Int. J. of Applied Mechanics and Engineering, 2021, vol.26, No.1, pp.76-87
DOI: 10.2478/ijame-2021-0005

REDUCED DIFFERENTIAL TRANSFORM METHOD FOR THERMOELASTIC PROBLEM IN HYPERBOLIC HEAT CONDUCTION DOMAIN

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In the present study, we have applied the reduced differential transform method to solve the thermoelastic problem which reduces the computational efforts. In the study, the temperature distribution in a two-dimensional rectangular plate follows the hyperbolic law of heat conduction. We have obtained the generalized solution for thermoelastic field and temperature field by considering non-homogeneous boundary conditions in the x and y direction. Using this method one can obtain a solution in series form. The special case is considered to show the effectiveness of the present method. And also, the results are shown numerically and graphically. The study shows that this method provides an analytical approximate solution in very easy steps and requires little computational work.

Key words*:* hyperbolic heat conduction, thermal stresses, thermal displacement, rectangular plate, reduced differential transform.

1. Introduction

The study of thermal stresses in a solid has received considerable attention in the field of research. With the advancement in technological and engineering fields, the study of thermal behaviour of solids. The thermal stresses and temperature distribution plays an important role. In the literature, two types of heat conduction theory are available, namely, the Fourier heat conduction or parabolic heat conduction and non-Fourier heat conduction or hyperbolic heat conduction depending on the infinite or finite speed of heat propagation. Cattaneo [1] and Vernetto [2] proposed the first non-Fourier heat conduction system based on finite speed of heat propagation with phase lag in the heat flux. In the case of no phase lag in the heat flux, the hyperbolic heat conduction model is shifted into the classical Fourier heat conduction model. The hyperbolic heat conduction model is more appropriate in the case of a higher temperature gradient or very short time duration.

In previous studies, most of the thermoelastic analyses for different materials have been done under the parabolic heat conduction model [3-10]. A few analyses of thermal stresses for a two-dimensional system have been made using the hyperbolic heat conduction model. Chen and Hu [11] studied the thermal stresses around a crack in a half-plane under the hyperbolic heat conduction model. They used an integral transform technique to find the solution of governing partial differential equations and also discussed a comparison between hyperbolic and parabolic heat conduction in the thermoelastic model. N. Sarkar [12] presented a model of thermoelasticity with non-local heat conduction based on generalized thermoelasticity to study the

*To whom correspondence should be addressed transient response of the finite rod of one dimension**.** Al-Qahtani and Yilbas [13] determined the solution of the temperature field and thermal stresses using a one-dimensional hyperbolic heat conduction model in the Laplace transform domain. Mohamed and Gepreel [14] used a reduced differential transform method to solve the non linear Kadomtsev-Petviashvili hierarchy differential equation and showed that the result obtained using RDTM converges very rapidly to the exact solutions. Taghizadeh and Noori [15] obtained the solution of heat-like and wave-like equations with variable coeffcients using RDTM. Recently, we [16] have found the solution to a thermoelastic problem in the context of hyperbolic heat conduction using the differential transform method.

In a literature survey, we found that most of the work on thermoelasticity has been done under Fourier heat conduction. In this article, we have determined the temperature distribution using the hyperbolic heat conduction model in a finite two-dimensional rectangular plate by a reduced differential transform method. Also, we have investigated the thermal stresses and thermal displacement by using the thermal stress function. The governing partial differential equations have been solved in the reduced differential transform domain. This method gave an approximate solution in series form. The effectiveness of the present method is illustrated numerically and graphically for a special case.

2. Mathematical problem formulation

The present investigation concerns a finite thin rectangular plate, initially kept at zero temperature and subjected to non-homogeneous heat transfer in spatial direction. A plate of rectangular shape with dimensions $0 \le x \le a,\ 0 \le y \le b$, has been taken into consideration. All material properties are assumed to be constant. As per CV theory, the unsteady state temperature distribution under the hyperbolic heat conduction in the plate with no internal heat generation is given as:

$$\frac{\partial T}{\partial t} + \tau_q \frac{\partial^2 T}{\partial t^2} = k\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)T \tag{2.1}$$

where $k = \dfrac{\lambda}{\rho c}$, ρ, c, λ are the thermal diffusivity, mass density, specific heat capacity and thermal conductivity of the material, respectively, and τ_q is called phase lag in the heat flux.

The plate subjected to boundary conditions, which are given as:

$$\begin{aligned} &T(0,y,t) = f(y,t), \quad \frac{\partial T(a,y,t)}{\partial x} = g(y,t), \\ &T(x,0,t) = l(x,t), \quad \frac{\partial T(x,b,t)}{\partial y} = j(x,t). \end{aligned} \tag{2.2}$$

Consider the thermal stress function χ defined as in Noda *et al*. [17], in the rectangular coordinate system for the plane thermoelastic problem. The fundamental equation is given as:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \chi + \alpha E\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Gamma = 0 . \tag{2.3}$$

The general solution of Eq.(2.3) may be expressed as a sum of the complementary function χ_c and the particular solution χ_p :

$$\chi = \chi_c + \chi_p \tag{2.4}$$

where χ_c and χ_p are satisfied by the equations

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \chi_c = 0, \tag{2.5}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\chi_p = -\alpha E \Gamma \tag{2.6}$$

where $\Gamma = T - T_0$ and α, E are the thermal expansion coefficient and Young's modulus of the material respectively.

Thermal stresses in terms of the stress function are given as follows:

$$\sigma_{xx} = \frac{\partial^2 \chi}{\partial y^2}, \quad \sigma_{yy} = \frac{\partial^2 \chi}{\partial x^2}, \quad \sigma_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y}. \tag{2.7}$$

Also, the fundamental equation for the displacement function defined as in Noda *et al*. [17] for the plane problem in a rectangular coordinate is given as:

$$U_x = \frac{1}{2G}\left[-\frac{\partial \chi}{\partial x} + \frac{1}{1+\nu}\frac{\partial \psi}{\partial y}\right], \tag{2.8}$$

$$U_y = \frac{1}{2G}\left[-\frac{\partial \chi}{\partial y} + \frac{1}{1+\nu}\frac{\partial \psi}{\partial x}\right] \tag{2.9}$$

where G and ν are the shear modulus of elasticity and Poisson's ratio, and ψ satisfies the following equation as:

$$\sigma_{xx} + \sigma_{yy} + \alpha E \Gamma \equiv \frac{\partial^2 \psi}{\partial x \partial y} \quad \text{and} \quad \frac{\partial^2}{\partial x \partial y}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \psi = 0. \tag{2.10}$$

3. Solution of the problem

3.1. Solution of temperature field

The solutions of the governing differential equations are obtained by using the reduced differential transform method [14].

If $T(x, y, t)$ is analytic and continuously differentiable function in x, y, t then the reduced differential transform function of $T(x, y, t)$ is defined as

$$T_r(x,y) = \frac{1}{r!}\left[\frac{\partial^r}{\partial t^r}T(x,y,t)\right]_{t=t_0}. \tag{3.1}$$

The inverse reduced differential function of $T_r(x,y)$ is given as

$$T(x,y,t) = \sum_{r=0}^{\infty}\frac{1}{r!}\left[\frac{\partial^r}{\partial t^r}T(x,y,t)\right]_{t=t_0}(t-t_0)^r. \tag{3.2}$$

Applying the reduced differential transform to Eq.(2.1) with respect to x we get,

$$k(r+1)(r+2)T_{r+2} = \frac{\partial T_r}{\partial t} + \tau_q\frac{\partial^2 T_r}{\partial t^2} - k\frac{\partial^2}{\partial y^2}T_r \tag{3.3}$$

and using $T_0(y,t) = f(y,t) = a_0$ and assuming, $T_1(y,t) = a_1$ in Eq.(3.3), we get:

$$T_r(y,t) = \begin{cases} \dfrac{a_r}{(k)^{r/2}(r!)} & \text{where } r \text{ is even,} \\[2ex] \dfrac{a_r}{(k)^{r-1/2}(r!)} & \text{where } r \text{ is odd} \end{cases} \tag{3.4}$$

where $a_r = \tau_q(a_{r-2})_{tt} + (a_{r-2})_t - k(a_{r-2})_{yy}$, $r = 2,3,4\ldots\ldots$

Applying the inverse reduced differential transform defined in Eq.(3.2), we get:

$$T(x,y,t) = a_0 + a_1 x + \sum_{r=2}^{\infty}T_r(y,t)(x)^r. \tag{3.5}$$

Now applying other boundary conditions from Eq.(2.2), one can calculate the value of a_1 and substituting it back into Eq.(3.5) we obtain the required solution of temperature field.

3.2. Determination of the thermal stress function

Assume the complementary function χ_c, which satisfies Eq.(2.5) as

$$\chi_c = b_0 + b_1 x + \frac{b_2}{2!}x^2 + \frac{b_3}{3!}x^3 + \sum_{r=4}^{\infty}\frac{b_r}{r!}x^r \tag{3.6}$$

where

$$[\chi_c]_{x=0} = b_0(y),\quad \left[\frac{\partial\chi_c}{\partial x}\right]_{x=0} = b_1(y),\quad \left[\frac{\partial^2\chi_c}{\partial x^2}\right]_{x=0} = b_2(y),\quad \left[\frac{\partial^3\chi_c}{\partial x^3}\right]_{x=0} = b_3(y)$$

and

$$b_r = -\left[2\left(b_{r-2}\right)_{yy} + \left(b_{r-4}\right)_{yyyy}\right],$$

also the function χ_p , which satisfies Eq.(2.6) given by

$$\chi_p = h_0 + h_1 x - \sum_{r=2}^{\infty}\left[\frac{\alpha E(r-2)!T_r(y,t) + \left(h_{r-2}\right)_{yy}}{r!}\right]x^r \tag{3.7}$$

where

$$\left[\chi_p\right]_{x=0} = h_0(y,t), \quad \left[\frac{\partial \chi_p}{\partial x}\right]_{x=0} = h_1(y,t).$$

From Eqs (2.4), (3.6), (3.7), we get

$$\begin{aligned}\chi = {} & b_0 + b_1 x + \frac{b_2}{2!}x^2 + \frac{b_3}{3!}x^3 + \sum_{r=4}^{\infty}\frac{b_r}{r!}x^r + \\ & + h_0 + h_1 x - \sum_{r=2}^{\infty}\left[\frac{\alpha E(r-2)!T_r(y,t) + \left(h_{r-2}\right)_{yy}}{r!}\right]x^r .\end{aligned} \tag{3.8}$$

3.3. Determination of thermal stresses

Now using Eq.(3.8) in Eq.(2.7) one can obtain the expression for thermal stresses as

$$\begin{aligned}\sigma_{xx} = {} & b_0'' + b_1'' x + \frac{b_2''}{2!}x^2 + \frac{b_3''}{3!}x^3 + \sum_{r=4}^{\infty}\frac{b_r''}{r!}x^r + \\ & + h_0'' + h_1'' x - \sum_{r=2}^{\infty}\left[\frac{\alpha E(r-2)!\left(T_r(y,t)\right)'' + \left(\left(h_{r-2}\right)_{yy}\right)''}{r!}\right]x^r ,\end{aligned} \tag{3.9}$$

$$\sigma_{yy} = b_2 + b_3 x + \sum_{r=4}^{\infty}\frac{b_r}{r-2!}x^{r-2} - \sum_{r=2}^{\infty}\left[\frac{\alpha E(r-2)!T_r(y,t) + \left(h_{r-2}\right)_{yy}}{r-2!}\right]x^{r-2} , \tag{3.10}$$

$$\sigma_{xy} = -\left[b_1' + b_2' x + \frac{b_3'}{2!} x^2 + \sum_{r=4}^{\infty} \frac{b_r'}{r-1!} x^{r-1} + h_1' - \right.$$
$$\left. -\sum_{r=2}^{\infty} \left[\frac{\alpha E(r-2)!\left(T_r(y,t)\right)' + \left((h_{r-2})_{yy}\right)'}{r-1!} \right] x^{r-1} \right]. \tag{3.11}$$

3.4. Determination of displacement function

From Eqs (2.8)-(2.10), one can obtain the thermal displacement in the plate without rigid deformation as:

$$U_x = \frac{1}{2G}\left[-\left(b_1 + b_2 x + \frac{b_3}{2!} x^2 + \sum_{r=4}^{\infty} \frac{b_r}{r-1!} x^{r-1} + h_1 - \right.\right.$$
$$\left. -\sum_{r=2}^{\infty} \left[\frac{\alpha E(r-2)! T_r(y,t) + (h_{r-2})_{yy}}{r-1!} \right] x^{r-1} \right) + \tag{3.12}$$
$$\left. + \frac{1}{1+\nu}\left(\left(b_0'' + b_2\right)x + \left(b_1'' + b_3\right)\frac{x^2}{2!} + \frac{b_2''}{3!}x^3 + \frac{b_3''}{4!}x^4 + \sum_{r=4}^{\infty}\left(\frac{x^2 b_r''}{r(r+1)} + b_r \right)\frac{x^{r-1}}{r-1!} \right) \right],$$

$$U_y = \frac{1}{2G}\left[-\left(b_0' + b_1' x + \frac{b_2'}{2!} x^2 + \frac{b_3'}{3!} x^3 + \sum_{r=4}^{\infty} \frac{b_r'}{r!} x^r + h_0' + h_1' x - \right.\right.$$
$$\left. -\sum_{r=2}^{\infty} \left[\frac{\alpha E(r-2)!\left(T_r(y,t)\right)' + \left((h_{r-2})_{yy}\right)'}{r!} \right] x^r \right) + \frac{1}{1+\nu}\left(\left(b_0' + \int b_2\right) + \right. \tag{3.13}$$
$$\left.\left. + \left(b_1' + \int b_3\right)x + \frac{b_2'}{2!}x^2 + \frac{b_3'}{3!}x^3 + \sum_{r=4}^{\infty}\left(\frac{x^2 b_r'}{r(r-1)} + \int b_r \right)\frac{x^{r-2}}{r-2!} \right) \right].$$

Equations (3.9)-(3.13) present the general solution of thermal stresses and thermal displacement for a finite rectangular plate.

4. Special case for numerical and graphical representation

For numerical purposes, we take

$$f(y,t) = a_0 = y^2 t^2, \quad g(y,t) = 2yt^2 - 2at^2, \quad l(x,t) = -x^2 t^2,$$

$$j(x,t) = 2yt^2 + 2bt^2, \quad b_2(y) = -3y^2 - y^6, \quad b_3(y) = -15y,^3$$

$$b_0(y) = b_1(y) = h_0(y,t) = h_1(y,t) = 0, \quad T_0 = 0.$$

From Eqs (3.3)-(3.5) we get,

$$T(x,y,t)=y^2t^2+2xyt^2-t^2x^2+\left(\frac{\tau_q}{k}-\frac{t}{k}\right)\times$$
$$\times\left(x^2y^2+\frac{2}{3}x^3y-\frac{x^4}{3}\right)+\frac{1}{k^2}\left(\frac{x^4y^2}{12}+\frac{x^5y}{30}-\frac{x^6}{60}\right). \tag{4.1}$$

From Eqs (3.6) - (3.8) one can obtain the stress function as:

$$\chi=-3x^2y^2-x^2y^6-15x^3y^3+x^4+5x^4y^4+9x^5y-3x^6y^2+\frac{x^8}{7}-\alpha E\left\{\frac{x^2y^2t^2}{2}+\right.$$
$$\left.+\frac{x^3yt^2}{3}-\frac{2t^2x^4}{12}+\left(\frac{\tau_q}{k}-\frac{t}{k}\right)\left(\frac{x^4y^2}{12}+\frac{4x^5y}{120}-\frac{x^6}{60}\right)+\frac{x^6y^2}{360k^2}+\frac{4x^7y}{k^2 7!}-\frac{2x^8}{k^2 7!}\right\}. \tag{4.2}$$

From Eq.(2.7) we get the thermal stresses as

$$\sigma_{xx}=-6x^2-30x^2y^4-6x^6-90x^3y+60x^4y^2-\alpha E\left\{x^2t^2+\left(\frac{\tau_q}{k}-\frac{t}{k}\right)\left(\frac{x^4}{6}\right)+\frac{x^6}{180k^2}\right\}, \tag{4.3}$$

$$\sigma_{yy}=-6y^2-2y^6-90xy^3+12x^2+60x^2y^4+180x^3y-90x^4y^2+8x^6+$$
$$-\alpha E\left\{y^2t^2+2xyt^2-2t^2x^2+\left(\frac{\tau_q}{k}-\frac{t}{k}\right)\left(x^2y^2+\frac{2x^3y}{3}-\frac{x^4}{2}\right)+\frac{x^4y^2}{12k^2}+\frac{x^5y}{30k^2}-\frac{x^6}{45k^2}\right\}, \tag{4.4}$$

$$\sigma_{xy}=12xy+12xy^5+135x^2y^2-80x^3y^3-45x^4+36x^5y+$$
$$+\alpha E\left\{2xyt^2+x^2t^2+\left(\frac{\tau_q}{k}-\frac{t}{k}\right)\left(\frac{2x^3y}{3}+\frac{x^4}{6}\right)+\frac{x^5y}{30k^2}-\frac{x^6}{180k^2}\right\} \tag{4.5}$$

and from Eqs (2.8)-(2.10) we get the thermal displacement as

$$U_x=\frac{1}{2G}\left[\left(-2+\frac{1}{1+\nu}\right)\left(2x^3+10x^3y^4+\frac{45x^4y}{2}\right)+\left(3-\frac{1}{1+\nu}\right)\left(6x^5y^2\right)+\right.$$
$$+\left(-4+\frac{1}{1+\nu}\right)\left(\frac{2x^7}{7}\right)+\left(1-\frac{1}{1+\nu}\right)\left(2xy^6+5xy^2+45x^2y^3\right)+$$
$$+\alpha E\left\{xy^2t^2+x^2yt^2-\frac{2x^3t^2}{3}+\left(\frac{\tau_q}{k}-\frac{t}{k}\right)\left(\frac{x^3y^2}{3}+\frac{x^4y}{6}-\frac{x^5}{10}\right)+\right.$$
$$\left.\left.+\frac{1}{k^2}\left(\frac{x^5y^2}{60}+\frac{4x^6y}{6!}-\frac{x^7}{315}\right)\right\}\right], \tag{4.6}$$

$$
\begin{aligned}
U_y = \frac{1}{2G}\Bigg[&\left(1+\frac{1}{1+\nu}\right)\left(6x^2y+6x^2y^5+45x^3y^2\right)-\left(2+\frac{1}{1+\nu}\right)\left(10x^4y^3\right)+ \\
&+\left(3+\frac{1}{1+\nu}\right)\left(2x^6y\right)-9x^5-\left(\frac{1}{1+\nu}\right)\left(2y^3+\frac{2y^7}{7}+\frac{45xy^4}{2}\right)+ \\
&+\alpha E\left\{x^2yt^2+\frac{x^3t^2}{3}+\left(\frac{\tau_q}{k}-\frac{t}{k}\right)\left(\frac{x^4y}{6}+\frac{x^5}{30}\right)+\frac{1}{k^2}\left(\frac{x^6y}{180}+\frac{4x^7}{7!}\right)\right\}\Bigg].
\end{aligned} \tag{4.7}
$$

For all numerical calculations, physical properties of copper material were taken as in Sherief and Anwar [18]:

$$\alpha = 17\times10^{-6}\,{}^{\circ}C^{-1},\quad k = 1.1283\times\frac{10^{-4}m^2}{s},\quad E = 1.19\times10^{11}\,N/m^2,$$
$\nu = 0.33, G = 4.5\times10^{10}$ and $\tau_q = 0.02s$.

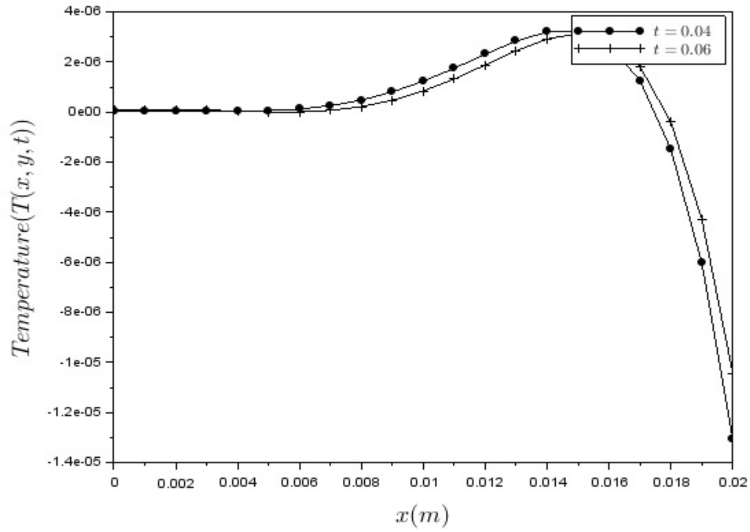

Fig.1. Temperature distribution versus *x*.

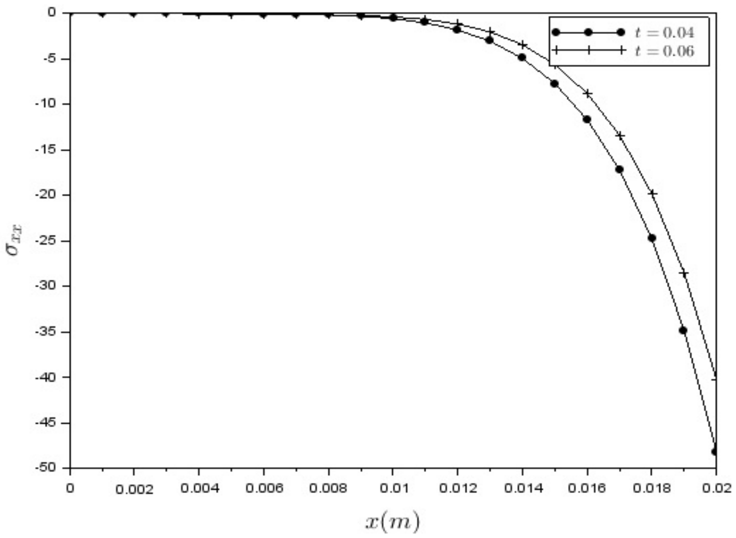

Fig.2(a). Thermal stress σ_{xx} versus *x*.

Figure 1 shows the temperature distribution along the *x*-direction for $t = 0.04$ and $t = 0.06$ at $y = 0.005$. It is observed that initially, the temperature increases slowly till it reaches the middle of the plate and then starts decreasing and shows the minimum temperature at the boundary surface. It is also observed that the magnitude of the temperature decreases with time.

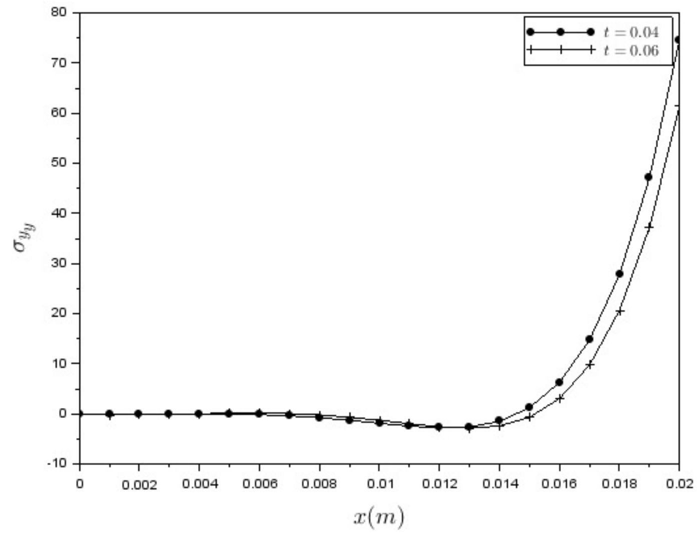

Fig.2(b). Thermal stress σ_{yy} versus *x*.

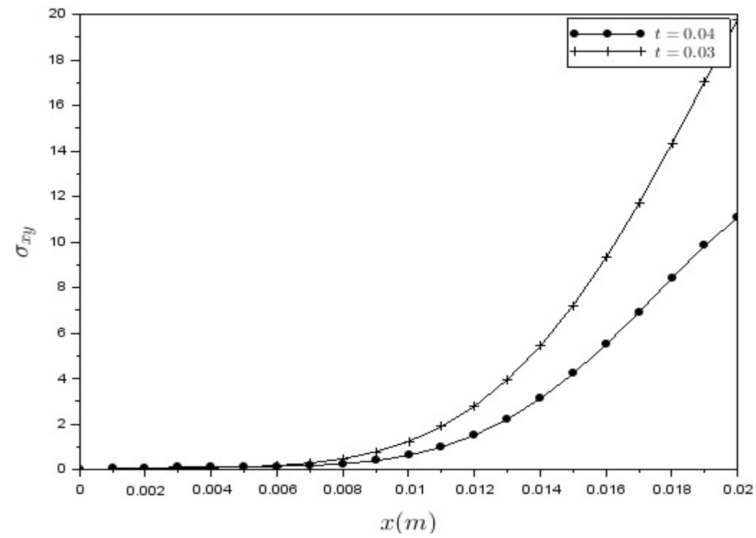

Fig. 2(c). Thermal stress σ_{xy} versus *x*.

Figure 2 describes the nature of thermal stress components σ_{xx}, σ_{yy} and σ_{xy} along the *x*-direction for different time intervals. Figure 2(a) shows the thermal stress σ_{xx} along the *x*-direction in the middle of the plane at $y = 0.005m$. It is clear from the graph that the compressive stress occurs from $x = 0.01$. The magnitude of this stress component is directly proportional to the time. Figure 2(b) describes the behaviour of thermal stress σ_{yy} along the *x*-direction in the middle plane at $y = 0.005m$. The figure shows that the plate experiences very little stress till the mid of the plate and then tensile stress occurs in the remaining part of the plate. The minimum stress occurs at $x = 0.014$ and then increases between $0.014 < x < 0.02$. The magnitude of σ_{yy} decreases as time increases. Figure 2(c) gives the distribution of thermal stress σ_{xy} along the *x*-direction in the middle of the plane at $y = 0.005m$. The nature of the curve is similar to the nature of σ_{yy} along the *x*-direction. The large variation in stress σ_{xy} was observed after $x = 0.008$ and it reaches its maximum value at $x = 0.02$.

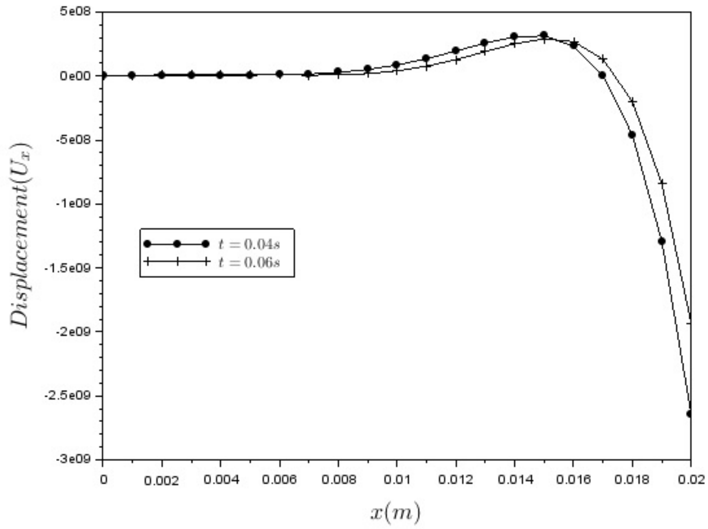

Fig.3(a). Displacement U_x versus *x*.

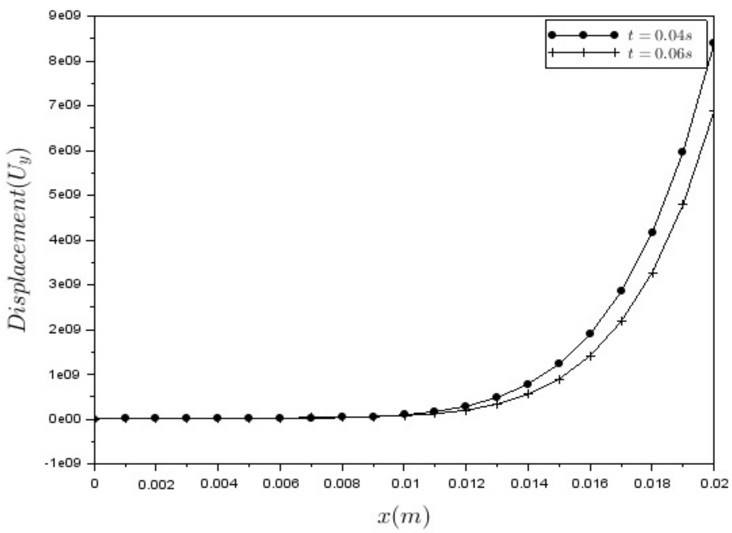

Fig.3(b). Displacement U_y versus *x*.

Figure 3 represents the nature of thermal displacement along the *x*-direction for different values of time. It is observed from Figure 3(a) that the nature of displacement U_x along the *x*-direction in the middle of the plate is similar to the temperature distribution along *x*-directions. There is a sudden variation in displacement after $x = 0.016$. It continuously decreases and shows the minimum value at the boundary $x = 0.02$. Figure 3(b) describes the nature of displacement U_y along the *x*-direction in the middle of the plate. The displacement U_y increases after $x = 0.01$ and reaches its maximum value at the boundary surface.

5. Conclusion

In this study, we considered a thermoelastic model under the hyperbolic heat conduction theory for a thin rectangular plate. The reduced differential transform method has been used to obtain the temperature field, thermal stresses and displacement. The method is simple to apply and it gives a solution in series form. We have presented a general solution for this model assuming non- homogeneous boundary conditions along a spatial direction. Also, we have discussed special case to show the effectiveness of this method. All the numerical and graphical solutions are presented for the copper plate. From the study, it is observed that there is a large variation in the temperature, normal stress, shear stress and the displacement along the *x*-direction after $x = 0.01$. The focus of the analysis is to present the solution of the hyperbolic heat conduction equation and thermal stresses using the reduced differential transform method. The results may be useful for different industrial and engineering applications related to the finite speed of heat propagation. It is also observed that the results obtained using the reduced differential transform method are more accurate and also reduce the computational work as compared to the differential transform method.

Nomenclature

T – temperature
t – time
$x,\ y$ – spatial coordinates
T_0 – initial temperature
τ_q – relaxation time
k – thermal diffusivity

ρ – mass density
c – specific heat capacity
λ – thermal conductivity
α – thermal expansion coefficient
E – Young's modulus
G – shear modulus of elasticity
ν – Poisson's ratio
r – positive integer

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Received: September 23, 2020
Revised: January 21, 2021