UNSTEADY ELECTRO-MAGNETO HYDRODYNAMIC FLOW AND HEAT TRANSFER OF TWO IONIZED FLUIDS IN A ROTATING SYSTEM WITH HALL CURRENTS

T. LINGA RAJU* and B. VENKAT RAO
Department of Engineering Mathematics, AUCE(A), Andhra University, Visakhapatnam
Pin-530003, INDIA
E-mail: prof.tlraju@andhrauniversity.edu.in

Abstract: An unsteady flow and heat transmission of ionized gases via a horizontal channel enclosed by non-conducting plates in a rotating framework with Hall currents is examined using electro-magnetohydrodynamic (EMHD) two-fluid heat flow. The Hall current impact is taken into account by assuming that the gases are totally ionized, the applied transverse magnetic field is very strong. For temperature and velocity distributions in two-fluid flow regions, the governing equations are solved analytically. For numerous physical parameters such as the Hartmann number, Hall parameter, rotation parameter, viscosity ratio, and so on, numerical solutions are visually displayed. It was discovered that an increase in temperature in the two regions is caused by the thermal conductivity ratio. It was also realized that an increase in rate of heat transfer coefficient at the plates is caused by either the Hartman number or the Hall parameter.

Keywords: unsteady flow, magnetic and electric fields, immiscible flow, plasma, Hall effect, rotating frame, heat transfer, insulating plates.

1. Introduction

The topic of electro-magneto hydrodynamics has been studied for over a century and the research in this field has advanced to the point where the development of an operational system is feasible. A number of excellent research articles have been published by several scientists. The authors of [1-9] have highlighted the various features of this rapidly expanding field and their findings have been adopted for various technological as well as in industrial applications. Plasma flow modelling in a rotating frame of reference can also reveal important information regarding electro-magneto hydrodynamic flows for single phase liquids due to its wide applications in many fields of science and engineering related to astrophysical, aeronautical, medical studies, and so on. In magnetized plasma, the effects of Hall current become significant when the strength of the magnetic field is very high. Many researchers focused on the simultaneous effect of Hall current and rotation on the MHD single phase fluid flows under the influence of a strong magnetic field in different geometries with various scenarios in view of its wide applications in many engineering problems, such as rotational viscometers, lubrication, centrifugal machinery, MHD rotating power generators, and in oceanography[10-22].

In many engineering problems, the quantity of heat flow and the pattern of temperature to which heat flow generates under unsteady conditions are of utmost importance. Unsteady electro-magneto hydrodynamics (EMHD) has drawn the attention of many researchers due to its extensive applications in science and engineering, especially in geophysical fluid dynamics, nuclear engineering control, plasma aerodynamics, mechanical engineering manufacturing, MHD energy generation systems, and so forth [see 23-31].

On the other hand, due to its numerous practical applications in aero-space science, engineering, technology, and a variety of industrial contexts such as energy conservation systems, oil extraction in geothermal regions, and designing thermonuclear fusion reactors, the EMHD two phase/ or 2-fluid flow

---

* To whom correspondence should be addressed

phenomenon under an unsteady situation has quickly become a significant domain of research for both academic and scientific communities. Evidently, two-fluid/two-phase flows have notable advantages in oil and gas production, gas liquid flows in boilers, ground water flow, ocean waves, inkjets, and aerosol deposition in spray medication, etc. [32-38].

Shail [39] made a study on the laminar 2- phase magnetohydrodynamics flows. Later, Lohrasbi and Sahai [40] looked at magnetohydrodynamic heat transfer in a two-phase flow between parallel plates. Magnetohydrodynamic heat transfer in two phase flow was studied by Malashetty and Leela [41]. Chamkha [42] examined hydromagnetic 2-phase flow in a channel. Umavathi *et al.* [43] investigated heat transfer flow of oscillatory Hartmann 2-fluid flows in a horizontal channel. The MHD two layered unsteady fluid flow and heat transfer through a horizontal channel between parallel plates in a rotating system was discussed by L.Raju and Valli [44]. Sharma and Kalpana [45] addressed the problem of an unsteady MHD two-fluid flow and heat transfer through a horizontal channel. An unsteady MHD flow of two immiscible fluids under chemical reaction in a horizontal channel was considered by Sivakamini and Govindarajan [46]. Magnetohydrodynamic heat transfer two-ionized fluid flow between two parallel plates with Hall currents was studied by L.Raju [47]. Elmaboud *et al.* [48] studied an electromagnetic flow for two layer immiscible fluids. The effect of Hall current on an unsteady magnetohydrodynamic two-ionized fluid flow and heat transfer in a channel was investigated by L.Raju and Gowri [49]. Recently, the problem of an electro-magnetohydrodynamic two ionized-gases with Hall and rotation effects was examined by L.Raju [50].

As can be seen from the aforementioned studies, there has been a significant amount of research work done on single-liquid flow phenomena in the literature, with only a few theoretical studies on channel flow system of electrically conducting fluids such as gases and liquids without or with Hall rotation effects under a steady and unsteady conditions. The time varying electro-magnetohydrodynamic (EMHD) two fluid plasma flows in a rotating frame of reference, on the other hand, is thought to be a vital model study with applications in the conceptual design of fusion reactors and liquid metal magnetohydrodynamic rotating power generators, among other things. Motivated by the above discussed literature and applications, the investigation aims at exploring the effects of Hall currents and Coriolis forces on an unsteady electro-magneto-hydrodynamic two fluid flow and heat transfer through a horizontal channel enclosed by non-conducting plates. Engineers may be interested in the combined effects of the proposed model in the design of energy extraction from the blanket of a thermonuclear fusion reactor, liquid metal MHD electrical rotating power generation models, Hall accelerators, the conceptual design of fusion reactors, design of liquid metal heat exchangers, and several other practical applications. For instance, four components of the MHD system that is the combustor, MHD channel, radiant boiler and the high temperature air pre-heater represent primary challenges to the thermal engineer.

2. Formulation and basic equations

The two-liquid EMHD flow of ionized gases controlled by a constant pressure gradient $-\dfrac{\partial p}{\partial x}$ is considered in a horizontal channel enclosed by two parallel rigid plates located at $y = h_1$ and $y = h_2$. The plates have an indefinitely long length in the x and z axes, with the origin placed midway between them. The x axis is oriented in the direction of the hydrodynamic pressure gradient in the plane parallel to the channel plates, but not toward the flow. In the y-direction, which is transverse to the flow field, a constant magnetic field B_0 is applied. The whole system is rotated with an angular velocity Ω about an axis perpendicular to the plates. The two liquid flows are supposed to be unsteady and laminar. The plates are considered to be electrically non-conducting. The Hall Effect is studied under the assumption that the gases are completely ionized and the magnetic field is extremely strong, and hence a velocity component along z-direction is initiated. In order to disregard the induced magnetic field, the magnetic Reynolds number is taken as small. The upper fluid region and lower fluid regions are considered as $0 \le y \le h_1$ and $-h_2 \le y \le 0$, also these fluid regions are labelled as Regions I, II. In these two regions there are two immiscible electrically conducting incompressible fluids with different densities ρ_1,

ρ_2, viscosities μ_1, μ_2, electrical conductivities σ_{01}, σ_{02} and thermal conductivities K_1 and K_2. The channel plates are kept up at uniform temperature T_w so that the temperature at the upper plate T_{w1} is equal to that at the lower plate T_{w2}. The thermal boundary conditions are thought to influence the infinite channel plates everywhere. This study also ignores thermal conduction and electron heating in the flow direction. The flat, stress-free, and undisturbed contact between the two immiscible fluids is preferable. With all aforesaid assumptions and in view of the available literature [11, 49, 50], governing equations of the motion, current and energy in two liquid regions as well as the boundary and interface conditions for an unsteady EMHD stream of ionized gases in the presence of Hall currents and rotation are simplified as:

Region-I (Fluid at the upper region $0 \leq y \leq h_1$):

$$\rho_1 \frac{\partial u_1}{\partial t} - \mu_1 \frac{\partial^2 u_1}{\partial y^2} + \left[1 - s \left(1 - \frac{\sigma_{11}}{\sigma_{01}} \right) \right] \frac{\partial p}{\partial x} +$$
$$- \left[-\sigma_{11} \left(E_{1z} + u_1 B_0 \right) - \sigma_{21} \left(E_{1x} - w_1 B_0 \right) \right] B_0 = -2\rho_1 \Omega w_1, \tag{2.1}$$

$$\rho_1 \frac{\partial w_1}{\partial t} - \mu_1 \frac{\partial^2 w_1}{\partial y^2} - s \frac{\sigma_{21}}{\sigma_{01}} \frac{\partial p}{\partial x} +$$
$$- \left\{ \sigma_{11} \left(E_{1x} - w_1 B_0 \right) - \sigma_{21} \left(E_{1z} + u_1 B_0 \right) \right\} B_0 = 2\rho_1 \Omega u_1, \tag{2.2}$$

$$\rho_1 c_{p_1} \frac{\partial T_1}{\partial t} - K_1 \frac{\partial^2 T_1}{\partial y^2} = \mu_1 \left[\left(\frac{\partial u_1}{\partial y} \right)^2 + \left(\frac{\partial w_1}{\partial y} \right)^2 \right] + \frac{J_1^2}{\sigma_{01}}, \tag{2.3}$$

$$J_{1x} = \sigma_{11} E_{1x} - B_0 \sigma_{11} w_1 + \sigma_{21} E_{1z} + B_0 \sigma_{21} u_1 + \frac{s\sigma_{21}}{\sigma_{01} B_0} \frac{\partial p}{\partial x}, \tag{2.4}$$

$$J_{1z} = \left(E_{1z} + B_0 u_1 \right) \frac{\sigma_{11}}{B_0} - \left(E_{1x} - B_0 w_1 \right) \frac{\sigma_{21}}{B_0} - \left(\sigma_{01} - \sigma_{11} \right) \frac{s}{B_0 \sigma_{01}} \frac{\partial p}{\partial x}. \tag{2.5}$$

Region-II (Fluid at the lower region $-h_2 \leq y \leq 0$):

$$\rho_2 \frac{\partial u_2}{\partial t} - \mu_2 \frac{\partial^2 u_2}{\partial y^2} + \left[1 - s \left(1 - \frac{\sigma_{12}}{\sigma_{02}} \right) \frac{\partial p}{\partial x} \right] +$$
$$- \left\{ -\sigma_{12} \left(E_{2z} + u_2 B_0 \right) - \sigma_{22} \left(E_{2x} - w_2 B_0 \right) \right\} B_0 = -2\rho_2 \Omega w_2, \tag{2.6}$$

$$\rho_2 \frac{\partial w_2}{\partial t} - \mu_2 \frac{\partial^2 w_2}{\partial y^2} - \frac{s\sigma_{22}}{\sigma_{02}} \frac{\partial p}{\partial x} +$$
$$- \left\{ \sigma_{12} \left(E_{2x} - w_2 B_0 \right) - \sigma_{22} \left(E_{2z} + u_2 B_0 \right) \right\} B_0 = 2\rho_2 \Omega u_2, \tag{2.7}$$

$$\rho_2 c_{p_2} \frac{\partial T_2}{\partial t} - K_2 \frac{\partial^2 T_2}{\partial y^2} = \mu_2 \left[\left(\frac{\partial u_2}{\partial y} \right)^2 + \left(\frac{\partial w_2}{\partial y} \right)^2 \right] + \frac{J_2^2}{\sigma_{02}}, \tag{2.8}$$

$$J_{2x} = \sigma_{12}E_{2x} - B_0\sigma_{12}w_2 + \sigma_{22}E_{2z} + B_0\sigma_{22}u_2 + \frac{s\sigma_{22}}{B_0\sigma_{02}}\frac{\partial p}{\partial x}, \tag{2.9}$$

$$J_{2z} = \left(E_{2z} + B_0u_2\right)\frac{\sigma_{12}}{B_0} - \left(E_{2x} - B_0w_2\right)\frac{\sigma_{22}}{B_0} - \left(\sigma_{02} - \sigma_{12}\right)\frac{s}{B_0\sigma_{02}}\frac{\partial p}{\partial x} \tag{2.10}$$

where, the subscripts *1* and *2* in the above equations refer to the quantities for Region-I and II individually. The symbols u_1, u_2 and w_1, w_2, are the velocity components in the x and z axes of the two liquids, and are called primary and secondary velocity distributions, respectively. The notations E_{ix} and E_{iz}; J_{ix} and J_{iz} are the components of the electric field, current densities in the x and z axes. The quantity $s = p_e/p$ is called the proportion of electron pressure to the total pressure. T_1 and T_2 are the temperatures of the two fluids, and $C_{pi}(i = 1, 2)$ is the specific heat at constant pressure. The symbol Ω stands for angular velocity, likewise the symbols σ_{11}, σ_{12} and σ_{21}, σ_{22} are the modified conductivities parallel and perpendicular to the direction of the electric field.

Corresponding boundary and interface conditions for fluid velocity in the two regions are:

$$u_1\left(h_1\right) \text{ and } w_1\left(h_1\right) = 0 \qquad \text{for } t \le 0,$$
$$\tag{2.11}$$
$$= \text{Real of } \varepsilon e^{i\omega t} \quad \text{for } t > 0,$$

$$u_2\left(-h_2\right) = 0 \text{ and } w_2\left(-h_2\right) = 0, \tag{2.12}$$

$$u_1(0) = u_2(0), \quad w_1(0) = w_2(0) \text{ for } h_1 = h_2. \tag{2.13}$$

$$\mu_1\frac{du_1}{dy} = \mu_2\frac{du_2}{dy} \text{ and } \mu_1\frac{dw_1}{dy} = \mu_2\frac{dw_2}{dy} \text{ at } y = 0. \tag{2.14}$$

The isothermal boundary conditions for temperature for both fluids are:

$$T_1\left(h_1\right) = T_{w_1}, \ T_2\left(-h_2\right) = T_{w_2}. \tag{2.15}$$

The continuity of temperature and heat flux at the interface are:

$$T_1(0) = T_2(0) \quad \text{for} \quad h_1 = h_2, \ K_1\frac{dT_1}{dy} = K_2\frac{dT_2}{dy} \quad \text{at} \quad y = 0. \tag{2.16}$$

Then non-dimensional quantities given below are utilized to formulate the flow equations (2.1)-(2.10) and conditions (2.11)-(2.16) as dimensionless:

$$u^{\bullet}_1 = \frac{u_1}{u_p}, \quad u^{\bullet}_2 = \frac{u_2}{u_p}, \quad w^{\bullet}_1 = \frac{w_1}{u_p}, \quad w^{\bullet}_2 = \frac{w_2}{u_p}, \quad y^{\bullet}_i = \frac{y_i}{h_i}, \quad u_p = -\frac{\partial p}{\partial x}\frac{h_1^2}{\mu_1},$$

$$t^\bullet = \frac{\mu_i t}{\rho_i h_i^2}, \qquad \omega^\bullet = \frac{\omega h_i^2 \rho_i}{\mu_i}, \qquad m_{ix} = \frac{E_{ix}}{B_0 u_p}, \qquad m_{iz} = \frac{E_{iz}}{B_0 u_p}, \qquad I_{ix} = \frac{J_{ix}}{\sigma_{0i} B_0 u_p},$$

$$I_{iz} = \frac{J_{iz}}{\sigma_{0i} B_0 u_p}, \; J_i^2 = J_{ix}^2 + J_{iz}^2 \; (i = 1, 2), \text{ the Hartmann number } M = \sqrt{\sigma_{01} B_0^2 h_1^2 / \mu_1},$$

the Taylor number or rotation parameter $K = \sqrt{h_1^2 \rho_1 \Omega / \mu_1}$,

ρ (density ratio) $= \dfrac{\rho_2}{\rho_1}$, α (viscosity ratio) $= \dfrac{\mu_1}{\mu_2}$, h (height ratio) $= \dfrac{h_2}{h_1}$,

β (thermal conductivity ratio) $= \dfrac{K_1}{K_2}$, σ_0 (electrical conductivity ratio) $= \dfrac{\sigma_{01}}{\sigma_{02}}$,

$$\sigma_{01} = \frac{\sigma_{12}}{\sigma_{11}}, \qquad \sigma_{02} = \frac{\sigma_{22}}{\sigma_{21}}, \qquad \frac{1}{1+m^2} = \frac{\sigma_{11}}{\sigma_{01}}, \qquad \frac{m}{1+m^2} = \frac{\sigma_{21}}{\sigma_{01}},$$

m (Hall parameter) $= \dfrac{\omega_e}{\left(\dfrac{1}{\tau} + \dfrac{1}{\tau_e}\right)}$, Prandtl number $P_{ri} = \dfrac{\mu_i C_{pi}}{K_i}$ and

the temperature distribution $\theta_i = \dfrac{T_i - T_{wi}}{(u_p^2 \mu_1 / K_i)}$ \hfill (2.17)

where ω_e is the gyration frequency of electrons, ε is the amplitude (a small constant quantity, $\varepsilon \ll 1$), τ, τ_e are the mean collision time amongst electron and ion, electron and neutral particles.

In view of Eq.(2.17) and for easiness, ignoring the asterisks, the non-dimensional form of the main equations (2.1)-(2.10) and conditions (2.11)-(2.16) become:

Region-I

$$\frac{\partial u_1}{\partial t} - \frac{\partial^2 u_1}{\partial y^2} + \frac{M^2}{1+m^2}(m_{1z} + u_1) - \frac{mM^2}{1+m^2}(m_{1x} - w_1) - P_1 = -2K^2 w_1, \tag{2.18}$$

$$\frac{\partial w_1}{\partial t} - \frac{\partial^2 w_1}{\partial y^2} - \frac{M^2}{1+m^2}(m_{1x} - w_1) - \frac{mM^2}{1+m^2}(m_{1z} + u_1) - P_2 = 2K^2 u_1, \tag{2.19}$$

$$\frac{d\theta_1}{dt} = \frac{1}{P_{r1}} \frac{d^2\theta_1}{dy^2} + \left[\left(\frac{du_1}{dy}\right)^2 + \left(\frac{dw_1}{dy}\right)^2\right] + M^2 I_1^2, \quad \text{where} \quad I_1^2 = I_{1x}^2 + I_{1z}^2, \tag{2.20}$$

$$I_{1x} = \frac{1}{1+m^2}(m_{1x} - w_1) + \frac{m}{1+m^2}(m_{1z} + u_1) - \frac{s}{M^2}\frac{m}{1+m^2}, \tag{2.21}$$

$$I_{1z} = \frac{1}{1+m^2}(m_{1z} + u_1) - \frac{m}{1+m^2}(m_{1x} - w_1) + \frac{s}{M^2}\left(1 - \frac{m}{1+m^2}\right), \tag{2.22}$$

Region-II

$$\frac{\partial u_2}{\partial t} - \frac{\partial^2 u_2}{\partial y^2} + \frac{\alpha\sigma_1 h^2 M^2}{1+m^2}(m_{2z} + u_2) - \frac{m\alpha\sigma_2 h^2 M^2}{1+m^2}(m_{2x} - w_2) - P_3\alpha h^2 =$$
$$= -2\rho\alpha h^2 K^2 w_2, \tag{2.23}$$

$$\frac{\partial w_2}{\partial t} - \frac{\partial^2 w_2}{\partial y^2} - \frac{\alpha\sigma_1 h^2 M^2}{1+m^2}(m_{2x} - w_2) - \frac{m\alpha\sigma_2 h^2 M^2}{1+m^2}(m_{2z} + u_2) - P_4\alpha h^2 =$$
$$= 2\rho\alpha h^2 K^2 u_2, \tag{2.24}$$

$$\frac{d\theta_2}{dt} = \frac{1}{P_{r2}}\frac{d^2\theta_2}{dy^2} + \frac{\beta}{\alpha}\left[\left(\frac{du_2}{dy}\right)^2 + \left(\frac{dw_2}{dy}\right)^2\right] + h^2\sigma\beta M^2 I_2^2, \quad \text{where} \quad I_2^2 = I_{2x}^2 + I_{2z}^2, \tag{2.25}$$

$$I_{2x} = \frac{\sigma_0\sigma_1}{1+m^2}(m_{2x} - w_2) + \frac{m\sigma_0\sigma_2}{1+m^2}(m_{2z} + u_2) - \frac{s\sigma_0^2\sigma_2}{M^2}\frac{m}{1+m^2}, \tag{2.26}$$

$$I_{2z} = \frac{\sigma_0\sigma_1}{1+m^2}(m_{2z} + u_2) - \frac{m\sigma_0\sigma_2}{1+m^2}(m_{2x} - w_2) + (1 - \frac{\sigma_0\sigma_1}{1+m^2})\frac{s\sigma_0}{M^2} \tag{2.27}$$

where

$$P_1 = 1 - \frac{m^2 s}{1+m^2}, \quad P_2 = \frac{-ms}{1+m^2}, \quad P_3 = 1 - \left(1 - \frac{\sigma_0\sigma_{01}}{1+m^2}\right)s, \quad P_4 = \frac{-\sigma_0\sigma_{02}ms}{1+m^2}.$$

The associated boundary and interface conditions for velocity are given by

$$u_1(1) = 0 \qquad \text{for } t \le 0,$$
$$\qquad = \varepsilon\cos\omega t \quad \text{for } t > 0, \tag{2.28}$$

$$w_1(1) = 0 \qquad \text{for} \quad t \le 0,$$
$$\qquad = \varepsilon\cos\omega t \quad \text{for} \quad t > 0, \tag{2.29}$$

$$u_2(-1) = 0, \qquad w_2(-1) = 0, \tag{2.30}$$

$$u_1(0) = u_2(0), \qquad w_1(0) = w_2(0), \tag{2.31}$$

$$\frac{du_1}{dy} = \left(\frac{1}{\alpha h}\right)\frac{du_2}{dy} \quad \text{and} \quad \frac{dw_1}{dy} = \left(\frac{1}{\alpha h}\right)\frac{dw_2}{dy} \quad \text{at interface} \quad y = 0. \tag{2.32}$$

The isothermal boundary conditions, continuity of temperature and heat flux at the interface are given by:

$$\theta_1(1)=0, \ \theta_2(-1)=0, \tag{2.33}$$

$$\theta_1(0)=\theta_2(0) \quad \text{and} \quad \frac{d\theta_1}{dy}=\frac{1}{\beta h}\frac{d\theta_2}{dy} \quad \text{at the interface} \quad y=0. \tag{2.34}$$

3. The solution process

The system of equations (2.18-2.19, 2.23-2.24) and (2.20, 2.25) is to be solved subject to the boundary and interface conditions (2.28)-(2.34) for the velocity and temperature distributions in the two liquid zones (regions). Since these equations are coupled partial differential equations, they cannot be solved in closed form. They can be reduced to ordinary linear differential equations with the assumption of the following 2-term series [49] so that the resulting equations can be solved easily:

$$u_1(y,t)=u_{01}(y)+\varepsilon\cos\omega t\,u_{11}(y), \qquad w_1(y,t)=w_{01}(y)+\varepsilon\cos\omega t\,w_{11}(y), \tag{3.1}$$

$$u_2(y,t)=u_{02}(y)+\varepsilon\cos\omega t\,u_{12}(y), \qquad w_2(y,t)=w_{02}(y)+\varepsilon\cos\omega t\,w_{12}(y), \tag{3.2}$$

$$\theta_1(y,t)=\theta_{01}(y)+\varepsilon\cos\omega t\,\theta_{11}(y), \qquad \theta_2(y,t)=\theta_{02}(y)+\varepsilon\cos\omega t\,\theta_{12}(y), \tag{3.3}$$

in which, the terms $u_{01}(y), u_{02}(y), w_{01}(y), w_{02}(y)$ and $\theta_{01}(y), \theta_{02}(y)$ are velocity and temperature distributions in the steady state part, while $u_{11}(y)$, $u_{12}(y)$, $w_{11}(y)$, $w_{12}(y)$, and $\theta_{11}(y), \theta_{12}(y)$ are the related time dependent components in the upper and lower fluid zones.

To begin, Eqs (2.18), (2.19), (2.23) and (2.24) are solved for the velocity employing the prescribed conditions (2.28)-(2.32). As a result, when the plates are formed of a non-conducting material, closed form solutions for temperature in the two-fluid areas and heat transfer rate at the side plates are obtained by considering equations (2.20, 2.25) and conditions (2.33) to (2.34).

Now, by utilizing the expressions from equations (3.1)-(3.3) in equations (2.18, 2.19, 2.23, 2.24) and (2.20 and 2.25), then separating the steady and time dependent parts the following differential equations in terms of the symbolized complex notations (namely, q_{01}, q_{11}, q_{02} and q_{12}) are obtained in the two regions as:

Region-I

$$\frac{d^2q_{01}}{dy^2}-a_1q_{01}=a_2, \tag{3.4}$$

$$\frac{d^2q_{11}}{dy^2}-(a_1-\omega\tan\omega t)q_{11}=0, \tag{3.5}$$

$$\frac{1}{P_{r1}}\frac{d^2\theta_{01}}{dy^2}=-\left(\frac{dq_{01}}{dy}\frac{d\overline{q}_{01}}{dy}\right)-\left(q_{01}a_9\overline{q_{01}}\,\overline{a_9}+q_{01}a_9\overline{a_{10}}+\overline{q_{01}}\,\overline{a_9}\,a_{10}+a_{10}\overline{a_{10}}\right)M^2, \tag{3.6}$$

$$\frac{1}{P_{r1}}\frac{d^2\theta_{11}}{dy^2} + \omega\tan\omega t\,\theta_{11} = -\left(\frac{dq_{11}}{dy}\frac{d\overline{q}_{01}}{dy} - \frac{dq_{01}}{dy}\frac{d\overline{q}_{11}}{dy}\right) - \varepsilon\cos\omega t\left(\frac{dq_{11}}{dy}\frac{d\overline{q}_{11}}{dy}\right) +$$

$$-\left(q_{01}\overline{q_{11}}\,\overline{a_9}\,a_9 + a_9\,\overline{a_9}\overline{q_{01}}\,q_{11} + \varepsilon\cos\omega t\,a_9\overline{a_9}\,q_{11}\overline{q_{11}} + q_{11}a_9\overline{a_{10}} + a_{10}\overline{a_9}\,\overline{q_{11}}\right)M^2. \tag{3.7}$$

Region-II

$$\frac{d^2q_{02}}{dy^2} - a_3q_{02} = a_4, \tag{3.8}$$

$$\frac{d^2q_{12}}{dy^2} - \left(a_3 - \omega\tan\omega t\right)q_{12} = 0, \tag{3.9}$$

$$\frac{1}{P_{r2}}\frac{d^2\theta_{02}}{dy^2} = -\left(\frac{dq_{02}}{dy}\frac{d\overline{q}_{02}}{dy}\right)\frac{\beta}{\alpha} +$$

$$-\left(a_{14}\overline{a_{14}}\,\overline{q_{02}}\,q_{02} + \overline{a_{15}}\,a_{14}q_{02} + a_{15}\overline{a_{14}}\,\overline{q_{02}} + a_{15}\overline{a_{15}}\right)h^2M^2\sigma\beta, \tag{3.10}$$

$$\frac{1}{P_{r2}}\frac{d^2\theta_{12}}{dy^2} + \omega\tan\omega t\,\theta_{12} = -\left[\left(\frac{dq_{02}}{dy}\frac{d\overline{q}_{12}}{dy} + \frac{dq_{12}}{dy}\frac{d\overline{q}_{02}}{dy}\right) + \varepsilon\cos\omega t\left(\frac{dq_{12}}{dy}\frac{d\overline{q}_{12}}{dy}\right)\right]\frac{\beta}{\alpha} +$$

$$-\left\{\overline{a_{14}}a_{14}q_{02}\overline{q_{12}} + \overline{a_{14}}\,a_{14}q_{12}\overline{q_{02}} + \varepsilon\cos\omega t\left(\overline{a_{14}}\,a_{14}\,q_{12}\overline{q_{12}}\right) + \right.$$

$$\left. + \overline{a_{15}}\,a_{14}q_{12} + \overline{a_{14}}\,a_{15}\,\overline{q_{12}}\right\}h^2M^2\sigma\beta. \tag{3.11}$$

The corresponding boundary and interface conditions are given by

For steady-state part:

$$q_{01}(1)=0,\quad q_{02}(-1)=0, \tag{3.12}$$

$$q_{01}(0)=q_{02}(0),\quad \frac{dq_{01}}{dy} = \frac{1}{\alpha h}\frac{dq_{02}}{dy}\quad \text{at}\quad y=0, \tag{3.13}$$

$$\theta_{01}(1)=0,\quad \theta_{02}(-1)=0, \tag{3.14}$$

$$\theta_{01}(0)=\theta_{02}(0)\ \text{and}\ \frac{d\theta_{01}}{dy} = \frac{1}{\beta h}\frac{d\theta_{02}}{dy}\quad \text{at}\quad y=0. \tag{3.15}$$

For transient time dependent part:

$$q_{11}(1)=1,\quad q_{12}(-1)=0, \tag{3.16}$$

$$q_{11}(0)=q_{12}(0),\quad \frac{dq_{11}}{dy} = \frac{1}{\alpha h}\frac{dq_{12}}{dy}\quad \text{at}\quad y=0, \tag{3.17}$$

$$\theta_{11}(1)=0,\quad \theta_{12}(-1)=0, \tag{3.18}$$

$$\theta_{11}(0) = \theta_{12}(0) \quad \text{and} \quad \frac{d\theta_{11}}{dy} = \frac{1}{\beta h} \frac{d\theta_{12}}{dy} \quad \text{at the interface } \left(y = 0\right) \tag{3.19}$$

where:

$$q_1(y,t) = q_{01}(y) + \varepsilon \cos \omega t q_{11}(y), \qquad q_2(y,t) = q_{02}(y) + \varepsilon \cos \omega t q_{12}(y),$$

$$\theta_1(y,t) = \theta_{01}(y) + \varepsilon \cos \omega t \theta_{11}(y), \qquad \theta_2(y,t) = \theta_{02}(y) + \varepsilon \cos \omega t \theta_{12}(y),$$

$$q_1(y,t) = u_1(y,t) + iw_1(y,t), \qquad q_2(y,t) = u_2(y,t) + iw_2(y,t),$$

$$A_1 = P_1 + iP_2, \qquad A_2 = P_3 + iP_4, \qquad M_1 = m_{1x} + im_{1z}, \qquad M_2 = m_{2x} + im_{2z},$$

$$a_1 = \left(\frac{1-mi}{1+m^2}\right)M^2 + 2iK^2, \qquad a_2 = -\left[A_1 + \left(\frac{i+m}{1+m^2}\right)M^2 M_1\right],$$

$$a_3 = \left(\sigma_1 - i\sigma_2 m\right)\frac{\alpha h^2 M^2}{1+m^2} - 2ih^2 K^2 \rho\alpha, \quad a_4 = -\left[A_2\alpha h^2 + \left(\sigma_1 i + m\sigma_2\right)\frac{h^2 M^2 M_2 \alpha}{1+m^2}\right],$$

$$a_5 = \frac{1}{1+m^2}, \qquad a_6 = \frac{m}{1+m^2}, \qquad a_7 = \frac{-ms}{\left(1+m^2\right)M^2}, \qquad a_8 = \frac{m^2 s}{\left(1+m^2\right)M^2},$$

$$a_9 = a_6 + ia_5, \quad a_{10} = -ia_9 M_1 + (1-i)a_7 - \frac{si}{M^2}, \qquad a_{11} = \frac{\sigma_0 \sigma_2 m}{1+m^2}, \quad a_{12} = \frac{\sigma_0 \sigma_1}{1+m^2},$$

$$a_{13} = \left\{\frac{-\sigma_0 \sigma_2 m}{\left(1+m^2\right)} + i\left[1 - \frac{\sigma_0 \sigma_1}{\left(1+m^2\right)}\right]\right\}\frac{s\sigma_0}{M^2}, \qquad a_{14} = \left(m\sigma_2 + i\sigma_1\right)\frac{\sigma_0}{1+m^2},$$

$$a_{15} = \left(M_2\sigma_1 - \frac{ms\sigma_0\sigma_2}{M^2}\right)\frac{\sigma_0}{1+m^2} + i\sigma_0\left[\frac{s}{M^2} + \left(\sigma_2 mM_2 - \frac{\sigma_0\sigma_1 s}{M^2}\right)\right]\frac{\sigma_0}{1+m^2}. \tag{3.20}$$

Since, the side plates are kept at a large distance from the *z* axis and are comprised of non-conducting (insulating) material, at that point the induced electric current does not leave the channel rather circulates in the fluid. So that the following conditions are obtained [11] as:

$$\int_0^1 I_{1z}dy = 0 \quad \text{and} \quad \int_0^1 I_{2z}dy = 0. \tag{3.21}$$

Similarly, at a large distance from along the *x* direction, other relations are obtained as:

$$\int_0^1 I_{1x}dy = 0 \quad \text{and} \quad \int_0^1 I_{2x}dy = 0. \tag{3.22}$$

Constants in the solution are found by the above conditions (3.12), (3.13), (3.16), (3.17), and then using the expressions of m_{ix} and m_{iz} ($i = 1, 2$) and conditions (3.21) and (3.22), as a result, the solutions are obtained for velocity distributions, namely the primary and secondary velocity distributions $u_1(y,t), u_2(y,t)$ and $w_1(y,t), w_2(y,t)$ of the researched problem in the two-fluid regions. Likewise their related mean velocity distributions u_{1m}, u_{2m} and w_{1m}, w_{2m} in the two regions are determined. Consequently, the energy equations for both steady and transient time-dependent components are similarly separated as in equations (3.6), (3.7), (3.10), (3.11). Ultimately, with the help of conditions (3.14), (3.15), (3.18), (3.19), the solutions for temperature distributions $\theta_1(y,t), \theta_2(y,t)$ in the two regions and the rate of heat transfer coefficients at the plates, that is the rate of heat transfer coefficient at the upper plate, $Nu_1 = -\dfrac{\partial \theta_1}{\partial y}$ at $y = 1$ and that at the lower plate, $Nu_2 = \dfrac{1}{\beta h}\dfrac{\partial \theta_2}{\partial y}$ at $y = -1$ are also obtained like in the study described in [49]. For simplicity, the specific calculations and usual expressions are omitted here as they are too lengthy to present.

4. Results and discussion

Numerical computations are performed for both the velocity and temperature fields in the two-ionized fluid regions in order to obtain the physical features of the problem and to discuss the results. In Figs 1-21, the estimations for various values of the governing parameters are graphically depicted. The plain lines show the velocity profiles for unsteady and dash-spot lines for the steady flow motions. For all computations, the specific parameters $\sigma_1 = 1.2, \sigma_2 = 1.5, \rho = 1$ and $P_{r1} = 1 = P_{r2}$ are fixed, and the impact of auxiliary imperative flow parameters on velocity and temperature is examined. The solutions are found to be independent of the ionization parameter s as expected (that is, the ratio of the electron pressure to the total pressure). It is found that the results of this study coincide with the results of L.Raju [50] under a steady condition, likewise the results of this investigation concur with the results of L.Raju and Gowri [49] when there is no rotation in the motion and also agree with the results obtained by Malashetty and Leela [41] for the MHD two liquid steady flow in the absence of Hall currents.

Figures 1-3 show the effect of various values of the Hartmann number M on the primary flow velocities u_1, u_2, secondary flow velocities w_1, w_2 and temperature distributions θ_1, θ_2 in the two fluid regions. Figure 1 indicates that the primary velocity distribution decreases to a certain value of the Hartman number, say $M = 4$ and thereafter it grows in both the regions as the Hartman number increases. From Fig.2, it is seen that a rise in the Hartman number rises the secondary velocity as M gets larger up to 4 and thereafter reduces. This tendency in velocity components could be caused by the existence of a magnetic field in an electrically conducting fluid flow along with the presence of Coriolis forces.

Figure 3 shows that the temperature distribution diminishes to a certain esteem of the Hartmann number say $M = 4$ and then rise above that esteem in the two regions as the Hartman number increases. The maximum temperature in the channel tends to move above the channel centre line towards region-I as the Hartmann number increases.

The effect of Hall parameter m on velocity and temperature fields in the two fluid regions is shown in Figs 4, 5, and 6. Increasing the Hall parameter values consistently boosts the primary and secondary flows in the two zones, as seen in Figs 4 and 5. It indicates that the Hall parameter accelerates velocity of the two fluids, and this is because of the fact that a rise in the Hall parameter m reduces the effective conductivity and hence the magnetic damping force is increased in the flow fields. Figure 6 indicates that a rise in the Hall parameter reduces the temperature field in the two regions. It is further noticed that the magnitude of temperature is higher in the lower region in comparison to that in the upper region with a rise in the Hall parameter.
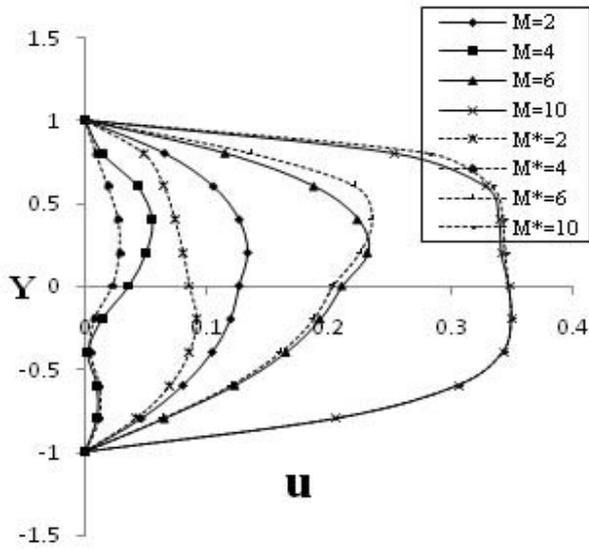
Fig.1. Primary velocity profiles u_1, u_2 (unsteady flow), u_1^*, u_2^* (steady flow) for different M and $m = 2$, $\alpha = 0.333$, $h = 1$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
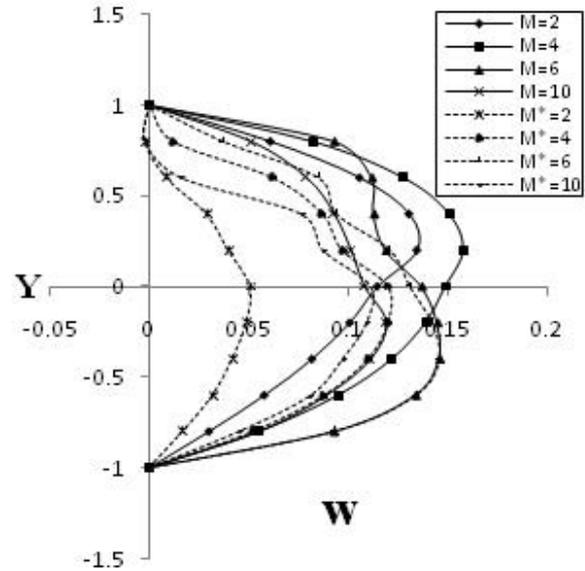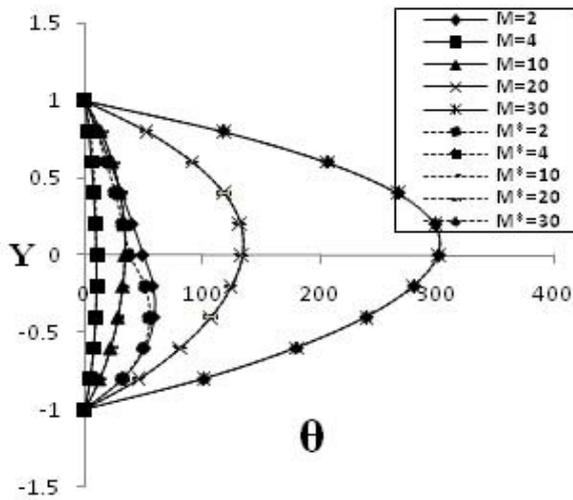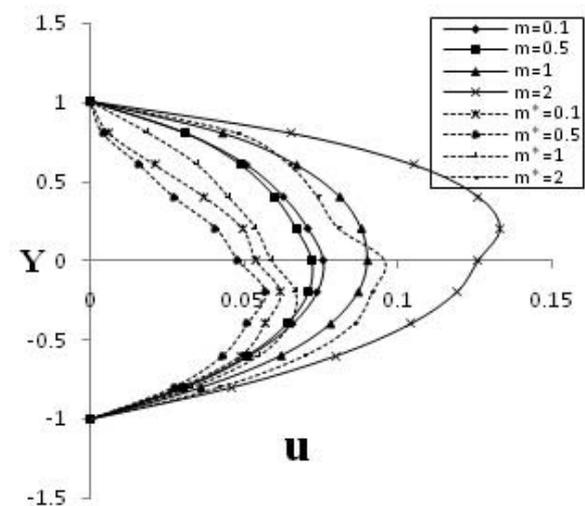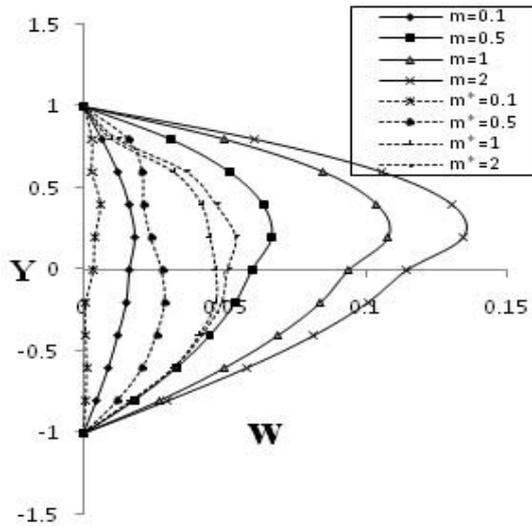

Fig.2. Secondary velocity profiles w_1, w_2 (unsteady flow), w_1^*, w_2^* (steady flow) for different M and $m = 2$, $\alpha = 0.333$, $h = 1$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).

Fig.3. Temperature profiles θ_1, θ_2 (unsteady flow), θ_1^*, θ_2^* (steady flow) for different M and $m = 2$, $\alpha = 0.333$, $h = 0.75$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\beta = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).

Fig.4. Primary velocity profiles u_1, u_2 (unsteady flow), u_1^*, u_2^* (steady flow) for different m and $M = 2$, $\alpha = 0.333$, $h = 1$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
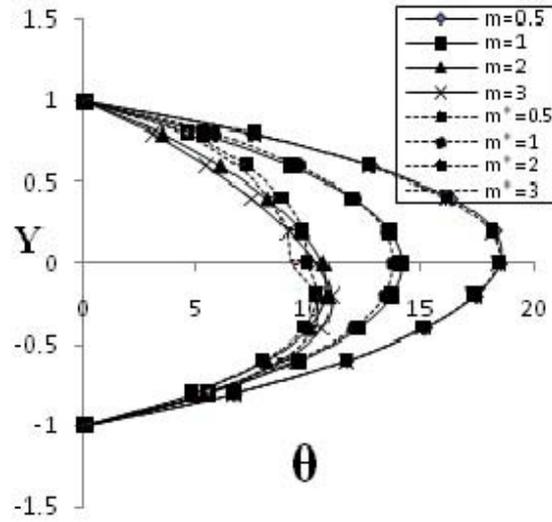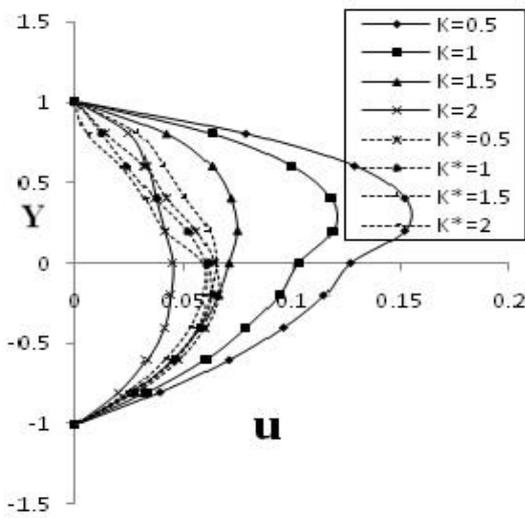
Fig.5. Secondary velocity profiles w_1, w_2 (unsteady flow), w_1^*, w_2^* (steady flow) for different m and $M = 2$, $\alpha = 0.333$, $h = 1$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).

Fig.6. Temperature profiles θ_1, θ_2 (unsteady flow), θ_1^*, θ_2^* (steady flow) for different m and $M = 4$, $\alpha = 0.333$, $h = 0.75$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\beta = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).

Fig.7. Primary velocity profiles u_1, u_2 (unsteady flow), u_1^*, u_2^* (steady flow) for different K and $M = 2$, $m = 2$, $\alpha = 0.333$, $h = 1$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
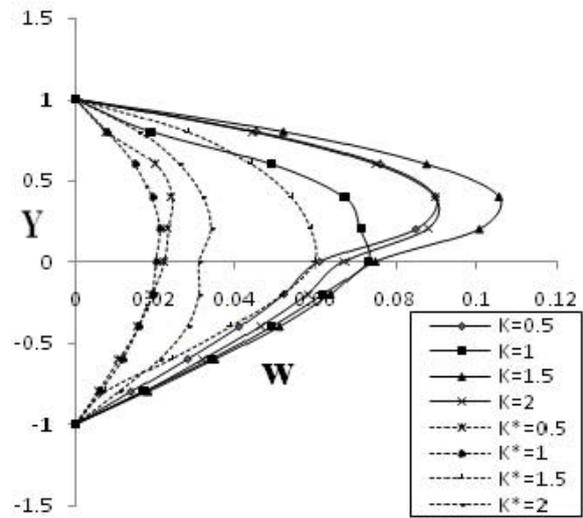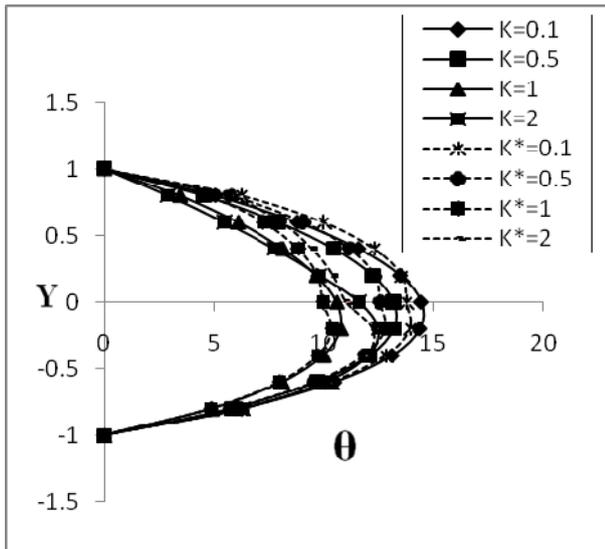

Fig.8. Secondary velocity profiles w_1, w_2 (unsteady flow), w_1^*, w_2^* (steady flow) for different K and $M = 2$, $m = 2$, $\alpha = 0.333$, $h = 1$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).

Fig.9. Temperature profiles θ_1, θ_2 (unsteady flow), θ_1^*, θ_2^* (steady flow) for different K and $M = 4$, $m = 1$, $\alpha = 0.333$, $h = 0.75$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\beta = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
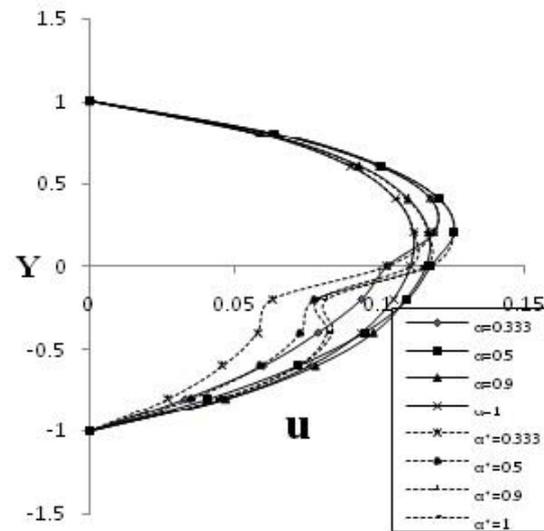
Fig.10. Primary velocity profiles u_1, u_2 (unsteady flow), u_1^*, u_2^* (steady flow) for different α and $M = 2$, $m = 2$, $h = 1$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
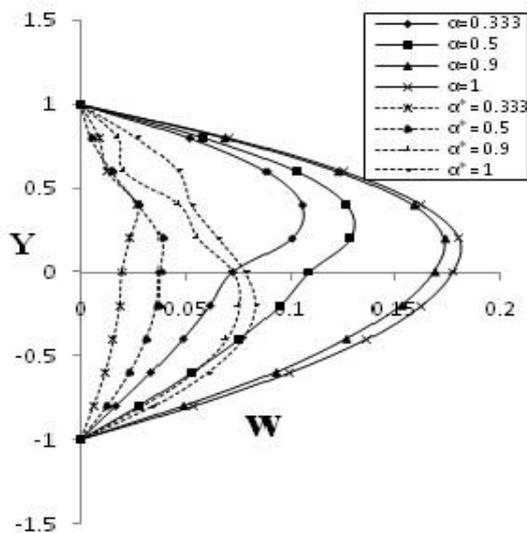
Fig.11. Secondary velocity profiles w_1, w_2 (unsteady flow), w_1^*, w_2^* (steady flow) for different α and $M = 2$, $m = 2$, $h = 1$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\varepsilon = 0.5$, $\rho = 1$, $k = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
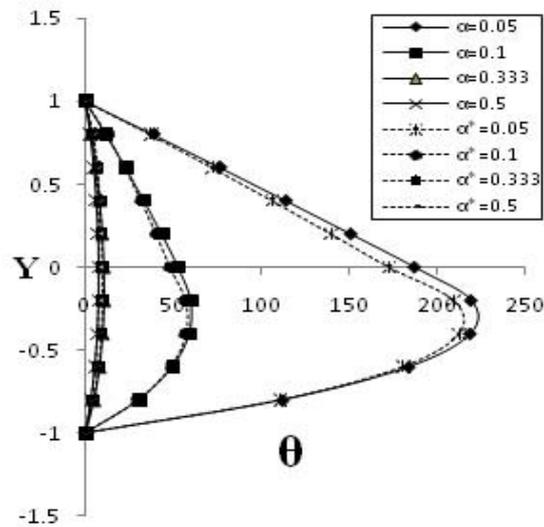
Fig.12. Temperature profiles θ_1, θ_2 (unsteady flow), θ_1^*, θ_2^* (steady flow) for different α and $M = 4$, $m = 1$, $h = 0.75$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\beta = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
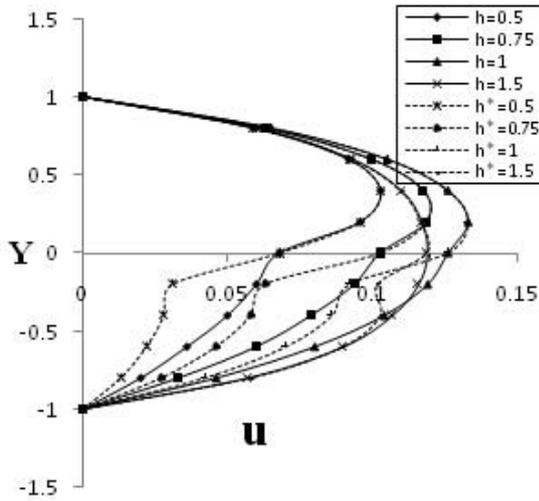
Fig.13. Primary velocity profiles u_1, u_2 (unsteady flow), u_1^*, u_2^* (steady flow) for different h and $M = 2$, $m = 2$, $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
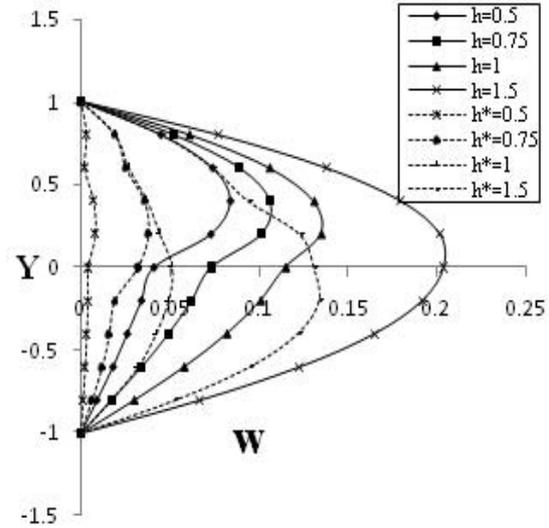

Fig.14. Secondary velocity profiles w_1, w_2 (unsteady flow), w_1^*, w_2^* (steady flow) for different h and $M = 2$, $m = 2$, $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
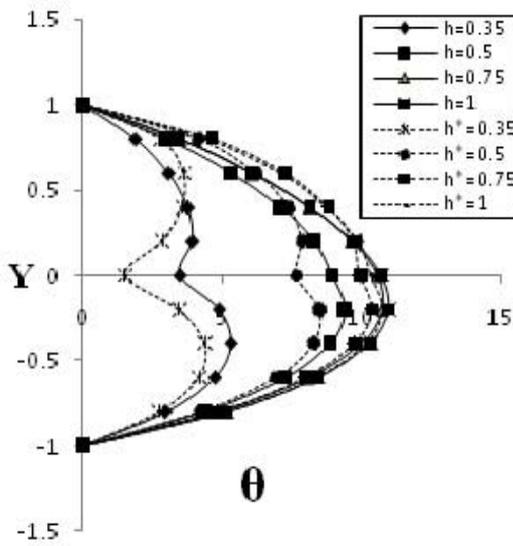
Fig.15. Temperature profiles θ_1, θ_2 (unsteady flow), θ_1^*, θ_2^* (steady flow) for different h and $M = 4$, $m = 1$, $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\beta = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
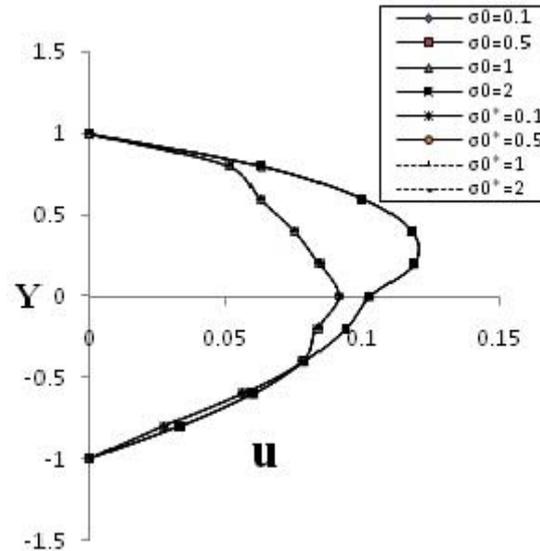

Fig.16. Primary velocity profiles u_1, u_2 (unsteady flow), u_1^*, u_2^* (steady flow) for different σ_0 and $M = 2$, $m = 2$, $\alpha = 0.333$, $h = 1$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).

　　　Figures 7 to 9 show the effect of the Taylor number K on the velocity and temperature distributions. As seen in Fig.7 the primary velocity in the two zones is found to decrease when the Taylor number increases. Figure 8 indicates that the secondary velocity distribution falls down for smaller estimates of the

Taylor number K, and then rises for certain estimate of the parameter; say $K = 1.5$ thereafter it diminishes as K increases. While in region-II the secondary velocity distribution increases up to an estimate of the Taylor number $K = 1.5$ beyond this number it decreases. For a rise in the Taylor number the maximum primary and secondary velocity distributions in the channel tend to move above the channel centre line towards region-I. The temperature distribution in region-I decreases as the Taylor number increases while in region-II it decreases for a certain value of the estimate, say $K = 1$ and thereafter it increases with an increase in K (as seen in Fig.9). As the Taylor number rises, the maximum temperature distribution in the channel tends to shift below the channel centreline and towards the lower area. That is, the temperature field is larger in the upper fluid region in comparison to the lower region for a rise in the Taylor number.

The influence of the viscosity ratio α on the velocity and temperature fields is seen in Figs 10-12. The primary velocity distribution grows up to a given value of the parameter, say $\alpha = 0.5$, as shown in Fig.10, and then falls, whereas in region-II it increases up to $\alpha = 0.9$ then reduces. The secondary velocity distribution reduces as the viscosity ratio α increases (as is seen in Fig.11.) It can be seen in Fig.12 that increasing the viscosity ratio α causes the temperature distributions in the two zones to drop. The positive viscosity ratio indicates viscous dissipation effect on the energy field. So as viscosity ratio α increases, the impact of viscous dissipation on the temperature profile is quicker and it causes the reduction of temperature. It is additionally seen that the magnitude of temperature is high in the lower region in comparison to the upper region as α increases. This means that the magnitude of temperature reduction is smaller in the lower fluid region when compared to the upper region as the viscosity ratio increases.
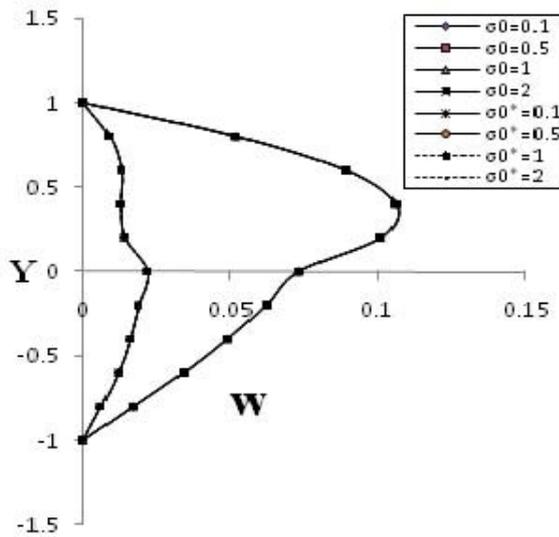

Fig.17. Secondary velocity profiles w_1, w_2 (unsteady flow), w_1^*, w_2^* (steady flow) for different σ_0 and $M = 2$, $m = 2$, $\alpha = 0.333$, $h = 1$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
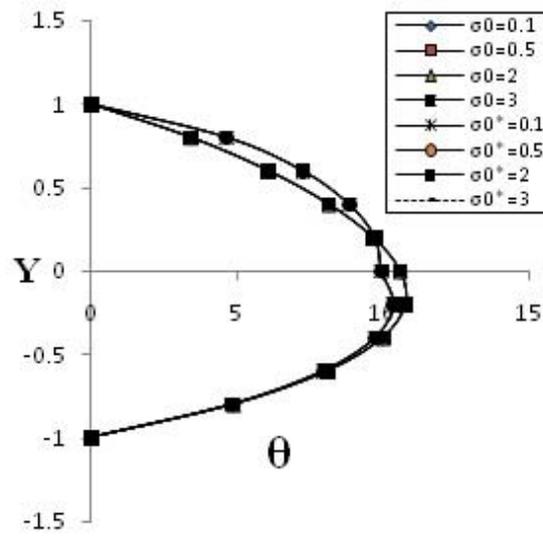
Fig.18. Temperature profiles θ_1, θ_2 (unsteady flow), θ_1^*, θ_2^* (steady flow) for different σ_0 and $M = 4$, $m = 1$, $\alpha = 0.333$, $h = 0.75$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\beta = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).

As it can be seen in Fig.13, a rise in the height ratio h increases the primary velocity for a certain value of the height ratio $h = 1$ and it decreases beyond this value in region-I but it grows in region-II. However, as shown in Fig.14, an increase in the height ratio, h increases the secondary velocity in both locations. The maximum primary and secondary velocity distributions in the channel tend to move above the channel centre line towards region-I as the height ratio augments. It is observed from Fig.15 that the temperature in the two regions increases with an increase in the height ratio. Also, the extreme temperature

in the channel tends to move beneath the channel centreline towards region-II as h increases. That is, the height of the upper fluid region is smaller compared to the lower region, larger in the magnitude of the temperature field.
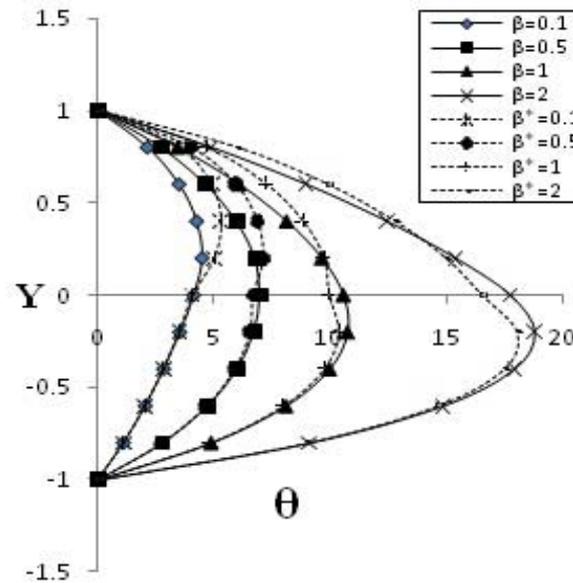

Fig.19. Temperature profiles θ_1, θ_2 (unsteady flow), θ_1^*, θ_2^* (steady flow) for different β and $M = 4$, $m = 1$, $\alpha = 0.333$, $h = 0.75$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).

Fig.20. Nusselt number Nu_1 for different M and $\alpha = 0.333$, $h = 0.75$, $\sigma_0 = 1$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\beta = 1$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
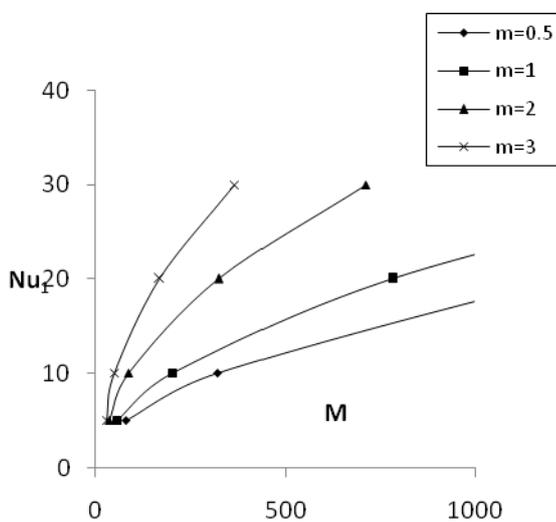
Fig.21. Nusselt number Nu_2 for different M and $\alpha = 0.333$, $h = 0.75$, $\sigma_0 = 1$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\beta = 1$, $K = 1$, $\varepsilon = 0.5$, $\rho = 1$, $\omega = 1$, $t = \Pi / \omega$ (non-conducting plates).
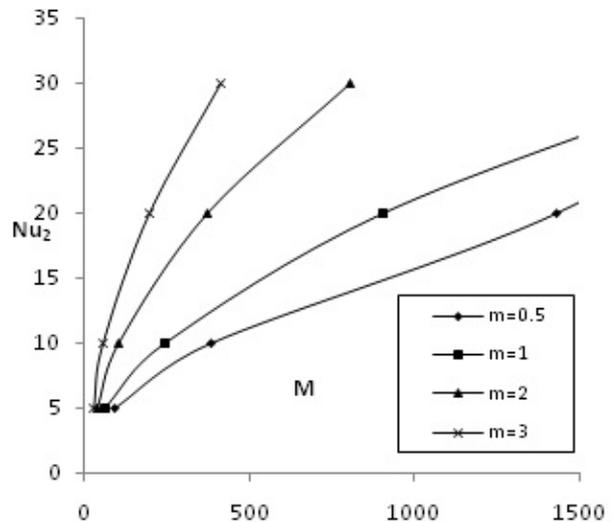
Figures 16-18 show the effect of the electrical conductivity ratio σ_0 on the primary and secondary velocity profiles, as well as the temperature distribution. It can be seen that there is not much of a difference

in primary and secondary velocities, and that the temperature field has a similar effect as σ_0 rises. However, the maximum temperature distribution in the channel tends to move above the channel centreline towards region-I as the electrical conductivity ratio augments.

The effect of the thermal conductivity proportion on the temperature distribution is depicted in Fig.19. It can be seen that as the thermal conductivity ratio β rises, the temperature rises. This shows that the ratio of thermal conductivity accelerates the fluid temperature in the two fluid regions. In like manner, the extreme temperature in the channel moves below the channel centreline towards region-II as the thermal conductivity ratio increases.

From Figs 20 and 21, it is seen that an increase either in the Hartmann number or in the Hall parameter increases the rate of the heat transfer coefficient at the two plates when all other governing parameters are fixed.

The foregoing results show that the velocity and temperature fields, as well as the heat transfer rates at the plates, are all significantly affected by the presence of rotation with applied magnetic field and Hall currents under an unsteady flow. As a result, the current theoretical model may be effective in addressing the real world engineering challenges.

5. Conclusion

An unsteady EMHD flow and its associated heat transfer of two ionized fluids through a channel bounded by two parallel plates in a rotating frame of reference with Hall effect is studied theoretically. The effect of flow characteristics such as the Hartmann number, Hall parameter, Taylor number, the ratios of the viscosity, density, height, electrical conductivity and thermal conductivity on the velocity and temperature fields in two-fluid regions are investigated. The following are the main findings of this analysis:

1. The primary velocity decreases for small values of the Harman number up to a specific value; beyond that estimate it increases, however the secondary velocity component develops for smaller values of the Hartman number, and diminishes when the Hartman number exceeds that value.
2. The primary velocity component decreases in both regions as the Taylor number increases. But the secondary velocity drops for smaller values of the Taylor number, then rises for a specific estimate of this parameter and beyond that value it again diminishes.
3. A rise in the Hall parameter rises the fluid velocity in the upper and lower fluid zones.
4. The flow field is broader when the viscosity of the fluid in the lower zone is lower than the viscosity of the fluid in the upper region.
5. The secondary velocity component grows in both regions as the height ratio increases, but the primary velocity component increases for a specific estimate of the height ratio and it decreases beyond this value in the upper zone but it grows in the lower fluid region.
6. The temperature diminishes until it reaches a certain level of the Hartmann number, after which it grows as the Hartman number increases.
7. The temperature is high in the lower fluid region in comparison to the upper one as the viscosity ratio augments.
8. The temperature distribution is enhanced by increasing the thermal conductivity ratio.
9. The rate of the heat transfer coefficient increases with growing values of the Hartman number and Hall parameter.

NOMENCLATURE

A_1, A_2 — notations used for simplicity: $A_1 = P_1 + iP_2$, $A_2 = P_3 + iP_4$

B_0 — applied uniform magnetic field

c_{p_i} $(i = 1,2)$ — specific heat at constant pressure in the two-fluid regions

$E_{ix}, E_{iz}, (i = 1,2)$ — applied electric fields in the x- and z- directions, where $\overline{E}_i = (E_{ix}, 0, E_{iz})$

h — ratio of the heights of the two regions

h_1 — height of the channel in the upper region (Region-I)

h_2 — height of the channel in the lower region (Region-II)

$I_{ix}, I_{iz}\, (i = 1,2)$ — dimensionless current densities along the x- and z-directions in region-I and region-II, that is,

$\qquad I_{1x}, I_{1z}; I_{2x}, I_{2z}$

I_1 and I_2 — $I_1 = I_{1x} + iI_{1z}$ and $I_2 = I_{2x} + iI_{2z}$ symbols for currents in two fluids, where current density

$\qquad \overline{J}_i = (J_{ix}, 0, J_{iz}), (i = 1,2)$

J_{ix}, J_{iz} — current densities along the x- and z- directions in two fluid regions

K_1, K_2 — thermal conductivities of the two fluids

K — $= \sqrt{\dfrac{h_1^2 \rho \Omega}{\mu_1}}$, rotation parameter (or Taylor number)

M — Hartman number $M = \sqrt{\sigma_{01} B_0^2 h_1^2 / \mu_1}$

M_1, M_2 — notations where $M_1 = m_{1x} + im_{1z}$, $M_2 = m_{2x} + im_{2z}$

m — Hall parameter where m $m = \omega_e \left/ \left(\dfrac{1}{\tau} + \dfrac{1}{\tau_e} \right) \right.$

$m_{ix}, m_{iz}, (i = 1,2)$ — dimensionless electric fields in region-I and-II as $m_{1x}, m_{1z}, m_{2x}, m_{2z}$

Nu_1, Nu_2 — rate of heat transfer coefficients at upper and lower plates

P_r — Prandtl number

P_1, P_2, P_3, P_4 — notations as: $P_1 = 1 - \dfrac{sm^2}{1+m^2}$, $P_2 = -\dfrac{sm}{1+m^2}$, $P_3 = 1 - \left(1 - \dfrac{\sigma_0 \sigma_{01}}{1+m^2} \right) s$, $P_4 = \dfrac{-\sigma_0 \sigma_{02} ms}{1+m^2}$

p — pressure

p_e — electron pressure

q_{01}, q_{02} and q_{11}, q_{12} — velocities in complex notation for steady and transient state in the two fluid regions:

$\qquad q_{01} = u_{01} + iw_{01}, q_{11} = u_{11} + iw_{11}, \; q_{02} = u_{02} + iw_{02}, q_{12} = u_{12} + iw_{12}$

$q_1(y,t), q_2(y,t)$ — solutions of velocity distributions for the two fluid in complex form:

$\qquad q_1(y,t) = q_{01}(y) + \varepsilon \cos \omega t . q_{11}(y), q_2(y,t) = q_{02}(y) + \varepsilon \cos \omega t . q_{12}(y)$

q_{1m}, q_{2m} — mean velocities as $q_{1m} = u_{1m} + iw_{1m}$ and $q_{2m} = u_{2m} + iw_{2m}$

subscripts *1, 2* — refers to the quantities in the upper and lower fluid regions: Region-I and Region-II.

s — $= \dfrac{p_e}{p}$ ionization parameter or the ratio of electron pressure to the total pressure

T — temperature

t — time,

$T_i (i = 1,2) : T_1, T_2$ — temperatures of the fluids in Region-I and Region-II

$u_i, (i = 1,2) : u_1, u_2$ — primary velocity distributions (velocity components along x-direction) in Region-I and -II

$u_{01}(y), u_{02}(y)$ — steady state primary velocities in the two fluid regions

$u_{11}(y), u_{12}(y)$ — transient primary velocity components in the two fluid regions

u_{1m}, u_{2m} — primary mean velocity distributions in the two fluid regions

u_p — $= \left(-h_1^2 / \mu_1 \right) \dfrac{\partial p}{\partial x}$: the characteristic velocity

$w_i, (i = 1, 2) : w_1, w_2$ − secondary velocity distributions (component of velocity field along *z*-direction) in the two fluid regions

$w_{01}(y), w_{02}(y)$ − steady state secondary velocity components in two fluid regions

$w_{11}(y), w_{12}(y)$ − transient secondary velocities in two regions

w_{1m}, w_{2m} − secondary mean velocity distributions in the two fluid regions

(x, y, z) − space co-ordinates in rectangular Cartesian co-ordinate system

$-\dfrac{\partial p}{\partial x}$ − common constant pressure gradient

α − $= \mu_1/\mu_2$, ratio of the viscosities

β − $= \dfrac{K_1}{K_2}$, thermal conductivity ratio

$\mu_i (i = 1, 2) : \mu_1, \mu_2$ − viscosities of the two fluids

$\sigma_{0i}(i = 1, 2), \sigma_{01}, \sigma_{02}$ − electrical conductivities of the two fluids

σ_0 − ratio of electrical conductivities $\sigma_0 = \dfrac{\sigma_{01}}{\sigma_{02}}$

$\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}$ − modified conductivities parallel and normal to the direction of electric fields

σ_1, σ_2 − symbols for the ratios $\sigma_1 = \dfrac{\sigma_{12}}{\sigma_{11}}, \sigma_2 = \dfrac{\sigma_{22}}{\sigma_{21}}$

ρ_1, ρ_2 − densities of the two fluids

ρ − density ratio of the two fluids $\rho = \dfrac{\rho_2}{\rho_1}$

θ_1, θ_2 − dimensionless temperature distributions for two-fluid regions

$\theta_{01}(y), \theta_{02}(y)$ − steady state temperature distributions in the two fluid regions

$\theta_{11}(y), \theta_{12}(y)$ − transient temperature distributions in the two fluid regions

τ, τ_e − mean collision time between electron and ion, electron and neutral particles

ε − amplitude (a small constant quantity, $\varepsilon \ll 1$)

ω − frequency of oscillation

ω_e − gyration frequency of electron

Ω − angular velocity, where $\bar{\Omega} = (0, \Omega, 0)$

References

[1] Hartmann J. (1937): *Theory of the laminar flow of an electrically conductive liquid in a homogeneous magnetic field.*− Mat.-Fys. Medd. Kgl. Danskevid Selskab, vol.15, No.6, pp.1-28.

[2] Mayer R.C. (1958): *On reducing aerodynamic heat-transfer rates by magnetohydrodynamic technique.*– J. Aerospace Sci., vol.25, pp.561-572.

[3] Cowling T.G. (1962): *Magnetohydrodynamics.*− Rep. Prog. Phys., vol.25, pp.244.

[4] Blum Ya.E. (1967): *Heat and Mass Transfer in Magnetic Field.*− Magnetohydrodynamics, vol.11, No.1, pp.27-36.

[5] Cramer K.R. and Pai Shih-I. (1973): *Magnetofluid dynamics for engineers and applied physicists*.– McGraw-Hill Company.

[6] Shercliff J.A. (1979): *Thermoelectric magnetohydrodynamics.*− J. Fluid Mechanics, vol.91, No.2, pp. 231-251.

[7] Aithal S.M. (2008): *Analysis of optimum power extraction in a MHD generator with spatially varying electrical conductivity.*− Int. J. of Thermal Science, vol.47, pp.1107-1112.

[8] Kabeel A.E., Emad EI-Said M.S. and Dafea S.A. (2015): *A review of magnetic fields effects on flow and heat transfer in liquids: present status and future potential for studies and applications.*– Renewable and Sustainable Energy Reviews, vol.45, pp.830-837.

[9] Selimli S., Resebli Z. and Arcakhoglu E. (2015): *Combined effects of magnetic and electric field on hydrodynamic and thermo-physical parameters of magneto-viscous fluid flow.* – Int. J. Heat Mass Trans., vol.86, pp.426-432.

[10] Broer L.J.F. and Peletier L.A. (1960): *A mechanical. Hall effect.* – Appl. Sci. Res., vol.8B, pp.259.

[11] Sato H. (1961): *The Hall effect in the viscous flow of ionized gas between parallel plates under transverse magnetic field.* – J. Phys. Soc. Japan, vol.16, No.7, pp.1427-1433.

[12] Sharma R.C. and Neela Rani (1988): *Hall effects of thermo-solute instability of a plasma.* – Indian J. Pure Appl. Math, vol.19, No.2, pp.202-207.

[13] Niranjan S.S., Soundalgekar V.M. and Takhar H.S. (1990): *Free convection effects on MHD horizontal channel flow with Hall currents.* – Plasma Sci. IEEE Trans, vol.18, No.2, pp.177-183.

[14] Linga Raju, T. and Ramana Rao V.V. (1992): *Hall Effect in the viscous flow of an ionized gas between two parallel walls under transverse magnetic field in a rotating system.* – Acta Physica Hungarica, vol.72, No.1, pp.23-45.

[15] Linga Raju, T. and Ramana Rao V.V.(1993): *Hall effects on temperature distribution in a rotating ionized hydromagnetic flow between parallel walls.* – Int. J. Engg. Sci., vol.31, No.7, pp.1073-1091.

[16] Attia H.A. (1998): *Hall current effects on the velocity and temperature fields of an unsteady Hartmann flow.* – Can. J. Phys., vol.76, No.9, pp.739-746.

[17] Macheret S.O., Shneider M.N. and Miles R.B. (2004): *Magnetohydrodynamic and electro-magnetic control of hypersonic flows of weakly ionized plasmas.* – AIAA J., vol.42, pp.1378-1387.

[18] Ghosh S.K. and Pop I. (2004): *Hall effects on MHD plasma Couette flow in rotating environment.* – Int. J. Appl. Mech. and Engng., vol.9, pp.293-305.

[19] Linga Raju T. and P.S.R. Murty. (2005): *Quasi-steady state solutions of MHD ionized flow and heat transfer with Hall currents between parallel walls in a rotating system.* – Bulletin of Pure and Applied sciences (An Int. Research Journal of Sciences), Section-E: Maths. & Stat., vol.24E, No.2, pp.467-490.

[20] Srivastava K.M. (2009): *Effect of Hall current on the instability of an anisotropic plasma Jet.* – J. Plasma Phys., vol.12, No.1, pp.33-43.

[21] Jha B.K. and Apere C.A. (2010): *Combined effects of Hall and ion-slip currents on unsteady MHD Couette flolws in a rotating system.* – J. Phys. Soc. Japan, vol.79, DOI:10.1143/JPSJ.79.104401.

[22] Singh J.K., Begum S.G. and Seth G.S. (2018): *Influence of Hall current and wall conductivity on hydromagnetic mixed convective flow in a rotating Darcian channel.* – Physics of fluids, vol.30, No.11, pp.113602-1 to 12.

[23] Ryabinin A.G. and Khozhainov A.I. (1967): *Exact and approximate formulations of problems for unsteady flows of conducting fluids in MHD channels.* – Fluid Dyn., vol.2, No.4, pp.107-109.

[24] Debnath L. (1975): *Inertial oscillations and hydromagnetic multiple boundary layers in a rotating fluid.* – ZAMM (Zeitschrift Fur Angewandte Mathematik and Mechanik), vol.52, pp.141-147.

[25] Takenouchi K.(1985): Transient *magnetohydrodynamic channel flow with an axial symmentry at a supersonic speed.* – J. Phys. Soc. Japan., vol.54, pp.1329-1338.

[26] Barmin, A.A and Uspenskii V.S. (1986): *Development of pulsations regimes in one dimensional unsteady MHD flows with switching off of the electrical conductivity.* – Fluid Dyn., vol.21, No.4, pp.18-30.

[27] Singh A.K., Sacheti N.C. and Chandran P.(1994): *Transient effects on magnetohdrodynamic Couette flow with rotation: Accelerated motion.* – Int. J. Engng. Sci., vol.32, pp.133-139.

[28] Chamkha A.J. (2000*): Unsteady laminar hydromagnetic fluid-particle flow and heat transfer in channels and circular pipes.* – Int. J. of Heat and Fluid Flow, vol.21, pp.740-746.

[29] Chen L., Gong J., Sun F. and Wu C. (2002): *Heat transfer effect on the performance of MHD power plant.* – Energy Conversion and Management, vol.43, No.15, pp.2085-2095.

[30] Morley N.B., Malang S. and Kirillov I. (2005): *Thermofluid magnetohydrodynamic isuues for liquid breeders.* – Fusion Science and Technology, vol.47, pp.488-501.

[31] Triwahju Hardianto T., Sakamoto N. and Harada N. (2008): *Computational study of diagonal channel magnetohydrodynamic power generation.* – Int. J. Energy Tech. Policy, vol.6, pp.96-111.

[32] Chwla T.C. and Ishii M. (1980): *Two-fluid model of two-phase flow in a pin bundle of a nuclear reactor.* – Int. J. Heat Mass Transfer, vol.23, pp.991-1001.

[33] Mitra P. (1982): *Unsteady flow of two electrically conducting fluids between two rigid parallel plates.* – Bulletin of the Calcutta Mathematical Society, vol.74, pp.87-95.

[34] Hasnain Qaisrani M., ZhenWei Xia and Dandan Zou (2005): *Statistical properties of three-dimensional two-fluid plasma model.* – Phys. Plasmas, vol. 22, pp.092303-1 to 9, doi.org/10.1063/1.4928900.

[35] Hyun S. and Kennel C.F. (2009): *Small amplitude waves in a hot relativistic two-fluid plasma.−* J. Plasma Phy., vol.20, No.2, pp.281-287.

[36] Linga Raju T. and Seedhar S. (2009): *Unsteady two-fluid flow and heat transfer of conducting fluids in channels under transverse magnetic field.−* Int. J. Appl. Mech. and Engg., vol.14, No.4, pp.1093-1114.

[37] Hyun S. and Kennel C.F. (2009): *Small amplitude waves in a hot relativistic two-fluid plasma.−* J. Plasma Phy., vol.20, No.2, pp.281-287.

[38] Naga Valli M., Linga Raju T. and Kameswaran P.K. (2022): *Two layered flow of ionized gases within a channel of parallel permeable plates under applied magnetic field with Hall effect.−* To Appear in Springer Proceedings of 8th International Conference on Mathematics and Computing, (ICMC-2022) held during January 6-8, 2022 at VIT, Vellore, India, Submission No.208.

[39] Shail R. (1973): *On laminar two-phase flows in magnetohydrodynamics.−* Int. J. Engg. Sci., vol.11, pp.1103-1109.

[40] Lohrasbi J. and Sahai V. (1988): *Magnetohydrodynamic heat transfer in two phase flow between parallel plates.−* Appl. Sci. Res., vol.45, pp.53-66.

[41] Malashetty M.S. and Leela V. (1992): *Magnetohydrodynamic heat transfer in two phase flow.−* Int. J. of Engg. Sci. vol.30, pp.371-377.

[42] Chamkha A.J. (1995): *Hydromagnetic two-phase flow in a channel.−* Int. J. Engg. Sci., vol.33, No.3, pp.437-446.

[43] Umavathi J.C., Mateen A., Chamkha A.J. and Al-Mudhaf A. (2006): *Oscillatory Hartmann two-fluid flow and heat transfer in a horizontal channel.−* Int. J. Appl. Mech. and Engg. vol.11, No.1, pp.155-178.

[44] Linga Raju T. and Nagavalli M. (2014): *MHD two-layered unsteady fluid flow and heat transfer through a horizontal channel between parallel plates in a rotating system.−* Int. J. Appl. Mech. and Engg., vol.19, No.1, pp.97-121.

[45] Sharma P.R. and Sharma Kalpana (2014): *Unsteady MHD two-fluid flow and heat transfer through a horizontal channel.−* Int. J. of Engineering Science Invention Research and Development, vol.1, No.3, pp.65-72.

[46] Sivakamini L and Govindarajan A. (2019): *Unsteady MHD flow of two immiscible fluids under chemical reaction in a horizontal channel.−* AIP conference proceedings 2112. pp.020157-1 to 9, doi.org/10.1063/1.5112342.

[47] Linga Raju T. (2019): *MHD heat transfer two-ionized fluids flow between two parallel plates with Hall currents.−* Results in Engineering, vol.4, 100043, p.15, doi.org/10.1016/j.rineng.2019.100043.

[48] Abd Elmaboud Y., Abdesalam Sara I., Mekheimer Kh.S. and Kambiz Vafai (2019): *Electromagnetic flow for two-layer immiscible fluids.−* Engineering Science and Technology, an International Journal, vol.22, pp.237-248.

[49] Linga Raju T. and Gowri Sankara Rao V. (2021): *Effect of Hall current on unsteady magnetohydro dynamic two-ionized fluid flow and heat transfer in a channel.−* Int. J. of Applied Mechanics and Engg., vol.26, No.2, pp. 84-106, DOI: 10.1515/ijame-2021-0038.

[50] Linga Raju T. (2021): *Electro-magnetohydrodynamic two fluid flow of ionized-gases with Hall and rotation effects.−* Int. J. Appl. Mech., vol.26, No.4. pp.128-144, DOI: 10.2478/ijame-2021-0054.