Int. J. of Applied Mechanics and Engineering, 2021, vol.26, No.2, pp.47-69
DOI: 10.2478/ijame-2021-0019

REFLECTION-REFRACTION COEFFICIENTS AND ENERGY RATIOS IN COUPLE STRESS MICROPOLAR THERMOVISCOUS ELASTIC SOLID

Ravinder KUMAR SAHRAWAT and POONAM*
Department of Mathematics, Deenbandhu Chhotu Ram University of Science and Technology
Murthal-131039, INDIA
E-mail: poonamsangwan2793@gmail.com

The reflection and refraction phenomenon of propagation of waves in couple stress micropolar thermoviscous elastic solid media with independent viscoelastic and micropolar properties have been studied. The structure of the model has been taken such that the plane interface is divides the given media into two half spaces in perfect contact. Here, we find that there are five waves, one of them is propagating independently while others are set of two coupled waves travelling with different speeds. Energy ratios, reflection and refraction coefficients relative to numerous reflected and refracted waves have been investigated when set of two coupled longitudinal waves and set of two coupled transverse waves strike at the interface through the solid medium. The inequality of energy ratios, refraction coefficients and reflection coefficients are evaluated numerically and presented graphically under three theories of thermoelasticity, namely, Green-Lindsay theory (GL), Lord- Shulman theory (LS), Coupled theory (CT) versus angular frequency and angle of incidence.

Key words: micropolar, thermoviscous, phase speed, energy ratio, reflection coefficient, refraction coefficient, couple stress.

1. Introduction

The micropolar theory developed by Eringen [1]. It is helpful in many structured materials like lattice micropolar structure. This theory explains the variation of micropolar elastic materials from classic elastic materials which are independent of translation. Eringen [2] and Nowacki [3] elaborated the micropolar thermoelasticity theory. The generalized micropolar theory was introduced by Dost and Taborrok [4] by using theory the Green and Lindsay theory [5]. Chandersekharaish [6] established the micropolar thermoelasticity dependence on heat flux.

In the literature, there are three theories of thermoelasticity, namely, generalized theory, coupled theory and uncoupled theory. The classical uncoupled theory is based on two drawbacks. The first drawback corresponds to the propagation of wave with infinite speed in the uncoupled thermoelasticity theory and was studied by many researchers like Chandrasekhariah [7], Ferkas and Szekeres [8], Szekers [9]. The second drawback corresponds to the propagation of thermal wave with infinite speed. Biot [10] proposed the theory of coupled thermoelasticity to eradicate the other drawback. Hetnarski and Ignaczak [11] investigated many generalizations of the coupled theory and achieved a number of significant results. Concerning the Green-Lindsay theory (GL), Lord-Shulman theory (LS) [12] and coupled theory (CT). Lord-Shulman determined the theory of generalized thermoelasticity to replace Fourier law by a new law named Maxwell-Catteneo law with one relaxation time. The emphasized theory confirmed the finite speed of propagation of elastic waves and heat waves when heat equations are converted into wave equations. The basic equations and relations remained same in this theory. In the Green-Lindsay theory is also recognized as the thermoelasticity theory depend on temperature rate or thermoelasticity theory with two relaxation times. In this theory, the equations for entropy and stress tensor are discussed with two relaxation times. Low temperature thermoelasticity theory is also

*-Corresponding author

called the coupled theory studied by Ignaczak and Hetnarski [13]. This representation is described by a system of non-linear field equations. The thermoelasticity theory without energy dissipation in the third generalization of the coupled theory was proposed by Green and Naghdi [14] in which the classical Fourier law replaced the heat flux rate temperature gradient.

The problems of reflection and refraction of set of two coupled transverse waves and a set of two coupled longitudinal waves at a plane interface dividing a given medium into two half-spaces which are useful in engineering and geophysics areas. Many problems related to reflection and refraction of waves for a plane split into two media have been discussed by Tomar and Khurana [15]-[17], Tomar and Gogna [18, 19], Parfitt and Eringen [20], Tomar [21], Singh *et al.* [22], Zhang and coworkers [23]-[25], Achenbach [26], Sarkar and Tomar [27], Sahrawat *et al.* [28] and Poonam *et al.* [29]. Sahrawat *et al.* [30] investigated the fundamental solution and plane wave propagation in nonlocal couple stress micropolar thermoelastic solid without energy dissipation.

The main aim of the paper is to study the effect of three theories of thermoelasticity, namely, Green-Lindsay theory (GL), Lord- Shulman theory (LS), Coupled theory (CT) on the propagation of a set of two coupled transverse waves and a set of two coupled longitudinal waves in a couple stress micropolar thermoviscous elastic solid medium. We study the reflection and refraction phenomenon of wave propagation at an interface of two couple stress microplolar thermoviscous elastic solid half spaces. It is found that there exist five waves, namely, longitudinal microrotational wave, a set of two coupled transverse waves and a set of two coupled longitudinal waves travelling with different speeds. The inequality of Energy ratios and reflection-refraction coefficients with respect to angle of incidence in both the half spaces has been studied numerically and graphically under the three theories of thermoelasticity, that is, LS, GL, CT.

2. Basic relations and equations

Following Tomar *et al.* [31], the Eqs for a couple stress micropolar thermoviscous elasticity are given by

$$\begin{aligned} t_{ij} &= \lambda' \boldsymbol{u}_{r,r}\delta_{ij} + K' e_{ijr}\phi_r - \vartheta\theta\delta_{ij} - \vartheta t_1 \delta_{k2}\dot{\theta}\delta_{ij} + K'^{*} e_{ijr}\dot{\phi}_r + \lambda'^{*}\dot{\boldsymbol{u}}_{r,r}\delta_{ij} + (\mu' + K')\boldsymbol{u}_{j,i} + \\ &+ \left(\mu'^{*} + K'^{*}\right)\dot{\boldsymbol{u}}_{j,i} + \mu' \boldsymbol{u}_{i,j} + \mu'^{*}\dot{\boldsymbol{u}}_{i,j}, \end{aligned} \tag{2.1}$$

$$\boldsymbol{m}_{ij} = \alpha'\phi_{r,r}\delta_{ij} + \alpha'^{*}\dot{\phi}_{r,r}\delta_{ij} + \beta'\phi_{i,j} + \beta'^{*}\dot{\phi}_{i,j} + \gamma'\phi_{j,i} + \gamma'^{*}\dot{\phi}_{j,i}, \tag{2.2}$$

$$\rho_0\eta = \vartheta u_{r,r}\delta_{ij} + a, \tag{2.3}$$

$$Y_i + t_0\delta_{k1}\dot{\theta} = K_t\theta_{,i} \tag{2.4}$$

where $\boldsymbol{u}$ denotes the displacement vector; ϕ denotes the microrotation vector; Dot (.) represents differentiation with respect to time; comma (,) represents the spatial derivative; ∇^2 denotes the Laplacian operator; λ' and μ' are Lame's constants; K', α', β', γ', λ'^{*}, μ'^{*}, α'^{*}, β'^{*}, γ'^{*}, ϑ, K_t and a are material constants; t_{ij} represents the stress tensor; m_{ij} represents the couple stress tensor; η denotes entropy per unit mass; ρ_0 denotes density of the medium; Y_i is the heat flux vector; θ denotes the change in temperature; T_0 is the ambient temperature and δ_{ij} denotes Kronecker's delta. Here t_0 and t_1 are the relaxation times and k identifies the different theories of thermoelasticity. The case $k=1$ corresponds to the LST and the case $k=2$ corresponds to the GLT. When both t_0 and t_1 vanish, it corresponds to the CT of thermoelasticity.

Now, we introduce the following dimensionless quantities:

$$x_i' = \frac{x_i}{l_0}, \quad u_i' = \frac{u_i}{l_0}, \quad \phi_i' = \frac{\phi_i}{l_0}, \quad t' = \frac{c_0 t}{l_0}, \quad T' = \frac{\theta}{T_0}, \quad t_{ij}' = \frac{t_{ij}}{l_0 \vartheta T_0}, \quad m_{ij}' = \frac{m_{ij}}{l_0 \vartheta T_0}$$

where c_0 and l_0 are the standard velocity and standard displacement, respectively.

With the help of these non-dimensional quantities, Eqs (2.1)-(2.4) become

$$\begin{gathered}(\lambda_2+\mu_2)\nabla(\nabla.\boldsymbol{u})+\left(\lambda_2^*+\mu_2^*\right)\nabla(\nabla.\dot{\boldsymbol{u}})+(\mu_2+K_2)\nabla^2\boldsymbol{u}+ \\ +\left(\mu_2^*+K_2^*\right)\nabla^2\dot{\boldsymbol{u}}+K_2\nabla\times\phi+K_2^*\nabla\times\dot{\phi}-\vartheta_1 T_{,i}-\vartheta_2\dot{T}_{,i}=\rho\ddot{\boldsymbol{u}},\end{gathered} \tag{2.5}$$

$$\begin{gathered}(\alpha_2+\beta_2)\nabla(\nabla.\phi)+\left(\alpha_2^*+\beta_2^*\right)\nabla(\nabla.\phi)+\gamma_2\nabla^2\phi+\gamma_2^*\nabla^2\dot{\phi}+ \\ +K_2\nabla\times\boldsymbol{u}+K_2^*\nabla\times\dot{\boldsymbol{u}}-2K_2^*\dot{\phi}-2K_2\phi=r_1\ddot{\phi},\end{gathered} \tag{2.6}$$

$$\left(1+n_1\delta_{lk}\frac{\partial}{\partial t}\right)\dot{\boldsymbol{T}}+\left(\frac{\vartheta_1}{\tau_3}+\frac{\vartheta_1}{\tau_2}\delta_{lk}\right)\nabla.\dot{\boldsymbol{u}}=k_1\nabla^2\boldsymbol{T} \tag{2.7}$$

where

$$\lambda_1=\lambda'+\lambda'^*\frac{\partial}{\partial t}, \quad \mu_1=\mu'+\mu'^*\frac{\partial}{\partial t}, \quad K_1=K'+K'^*\frac{\partial}{\partial t}, \quad \alpha_1=\alpha'+\alpha'^*\frac{\partial}{\partial t},$$

$$\beta_1=\beta'+\beta'^*\frac{\partial}{\partial t}, \quad \gamma_1=\gamma'+\gamma'^*\frac{\partial}{\partial t}, \quad \lambda_2=\frac{\lambda_1}{\rho_0 c_0^2}, \quad \lambda_2^*=\frac{\lambda_1^*}{\rho_0 c_0 l_0}, \quad \mu_2=\frac{\mu_1}{\rho_0 c_0^2},$$

$$\mu_2^*=\frac{\mu_1^*}{\rho_0 c_0 l_0}, \quad K_2=\frac{K_1}{\rho_0 c_0^2}, \quad K_2^*=\frac{K_1^*}{\rho_0 c_0 l_0}, \quad \vartheta_1=\frac{\vartheta T_0}{\rho_0 c_0^2}, \quad \vartheta_2=\frac{\vartheta T_0 t_1}{\rho_0 c_0 l_0},$$

$$k_1=\frac{k_t}{c c_0 l_0}, \quad n_1=\frac{t_0 c_0}{l_0}, \quad r_1=\frac{j}{l_0^2}, \quad \tau_2=\frac{T_0 l_0 c}{\rho_0 c_0 l_0^3}, \quad \tau_3=\frac{T_0 c}{\rho_0 c_0^2}, \quad \alpha_2=\frac{\alpha_1}{\rho_0 c_0^2},$$

$$\alpha_2^*=\frac{\alpha_1^*}{\rho_0 c_0 l_0}, \quad \beta_2=\frac{\beta_1}{\rho_0 c_0^2 l_0^2}, \quad \beta_2^*=\frac{\beta_1^*}{\rho_0 c_0 l_0^3}, \quad \gamma_2=\frac{\gamma_1}{\rho_0 c_0^2}, \quad \gamma_2^*=\frac{\gamma_1^*}{\rho_0 l_0 c_0},$$

$$c_1'^2=\left(\vartheta_1+\vartheta_2\delta_{2k}\frac{\partial}{\partial t}\right), \quad c_3'^2=\left(\frac{\vartheta_1}{\tau_3}+\frac{\vartheta_1\delta_{1k}}{\tau_2}\frac{\partial}{\partial t}\right), \quad c_4'^2=\left(1+n_1\delta_{1k}\frac{\partial}{\partial t}\right).$$

3. Wave propagation

By the Helmholtz decomposition theorem on vectors, we introduce a scalar potential (σ, ξ) and a vector potential $(\boldsymbol{U}, \boldsymbol{\Pi})$, given by

$$\boldsymbol{u}=\nabla\sigma+\nabla\times\boldsymbol{U}, \quad \nabla\cdot\boldsymbol{U}=0, \quad \phi=\nabla\xi+\nabla\times\boldsymbol{\Pi}, \quad \nabla\cdot\boldsymbol{\Pi}=0. \tag{3.1}$$

In the absence of body couples, body forces and heat equation, the Eqs of motion are obtained by using Eq.(3.1) in Eqs (2.5)-(2.7) as

$$-c_1'^2 T + (\lambda_3 + 2\mu_3 + K_3)\nabla^2\sigma - \ddot{\sigma} = 0, \tag{3.2}$$

$$(\mu_3 + K_3)\nabla^2 \mathbf{U} + K_3 \nabla \times \mathbf{\Pi} - \ddot{\boldsymbol{U}} = 0, \tag{3.3}$$

$$(\alpha_3 + \beta_3 + \gamma_3)\nabla^2\xi - 2K_3\xi - \ddot{\xi} = 0, \tag{3.4}$$

$$\gamma_3 \nabla^2 \mathbf{\Pi} + K_3 \nabla \times \boldsymbol{U} - 2K_3 \mathbf{\Pi} - \ddot{\mathbf{\Pi}} = 0, \tag{3.5}$$

$$k_1 \nabla^2 T - c_3'^2 \nabla^2 \dot{\sigma} - c_4'^2 \dot{T} = 0 \tag{3.6}$$

where

$$\lambda_3 = \lambda_2 + \lambda_2^* \frac{\partial}{\partial t}, \qquad \mu_3 = \mu_2 + \mu_2^* \frac{\partial}{\partial t}, \qquad K_3 = K_2 + K_2^* \frac{\partial}{\partial t}, \qquad \alpha_3 = \frac{1}{r}\alpha_2 + \alpha_2^* \frac{\partial}{\partial t},$$

$$\beta_3 = \frac{1}{r}\beta_2 + \beta_2^* \frac{\partial}{\partial t}, \qquad \gamma_3 = \frac{1}{r}\gamma_2 + \gamma_2^* \frac{\partial}{\partial t}, \qquad c_1'^2 = \left(\vartheta_1 + \vartheta_2 \delta_{2k} \frac{\partial}{\partial t}\right),$$

$$c_3'^2 = \left(\frac{\vartheta_1}{\tau_3} + \frac{\vartheta_1 \delta_{1k}}{\tau_2}\frac{\partial}{\partial t}\right), \qquad c_4'^2 = \left(1 + \mathrm{n}_1 \delta_{1\mathrm{k}} \frac{\partial}{\partial \mathrm{t}}\right).$$

Eq.(3.2) and Eq.(3.6) show that σ and T are coupled; Eq.(3.3) and Eq.(3.5) show that $\boldsymbol{U}$ and $\mathbf{\Pi}$ are coupled; But Eq.(3.4) shows ξ is uncoupled. The propagation of plane waves for Eqs (3.2)-(3.6) in the linear couple stress thermoviscous micropolar elasticity are given by

$$\{\sigma, \xi, \boldsymbol{U}, \mathbf{\Pi}, T\} = \{a_1, b_1, \boldsymbol{A}, \boldsymbol{B}, c_1\}\exp\{\iota l(\boldsymbol{n}.\boldsymbol{r} - \mathrm{S}t)\}, \tag{3.7}$$

travelling with speed S in Eqs (2.5)-(2.7), where $a_1, b_1, c_1, \boldsymbol{A}, \boldsymbol{B}$ are scalars, 'ι' denotes the iota, $\boldsymbol{n}$ is the unit vector and $\boldsymbol{r}$ is position vector. Putting the value of σ and T from Eq.(3.7) in Eq.(3.2) and Eq.(3.6), we will obtain

$$X_1 S^4 + X_2 S^2 + X_3 = 0. \tag{3.8}$$

The roots of the Eq. (3.8) are given by

$$S_{1,2}^2 = \frac{-Y_2 \pm \sqrt{Y_2^2 - 4Y_1 Y_3}}{2Y_1} \tag{3.9}$$

where Y_s $(s = 1,2,3)$ are provided in Appendix A_1.

From Eq.(3.2), we will obtain the relation between a_1 and c_1

$$a_1 = \frac{-c_1'^2}{\omega^2 - l^2\left(\zeta_3 - \iota\omega\zeta_3^*\right)} c_1 \tag{3.10}$$

where ζ_3 and ζ_3^* are provided in Appendix A_1.

Using Eq.(3.7) in Eq.(3.1), we will obtain the displacement vector as follows

$$\boldsymbol{u} = \iota l a_1 \boldsymbol{n} \exp\left\{\iota l\left(\boldsymbol{n}.\boldsymbol{r} - \mathrm{St}\right)\right\}, \tag{3.11}$$

which explains that the directions of $\boldsymbol{u}$ and $\boldsymbol{n}$ are same. The directions of the displacement of wave and travelling of wave are same, so this is longitudinal in nature and known as longitudinal displacement wave. Also, the direction of wave related to T is longitudinal in nature (using Eq.(3.10) and Eq.(3.11)) and known as longitudinal thermal wave. Since, both the waves are coupled and hence, known as "coupled longitudinal waves".

Putting the value of ξ from Eq. (3.7) into Eq. (3.4), we will obtain the longitudinal microrotational wave

$$l^2 = \frac{\omega^2 - 2\zeta_2}{C} \tag{3.12}$$

where C and ζ_2 are provided in Appendix A_1.

Using Eq.(3.7) in Eq.(3.3) and Eq.(3.5), we will obtain a quadratic equation in terms of S^2 such that

$$X_1 S^4 + X_2 S^2 + X_3 = 0. \tag{3.13}$$

The roots of the Eq.(3.13) are given by

$$S_{3,4}^2 = \frac{-X_2 \pm \sqrt{X_2^{\,2} - 4X_1 X_3}}{2X_1} \tag{3.14}$$

where X_s (s=1, 2, 3) are provided in Appendix A_1.

Using Eq.(3.7) in Eq.(3.1), we will obtain

$$\boldsymbol{n}.\boldsymbol{A} = \boldsymbol{n}.\boldsymbol{B}\,, \tag{3.15}$$

$$\boldsymbol{A} = \frac{-\iota l \zeta_2}{\left[l^2\left(\zeta_1 - \iota\omega\zeta_1^*\right) - \omega^2\right]}\boldsymbol{B} \tag{3.16}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ lie in the same plane and mutually perpendicular to each other. From Eq.(3.16), we observe that if $\boldsymbol{A} \equiv 0$, then $\boldsymbol{B} \equiv 0$, making both $\boldsymbol{U}$ and $\boldsymbol{\Pi}$ vanish. These two waves vanish together. Eq.(3.1) shows that $\boldsymbol{U}$ and $\boldsymbol{\Pi}$ are normal to each other and also both are normal to the direction of propagation $\boldsymbol{n}$ and known as transverse waves. Since both the waves are coupled in nature and hence, known as "coupled transverse waves". ζ_2, ζ_1 and ζ_1^* are provided in Appendix A_1.

4. Model

We consider two distinct couple stress micropolar thermoviscous elastic half-spaces, namely, M_1 and M_2, respectively, in perfect contact in the Cartesian co-ordinate system $Ox_1x_2x_3$ in which the plane interface coincides with x_1x_3- plane. M_1 and M_2 occupy the region $x_3 \geq 0$ and $x_3 \leq 0$, respectively, as shown in Fig.1. The elastic moduli corresponding to region M_1 are $\lambda_2, \mu_2, K_2, \alpha_2, \beta_2, \gamma_2^*, \lambda_2^*, \mu_2^*, K_2^*, \alpha_2^*, \beta_2^*, \gamma_2^*$ those corresponding to region M_2 are λ_2', μ_2', K_2', α_2', β_2', $\gamma_2'^*$, $\lambda_2'^*, \mu_2'^*$, $K_2'^*, \alpha_2'^*, \beta_2'^*, \gamma_2'^*$.

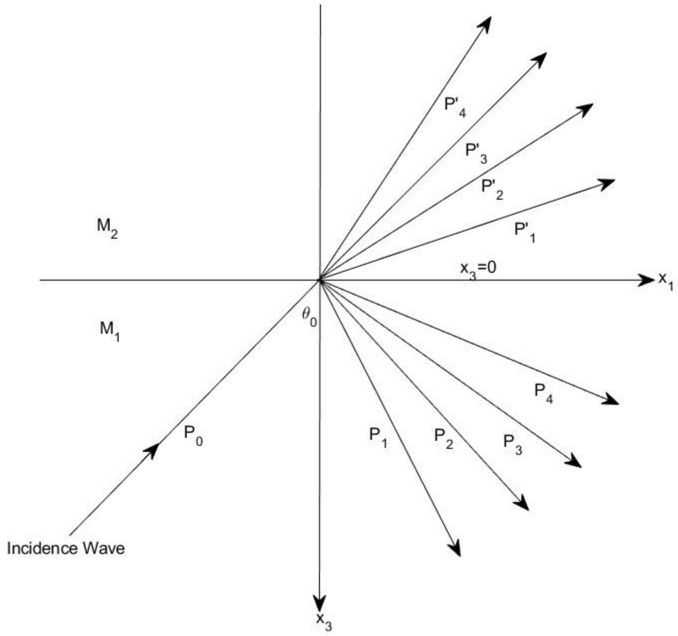

Fig.1. The geometry of the problem.

5. Boundary conditions

Solving Eqs (2.1)-(2.4) and Eq.(3.1), we will obtain the components of stress tensor, couple stress tensor, displacement and microrotation for mediums M_1 and M_2 are given by

$$u_1 = \sigma_{,1} - U_{2,3}, \qquad u_3 = \sigma_{,3} + U_{2,1}, \qquad \phi_2 = \Pi_{1,3} - \Pi_{3,1},$$

$$t_{33} = -C_1'^2 T + \lambda_3 \sigma_{,11} + (\lambda_3 + 2\mu_3 + K_3)\sigma_{,33} + (2\mu_3 + K_3)U_{2,13},$$

$$t_{13} = (2\mu_3 + K_3)\sigma_{,13} + \mu_3 U_{2,11} - (\mu_3 + K_3)U_{2,33} k_1 \phi_2, \qquad q_k = k_1 T_{,k}, \qquad m_{32} = \gamma_3 \phi_{2,3},$$

$$u_1' = \sigma_{,1}' - U_{2,3}', \qquad u_3' = \sigma_{,3}' + U_{2,1}', \qquad \phi_2' = \Pi_{1,3}' - \Pi_{3,1}',$$

$$t_{33}' = -C_1''^2 T' + \lambda_3' \sigma_{,11}' + \left(\lambda_3' + 2\mu_3' + K_3'\right)\sigma_{,33}' + \left(2\mu_3' + K_3'\right)U_{2,13}',$$

$$t_{13}' = \left(2\mu_3' + K_3'\right)\sigma_{,13}' + \mu_3' U_{2,11}' - \left(\mu_3' + K_3'\right)U_{2,33}' k_1' \phi_2',$$

$$q_k' = k_1' T_{,k}', \qquad m_{32}' = \gamma_3' \phi_{2,3}'.$$

The boundary conditions at the interface of two half spaces are given by

$$\begin{aligned} &t_{33} = t_{33}', \quad t_{31} = t_{31}', \quad m_{32} = m_{32}', \quad q_k = q_k', \\ &u_1 = u_1', \quad u_3 = u_3', \quad \phi_2 = \phi_2', \quad T = T'. \end{aligned} \tag{5.1}$$

6. Incidence of coupled longitudinal waves

6.1. Incidence of longitudinal displacement wave

At the interface $x_3 = 0$, let us consider a longitudinal wave of amplitude P_0 with speed S_1, propagating through the half space M_1 with wave number l_1 and making an angle θ_0.

The boundary conditions for the problem are given by:

Reflected waves in medium M_1 : A set of two coupled longitudinal waves and a set of two coupled transverse waves with amplitude P_1, P_2, P_3 and P_4 propagating with speeds S_1, S_2, S_3 and S_4 making an angle $\theta_1, \theta_2, \theta_3$ and θ_4 respectively with the normal.

Reflected waves in medium M_2 : A set of two coupled longitudinal waves and a set of two coupled transverse waves with amplitude P_1', P_2', P_3' and P_4' propagating with speeds S_1', S_2', S_3' and S_4' making an angle $\theta_1', \theta_2', \theta_3'$ and θ_4' respectively with the normal.

Equations (3.9) and Eq. (3.14) show the speeds S_1, S_2, S_3 and S_4, respectively. The equations for speeds S_1', S_2', S_3' and S_4' are same as for S_1, S_2, S_3 and S_4, respectively, with the primes at significant places. The total wave field is given by (Tomar *et al.* [19])

$$\sigma = P_0 \exp\{\iota l_1 (\sin\theta_0 x_1 - \cos\theta_0 x_3) - \iota\omega_1 t\} + \\ + \sum_{r=1,2} \mathrm{P_r} \exp\{\iota l_r (\sin\theta_r x_1 + \cos\theta_r x_3) - \iota\omega_r t\}, \tag{6.1}$$

$$T = \varkappa_1 P_0 \exp\{\iota l_1 (\sin\theta_0 x_1 - \cos\theta_0 x_3) - \iota\omega_1 t\} + \\ + \sum_{r=1,2} \varkappa_r \mathrm{P_r} \exp\{\iota l_r (\sin\theta_r x_1 + \cos\theta_r x_3) - \iota\omega_r t\}, \tag{6.2}$$

$$U = \sum_{r=3,4} \mathrm{P_r} \exp\{\iota l_r (\sin\theta_r x_1 + \cos\theta_r x_3) - \iota\omega_r t\}, \tag{6.3}$$

$$\Pi = \sum_{r=3,4} \eta_r \mathrm{P_r} \exp\{\iota l_r (\sin\theta_r x_1 + \cos\theta_r x_3) - \iota\omega_r t\}, \tag{6.4}$$

$$\sigma' = \sum_{r=1,2} \mathrm{P_r'} \exp\{\iota l_r (\sin\theta_r x_1 + \cos\theta_r x_3) - \iota\omega_r t\}, \tag{6.5}$$

$$T' = \sum_{r=1,2} \varkappa_r' \mathrm{P_r'} \exp\{\iota l_r (\sin\theta_r x_1 + \cos\theta_r x_3) - \iota\omega_r t\}, \tag{6.6}$$

$$\boldsymbol{U}' = \sum_{r=3,4} \mathrm{P_r'} \exp\{\iota l_r (\sin\theta_r x_1 + \cos\theta_r x_3) - \iota\omega_r t\}, \tag{6.7}$$

$$\boldsymbol{\Pi}' = \sum_{r=3,4} \eta_r' \mathrm{P_r'} \exp\{\iota l_r (\sin\theta_r x_1 + \cos\theta_r x_3) - \iota\omega_r t\} \tag{6.8}$$

where $(l_q, l_q^{'})$ and $(S_q, S_q^{'})$ are the wave numbers and phase speeds corresponding media M_1 and M_2, respectively. $\left(\omega_l = l_l S_l,\ \omega_l^{'} = l_l^{'} S_l^{'}\right)$ denotes the angular frequencies of the mediums M_1 and M_2, respectively. $\varkappa_{1,2}$ and $\eta_{3,4}$ are the coupling parameters obtained by the Eq.(3.11) and Eq.(3.16) respectively

$$\varkappa_{1,2} = \frac{-c_1'^2}{\omega^2 - l^2\left(\zeta_3 - \iota\omega\zeta_3^*\right)}, \qquad \eta_{3,4} = \frac{-\iota l\zeta_2}{\left[l^2\left(\zeta_1 - \iota\omega\zeta_1^*\right) - \omega^2\right]}.$$

Using Eq. (5.1) in Eqs (6.1) -(6.8), we will obtain

$$\begin{aligned}&\left[\zeta_4 + \zeta_5\cos^2\theta_0 + \frac{\nabla_4\varkappa_1}{l_1^2}\right]l_1^2 P_0 + \sum_{r=1,2}\left[\zeta_4' + \zeta_5'\cos^2\theta_r + \frac{\nabla_4\varkappa_r}{l_r^2}\right]l_r^2\,\mathrm{P_r} + \\ &+ \sum_{r=3,4}\zeta_5\sin\theta_r\cos\theta_r l_r^2\,\mathrm{P_r} - \sum_{r=1,2}\left[\zeta_4' + \zeta_5'\cos^2\theta_r' + \frac{\nabla_4'\varkappa_r'}{l_r'^2}\right]l_r'^2\,\mathrm{P_r'} + \\ &+ \sum_{r=3,4}\zeta_5'\sin\theta_r'\cos\theta_r' l_r'^2\,\mathrm{P_r'} = 0,\end{aligned} \tag{6.9}$$

$$\begin{aligned}&\zeta_5\sin\theta_0\cos\theta_0 l_1^2 P_0 - \sum_{r=1,2}\zeta_5\sin\theta_r\cos\theta_r l_r^2\,\mathrm{P_r} + \\ &+\sum_{3,4}\left[\zeta_6\cos 2\theta_r + \zeta_7\cos^2\theta_r - \frac{\zeta_7\eta_r}{l_r^2}\right]l_r^2 P_r + \\ &-\sum_{r=1,2}\zeta_5'\sin\theta_r'\cos\theta_r' l_r'^2\,\mathrm{P_r'} - \sum_{3,4}\left[\zeta_6'\cos 2\theta_r' + \zeta_7'\cos^2\theta_r' - \frac{\zeta_7'\eta_r'}{l_r'^2}\right]l_r'^2\,\mathrm{P_r'} = 0,\end{aligned} \tag{6.10}$$

$$\sum_{3,4}\zeta_2^*\eta_r\cos\theta_r l_r P_r + \sum_{3,4}\zeta_2^{'*}\eta_r^{'}\cos\theta_r^{'} l_r^{'} P_r^{'} = 0, \tag{6.11}$$

$$k_1\varkappa_1\cos\theta_0 l_1 P_0 - \sum_{r=1,2}k_1\varkappa_r\cos\theta_r l_r\,\mathrm{P_r} - \sum_{r=1,2}k_1'\varkappa_r'\cos\theta_r' l_r'\,\mathrm{P_r'} = 0, \tag{6.12}$$

$$\sin\theta_0 l_1 P_0 + \sum_{r=1,2}\sin\theta_r l_r\,\mathrm{P_r} - \sum_{r=3,4}\cos\theta_r l_r\,\mathrm{P_r} - \sum_{r=1,2}\sin\theta_r' l_r'\,\mathrm{P_r'} - \sum_{r=3,4}\cos\theta_r' l_r'\,\mathrm{P_r'} = 0, \tag{6.13}$$

$$\cos\theta_0 l_1 P_0 - \sum_{r=1,2}\cos\theta_r l_r\,\mathrm{P_r} - \sum_{r=3,4}\sin\theta_r l_r\,\mathrm{P_r} - \sum_{r=1,2}\cos\theta_r' l_r'\,\mathrm{P_r'} + \sum_{r=3,4}\sin\theta_r' l_r'\,\mathrm{P_r'} = 0, \tag{6.14}$$

$$\eta_3 P_3 + \eta_4 P_4 - \eta_3' P_3' - \eta_4' P_4' = 0, \tag{6.15}$$

$$\varkappa_1 P_0 + \varkappa_1 P_1 + \varkappa_2 P_2 - \varkappa_1' P_1' - \varkappa_2' P_2' = 0. \tag{6.16}$$

Equations (6.9)-(6.16) in the matrix form can be written as

$$[H][Q]=[Z] \tag{6.17}$$

where $[H]=\left[h_{ij}\right]$ is a 8×8 square matrix, $[Q]=[Q_1,Q_2,Q_3,Q_4,Q_5,Q_6,Q_7,Q_8]'$ is a 8×1 column matrix, 'dash' denotes the transpose of the matrix. $Q_r=\dfrac{P_r}{P_0}(r=1,2,3,4)$ and $Q_t=\dfrac{P'_{t-4}}{P_0}(t=5,6,7,8)$ are the reflection and refraction coefficients. All non-zero entries of the matrices $[H]$ and $[Z]$ are defined in Appendix A_1.

6.2. Energy partitioning

In this problem, we must show the energy balance to establish the reflection and refraction coefficients. The rate of transmission $\left\langle O^*\right\rangle$ per unit area is (Achenbach [26])

$$O^*=t_{33}\dot{u}_3+t_{31}u_1+m_{32}\dot{\phi}_2. \tag{6.18}$$

The energy carried along the incident wave, reflected and refracted waves is denoted by $\langle O_0\rangle,\langle O_i\rangle$ and $\left\langle O'_i\right\rangle$ respectively and the energy ratios $\langle E_i\rangle$ $(i=1,2....8)$ such that

$$E_i=\frac{\langle O_i\rangle}{\langle O_0\rangle}. \tag{6.19}$$

The energy ratios for the reflected and refracted waves are given in Appendix A_1.

When the coupled longitudinal wave of amplitude P_0 with S_2 and l_2 as the speed and wave number propagating through medium M_1 by making angle θ_0, then we follow the same process and obtain a matrix similar to Eq.(6.17), with the modified values given in Appendix A_2. The matrix [Q] and [Z] remains same. The expressions of Energy ratios are also given in Appendix A_2.

7. Incidence of coupled transverse waves

We examine a coupled transverse wave having amplitude P_3 travelling with speed S_3 for an angle θ_0, at the interface of two half-spaces using boundary conditions as given in Eq.(5.1) The reflected and refracted waves obtained in incidence of a set of two coupled longitudinal waves are same.

The total wave field equations in Medium M_1 are given by (Tomar *et al.* [19])

$$\sigma=\sum_{r=1,2}\mathrm{P_r}\exp\left\{\iota l_r\left(\sin\theta_r x_1+\cos\theta_r x_3\right)-\iota\omega_r t\right\}, \tag{7.1}$$

$$T=\sum_{r=1,2}\varkappa_r\,\mathrm{P_r}\exp\left\{\iota l_r\left(\sin\theta_r x_1+\cos\theta_r x_3\right)-\iota\omega_r t\right\}, \tag{7.2}$$

$$U = P_0 \exp\left\{\iota l_3 \left(\sin\theta_0 x_1 + \cos\theta_0 x_3\right) - \iota\omega_3 t\right\} + \\ + \sum_{q=3,4} P_{qx_2} e_{x_2} \exp\left\{\iota l_q \left(\sin\theta_q x_1 - \cos\theta_q x_3\right) - \iota\omega_q t\right\}, \tag{7.3}$$

$$\Pi = \eta_3 P_0 \exp\left\{\iota l_3 \left(\sin\theta_0 x_1 + \cos\theta_0 x_3\right) - \iota\omega_3 t\right\} + \\ + \sum_{q=3,4} \left(Q_{qx_1} e_{x_1} + Q_{qx_3} e_{X_3}\right) \exp\left\{\iota l_q \left(\sin\theta_q x_1 - \cos\theta_q x_3\right) - \iota\omega_q t\right\}. \tag{7.4}$$

The total wave field equations in medium M_2 are same as given in the case of a longitudinal displacement wave. Using boundary conditions (5.1) in Eqs (6.5)-(6.8) and Eqs (7.1)-(7.4), we will obtain

$$-\zeta_5 \sin\theta_0 \cos\theta_0 l_3^2 P_0 + \sum_{r=1,2}\left[\zeta_4' + \zeta_5' \cos^2\theta_r + \frac{\nabla_4 \varkappa_r}{l_r^2}\right] l_r^2 \mathrm{P_r} + \\ + \sum_{r=3,4} \zeta_5 \sin\theta_r \cos\theta_r l_r^2 \mathrm{P_r} - \sum_{r=1,2}\left[\zeta_4' + \zeta_5' \cos^2\theta_r' + \frac{\nabla_4' \varkappa_r'}{l_r'^2}\right] l_r'^2 \mathrm{P_r'} + \\ + \sum_{r=3,4} \zeta_5' \sin\theta_r' \cos\theta_r' l_r'^2 \mathrm{P_r'} = 0, \tag{7.5}$$

$$\left[\zeta_6 \cos 2\theta_r + \zeta_7 \cos^2\theta_r - \frac{\zeta_7 \eta_r}{l_r^2}\right] l_3^2 P_0 - \sum_{r=1,2} \zeta_5 \sin\theta_r \cos\theta_r l_r^2 \mathrm{P_r} + \\ + \sum_{3,4}\left[\zeta_6 \cos 2\theta_r + \zeta_7 \cos^2\theta_r - \frac{\zeta_7 \eta_r}{l_r^2}\right] l_r^2 \mathrm{P_r} - \sum_{r=1,2} \zeta_5' \sin\theta_r' \cos\theta_r' l_r'^2 \mathrm{P_r'} + \\ - \sum_{3,4}\left[\zeta_6' \cos 2\theta_r' + \zeta_7' \cos^2\theta_r' - \frac{\zeta_7' \eta_r'}{l_r'^2}\right] l_r'^2 \mathrm{P_r'} = 0, \tag{7.6}$$

$$\zeta_2^* \eta_r \cos\theta_r l_r \mathrm{P_r} - \sum_{3,4} \zeta_2^* \eta_r \cos\theta_r l_r \mathrm{P_r} + \sum_{3,4} \zeta_2'^* \eta_r' \cos\theta_r' l_r' \mathrm{P_r'} = 0, \tag{7.7}$$

$$-\sum_{r=1,2} k_1 \varkappa_r \cos\theta_r l_r \mathrm{P_r} - \sum_{r=1,2} k_1' \varkappa_r' \cos\theta_r' l_r' \mathrm{P_r'} = 0, \tag{7.8}$$

$$\cos\theta_0 l_3 P_0 + \sum_{r=1,2} \sin\theta_r l_r \mathrm{P_r} - \sum_{r=3,4} \cos\theta_r l_r \mathrm{P_r} - \sum_{r=1,2} \sin\theta_r' l_r' \mathrm{P_r'} - \sum_{r=3,4} \cos\theta_r' l_r' \mathrm{P_r'} = 0, \tag{7.9}$$

$$\sin\theta_0 l_3 P_0 - \sum_{r=1,2} \cos\theta_r l_r P_r - \sum_{r=3,4} \sin\theta_r l_r P_r - \sum_{r=1,2} \cos\theta_r' l_r' \mathrm{P_r'} + \sum_{r=3,4} \sin\theta_r' l_r' \mathrm{P_r'} = 0, \tag{7.10}$$

$$\eta_3 P_0 + \eta_3 P_3 + \eta_4 P_4 - \eta_3' P_3' - \eta_4' P_4' = 0, \tag{7.11}$$

$$\varkappa_1 P_1 + \varkappa_2 P_2 - \varkappa_1' P_1' - \varkappa_2' P_2' = 0 . \tag{7.12}$$

Equations (7.5)-(7.12) in the matrix form can be written as

$$[U][V] = [W] \tag{7.13}$$

where $[U] = [U_{ij}]$ is a 8×8 square matrix, $[V] = [V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8]'$ is a 8×1 column matrix. $V_r = \frac{P_r}{P_0}(r = 1,2,3,4)$ and $V_t = \frac{P'_{t-4}}{P_0}(t = 5,6,7,8)$ are the reflection and refraction coefficients. All non-zero entries of the matrices $[U]\, and\, [W]$ are provided in Appendix B_1.

7.2. Energy partitioning

The logical expressions of the energy ratios $E_i\,(i = 1,2 \ldots ..8)$ in the medium M_1 and M_2 for the coupled transverse waves are given in Appendix B_1. When the coupled transverse wave of amplitude P_4 with S_4 and l_4 as the speed and wave number propagating through medium M_1 by making angle θ_0, then we follow the same process and obtain a matrix similar to (7.13), with the modified values given in Appendix B_2. The matrix [V] and [W] remains same. The expressions of Energy ratios are also given in Appendix B_2.

8. Special cases

In this section, the speeds of propagating plane waves are reduced by using the conditions of different theories of thermoviscous elasticity.

8.1. Lord-Shulman theory

If we take $k = 1,$ all the equations remain same except those which contain Kronecker's delta. Therefore, we have

$$c_1'^2 = \vartheta_1, \qquad c_3'^2 = \left(\frac{\vartheta_1}{\tau_3} + \frac{\vartheta_1}{\tau_2}\frac{\partial}{\partial t}\right), \qquad c_4'^2 = \left(1 + n_1\frac{\partial}{\partial t}\right).$$

Using these expressions, we can find the speeds of various waves in terms of the LS theory.

8.2. Green-Lindsay theory

If we put $k = 2$, all the terms which contain Kronecker's delta can be reduced in the form

$$c_1'^2 = \left(\vartheta_1 + \vartheta_2\frac{\partial}{\partial t}\right), \qquad c_3'^2 = \frac{\vartheta_1}{\tau_3}, \qquad c_4'^2 = 1.$$

Using these expressions, we can find the speed of various waves in terms of GL theory.

8.3. Coupled theory

When we take $t_0 = t_1 = 0$, then the expressions which contains the terms t_0, t_1 become

$$c_1'^2 = \left(\vartheta_1 + \vartheta_2 \frac{\partial}{\partial t}\right), \qquad c_3'^2 = \frac{\vartheta_1}{\tau_3}, \qquad c_4'^2 = 1.$$

Now, the equations of the given waves are reduced into classical elastic medium.

9. Numerical analysis

The reflection and refraction coefficients with their energy ratios and also with a set of two couple longitudinal wave and a set of two coupled transverse wave under three theories of thermoelasticity are calculated and drawn graphically using numerical values of different parameters from Kumar *et al.* [32] specified in Tab.1 and Tab.2 for both medium M_1 and M_2 with phase speeds S_r and S_r' $(r = 1,2,3,4)$ respectively by using MATLAB software.

Table 1. Numerical values of parameters.

symbol	value	symbol	value
λ_2	$9.4\times 10^{10}\, Nm^{-2}$	μ_2	$4.0\times 10^{10}\, Nm^{-2}$
K_2	$1.0\times 10^{10}\, Nm^{-2}$	ρ_0	$1.73\times 10^{3}\, Kgm^{-3}$
j	$0.2\times 10^{-19}\, m^2$	γ_2	$0.779\times 10^{-9}\, N$
α_2	$2.33\times 10^{-5}\, N$	β_2	$2.48\times 10^{-5}\, N$
ϑ	$0.9\times 10^{-17}\, N$	K_t	$1.7\times 10^{6}\, jKm^{-1}s^{-1}$
t_0	1.03	t_1	1.04
λ_1^*	$0.5\times 10^{1}0 Nm^{-2}$	μ_1^*	$0.1\times 10^{9}\, Nm^{-2}$
α_1^*	$5.27\times 10^{3}\, N$	β_1^*	$3.17\times 10^{-3}\, N$
γ_1^*	$0.5\times 10^{3}\, N$	K_1^*	$0.5\times 10^{10}\, N$
T_0	$0.293\times 10^{3}\, K$	c	$1.04\times 10^{3}\, JKg^{-1}K^{-1}$

Table 2. Numerical values of parameters.

symbol	value	symbol	value
λ_1'	$75900\times 10^{6}\, Nm^{-2}$	μ_1'	$13500\times 10^{6}\, Nm^{-2}$
K_1'	$149\times 10^{6}\, Nm^{-2}$	ρ_0	$2.65\times 10^{3}\, Kgm^{-3}$
j	$0.00000196 m^2$	γ_1'	$0.0268\times 10^{6}\, N$
α_1'	$0.01\times 10^{6}\, N$	β_1'	$0.015\times 10^{6}\, N$
ϑ	$0.9\times 10^{6}\, N$	K_t	$1.3\times 10^{6}\, jKm^{-1}s^{-1}$
t_0	0.03	t_1	0.04
$\lambda_1'^*$	$500\times 10^{6}\, Nm^{-2}$	$\mu_1'^*$	$300\times 10^{6}\, Nm^{-2}$
$\alpha_1'^*$	$2.0\times 10^{6}\, N$	$\beta_1'^*$	$4.0\times 10^{6}\, N$
$\gamma_1'^*$	$5.0\times 10^{6}\, N$	$K_1'^*$	$0.0149\times 10^{6}\, N$
T_0	$1.293\times 10^{3}\, K$	c	$2.04\times 10^{3}\, JKg^{-1}K^{-1}$

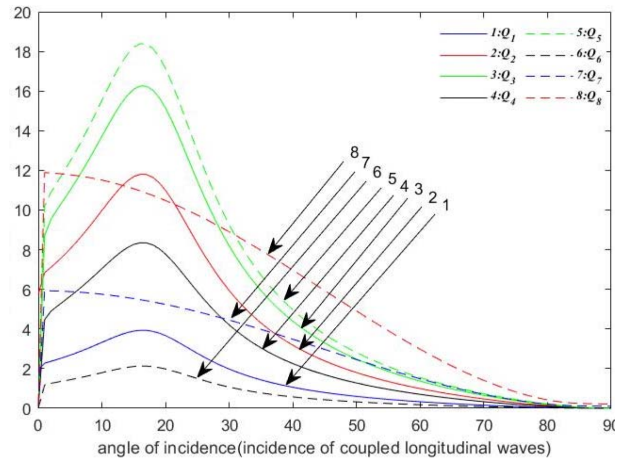

Fig.2. The magnitude of reflection coefficients versus the angle of incidence (incidence of coupled longitudinal waves) under the LS-theory.

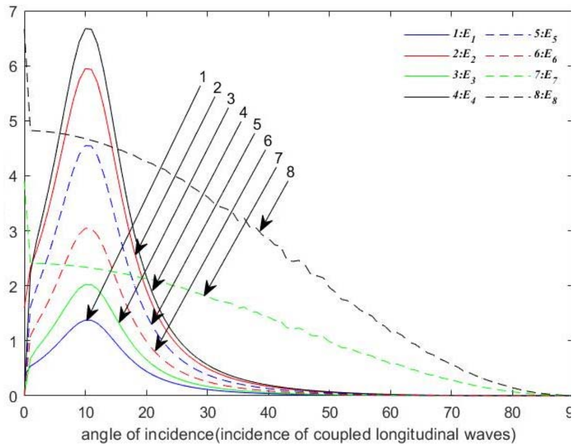

Fig.3. The magnitude of energy coefficients versus the angle of incidence (incidence of coupled longitudinal waves) under the LS-theory.

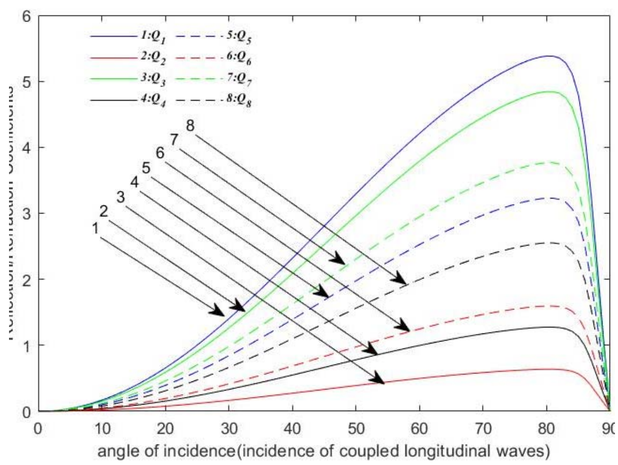

Fig.4. The magnitude of reflection coefficients versus the angle of incidence (incidence of coupled longitudinal waves) under the GL-theory.

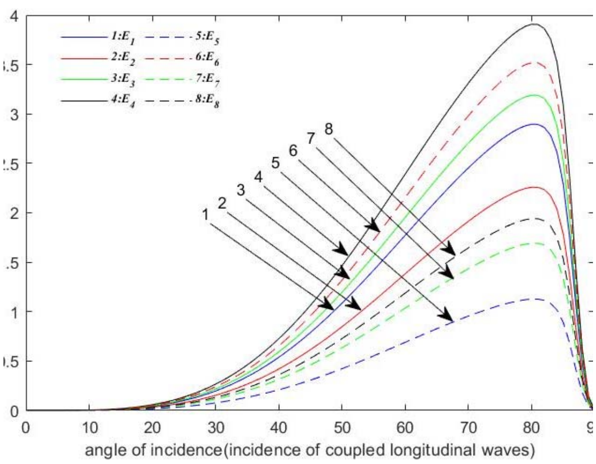

Fig.5. The magnitude of energy coefficients versus the angle of incidence (incidence of coupled longitudinal waves) under the GL-theory.

Figure 2 illustrates that the $Q_s\left(s=1,2\ldots 6\right)$ increase sharply for $0^\circ \le \theta_0 \le 10$ and after obtaining maxima at $\theta_0 = 10^\circ$, decrease gradually as θ_0 increases for $10^\circ \le \theta_0 \le 90^\circ$. Q_7 and Q_8 decrease continuously as θ_0 increases for $0^\circ \le \theta_0 \le 90^\circ$ and attain their maximum value at grazing incidence and vanish at grazing incidence. Figures 2 and 3 illustrate that the graphical behaviour of the reflection and refraction coefficients $\left(Q_s\right)$ is similar to their corresponding energy coefficients (E_s).

Figure 4 illustrates that the absolute value of reflection and refraction coefficients $\left(Q_s\right)$ $\left(s=1,2\ldots 8\right)$ increase continuously and after that attain their maximum value at $\theta_0 = 85^\circ$, it decreases sharply for $85^\circ \le \theta_0 \le 90^\circ$. Figures 4 and 5 illustrate that the graphical behaviour of the reflection and refraction coefficients $\left(Q_s\right)$ is similar to their corresponding energy coefficients (E_s). The reflection and refraction coefficients with their corresponding energy ratios are vanishing at grazing as well as normal incidence under GL-theory.

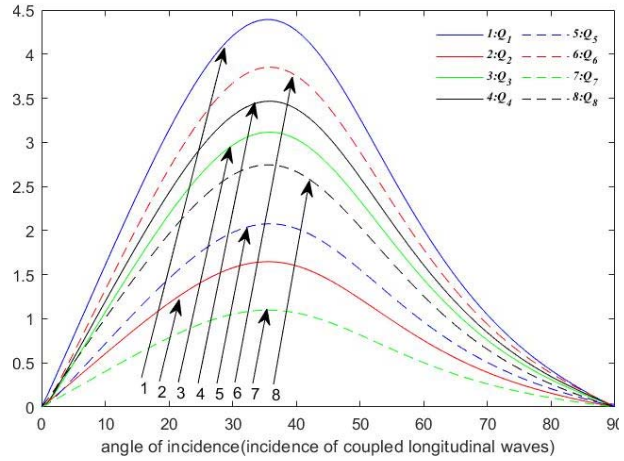

Fig.6. The magnitude of reflection coefficients versus the angle of incidence (incidence of coupled longitudinal waves) under the CT-theory.

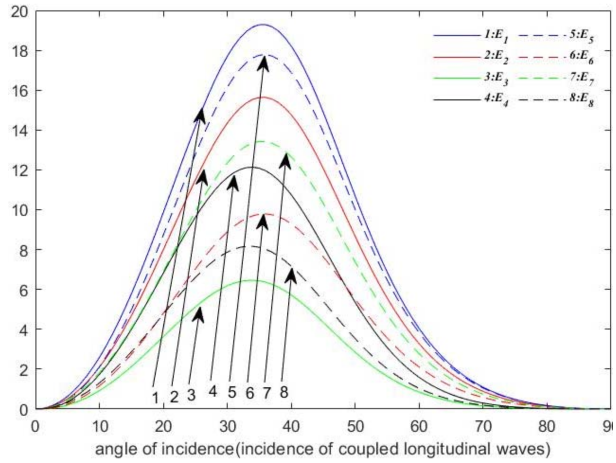

Fig.7. The magnitude of energy coefficients versus the angle of incidence (incidence of coupled longitudinal waves) under the CT-theory.

Figure 6 illustrates that that the absolute value of reflection and refraction coefficients $\left(Q_s\right)$ $\left(s = 1, 2\ldots 8\right)$ increase continuously for $0^\circ \le \theta_0 \le 60^\circ$ and after attaining maxima at $\theta_0 = 60^\circ$, the curves go down and further increase as θ_0 increases for $60^\circ \le \theta_0 \le 90^\circ$. Figures 6 and 7 illustrate that the graphical behaviour of the reflection and refraction coefficients $\left(Q_s\right)$ is similar to their corresponding energy coefficients (E_s). The reflection and refraction coefficients with their corresponding energy ratios are vanishing at grazing as well as normal incidence under CT-theory.

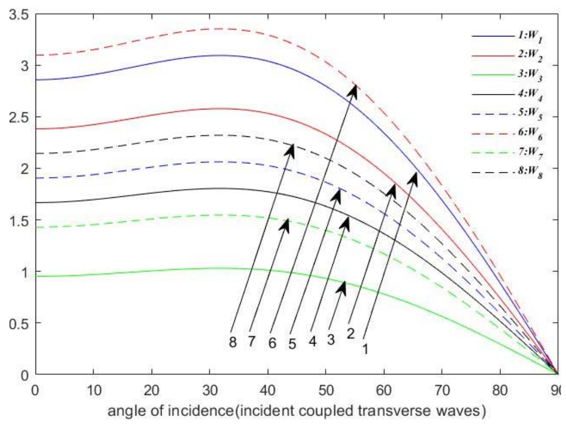

Fig.8. The magnitude of reflection coefficients versus the angle of incidence (incidence of coupled transverse wave) under the LS-theory.

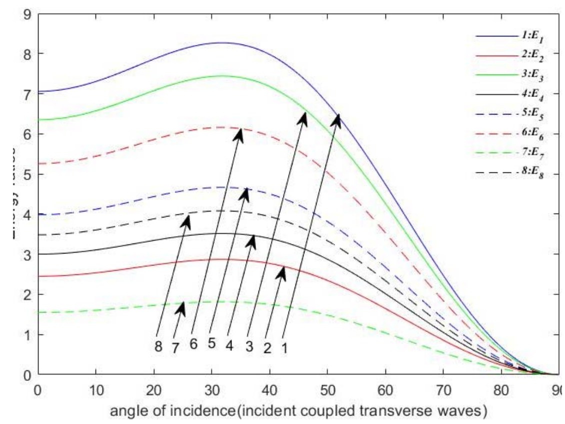

Fig.9. The magnitude of energy coefficients versus the angle of incidence (incidence of coupled transverse wave) under the LS-theory.

Figures 8 and 9 illustrate that the absolute value of W_s with their corresponding energy ratios E_s $(s = 1, 2 \ldots 8)$ decrease continuously as θ_0 increases for $0^\circ \le \theta_0 \le 90^\circ$ and attaining their maximum values at normal incidence. All the reflection and refraction coefficients with their corresponding energy coefficients are disappear at normal incidence under LS-theory.

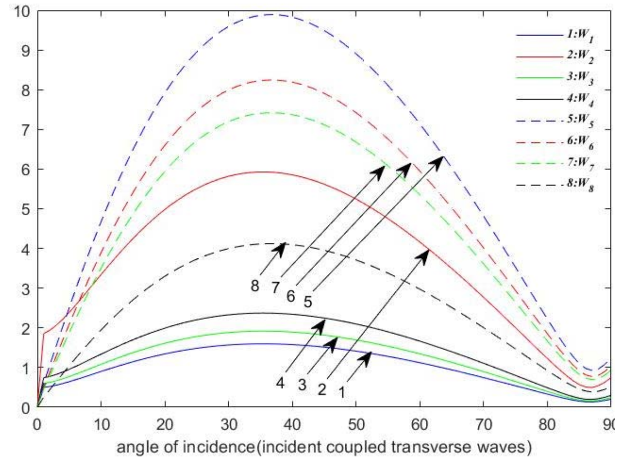

Fig.10. The magnitude of reflection coefficients versus the angle of incidence (incidence of coupled transverse wave) under the GL-theory.

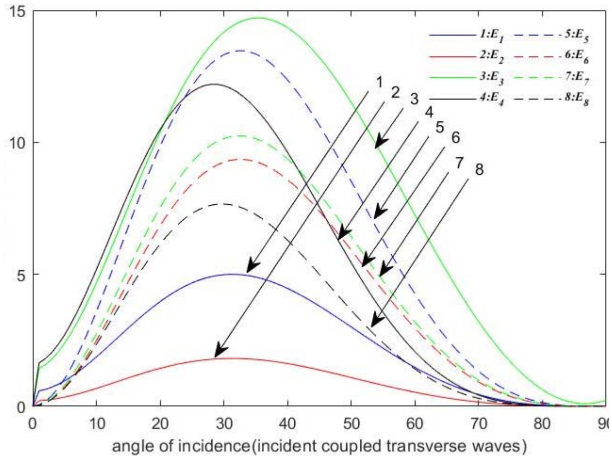

Fig.11. The magnitude of energy coefficients versus the angle of incidence (incidence of coupled transverse wave) under the GL-theory.

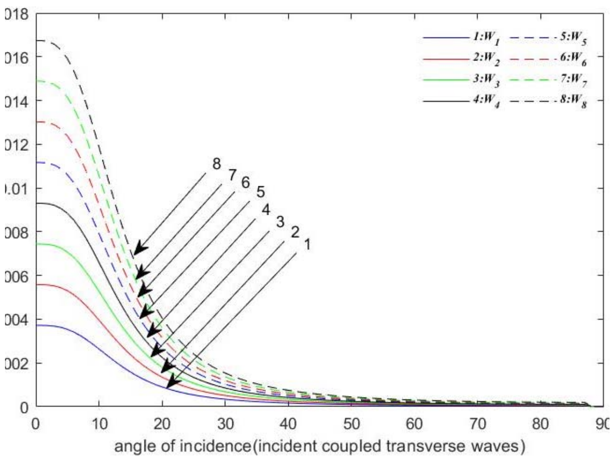

Fig.12. The magnitude of reflection coefficients versus the angle of incidence (incidence of coupled transverse wave) under the CT-theory.

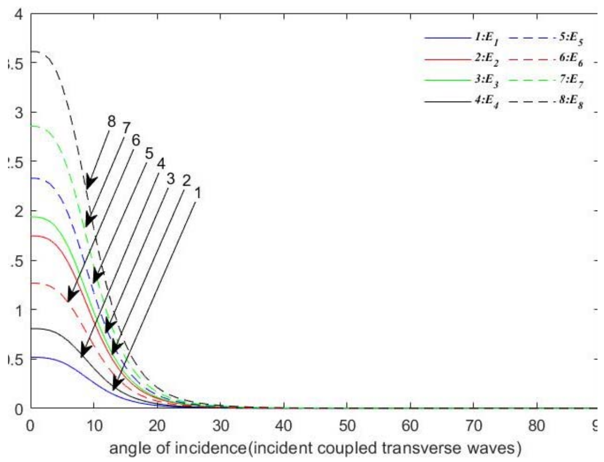

Fig.13. The magnitude of energy coefficients versus the angle of incidence (incidence of coupled transverse wave) under the CT-theory.

Figures 10-11 illustrates that the magnitude of W_s with their corresponding energy ratios E_s $(s = 1, 2 \ldots 8)$ against angle of incidence θ_0 moving continuously and after attaining their maximum value in the middle of angle of incidence, it decreases gradually. All the reflection and refraction coefficients with their corresponding energy coefficients are disappear at grazing as well as normal incidence under GL-theory.

Figures 12-13 illustrates that the magnitude of W_s with their corresponding energy ratios E_s $(s = 1, 2 \ldots 8)$ are decreases continuously with an increasing value of θ_0 for $0^\circ \le \theta_0 \le 90^\circ$. All the reflection and refraction coefficients with their corresponding energy coefficients disappear at normal incidence and attain their maximum value at grazing incidence under CT-theory.

10. Conclusions

We have examined the effect of three theories of thermoelasticity, namely, Green-Lindsay theory (GL), Lord- Shulman theory (LS) and Coupled theory (CT) on the propagation of set of two coupled transverse waves and set of two coupled longitudinal waves in a couple stress micropolar thermoviscous elastic solid medium. We have also examined the reflection and refraction phenomenon of wave propagation in which the plane interface divides a given medium into two half spaces. Here, two relaxation times have been set forth via Kronecker's delta in the equations and constitutive relations for the three theories of thermoelasticity. After reflection and refraction, we find that there exist five waves, namely, longitudinal microrotational wave, set of two coupled transverse waves and set of two coupled longitudinal waves that transmit with various speeds. The major consequences are obtained as follows

1. The velocity, reflection-refraction coefficients with their corresponding energy ratios of coupled waves are calculated.
2. When coupled longitudinal waves are incident, the coefficients of all the coupled longitudinal and transverse waves are vanishing at normal as well as grazing incidence under GL- and CT- theory. But under LS-theory, the refraction coefficients of coupled transverse waves with their energy ratios are vanishing at normal incidence only.
3. When coupled transverse waves are incident, the reflection and refraction coefficients with their corresponding energy ratios are vanishing at normal incidence only and attain their maximum value at grazing incidence under LS- and CT- theory. But under GL- theory, all the coefficients are vanishing at normal as well as grazing incidence and attain their maximum value in the middle of angle of incidence.

Acknowledgement

One of the authors (1, b) is grateful to Deenbandhu Chhotu ram University of Science and Technology for awarding a scholarship in the form of URS. The authors are grateful to reviewers for their suggestions for the improvement of the paper.

NOMENCLATURE

$a K_t$, K'	– material constants
$e_{kl} = u_{l.k} + e_{lkm}\dot{\phi}_{,i}$	– relative tensor
m_{ij}	– couple stress tensor
t_{ij}	– stress tensor
T_0	– ambient temperature
t_0 , t_1	– relaxation times
u	– displacement vector
Y_i	– heat flux vector
$\alpha', \beta', \gamma', \lambda'^{*}, \mu'^{*}$	– material constants

$\alpha'^{*},\beta'^{*},\gamma'^{*},\vartheta$ – material constants

$\gamma_{kl}=\phi_{k,l}$ – wryness tensor

$\lambda',\ \mu'$ – Lame's constants

δ_{ij} – Kronecker's delta

ϕ – microrotation vector

∇^2 – Laplacian operator

η – entropy per unit mass

θ – change in temperature

ρ_0 – density of the medium

Appendix A_1

$$X_1=2\iota\omega K_2-2K_2,\quad X_2=2\zeta_1\zeta_2-\nabla_1\omega^3-2\iota K_2\omega\zeta^*-2\omega^2\zeta_2^*-2\omega^2K_2^*-\omega^2\zeta_2+K_2\zeta_2,$$

$$X_3=\nabla_1\omega^3-\nabla_2\omega^3+\iota\nabla_1^*\omega^4\quad Y_1=1-\Omega,\quad Y_2=(1-\Omega)(\zeta_3-\iota\omega\zeta_3^*)+\nabla_3\nabla_4,$$

$$Y_3=k_1(\zeta_3-\iota\omega\zeta_3^*),\quad \zeta_1=\mu_2+K_2\quad \zeta_1^*=\mu_2^*+K_2^*,\quad \nabla_1=\gamma_2\zeta_1\quad \nabla_1^*=\gamma_2^*\zeta_1^*,\nabla_2=\gamma_2\zeta_1^*,$$

$$\nabla_2^*=\gamma_2^*\zeta_1,\quad \zeta_2=K_2+\iota\omega K_2^*,\quad \zeta_2^*=\gamma_2+\iota\omega\gamma_2^*,\quad \Omega=1-\iota n_1\delta_{1k}\omega,\quad \zeta_3=\lambda_2+\mu_2+K_2,$$

$$\zeta_3^*=\lambda_2^*+\mu_2^*+K_2^*,\quad \nabla_3=\iota\left(\frac{\vartheta_1^2}{\tau_2}+\frac{\vartheta_1^2}{\tau_3}\right)\delta_{1k}\omega^4,\quad \nabla_4=\vartheta_1+\vartheta_2\delta_{2k}\omega^3,\ C=(\alpha_2+\beta_2+\gamma_2)-\iota\omega\left(\alpha_2^*+\beta_2^*+\gamma_2^*\right)$$

$$\zeta_2'^*=\gamma_2'+\iota\omega\gamma_2'^*,\ \zeta_4=\lambda_2+\iota\omega\lambda_2^*,\quad \zeta_5=2\left(\mu_2+\iota\omega\mu_2^*\right)+K_2+\iota\omega K_2^*,\ \zeta_4'=\lambda_2'+\iota\omega\lambda_2'^*,$$

$$h_{11}=1,\quad h_{12}=\left[\zeta_4+\zeta_5\left(1-s_{21}^2\sin^2\theta_0\right)+\frac{\nabla_4\varkappa_2}{l_2^2}\right]/D_1s_{21}^2,\qquad h_{1s}=\zeta_5\sin\theta_0\sqrt{\left(1-s_{s1}^2\sin^2\theta_0\right)}/D_1s_{s1},$$

$$h_{1t}=-\left[\ \zeta_4'+\ \zeta_5'\left(1-S_{t1}'^2\sin^2\theta_0\right)+\nabla'_4\varkappa_t'\right]/D_1S_{t1}'^2,\qquad h_{1q}=\ \zeta_5'\sin\theta_0\sqrt{\left(1-S_{q1}'^2\sin^2\theta_0\right)}/D_1S_{q1}',$$

$$h_{21}=\sin\theta_0\cos\theta_0,h_{22}=\sin\theta_0\sqrt{\left(1-s_{21}^2\sin^2\theta_0\right)}/s_{21},$$

$$h_{2s}=-\left[\zeta_6\left(1-2s_{s1}^2\sin^2\theta_0\right)+\zeta_7\left(1-s_{s1}^2\sin^2\theta_0\right)-\frac{\zeta_7\eta_s}{l_s^2}\right]/D_2s_{s1}^2,\qquad h_{2t}=\zeta_5'\sin\theta_0\sqrt{\left(1-S_{t1}'^2\sin^2\theta_0\right)}/D_2S_{t1}',$$

$$h_{2q}=\left[\ \zeta_6'\left(1-2S_{q1}'^2\sin^2\theta_0\right)+\zeta_7'\left(1-S_{q1}'^2\sin^2\theta_0\right)-\zeta_7'\eta_q'\ \right]/D_2S_{q1}'^2,\qquad h_{33}=\sqrt{1-s_{31}^2\sin^2\theta_0}/s_{31},$$

$$h_{34}=\eta_4\sqrt{1-s_{41}^2\sin^2\theta_0}/\eta_3s_{41},\qquad h_{37}=\zeta_2'^*\eta_3'\sqrt{1-s_{31}'^2\sin^2\theta_0}/s_{31}'\zeta_2^*\eta_3,$$

$$h_{41}=\cos\theta_0,\quad h_{42}=\varkappa_2\sqrt{\left(1-s_{21}^2\sin^2\theta_0\right)}/\varkappa_1s_{21}\quad h_{45}=k_1'\varkappa_1'\sqrt{1-s_{11}'^2\sin^2\theta_0}/k_1\varkappa_1s_{11}',$$

$$h_{46} = k_1' \varkappa_2' \sqrt{1 - s_{21}'^2 \sin^2 \theta_0} \,/\, k_1 \varkappa_1 s_{21}', \quad h_{51} = \sin\theta_0 = h_{52},\ h_{5s} = -\sqrt{\left(1 - s_{s1}^2 \sin^2 \theta_0\right)} \,/\, s_{s1},$$

$$h_{55} = -\sin\theta_0 = h_{56}, \quad h_{5q} = -\sqrt{\left(1 - S_{q1}'^2 \sin^2 \theta_0\right)} \,/\, S_{q1}', \quad h_{61} = \cos\theta_0, \quad h_{62} = \sqrt{\left(1 - s_{21}^2 \sin^2 \theta_0\right)} \,/\, s_{21},$$

$$h_{63} = \sin\theta_0 = h_{64}, \quad h_{6t} = \sqrt{\left(1 - S_{t1}'^2 \sin^2 \theta_0\right)} \,/\, S_{t1}', \quad h_{67} = -\sin\theta_0 = h_{68}, \quad h_{73} = 1,\ h_{74} = \eta_4 / \eta_3,$$

$$h_{77} = -\eta_3' / \eta_3, \quad h_{78} = -\eta_4' / \eta_3, \quad h_{81} = 1, \quad h_{82} = \varkappa_2 / \varkappa_1, \quad h_{85} = -\varkappa_1' / \varkappa_1, \quad h_{86} = -\varkappa_2' / \varkappa_1,$$

$$s = 3,4, \quad t = 5,6, \quad q = 7,8, \quad s_{p1} = \frac{S_p}{S_1} \ (p = 2,3,4), \quad s_{r1}' = \frac{S_r'}{S_1} \ (r = 1,2,3,4),$$

$$S_{51}' = s_{11}', \quad S_{61}' = s_{21}', \quad S_{71}' = s_{31}', \quad S_{81}' = s_{41}', \quad \varkappa_5' = \frac{\varkappa_1'}{l_1'^2}, \quad \varkappa_6' = \frac{\varkappa_2'}{l_2'^2}, \quad \eta_7' = \frac{\eta_3'}{l_3'^2}, \quad \eta_8' = \frac{\eta_4'}{l_4'^2},$$

$$\nabla_4' = \vartheta_1' + \vartheta_2' \delta_{2k} \omega^3, \quad D_1 = \left[\zeta_4 + \zeta_5 \cos^2\theta_0 + \frac{\nabla_4 \varkappa_1}{l_1^2}\right], \quad D_2 = \zeta_5, \quad Z_1 = -1, \quad Z_2 = \sin\theta_0 \cos\theta_0$$

$$Z_3 = 0, \quad Z_4 = \cos\theta_0, \quad Z_5 = -\sin\theta_0, \quad Z_6 = \cos\theta_0, \quad Z_7 = 0, \quad Z_8 = -1, \quad \zeta_6 = \mu_2 + \iota\omega\mu_2^*$$

$$\zeta_5' = 2\left(\mu_2' + \iota\omega'\mu_2'^*\right) + K_2' + \iota\omega' K_2'^*, \quad \zeta_6' = \mu_2' + \iota\omega\mu_2'^*, \quad \zeta_7 = K_2 + \iota\omega K_2^*, \quad \zeta_7' = K_2' + \iota\omega K_2'^*,$$

$$E_1 = -Q_1^2,\ E_2 = Q_2^2 L_1 \left[\zeta_4 + \zeta_5 + \frac{\nabla_4 \varkappa_2}{l_2^2} - \frac{\zeta_7 \varkappa_2^2}{l_2^2}\right] l_2^3 \cos\theta, \quad h_{38} = \zeta_2'^* \eta_4' \sqrt{1 - s_{41}'^2 \sin^2\theta_0} \,/\, s_{41}' \zeta_2^* \eta_3,$$

$$E_{3,4} = Q_{3,4}^2 L_1 \left[\zeta_6 + \zeta_7 - \frac{\eta_{3,4}}{l_{3,4}^2}\left(\zeta_2^* \eta_{3,4} + \zeta_7\right)\right] l_{3,4}^3 \cos\theta_{3,4},$$

$$E_{5,6} = Q_{5,6}^2 L_1 \left[\zeta_4' + \zeta_5' + \frac{\nabla_4' \varkappa_{1,2}'}{l_{1,2}'^2} - \frac{k_1' \varkappa_{1,2}'^2}{l_{1,2}'^2}\right] l_{1,2}'^3 \cos\theta_{1,2}'.$$

$$E_{7,8} = Q_{7,8}^2 L_1 \left[\zeta_6' + \zeta_7' - \frac{\eta_{3,4}'}{l_{3,4}'^2}\left(\zeta_2^* \eta_{3,4}' + \zeta_7'\right)\right] l_{3,4}'^3 \cos\theta_{3,4}', \quad L_1 = \left[\left(\zeta_4 + \zeta_5 + \frac{\nabla_4 \varkappa_1}{l_2^2} - \frac{\zeta_7 \varkappa_1^2}{l_2^2}\right) l_1^3 \cos\theta_1\right]^{-1}.$$

Appendix A_2

$$h_{11} = \left[\zeta_4 + \zeta_5\left(1 - s_{12}^2 \sin^2\theta_0\right) + \frac{\nabla_4 \varkappa_1}{l_2^2}\right] / D_3 s_{12}^2, \quad h_{12} = 1, \quad h_{1s} = \zeta_5 \sin\theta_0 \sqrt{\left(1 - s_{s2}^2 \sin^2\theta_0\right)} \,/\, D_3 s_{s2},$$

$$h_{1t} = -\left[\zeta_4' + \zeta_5'\left(1 - S_{t2}'^2 \sin^2\theta_0\right) + \nabla'_4 \varkappa_t'\right] / D_3 S_{t2}'^2, \quad h_{1q} = \zeta_5' \sin\theta_0 \sqrt{\left(1 - S_{q2}'^2 \sin^2\theta_0\right)} \,/\, D_1 S_{q2}'$$

$$h_{21} = \sin\theta_0 \sqrt{\left(1 - s_{12}^2 \sin^2\theta_0\right) \backslash Big} \,/\, s_{12}, \quad h_{22} = \sin\theta_0 \cos\theta_0$$

$$h_{2s} = -\left[\zeta_6\left(1-2s_{s2}^2\sin^2\theta_0\right)+\zeta_7\left(1-s_{s2}^2\sin^2\theta_0\right)-\frac{\zeta_7\eta_s}{l_s^2}\right]/D_2 s_{s2}^2,$$

$$h_{2t} = \zeta_5'\sin\theta_0\sqrt{\left(1-S_{t2}'^2\sin^2\theta_0\right)}/D_2 S_{t2}', \qquad h_{33} = \sqrt{1-s_{32}^2\sin^2\theta_0}/s_{32},$$

$$h_{2q} = \left[\zeta_6'\left(1-2S_{q2}'^2\sin^2\theta_0\right)+\zeta_7'\left(1-S_{q2}'^2\sin^2\theta_0\right)-\zeta_7'\eta_q'\right]/D_2 S_{q2}'^2,$$

$$h_{34} = \eta_4\sqrt{1-s_{42}^2\sin^2\theta_0}/\eta_3 s_{42}, \qquad h_{37} = \zeta_2'^*\eta_3'\sqrt{1-s_{32}'^2\sin^2\theta_0}/s_{32}'\zeta_2^*\eta_3,$$

$$h_{41} = \varkappa_1\sqrt{\left(1-s_{12}^2\sin^2\theta_0\right)}/\zeta_2 s_{12}, \quad h_{42} = \cos\theta_0, \quad h_{45} = k_1'\varkappa_1'\sqrt{1-s_{12}'^2\sin^2\theta_0}/k_1\varkappa_1 s_{12}',$$

$$h_{46} = k_1'\varkappa_2'\sqrt{1-s_{22}'^2\sin^2\theta_0}/k_1\varkappa_1 s_{21}', \qquad h_{51} = \sin\theta_0 = h_{52}, \qquad h_{5s} = -\sqrt{\left(1-s_{s2}^2\sin^2\theta_0\right)}/s_{s2},$$

$$h_{55} = -\sin\theta_0 = h_{56}, \qquad h_{5q} = -\sqrt{\left(1-S_{q2}'^2\sin^2\theta_0\right)}/S_{q2}', \quad h_{61} = \sqrt{\left(1-s_{12}^2\sin^2\theta_0\right)}/s_{12}, \qquad h_{62} = \cos\theta_0$$

$$h_{63} = \sin\theta_0 = h_{64}, \qquad h_{6t} = \sqrt{\left(1-S_{t2}'^2\sin^2\theta_0\right)}/S_{t2}', \qquad h_{67} = -\sin\theta_0 = h_{68}, \qquad h_{73} = 1, \qquad h_{74} = \eta_4/\eta_3,$$

$$h_{77} = -\eta_3'/\eta_3, \quad h_{78} = -\eta_4'/\eta_3, \quad h_{81} = \varkappa_1/\varkappa_2, \quad h_{82} = 1, \qquad h_{38} = \zeta_2'^*\eta_4'\sqrt{1-s_{42}'^2\sin^2\theta_0}/s_{42}'\zeta_2^*\eta_3,$$

$$h_{85} = -\varkappa_1'/\varkappa_1, \qquad h_{86} = -\varkappa_2'/\varkappa_1 \qquad s_{p2} = \frac{S_p}{S_2} \quad (p=1,3,4), \quad s_{r2}' = \frac{S_r'}{S_2} \quad (r=1,2,3,4),$$

$$S_{52}' = s_{12}', \qquad S_{62}' = s_{22}', \qquad S_{72}' = s_{32}', \qquad S_{82}' = s_{42}', \qquad D_3 = \left[\zeta_4+\zeta_5\cos^2\theta_0+\frac{\nabla_4\varkappa_2}{l_2^2}\right],$$

$$E_1 = Q_1^2 L_2\left[\zeta_4+\zeta_5+\frac{\nabla_4\varkappa_1}{l_1^2}-\frac{\zeta_7\varkappa_1^2}{l_1^2}\right]l_1^3\cos\theta_1, \qquad E_{3,4} = Q_{3,4}^2 L_2\left[\zeta_6+\zeta_7-\frac{\eta_{3,4}}{l_{3,4}^2}\left(\zeta_2^*\eta_{3,4}+\zeta_7\right)\right]l_{3,4}^3\cos\theta_{3,4}$$

$$E_2 = Q_2^2, \qquad E_{5,6} = Q_{5,6}^2 L_2\left[\zeta_4'+\zeta_5'+\frac{\nabla_4'\varkappa_{1,2}'}{l_{1,2}'^2}-\frac{k_1'\varkappa_{1,2}'^2}{l_{1,2}'^2}\right]l_{1,2}'^3\cos\theta_{1,2}',$$

$$E_{7,8} = Q_{7,8}^2 L_2\left[\zeta_6'+\zeta_7'-\frac{\eta_{3,4}'}{l_{3,4}'^2}\left(\zeta_2^*\eta_{3,4}'+\zeta_7'\right)\right]l_{3,4}'^3\cos\theta_{3,4}', \qquad L_2 = \left[\left(\zeta_4+\zeta_5+\frac{\nabla_4\varkappa_2}{l_2^2}-\frac{\zeta_7\varkappa_2^2}{l_2^2}\right)l_2^3\cos\theta_2\right]^{-1}.$$

Appendix B_1

$$U_{1s} = \left[\zeta_4+\zeta_5\left(1-s_{s3}^2\sin^2\theta_0\right)+\frac{\nabla_4\varkappa_s}{l_s^2}\right]/D_2 s_{s3}^2, \qquad U_{13} = \sin\theta_0\cos\theta_0, \qquad U_{14} = \sin\theta_0\sqrt{\left(1-s_{43}^2\sin^2\theta_0\right)}/s_{43},$$

$$U_{1t} = -\left[\zeta_4'+\zeta_5'\left(1-S_{t3}'^2\sin^2\theta_0\right)+\nabla'_4\varkappa_t'\right]/D_2 S_{t3}'^2, \qquad U_{1q} = \zeta_5'\sin\theta_0\sqrt{\left(1-S_{q3}'^2\sin^2\theta_0\right)}/D_2 S_{q3}',$$

$$U_{2s} = \zeta_5 \sin\theta_0 \sqrt{\left(1 - s_{s3}^2 \sin^2\theta_0\right)} / D_4 s_{s3}, \quad U_{23} = -1, \quad U_{2t} = \zeta_6' \sin\theta_0 \sqrt{\left(1 - S_{t3}'^2 \sin^2\theta_0\right)} / D_4 S_{t3}',$$

$$U_{24} = -\left[\zeta_6\left(1 - 2s_{43}^2 \sin^2\theta_0\right) + \zeta_7\left(1 - s_{43}^2 \sin^2\theta_0\right) - \frac{\zeta_7\eta_4}{l_4^2}\right] / D_4 s_{43}^2, \quad U_{33} = \cos\theta_0,$$

$$U_{2q} = \left[\zeta_6'\left(1 - 2S_{q3}'^2 \sin^2\theta_0\right) + \zeta_7'\left(1 - S_{q3}'^2 \sin^2\theta_0\right) - \zeta_7'\eta_q'\right] / D_4 S_{q3}'^2, \quad U_{34} = \eta_4 \sqrt{1 - s_{43}^2 \sin^2\theta_0} / \eta_3 s_{43},$$

$$U_{37} = \zeta_2'^{*}\eta_3' \sqrt{1 - s_{33}'^2 \sin^2\theta_0} / s_{33}'\zeta_2^{*}\eta_3, \qquad U_{38} = \zeta_2'^{*}\eta_4' \sqrt{1 - s_{43}'^2 \sin^2\theta_0} / s_{43}'\zeta_2^{*}\eta_3,$$

$$U_{51} = \sin\theta_0 = U_{52}, \qquad U_{4s} = \sqrt{\left(1 - s_{s3}^2 \sin^2\theta_0\right)} / s_{s3}, \qquad U_{45} = \zeta_2'^{*}\varkappa_1' \sqrt{1 - s_{13}'^2 \sin^2\theta_0} / \zeta^{*}\varkappa_1 s_{13}',$$

$$U_{53} = -\cos\theta_0, \qquad U_{46} = k_1'\zeta_2' \sqrt{1 - s_{23}'^2 \sin^2\theta_0} / k_1\zeta_1 s_{23}', \qquad U_{54} = -\sqrt{\left(1 - s_{43}^2 \sin^2\theta_0\right)} / s_{43},$$

$$U_{55} = -\sin\theta_0 = U_{56}, \qquad U_{5q} = -\sqrt{\left(1 - S_{q3}'^2 \sin^2\theta_0\right)} / S_{q3}', \qquad U_{6s} = -\sqrt{\left(1 - s_{s3}^2 \sin^2\theta_0\right)} / s_{s3},$$

$$h_{63} = -\sin\theta_0 = U_{64}, \qquad U_{6t} = -\sqrt{\left(1 - S_{t3}'^2 \sin^2\theta_0\right)} / S_{t3}', \qquad U_{67} = \sin\theta_0 = U_{68},$$

$$U_{73} = 1, \quad U_{74} = \eta_4 / \eta_3, \quad U_{77} = -\eta_3' / \eta_3, \qquad U_{78} = -\eta_4' / \eta_3, \qquad U_{81} = 1, \qquad U_{82} = \varkappa_2 / \varkappa_1,$$

$$U_{85} = -\varkappa_1' / \varkappa_1, \qquad U_{86} = -\varkappa_2' / \varkappa_1, \qquad W_1 = \sin\theta_0 \cos\theta_0, \qquad W_2 = 1, \qquad W_3 = \cos\theta_0,$$

$$W_4 = 0, \qquad W_5 = -\cos\theta_0, \qquad W_6 = \sin\theta_0, \quad W_7 = -1, \quad W_8 = 0.$$

Here now

$$s = 1,2; \qquad s_{p3} = \frac{S_p}{S_3} \quad (p = 1,2,4), \qquad s_{r3}' = \frac{S_r'}{S_3} \quad (r = 1,2,3,4), \qquad S_{53}' = s_{13}', \quad S_{63}' = s_{23}',$$

$$S_{73}' = s_{33}', \qquad S_{83}' = s_{43}', \qquad D_4 = -\left[\zeta_6 \cos 2\theta_0 + \zeta_7 \cos^2\theta_0 - \frac{\zeta_7\eta_3}{l_3^2}\right],$$

$$E_{1,2} = W_{1,2}^2 L_3\left[\zeta_4 + \zeta_5 + \frac{\nabla_4\varkappa_{1,2}}{l_{1,2}^2} - \frac{\zeta_7\varkappa_{1,2}^2}{l_{1,2}^2}\right] l_{1,2}^3 \cos\theta_{1,2}, \qquad E_3 = W_3^2,$$

$$E_4 = W_4^2 L_3\left[\zeta_6 + \zeta_7 - \frac{\eta_4}{l_4^2}\left(\zeta_2^{*}\eta_4 + \zeta_7\right)\right] l_4^3 \cos\theta_4, \qquad E_{5,6} = W_{5,6}^2 L_3\left[\zeta_4' + \zeta_5' + \frac{\nabla_4'\varkappa_{1,2}'}{l_{1,2}'^2} - \frac{k_1'\varkappa_{1,2}'^2}{l_{1,2}'^2}\right] l_{1,2}'^3 \cos\theta_{1,2}',$$

$$E_{7,8} = W_{7,8}^2 L_3\left[\zeta_6' + \zeta_7' - \frac{\eta_{3,4}'}{l_{3,4}'^2}\left(\zeta_2^{*}\eta_{3,4}' + \zeta_7'\right)\right] l_{3,4}'^3 \cos\theta_{3,4}', \qquad L_3 = \left[\left(\zeta_6 + \zeta_7 - \frac{\eta_3}{l_3^2}\left(\zeta_2^{*}\eta_3 + \zeta_7\right)\right) l_3^3 \cos\theta_0\right]^{-1}.$$

Appendix B_2

$$U_{1s} = \left[\zeta_4 + \zeta_5\left(1 - s_{s4}^2 \sin^2\theta_0\right) + \frac{\nabla_4 \varkappa_s}{l_s^2}\right] / D_2 s_{s4}^2, \qquad U_{13} = \sin\theta_0 \sqrt{\left(1 - s_{34}^2 \sin^2\theta_0\right)} / s_{34},$$

$$U_{14} = \sin\theta_0 \cos\theta_0, \qquad U_{1t} = -\left[\zeta_4' + \zeta_5'\left(1 - S_{t4}'^2 \sin^2\theta_0\right) + \nabla'_4 \varkappa_t'\right] / D_2 S_{t4}'^2, \qquad U_{63} = -\sin\theta_0 = U_{64},$$

$$U_{1q} = \zeta_5' \sin\theta_0 \sqrt{\left(1 - S_{q4}'^2 \sin^2\theta_0\right)} / D_2 S_{q4}', \qquad U_{2s} = \zeta_5 \sin\theta_0 \sqrt{\left(1 - s_{s4}^2 \sin^2\theta_0\right)} / D_4 s_{s4},$$

$$U_{23} = -\left[\zeta_6\left(1 - 2s_{34}^2 \sin^2\theta_0\right) + \zeta_7\left(1 - s_{34}^2 \sin^2\theta_0\right) - \frac{\zeta_7 \eta_3}{l_3^2}\right] / D_3 s_{34}^2, \qquad U_{4s} = \sqrt{\left(1 - s_{s3}^2 \sin^2\theta_0\right)} / s_{s3},$$

$$U_{24} = -1, \; U_{2t} = \zeta_6' \sin\theta_0 \sqrt{\left(1 - S_{t4}'^2 \sin^2\theta_0\right)} / D_4 S_{t4}', \qquad U_{33} = \eta_3 \sqrt{1 - s_{34}^2 \sin^2\theta_0} / \eta_4 s_{44}, \qquad U_{34} = \cos\theta_0$$

$$U_{2q} = \left[\zeta_6'\left(1 - 2S_{q4}'^2 \sin^2\theta_0\right) + \zeta_7'\left(1 - S_{q4}'^2 \sin^2\theta_0\right) - \zeta_7' \eta_q'\right] / D_4 S_{q4}'^2, \qquad U_{51} = \sin\theta_0 = U_{52},$$

$$U_{37} = \zeta_2'^{*} \eta_3' \sqrt{1 - s_{34}'^2 \sin^2\theta_0} / s_{34}' \zeta_2^{*} \eta_4, \qquad U_{38} = \zeta_2'^{*} \eta_4' \sqrt{1 - s_{44}'^2 \sin^2\theta_0} / s_{44}' \zeta_2^{*} \eta_4, \qquad U_{54} = -\cos\theta_0,$$

$$U_{45} = \zeta_2'^{*} \varkappa_1' \sqrt{1 - s_{14}'^2 \sin^2\theta_0} / \zeta^{*} \varkappa_1 s_{14}', \qquad U_{46} = k_1' \zeta_2' \sqrt{1 - s_{24}'^2 \sin^2\theta_0} / k_1 \zeta_1 s_{24}', \qquad U_{55} = -\sin\theta_0 = U_{56},$$

$$U_{53} = -\sqrt{\left(1 - s_{34}^2 \sin^2\theta_0\right)} / s_{34}, \qquad U_{5q} = -\sqrt{\left(1 - S_{q4}'^2 \sin^2\theta_0\right)} / S_{q4}', \qquad U_{6s} = -\sqrt{\left(1 - s_{s4}^2 \sin^2\theta_0\right)} / s_{s4},$$

$$U_{6t} = -\sqrt{\left(1 - S_{t4}'^2 \sin^2\theta_0\right)} / S_{t4}', \qquad U_{67} = \sin\theta_0 = U_{68}, \qquad U_{73} = \eta_3 / \eta_4, \qquad U_{74} = 1,$$

$$U_{77} = -\eta_3' / \eta_4, \qquad U_{78} = -\eta_4' / \eta_4, \; U_{81} = 1, \qquad U_{82} = \varkappa_2 / \varkappa_1, \qquad U_{85} = -\varkappa_1' / \varkappa_1, \qquad U_{86} = -\varkappa_2' / \varkappa_1.$$

Here now $s = 1,2; \quad s_{p4} = \dfrac{S_p}{S_4} \quad (p = 1,2,3),$

$$s_{r4}' = \frac{S_r'}{S_4}, (r = 1,2,3,4), \quad S_{54}' = s_{14}', \quad S_{64}' = s_{24}', \quad S_{74}' = s_{34}', \quad S_{84}' = s_{44}',$$

$$D_3 = -\left[\zeta_6 \cos 2\theta_0 - \zeta_7 \cos^2\theta_0 - \frac{\zeta_7 \eta_4}{l_4^2}\right], \qquad E_{1,2} = W_{1,2}^2 \mathrm{L}_4 \left[\zeta_4 + \zeta_5 + \frac{\nabla_4 \varkappa_{1,2}}{l_{1,2}^2} - \frac{\zeta_7 \varkappa_{1,2}^2}{l_{1,2}^2}\right] l_{1,2}^3 \cos\theta_{1,2},$$

$$E_3 = W_3^2 \mathrm{L}_4 \left[\zeta_6 + \zeta_7 - \frac{\eta_4}{l_4^2}\left(\zeta_2^{*} \eta_4 + \zeta_7\right)\right] l_4^3 \cos\theta_4, \qquad E_4 = W_4^2, \qquad L_4 = \left[\left(\zeta_6 + \zeta_7 - \frac{\eta_4}{l_4^2}\left(\zeta_2^{*} \eta_4 + \zeta_7\right)\right) l_4^3 \cos\theta_0\right]^{-1},$$

$$E_{5,6} = W_{5,6}^2 L_4 \left[\zeta_4' + \zeta_5' + \frac{\nabla_4' \varkappa_{1,2}'}{l_{1,2}'^2} - \frac{k_1' \varkappa_{1,2}'^2}{l_{1,2}'^2}\right] l_{1,2}'^3 \cos\theta_{1,2}'$$

$$E_{7,8} = W_{7,8}^2 L_4 \left[\zeta_6' + \zeta_7' - \frac{\eta_{3,4}'}{l_{3,4}'^2} \left(\zeta_2^* \eta_{3,4}' + \zeta_7' \right) \right] l_{3,4}'^3 \cos\theta_{3,4}' .$$

Reference

[1] Eringen A.C. (1966): *Linear theory of micropolar elasticity.*– J. of Mech. and Mat. Struct., vol.15, pp.909-923.

[2] Eringen A.C. (1970): *Foundations of micropolar thermoelasticity.*– Int. J. Eng. Sci., vol.23, Springen-Verlag, Berlin.

[3] Nowacki W. (1986): *Theory of asymmetric elasticity.*– Oxford, Pergamon.

[4] Dost S. and Taborrok B. (1978): *Generalised micropolar thermoelasticity.*– Int. J. Eng. Sci., vol.16, pp.173-178.

[5] Green A.E. and Lindsay K.A. (1972): *Thermoelasticity.*– J. of Elas., vol.2, pp.1-7

[6] Chandersekharaiah D.S. (1986): *Heat flux dependent micropolar thermoelasticity.*– Int. J. Eng. Sci., vol.24, pp.1389-1395.

[7] Chandrasekhariah D.S. (1998): *Hyperbolic thermoelasticity.*– Appl. Mech. Rev., vol.51, pp.705-729.

[8] Farka I.s and Szekers A. (1984): *Application of modified law of heat conduction and state equation to dynamical problem of thermoelasticity.*– Periodica Poly. Mech. Eng., vol.28, pp.163-170.

[9] Szekers A. (1980): *Equation system of thermoelasticity using the modified law of thermal conductivity.*– Periodica Poly. Mech. Eng., vol.24, pp.253-261.

[10] Biot A.M. (1965): *Mechanics of incremental deformations.*– John Wiley and Sons, New York, p.516.

[11] Hetnarski R.B. and Ignaczak J. (1999): *Generalised thermoelasticity.*– J. of Ther. Str., vol.22, pp.451-476.

[12] Lord H. and Shulman Y. (1967): *A generalised dynamical theory of thermoelasticity.*– J. of Mech. and Phy. of Solids, vol.15, pp.299-309.

[13] Hetnarski R.B. and Ignaczak J. (1996): *Solution like wave in a low temperature nonlinear thermoelastic solid.*– Int. J. Eng. Sci., vol.4, pp.1767-1787.

[14] Green A.E. and. Naghdi P.M. (1993): *Thermoelasticity without energy dissipation.*– J. Elast., vol.31, pp.189-208.

[15] Tomar S.K. and Khurana A. (2008): *Transmission of longitudinal wave at plane interface between micropolar elastic and chiral solid half-spaces: incident from micropolar half-spaces.*– J. Of Sound and Vib., vol.311, pp. 973-990.

[16] Tomar S. K. and Khurana A. (2017): *Waves in non-elastic solids with voids*.– Int. J. of Elasticity, vol.128, pp.85-114.

[17] Tomar S. K. and Khurana A. (2018): *Waves at the interface of dissimilar nonlocal micropolar elastic half-spaces.*– Mech. of ad. Mater. and struct., vol.26, pp.825-833.

[18] Tomar S.K. and Gogna M.L. (1992): *Reflection and refraction of longitudinal wave micro-rotational at the interface between two micropolar elastic media in welded contact.*– Int. J. Engng. Sci., vol.30, pp.1637-1646.

[19] Tomar S.K. and Gogna M.L. (1995): *Reflection and refraction of longitudinal waves at the interface between two micropolar elastic media in welded contact.*– J. Acoust. Soc. Am., vol.97, pp.822-830.

[20] Parfitt V.R. and. Eringen A.C. (1969): *Reflection of plane waves from the flat boundary of a micropolar elastic half-space.*– J. Acoust. Soc. Am., vol.45, pp.1258-1272.

[21] Tomar S.K. (2015): *Wave propagation in local and nonlocal microstretch elastic media.*– Mec. of ad. Mater. and Struct., vol.84, pp.1-23.

[22] Singh D., Rani N. and Tomar S.K. (2016): *Dilatational waves at a microstretch solid/fluid interface.*– J. Vibr. Cont., vol.23, pp.3448-3467.

[23] Zhang P., Wei P. and Tang Q. (2015): *Reflection of micropolar elastic waves at the non-free surface of a micropolar elastic half-space.*– Acta Mech., vol.226, pp.2925-2937.

[24] Zhang P., Wei P. and Tang Q. (2015): *Reflection of micropolar elastic waves at the non-free surface of a micropolar elastic half-space.*– Acta mech., vol.226, pp.2925-2937.

[25] Zhang P., Wei P. and Tang Q. (2017): *Reflection of longitudinal displacement wave at the visco-elastically supported boundary of a micropolar half space.*– Meccanica, vol.52, pp.1641-1654.

[26] Achenbach J.D. (1973): *Wave propagation in elastic solids.*– North Holland, Amsterdam.

[27] Sarkar N. and Tomar S.K. (2019): *Plane waves in nonlocal thermoelastic solid with voids.*– J. of Thermal Stresses, vol.42, pp.580-606.

[28] Sahrawat R.K., Poonam and Kumar K. (2020): *Wave propagation in nonlocal couple stress thermoelastic solid*.– AIP Conf. Proceed., vol.2253, pp.1-14.

[29] Poonam, Sahrawat R.K., Kumar K. and Arti (2021): *Plane wave propagation in functionally graded isotropic couple stress thermoelastic solid media under initial stress and gravity*.– Eur. Phys. J. Plus, vol.136, pp.1-32.

[30] Sahrawat R.K., Poonam and Kumar K. (2021): *Plane wave and fundamental solution in non-local couple stress micropolar thermoelastic solid without energy dissipation*.– J. of Ther. Stress, vol.44, pp.295-314.

[31] Tomar S.K., Goyal N. and Szekers A. (2019): *Plane waves in thermo-viscoelastic material with voids under different theories of thermoelasticity*.– Int. J. of Applied Mechanics and Engineering, vol.24, No.3, pp.691-708.

[32] Kumar B., Kumar R. and Kaushal S. (2017): *Viscosity effect on reflection and transmission coefficients between two micropolar viscothermoelastic half spaces with three-phase legs*.– Int. J. of Eng. Sci. and Res. Tech., vol.9, pp.2348-2369.

Received: September 19, 2020

Revised: November 24, 2020