściwości materiałów. Konstrukcje zostały wyposażone przez układy nośne sprężysto-plastycznych i kruchych elementów, które absorbują energię sejsmicznych działań. Zaproponowano model matematyczny tego problemu na podstawie teorii nośności granicznej przy ograniczonej redystribucji sił wewnętrznych resztkowych w takich konstrukcjach. Założono że obciążenia zmieniają się losowo w zadanych obszarach, zależnych od kierunków i amplitud sejsmicznych działań. Podano ilustratywny przykład takiej analizy. Została zwrócona uwaga na praktyczne zastosowania zaproponowanej modeli matematycznej.

Słowa kluczowe: nośność graniczna i przystosowanie, ramy zespolone stalowo-betonowe, sprężysto-plastyczne i kruche elementy, ochrona sejsmiczna

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