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A NEW APPROACH FOR STUDY THE ELECTROHYDRODYNAMIC OSCILLATORY FLOW THROUGH A POROUS MEDIUM IN A HEATING COMPLIANT CHANNEL

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The governing equations of an electrohydrodynamic oscillatory flow were simplified, using appropriate non-dimensional quantities and the conversion relationships between fixed and moving frame coordinates. The obtained system of equations is solved analytically by using the regular perturbation method with a small wave number. In this study, modified non-dimensional quantities were used that made fluid pressure in the resulting equations dependent on both axial and vertical coordinates. The current study is more realistic and general than the previous studies in which the fluid pressure is considered functional only in the axial coordinate. A new approach enabled the author to find an analytical form of fluid pressure while previous studies have not been able to find it but have found only the pressure gradient. Analytical expressions for the stream function, electrical potential function and temperature distribution are obtained. The results show that the electrical potential function decreases by the increase of the Prandtl number, secondary wave amplitude ratio and width of the channel.

Key words: oscillatory flow, electrohydrodynamic, porous medium, heat transfer, compliant channel.

1. Introduction

Studying the movement of a viscous liquid through a channel with moving walls is one of the important topics that have attracted the attention of scientists for a long time due to many applications in different areas of life. This movement appears in transporting many physiological fluids in the body in various situations such as transport of urine through the ureter, transport of spermatozoa, transport of contents of the gastrointestinal passage and blood pumps in heart-lung machines. Latham [1] was the first to introduce the experimental study of this movement in which he studied the movement of urine passing through the ureter from kidneys to the bladder. He determined the critical value of the gradient in pressure (the value at which the urine moves in the opposite direction from the bladder to the kidney). A year after this study, Fung and Yih [2] presented the theoretical framework for this study. They examined the movement of a viscous fluid inside a channel with periodically contracting and expanding walls. The expansion and contraction of the walls generate successive sine waves that move the fluid inside the channel. They considered the amplitude of the channel wave to be much less than half the width of the channel and were able by this approximation to use the perturbation method to find the critical value of the pressure gradient. They also found that it depends on both the Reynolds number and wave number. In the same year, Shapiro *et al.* [3] studied the same problem, but assumed that the wavelength of the wave generated on the wall is large and the Reynolds number is small. They described the problem using two sets of coordinates; one fixed in the vacuum and the other moving at the same speed of the wall. They were able to set the equations of conversion between these coordinates and were able to study the issue using this new type of coordinates, where the problem does not depend on time, and using the long wavelength approximation. They studied new natural phenomena that were not studied before such as the phenomenon of trapping and the pressure rise. The studies were then diversified and new studies emerged as a generalization of this study by introducing some additions and modifications. Some of these studies [4-6] dealt with different forms of channel walls. Some of them considered the wall a flexible membrane and others considered the wall a compliant wall so that it would be closer to the surface of the vital vessels. These studies have shown the effect of wall parameters such as wall mass, wall elasticity and wall hardness on fluid movement. Other studies [7-9] have examined the effect of the existence of external fields on this movement such as the effect of the existence of constant or variable magnetic field on the movement of fluid inside a horizontal, vertical or inclined channel and some of these studies have examined also the effect of the existence of thermal and mass transmission. Recently, there have been few studies [10-12] that dealt with this issue in the presence of an electric field. In some of them, the researcher studied the effect of a variable electric field on the motion of a Newtonian fluid through a channel whose walls generate successive waves and in the presence of heat transfer. Due to the difficulty of such type of study, we considered that the amplitude of the wall wave is smaller in relation to half the width of the channel and therefore the ratio between them is a very small parameter called the amplitude ratio and the method of perturbation was used in terms of this small parameter to solve the differential equations that govern the motion. Then, the researcher also used the method of perturbation was used again in terms of the wave number this time to solve the resulting system of the first order and the solution method was very stressful and difficult. This has led the current research to develop the method of solving the previous issues by selecting new non-dimensional quantities that enabled us to use the method of perturbation in terms of wave number only once. The new method of solution is much simpler than the previous two studies, although in this study we took into account the existence of porous media and the channel wall of the compliant type. We hope that this new approach will simplify the study of difficult problems that we could not have been studied before.

2. Formulation of the problem

We consider a symmetric two-dimensional compliant channel of uniform width 2d filled with a viscous incompressible dielectric fluid. We assume an infinite sinusoidal wave train traveling along the walls with velocity C (see Fig.1).

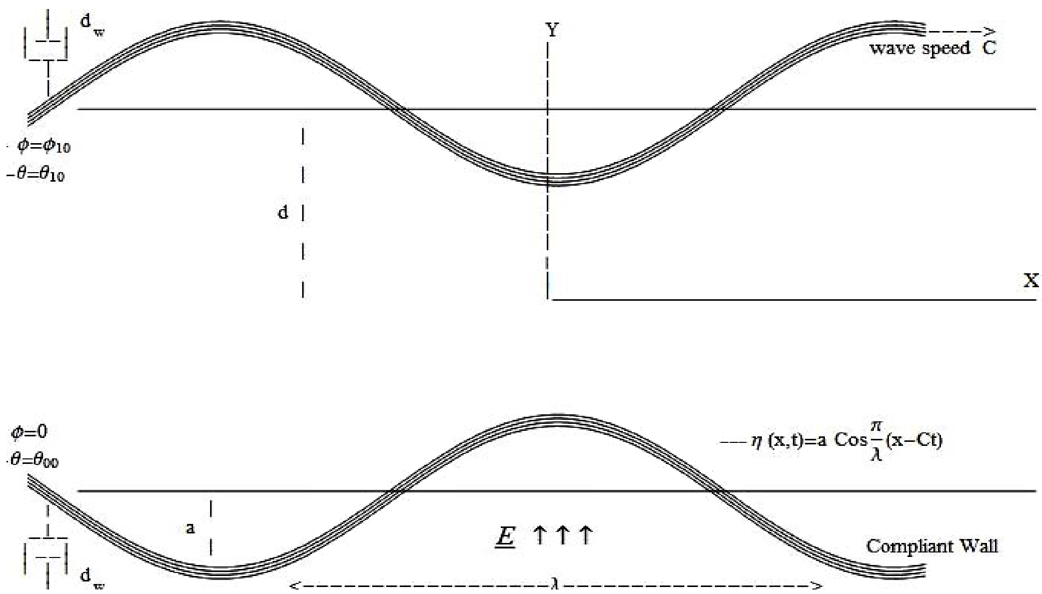

Fig.1. Geometry of the problem.

The lower and the upper walls are maintained at constant temperatures θ_{00} and θ_{10}, respectively. In addition to a temperature gradient, a vertical variable electric field is also imposed across the channel. The lower wall is grounded and the upper wall is kept at the electrical potential $\varnothing_{10}$. If we assume that the density of the fluid ρ is constant (corresponding to the assumption that the vertical flow due to buoyancy is neglected), the continuity equation is

$$\nabla \cdot \underline{V} = 0. \tag{2.1}$$

The equation of motion is

$$\rho\left(\frac{\partial \underline{V}}{\partial t} + \underline{V} \cdot \nabla \underline{V}\right) = -\nabla P + \mu \nabla^2 \underline{V} - \frac{\mu}{k1}\underline{V} + \underline{f_e}, \tag{2.2}$$

where P is the pressure, μ is the viscosity, $k1$ is the permeability of the medium, $\underline{V}$ =(U, V, 0)is the velocity of the fluid and $\underline{f_e}$ is the body force of electrical origin per unit volume which may be expressed as [13]

$$\overline{f_e} = \rho_e \underline{E} - \frac{1}{2}E^2 \nabla \epsilon + \frac{1}{2}\nabla\left(\rho \frac{\partial \epsilon}{\partial \rho} E^2\right) \tag{2.3}$$

where $\underline{E}$ is the electric field, ρ_e is the free charge density and ϵ is the dielectric constant. The first term in (3) has been neglected, since the free charge density can be assumed to be zero and the last term can be included in the pressure term.

The equation of energy, neglecting the dissipation terms [14], is

$$\frac{\partial T}{\partial t} + \underline{V} \cdot \nabla T = k \nabla^2 T \tag{2.4}$$

where T is the temperature and k is the thermometric conductivity.

Since there is no free charge, Maxwell's equations are

$$\nabla \cdot (\epsilon \underline{E}) = 0 , \tag{2.5}$$

$$\nabla \times \underline{E} = 0 \qquad \text{or} \qquad \underline{E} = -\nabla \varnothing , \tag{2.6}$$

where $\varnothing$ is the electrical potential function. The dielectric constant ϵ is assumed to be a function of temperature as follows [15]

$$\epsilon = \epsilon_0 \left[1 - e(T - \theta_{00})\right]. \tag{2.7}$$

Substituting into (2.3) and (2.5) we get

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{1}{\rho}\frac{\partial P^*}{\partial X} + \upsilon \nabla^2 U - \frac{\upsilon}{k1}U + \frac{\epsilon_0 e}{2\rho}\left(\frac{\partial \varnothing}{\partial Y}\right)^2 \frac{\partial T}{\partial X} , \tag{2.8}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{1}{\rho}\frac{\partial P^*}{\partial Y} + \upsilon \nabla^2 V - \frac{\upsilon}{k1}V + \frac{\epsilon_0 e}{2\rho}\left(\frac{\partial \varnothing}{\partial Y}\right)^2 \frac{\partial T}{\partial Y} , \tag{2.9}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=k\nabla^2 T, \tag{2.10}$$

$$\frac{\partial}{\partial Y}\left(\left[1-e\left(T-\theta_{00}\right)\right]\left[\frac{\partial \varnothing}{\partial Y}\right]\right)=0, \tag{2.11}$$

where $P^*=P-\frac{1}{2}\left(\rho\frac{\partial\epsilon}{\partial\rho}E^2\right)$ is the modified pressure.

The compliant wall is modelled as a spring-backed plate, it is constrained to move only in the vertical direction. Letting the vertical displacement of the compliant wall be $\eta(X,t)$, the equation of motion of the compliant wall can be written as [16].

$$\left(m_w\frac{\partial^2}{\partial t^2}+d_w\frac{\partial}{\partial t}+b_w\frac{\partial^4}{\partial X^4}-t_w\frac{\partial^2}{\partial X^2}+k_w\right)\eta(X,t)=P-P_{00}\ . \tag{2.12}$$

The vertical movement of the compliant wall will result in a progressive wave of area contraction or expansion along the length of the flexible channel containing the fluid. This will generate sine waves along the channel walls. Therefore, the vertical displacement of the compliant wall $\eta(X,t)$ is assumed to be in the form of a sinusoidal wave of amplitude a, wavelength λ and wave speed C. Thus, $\eta(X,t)$ may be expressed as

$$\eta(X,t)=a\cos\left[\frac{\pi}{\lambda}(X-\mathrm{C}t)\right]. \tag{2.13}$$

The horizontal displacement will be assumed to be zero. Hence the boundary conditions are

$$\begin{aligned} &U=0, \quad V=\frac{\partial\eta}{\partial t}, \quad T=\theta_{10}, \quad \varnothing=\varnothing_{10}, \quad \text{at} \quad Y=d+\eta, \\ &U=0, \quad V=-\frac{\partial\eta}{\partial t}, \quad T=\theta_{00}, \quad \varnothing=0, \quad \text{at} \quad Y=-d-\eta. \end{aligned} \tag{2.14}$$

We shall carry out this investigation in a coordinate system moving with the wave speed, in which the boundary shape is stationary. The coordinates and velocities in the laboratory frame (X,Y) and the wave frame (x,y) are related by

$$x=X-\mathrm{C}t, \quad y=Y, \quad u=U-C, \quad \text{and} \quad v=V \tag{2.15}$$

where (u,v) and (U,V) are velocity components in the wave and fixed frame of reference respectively. We employ these transformations in the governing equations of motion and then introduce the following dimensionless variables and parameters

$$\bar{x}=\frac{\pi x}{\lambda};\quad \bar{y}=\frac{y}{d};\quad \bar{u}=\frac{u}{C};\quad \bar{v}=\frac{v}{C};\quad \bar{\eta}=\frac{\eta}{d};\quad \bar{t}=\frac{\pi Ct}{\lambda};\quad \overline{p^*}=\frac{p^*}{\rho C^2};\quad \bar{\psi}=\frac{\psi}{Cd};$$

$$\bar{\theta}=\frac{T}{\beta d};\quad \bar{\varnothing}=\frac{\varnothing}{E_0 d};\quad \gamma=\frac{L}{L_1 RR};\quad K1=\frac{k1}{d^2};\quad L_1=e\beta d;\quad A0=\frac{a}{d};\quad \delta=\frac{\pi d}{\lambda}; \tag{2.16}$$

$$\Pr=\frac{\mu}{\rho k};\quad RR=\frac{Cd}{\upsilon};\quad L=\frac{\epsilon_0 E_0^2 d^3 e^2\beta^2}{C};\quad \beta=\frac{\theta_{00}-\theta_{10}}{2d}\ .$$

Using the stream function $\psi(x,y)$, the governing equations become

$$\delta\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial^2\psi}{\partial x\partial y}\right)-\delta\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)=$$
$$=-\delta\left(\frac{\partial p^*}{\partial x}\right)+\frac{1}{RR}\left(\delta^2\frac{\partial^3\psi}{\partial x^2\partial y}+\frac{\partial^3\psi}{\partial y^3}\right)+\frac{\gamma\delta}{2}\left(\frac{\partial\varnothing}{\partial y}\right)^2\left(\frac{\partial\theta}{\partial x}\right)-\frac{1}{RR\,K1}\left(1+\frac{\partial\psi}{\partial y}\right), \tag{2.17}$$

$$\delta^2\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial^2\psi}{\partial x\partial y}\right)-\delta^2\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial^2\psi}{\partial x^2}\right)=$$
$$=-\left(\frac{\partial p^*}{\partial y}\right)-\frac{1}{RR}\left(\delta^3\frac{\partial^3\psi}{\partial x^3}+\delta\frac{\partial^3\psi}{\partial x\partial y^2}\right)+\frac{\gamma}{2}\left(\frac{\partial\varnothing}{\partial y}\right)^2\left(\frac{\partial\theta}{\partial y}\right)-\frac{\delta}{RR\,K1}\left(\frac{\partial\psi}{\partial x}\right), \tag{2.18}$$

$$\delta\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial\theta}{\partial x}\right)-\delta\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial\theta}{\partial y}\right)=\frac{1}{RRP_r}\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right), \tag{2.19}$$

$$\frac{\partial}{\partial y}\left(\left[1+e\theta_{00}-L_1\theta\right]\left(\frac{\partial\varnothing}{\partial y}\right)\right)=0, \tag{2.20}$$

with the boundary conditions

$$\frac{\partial\psi}{\partial y}=-1,\qquad \frac{\partial\psi}{\partial x}=\frac{\partial\eta}{\partial x},\qquad \theta=\frac{\theta_{10}}{\beta d},\qquad \varnothing=\frac{\varnothing_{10}}{E_0 d},\qquad \text{at}\quad y=h(x), \tag{2.21}$$

$$\frac{\partial\psi}{\partial y}=-1,\qquad \frac{\partial\psi}{\partial x}=-\frac{\partial\eta}{\partial x},\qquad \theta=\frac{\theta_{00}}{\beta d},\qquad \varnothing=0,\qquad \text{at}\quad y=-h(x), \tag{2.22}$$

$$\delta\Gamma(D)\eta(x)=-\delta\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial^2\psi}{\partial x\partial y}\right)+\delta\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)+\frac{1}{RR}\left(\delta^2\frac{\partial^3\psi}{\partial x^2\partial y}+\frac{\partial^3\psi}{\partial y^3}\right)+$$
$$+\frac{\gamma\delta}{2}\left(\frac{\partial\varnothing}{\partial y}\right)^2\left(\frac{\partial\theta}{\partial x}\right)-\frac{1}{RR\,K1}\left(1+\frac{\partial\psi}{\partial y}\right)\qquad \text{at}\quad y=\begin{Bmatrix}h(x)\\-h(x)\end{Bmatrix}, \tag{2.23}$$

where $\Gamma(D)=\left(BB_w\frac{\partial^5}{\partial x^5}+\left(M_w-T_w\right)\frac{\partial^3}{\partial x^3}-D_w\frac{\partial^2}{\partial x^2}+K_w\frac{\partial}{\partial x}\right)$, $h(x)=1+\eta(x)$ and BB_w, M_w, T_w, D_w and K_w are the non-dimensional wall compliance parameters.

3. Method of solution

We assume that the stream function ψ, the modified pressure p^*, the temperature θ, the electric potential $\varnothing$ and the wall vertical displacement η can be expanded as [17]

$$\psi=\psi_0+\delta\psi_1+\ldots,\qquad p^*=p_0^*+\delta p_1^*+\ldots,\qquad \theta=\theta_0+\delta\theta_1+\ldots,$$
$$\varnothing=\varnothing_0+\delta\varnothing_1+\ldots,\qquad \eta=\eta_0+\delta\eta_1+\ldots\ . \tag{3.1}$$

Substituting Eq.(3.1) into Eqs (2.17)-(2.20) and the boundary conditions (2.21)-(2.23) and collecting terms of like powers of δ, and then solving the resulting systems up to first order of δ we get the solutions of the following form:

Zeroth-order solution

The solution of the problem at this order is given by

$$\theta_0(x,y)=R01+S1(x)y, \tag{3.2}$$

$$\psi_0(x,y)=S11(x)\sinh\left[\frac{y}{\sqrt{k1}}\right]-y, \tag{3.3}$$

$$\varnothing_0(x,y)=S5(x)Log\left[y-S2(x)\right]+S6(x), \tag{3.4}$$

$$p_0^*(x,y)=\frac{S7(x)}{y-S2(x)}+S8(x), \tag{3.5}$$

where

$$S1(x)=\frac{R02}{1+A0\cos[x]},\qquad S2(x)=\frac{R00-R01}{S1(x)},$$

$$S3(x)=\frac{A0\cos[x]}{1-S2(x)}-\frac{(A0)^2(\cos[x])^2}{2(1-S2(x))^2}+Log\left[1-S2(x)\right], \tag{3.6}$$

$$S4(x)=\frac{A0\cos[x]}{1+S2(x)}-\frac{(A0)^2(\cos[x])^2}{2(1+S2(x))^2}+Log\left[-1-S2(x)\right],$$

$$S5(x)=\frac{\varnothing_{10}}{A01\big(S3(x)-S4(x)\big)},\quad S6(x)=-S4(x)S5(x),\ \ S7(x)=\frac{-\gamma S1(x)\big(S5(x)\big)^2}{2},$$

$$S8(x)=G00(x)-\frac{S7(x)}{\big(1-S2(x)\big)}+\frac{A0\,S7(x)\cos[x]}{\big(1-S2(x)\big)^2}-\frac{\big(A0\cos[x]\big)^2 S7(x)}{\big(1-S2(x)\big)^2}+P_{00},$$

(cont. 3.6)

$$S9(x)=-\frac{\Big(R03+R04\,A0\cos[x]+R05\big(A0\,Cos[x]\big)^2\Big)}{1+A0\cos[x]},\qquad S10(x)=\frac{A0\cos[x]}{1+A0\cos[x]},$$

$$S11(x)=-\frac{1+S10(x)}{S9(x)},\qquad R00=\frac{e\theta_{00}+1}{L_1},\quad R01=\frac{\theta_{00}+\theta_{10}}{2\beta d},$$

$$R02=\frac{\theta_{10}-\theta_{00}}{2\beta d},\ \ R03=\sinh\left[\frac{1}{\sqrt{k1}}\right],\ R04=\frac{\cosh\left[\frac{1}{\sqrt{k1}}\right]}{\sqrt{k1}},\ \ R05=\frac{\sinh\left[\frac{1}{\sqrt{k1}}\right]}{2k1},\ \ A01=E_0 d.$$

The non-dimensional pressure rise per wavelength at this order is defined as

$$\Delta P_{\lambda 0}=\int_0^{\pi}\frac{\partial p_0^*}{\partial x}dx=$$
$$=-\frac{1}{2}\left(\frac{d3\big(d1+R02\big)Log\big[d1\big]}{d4\,R02}+\frac{\gamma(\varnothing_{10})^2\,R02\ Log\big[d6\big]}{d7}+\frac{d3\big(1+A0\big)}{d4}-\big(1+A0\big)d8\right)+\quad(3.7)$$
$$+\frac{1}{2}\left(\frac{d3\big(d1+R02\big)Log\big[d1+\big(1+A0\big)R02\big]}{d4\,R02}+\frac{\gamma(\varnothing_{10})^2\,R02\ Log\big[d6+\big(1+A0\big)R02\big]}{d7}\right).$$

First-order solution

The solution of the problem at this order is given by

$$\theta_1(x,y)=\sqrt{k1}\ \Pr RR\left(\frac{\big(A0\,R02\,S11(x)\sin[x]\big)y\cosh\left[\frac{y}{\sqrt{k1}}\right]}{\big(1+A0\cos[x]\big)^2}-S13(x)\sinh\left[\frac{y}{\sqrt{k1}}\right]\right)+\quad(3.8)$$
$$+\Pr RR\left(S14(x)\,y-\frac{\big(A0\,R02\sin[x]\big)y^3}{6\big(1+A0\cos[x]\big)^2}\right),$$

$$\psi_1(x,y)=C1(x)\left(\sinh\left[\frac{y}{\sqrt{k1}}\right]+\cosh\left[\frac{y}{\sqrt{k1}}\right]\right)-C2(x)\left(\cosh\left[\frac{y}{\sqrt{k1}}\right]-\sinh\left[\frac{y}{\sqrt{k1}}\right]\right)+$$
$$+S57(x)\left(\cosh\left[\frac{y}{\sqrt{k1}}\right]-\sinh\left[\frac{y}{\sqrt{k1}}\right]\right)Ei\left[\frac{y-S2(x)}{\sqrt{k1}}\right]+S59(x)Log\left[\frac{y-S2(x)}{\sqrt{k1}}\right]+$$
$$+S58(x)\left(\sinh\left[\frac{y}{\sqrt{k1}}\right]+\cosh\left[\frac{y}{\sqrt{k1}}\right]\right)Ei\left[\frac{S2(x)-y}{\sqrt{k1}}\right]-S60(x)(y-S2(x))+ \tag{3.9}$$
$$+\frac{1}{4}k1S52(x)\left(3\sqrt{k1}\sinh\left[\frac{y}{\sqrt{k1}}\right]-2y\cosh\left[\frac{y}{\sqrt{k1}}\right]\right)+C3(x),$$

$$\varnothing_1(x,y)=\Pr RR\left(-\frac{B1(x)\,y\,Log[y-S2(x)]}{S1(x)(y-S2(x))}+\frac{S17(x)y^3}{2(y-S2(x))}-\frac{S26(x)\,y\,Log[y-S2(x)]}{y-S2(x)}\right)+$$
$$+\frac{\Pr RR}{y-S2(x)}\left(B1(x)S27(x)Log[y-S2(x)]+S20(x)+S28(x)\ Log[y-S2(x)]\right)+$$
$$+\frac{\Pr RR}{y-S2(x)}\left(-S16(x)\sinh\left[\frac{y}{\sqrt{k1}}\right]+S22(x)\cosh\left[\frac{y}{\sqrt{k1}}\right]+y^2S21(x)-yS23(x)\right)+ \tag{3.10}$$
$$+\frac{\Pr RR}{y-S2(x)}\left(yS24(x)-S29(x)\right)Chi\left[\frac{y-S2(x)}{\sqrt{k1}}\right]+\Pr RR\ B2(x)+$$
$$+\frac{\Pr RR}{y-S2(x)}\left(S30(x)-yS25(x)\right)Shi\left[\frac{y-S2(x)}{\sqrt{k1}}\right],$$

$$p_1^*(x,y)=S108(x)Chi\left[\frac{y-S2(x)}{\sqrt{k1}}\right]+\left(S111(x)-\frac{S103(x)}{2(1-S2(x))^2}\right)Shi\left[\frac{y-S2(x)}{\sqrt{k1}}\right]+\frac{S92(x)}{(y-S2(x))^2}+$$
$$+\frac{S103(x)\cosh\left[\frac{y-S2(x)}{\sqrt{k1}}\right]}{4\sqrt{k1}(y-S2(x))}-\frac{S106(x)\sinh\left[\frac{y}{\sqrt{k1}}\right]}{y-S2(x)}-\frac{(S109(x)+yS81(x))\cosh\left[\frac{y-S2(x)}{\sqrt{k1}}\right]}{(y-S2(x))^2}+$$
$$-\frac{S113(x)\cosh\left[\frac{y}{\sqrt{k1}}\right]}{y-S2(x)}-\frac{(S112(x)-4yS83(x))\sinh\left[\frac{y-S2(x)}{\sqrt{k1}}\right]}{4(y-S2(x))^2}+$$
$$+\frac{S110(x)\sinh\left[\frac{y-S2(x)}{\sqrt{k1}}\right]}{y-S2(x)}-\frac{S86(x)\sinh\left[\frac{y}{\sqrt{k1}}\right]}{2(y-S2(x))^2}-\frac{S94(x)}{y-S2(x)}+ \tag{3.11}$$
$$-\sqrt{k1}\,S90(x)\cosh\left[\frac{y}{\sqrt{k1}}\right]-S93(x)(y-S2(x))+S95(x)Log[y-S2(x)]+$$
$$-\frac{S96(x)(2Log[y-S2(x)]+1)}{(y-S2(x))^2}-\frac{2S102(x)Chi\left[\frac{y-S2(x)}{\sqrt{k1}}\right]}{(y-S2(x))^2}+C4(x),$$

where $Shi[Z]$ is the hyperbolic sine integral function, $Chi[Z]$ is the hyperbolic cosine integral function, $Ei[Z]$ is the exponential integral function

$$B1(x)=\frac{S50(x)-S48(x)}{S47(x)-S49(x)}, \qquad B2(x)=\frac{S48(x)S49(x)-S47(x)S50(x)}{S47(x)-S49(x)}, \quad (3.12)$$

$$C1(x)=\sqrt{k1}\left(\frac{-S61(x)S64(x)+S61(x)S67(x)+S65(x)S67(x)-S64(x)S68(x)}{S64(x)S66(x)-S63(x)S67(x)}\right),$$

$$C2(x)=\sqrt{k1}\left(\frac{-\big(S61(x)+S65(x)\big)S66(x)+S63(x)\big(S61(x)+S68(x)\big)}{S64(x)S66(x)-S63(x)S67(x)}\right),$$

$$(3.13)$$

$$C3(x)=\sqrt{k1}\left(\frac{-S61(x)S64(x)+S61(x)S67(x)+S65(x)S67(x)-S64(x)S68(x)}{-S64(x)S66(x)+S63(x)S67(x)}\right)+$$
$$-\sqrt{k1}\left(\frac{-\big(S61(x)+S65(x)\big)S66(x)+S63(x)\big(S61(x)+S68(x)\big)}{-S64(x)S66(x)+S63(x)S67(x)}\right)-S72(x)\ ,$$

$$C4(x)=G_{01}(x)-S114(x)-\eta_0(x)S115(x)-\eta_1(x)S116(x)-\frac{1}{2}\big(\eta_0(x)\big)^2 S117(x)+$$
$$-\eta_0(x)\eta_1(x)S118(x),$$

$$\eta_0(x)=A0\cos[x], \qquad \eta_1(x)=A1\big(\cos[x]\big)^2,$$

$$(3.14)$$

$$G_{01}(x)=2\,A1\big(4BB_w+T_w-M_w\big)\cos[2x]+A1\,D_w\sin[2x]+A1\,K_w\big(\cos[x]\big)^2.$$

The non-dimensional pressure rise per wavelength at this order is defined as

$$\Delta P_{\lambda 1}=\int_0^{\pi}\frac{\partial p_1^*}{\partial x}dx=d9\big(1+A0\big)+d10\,Log\big[1+A0+d12\big]+$$
$$-d10\,Log\big[d12\big]+d11\,Log\big[d13\big]-d11\,Log\big[1+A0+d13\big]. \quad (3.15)$$

The constants $R06-R13$, $d1-d13$ and the functions $S12(x)-S118(x)$ were obtained and are not regarded here.

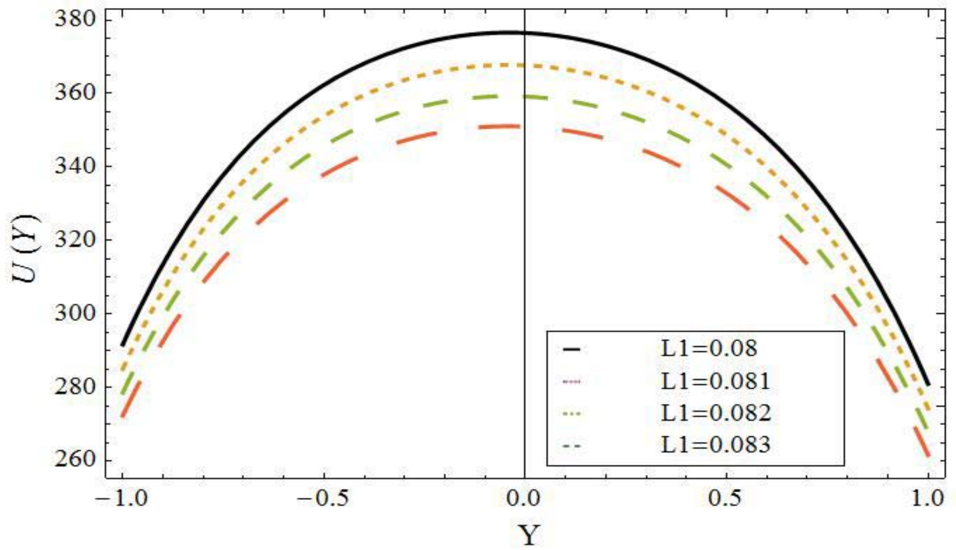

Fig.2. Variation of $u(y)$ with y for various values of $L1$ when $d=1$, $L=1000$, $A0=0.5, A1=0.1$, $RR=50, \delta=0.1, k1=0.24$.

4. Discussion of the results

In this paper, we take $BB_w=20$, $M_w=0.01$, $T_w=10$, $K_w=10$, $D_w=0.5$, $P_{00}=1$, $x=\frac{\pi}{3}$. The effects of the temperature parameter $L1$ and electrical Rayleigh number L on the axial velocity $u(y)$ are shown in Figs 2-3. It is observed that the axial velocity increases with an increase in the electrical Rayleigh number, whereas it decreases with an increase of the temperature parameter. Figures 4-6 depict the variation of the electrical potential function $\varnothing$ with y for various values of the permeability of the medium $K1$, primary amplitude ratio $A0$ and secondary amplitude ratio $A1$. As illustrated in these figures, the electrical potential increases by increasing $K1$ and $A0$ while it decreases with an increase of $A1$.

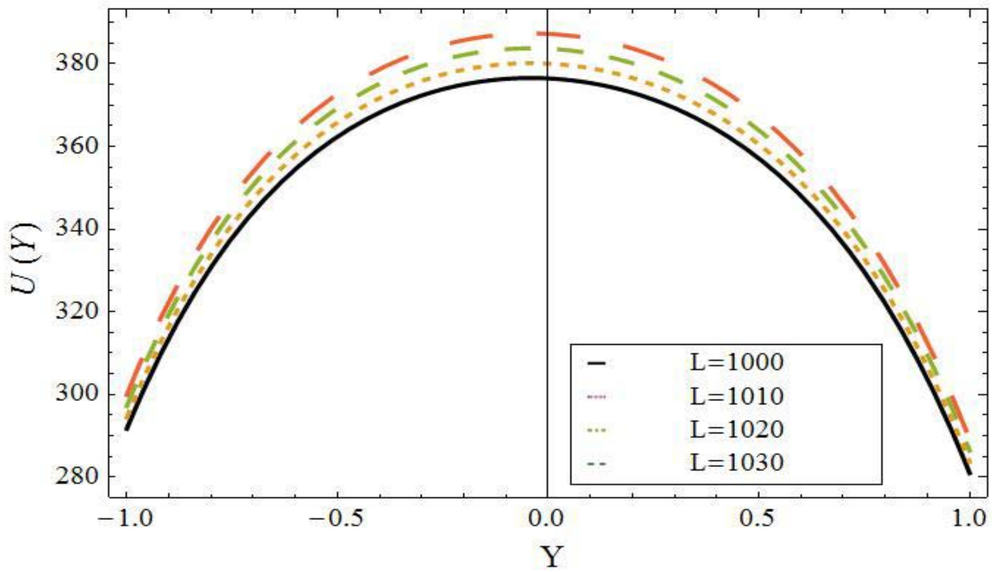

Fig.3. Variation of $u(y)$ with y for various values of L when $d=1$, $L1=0.08$, $A0=0.5, A1=0.1$, $RR=50, \delta=0.1, k1=0.24$.

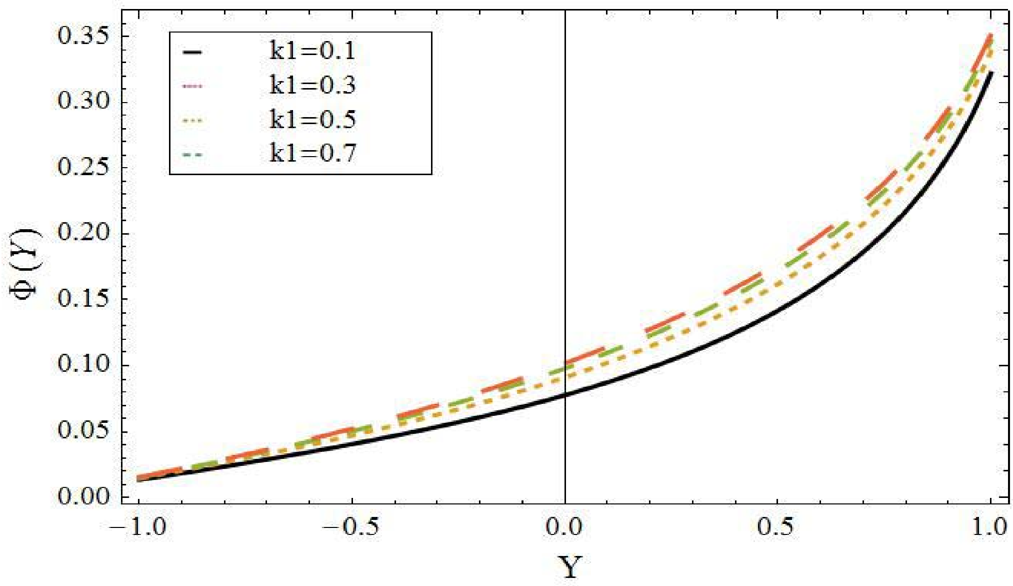

Fig.4. Variation of $\varnothing(y)$ with y for various values of $k1$ when $d=1$, $P_r=0.7$, $A0=0.3, A1=0.1$, $RR=50, \delta=0.1$.

The pressure rises against the flow direction ΔP_λ are illustrated in terms of the dimensionless wave number δ with various values of the temperature parameter $L1$ (Fig.7) and the electrical Rayleigh number L (Fig.8), respectively. It is obvious that increasing the wave number causes an increase of the pressure rises. The pressure rises ΔP_λ for different values of the temperature parameter $L1$ are illustrated in Fig.7. It is shown that ΔP_λ increases with an increase in $L1$ for small values of $\delta(0 \le \delta \le 0.34)$ and after that ΔP_λ decreases. The graphs of ΔP_λ for different values of the electrical Rayleigh number L are presented in Fig.8. It is observed that the pressure rise decreases for small values of $\delta(0 \le \delta \le 0.36)$ with an increase in the electrical Rayleigh number L and for large $\delta(0.36 \le \delta \le 0.50)$, the pressure rise increases.

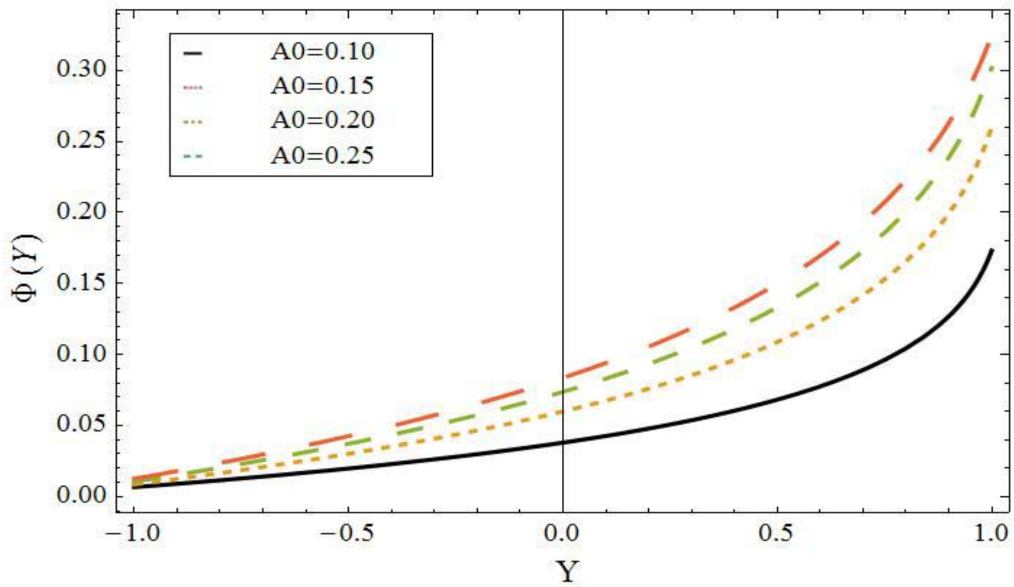

Fig.5. Variation of $\varnothing(y)$ with y for various values of $A0$ when $d=1$, $\Pr=0.7$, $k1=0.3, A1=0.1$, $RR=50, \delta=0.1$.

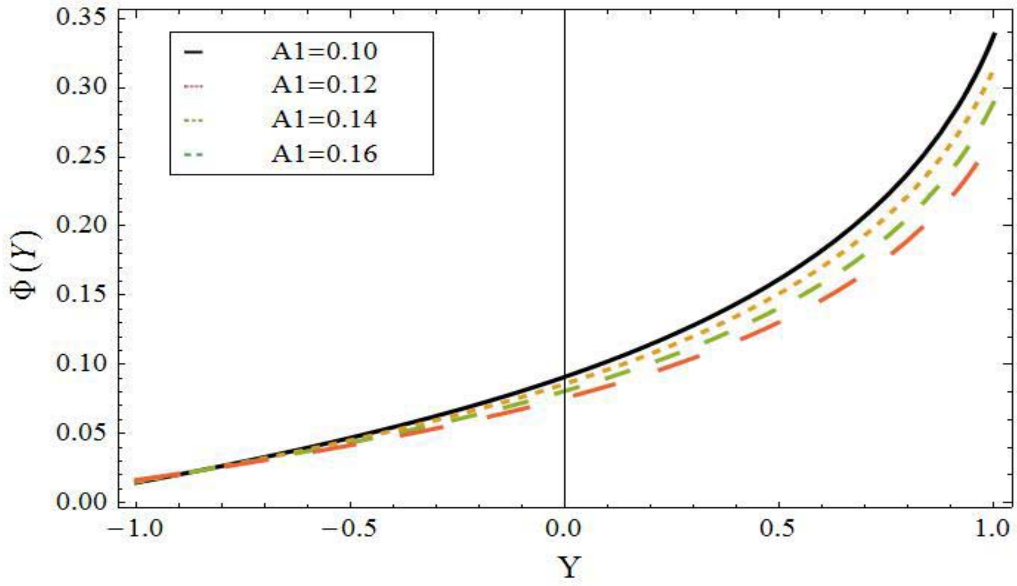

Fig.6. Variation of $\varnothing(y)$ with y for various values of $A1$ when $d=1$, $\Pr = 0.7$, $k1 = 0.3, A0 = 0.3$, $k1 = 0.3, A0 = 0.3$, $RR = 50, \delta = 0.1$.

The formation of an internally circulating bolus of fluid by closed streamlines is called trapping and this trapped bolus is pushed ahead along with the peristaltic wave. The effects of the temperature parameter $L1$ and the electrical Rayleigh number L on trapping are illustrated in Fig.9. It is observed that the bolus decreases in size with an increase of the temperature parameter $L1$, whereas it increases in size with an increase of the electrical Rayleigh number L. Also, it is observed that the trapped bolus in the case of a channel with compliant walls is less than these for a channel with no compliant walls.

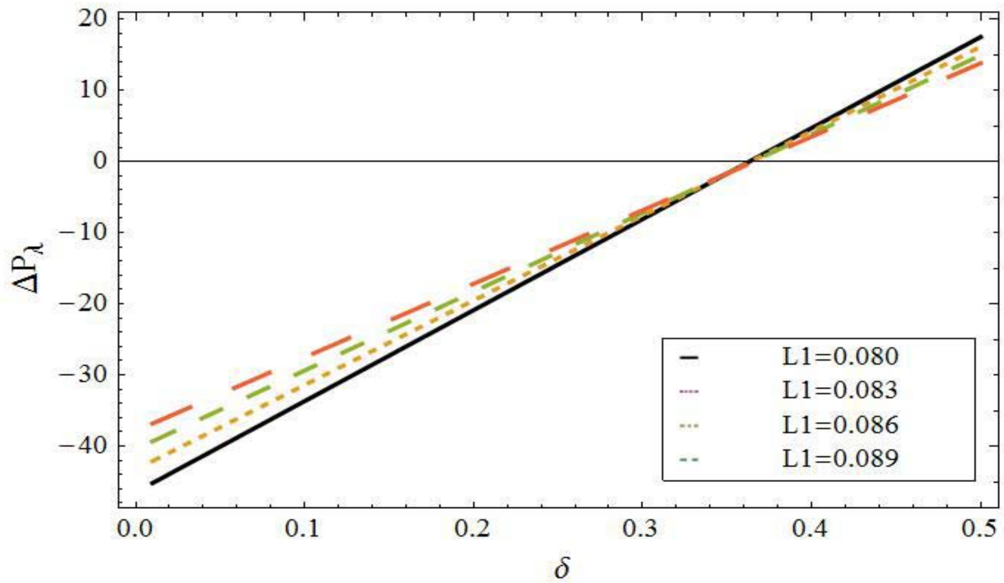

Fig.7. Pressure rise per wavelength ΔP_{λ} plotted against the dimensionless wavenumber δ for different values of $L1$ when $d=1$, $\Pr = 0.01, A0 = 0.02$, $A1 = 0.01, k1 = 0.1$, $RR = 150, L = 500$.

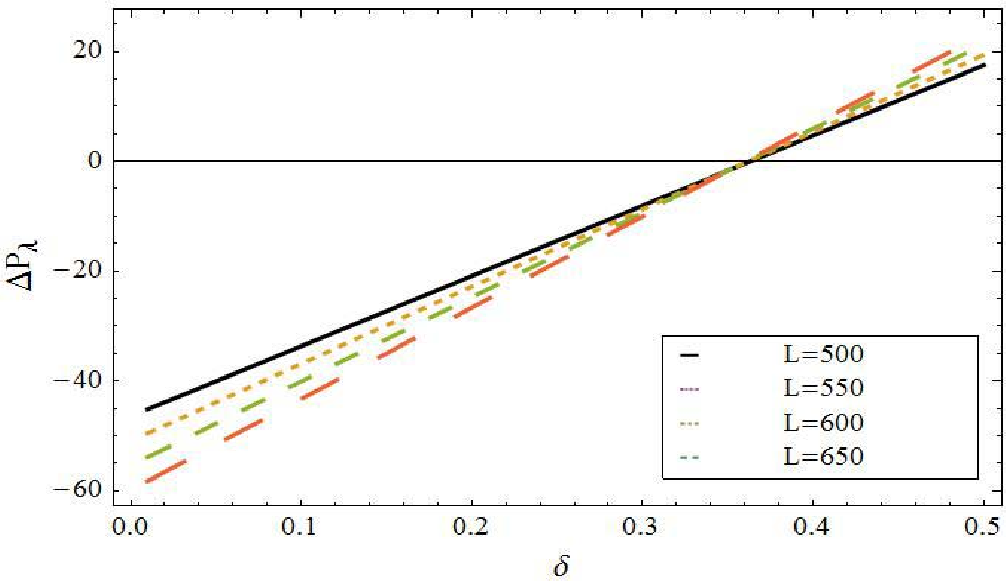

Fig.8. Pressure rise per wavelength ΔP_{λ} plotted against the dimensionless wavenumber δ for different values of L when $d=1$, $\Pr=0.01, A0=0.02$, $A1=0.01, k1=0.1$, $RR=150, L1=0.08$.

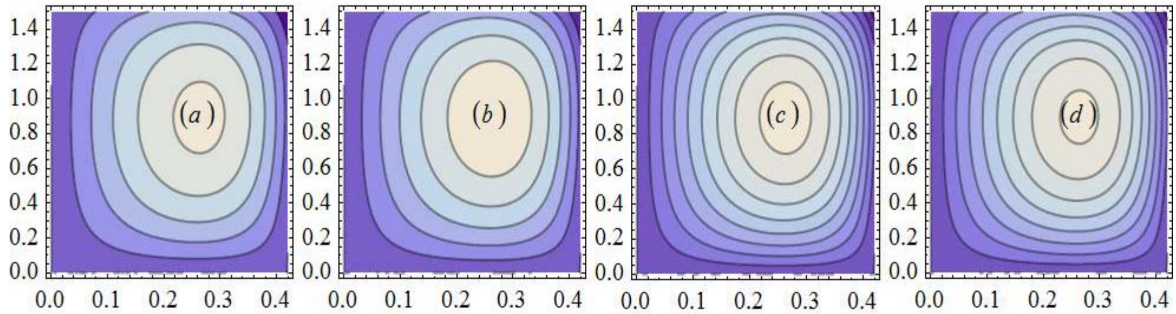

Fig.9. Streamlines at $d=1$, $\Pr=0.7, A0=0.3$, $A1=0.1, k1=0.3, RR=50, \delta=0.01$ for different L: (a) $L1=0.01, L=1000$; (b) $L1=0.01, L=1100$; for different $L1$: (c) $L1=0.5, L=10000$; (d) $L1=0.8, L=10000$;

5. Conclusions

We have presented a new technique to study the influence of the electrical Rayleigh number and temperature parameter on an oscillatory flow through a porous medium in the presence of compliant walls. In the current study, we considered modified non-dimensional quantities that made fluid pressure in the resulting equations dependent on both axial and vertical coordinates. The current study seens to be more realistic and more comprehensive than the previous ones which considered the fluid pressure to be function only in the axial coordinate. The following interesting observations are made:

(1) The axial velocity increases with an increase in the permeability of the medium and the electrical Rayleigh number, whereas it decreases with an increase of the temperature parameter.
(2) The electrical potential increases by increasing the permeability of the medium and the primary amplitude ratio, while it decreases with an increase of the Prandtl number, thermal expansion

coefficient, the root mean square value of the electric field (at $Y = 0$), the channel half width and the secondary amplitude ratio .

(3) For small values of δ, the pressure rise decreases with an increase in the electrical Rayleigh number, whereas it increases with an increase in the temperature parameter.

(4) The bolus decreases in size with an increase of the temperature parameter, whereas it increases in size with an increase of the electrical Rayleigh number.

(5) The trapped bolus, in the case of a channel with compliant walls, is less than these for a channel with no compliant walls.

Nomenclature

$A0$ – primary amplitude ratio
$A1$ – secondary amplitude ratio
b_w – flexural rigidity of the plate
d_w – wall damping coefficient
e – thermal expansion coefficient of dielectric constant
$K1$ – permeability of the medium
k_w – spring stiffness
L – electrical Rayleigh number
L_1 – temperature parameter
m_w – plate mass per unit area
ϵ_0 – permittivity at vacuum
P_{00} – pressure on the outside surface of the wall
Pr – Prandtl number
RR – Reynolds number
t_w – longitudinal tension per unit width
β – adverse temperature gradient
δ – wave number

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