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A GENERAL STUDY OF FUNDAMENTAL SOLUTIONS IN ANIOTROPICTHERMOELASTIC MEDIA WITH MASS DIFFUSION AND VOIDS

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The present paper deals with the study of a fundamental solution in transversely isotropic thermoelastic media with mass diffusion and voids. For this purpose, a two-dimensional general solution in transversely isotropic thermoelastic media with mass diffusion and voids is derived first. On the basis of the obtained general solution, the fundamental solution for a steady point heat source on the surface of a semi-infinite transversely isotropic thermoelastic material with mass diffusion and voids is derived by nine newly introduced harmonic functions. The components of displacement, stress, temperature distribution, mass concentration and voids are expressed in terms of elementary functions and are convenient to use. From the present investigation, some special cases of interest are also deduced and compared with the previous results obtained, which prove the correctness of the present result.

Key words: general solution, fundamental solution, thermoelastic, voids, mass diffusion.

1. Introduction

Fundamental solutions play a crucial role in the theory of partial differential equations. They can be used to derive many analytical solutions of practical problems when boundary conditions are imposed. Fundamental solutions play a key role in an integral equation representation of a boundary value problem and are more easily solved by analytical methods in comparison to a differential equation with specified initial and boundary conditions. This type of situation (numerical methods technique) makes the subject more attractive mainly for these researchers whose interest is in numerical methods. The fundamental solution also provides a wonderful platform to overcome the main drawbacks in the boundary element method which also uses the fundamental solution to satisfy the governing equation. Consequently, we can say that with the latest technological demand, no boundary element method can be made more advanced without further developments in the area of fundamental solutions or in other words we can say that fundamental solution is the basis for many further works.

Ding *et al*. [1] constructed the general solutions for coupled equations in transversely isotropic piezoelectric media by using the operator theory. Dunn and Wienecke [2] derived the half space Green's functions for a transversely isotropic piezoelectric solid and also obtained closed-form expressions for the half-space Green's functions. Pan and Tanon [3] presented Green's functions for a three dimensional problem in anisotropic piezoelectric solids and also presented the applications. Chen [4] derived a general solution for transverse isotropic thermo-piezo-elastic media in dynamic as well as in static case and derived an exact

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solution for a penny shaped cracked subjected to uniform temperature load. Chen *et al*. [5] presented three-dimensional exact solution for a penny-shaped crack in an infinite piezoelectric medium subjected to an arbitrarily point temperature load by using the potential theory method for both impermeable and permeable cracks.

After consideration of thermal effects, Sharma [6] derived the fundamental solution for a transversely isotropic thermoelastic material in an integral form. Ciarletta *et al*. [7] derived the fundamental solution for a micropolar isotropic thermoelastic material with voids by the potential method. Hou *et al.* [8] constructed Green’s function for a three-dimensional problem for transversely isotropic biomaterials by using the operator theory. Hou *et al*. [9] studied Green’s functions for a two dimensional problem for semi-infinite orthotropic thermoelastic media by introducing new harmonic functions. Xiong *et al.* [10] discussed Green’s functions for a two dimensional problem for orthotropic piezothermoelastic material by trial and error method. Hou *et al.* [12] constructed the general solution and fundamental solution a two dimensional problem for orthotropic thermoelastic material. Seremet [13] constructed an exact Green’s function and integral formula for a boundary-value problem (BVP) for a thermoelastic wedge in terms of elementary functions. Seremet [14] derived a new Green’s function and a new Green-type integral formula for a boundary value problem (BVP) in thermoelastic quadrant. Kumar and Kansal [15] studied the plane wave propagation and fundamental solution in generalized theory of thermoelastic diffusion.

Kumar and Chawla [16, 17] derived the fundamental solution and Green’s function for a two dimensional problem in orthotropic thermoelastic diffusion media by using the operator theory and also presented the result graphically. Also, Kumar and Chawla [18, 19] derived the fundamental solution and Green’s function in orthotropic piezothermoelastic diffusion media by trial and error method. Kumar and Chawla [20] discussed the problem of reflection and transmission in thermoelastic media with three-phase-lag model for isotropic case. Kumar and Vandna [21] derived a Green's function for a three dimensional problem in transversely isotropic thermoelastic biomaterial for concentrated heat source. Kumar and Chawla [22] presented the fundamental solution for a two-dimensional problem in orthotropic thermoelastic media with voids by introducing nine new harmonic functions. Şeremet [23] derived new constructive formulas in thermoelastic Green’s functions for a boundary value problem of thermoelasticity in a steady state case and also expressed the constructive formulas in terms of Green’s functions for Poisson’s equation. Pan *et al*. [24] derived the general solution and fundamental solution for fluid-saturated, orthotropic, poroelastic materials in case of a steady state problem. Chawla *et al.* [25] constructed a general solution and fundamental solution for a two dimensional problem in micropolar thermoelastic material. Dang *et al.* [26] investigated a planar crack of an arbitrary shape embedded in three-dimensional isotropic hygrothermoelastic media by using the Hankel transform technique. Zhao *et al.* [27] derived the three dimensional general solution and fundamental solution in hygrothermoelastic media by using the operator theory. Tomar *et al.* [28] studied plane waves in thermo-viscoelastic material with voids under different theories of thermoelasticity. Biswas [29] investigated the fundamental solution in steady oscillations equations for nonlocal thermoelastic medium with voids.

However, the important general solution and fundamental solution for a two-dimensional problem for a steady point heat source in an anisotropic thermoelastic material with mass diffusion and voids has not been discussed so far in the literature.

2. Basic equations

Following Aouadi [11] the basic equations for an anisotropic thermoelastic material with mass diffusion and voids, in the absence of body forces, extrinsic equilibrated body force and heat sources, are

Constitutive relations

$$\sigma_{ij} = c_{ijkm} e_{km} + B_{ij}\varphi - \beta_{ij} T - \gamma_{ij} C. \tag{2.1}$$

Equations of motion

$$\rho \ddot{u}_i = c_{ijkm} e_{km,j} + B_{ij}\varphi_{,j} - \beta_{ij} T_{,j} - \gamma_{ij} C_{,j}. \tag{2.2}$$

Equilibrated equation

$$\rho\chi\ddot{\varphi} = A_{ij}\varphi_{,ij} - \omega_0\dot{\varphi} - \xi\varphi - B_{ij}u_{i,j} + b_1^* T + b_2^* C. \tag{2.3}$$

Equation of heat conduction

$$\rho C^* \dot{T} + T_0(\beta_{ij}\dot{u}_{i,j} + b_1^*\dot{\varphi}) + aT_0\dot{C} = K_{ij}T_{,ij}. \tag{2.4}$$

(iv) Equation of mass diffusion

$$\alpha_{ij}^*[-\gamma_{ij}u_{i,j} - b_2^*\varphi - aT + dC]_{,ij} = \dot{C}\,. \tag{2.5}$$

Here, $c_{ijkm}\left(= c_{kmij} = c_{jikm}\right)$ is the tensor of elastic tensor $k_{ij}\left(= k_{ji}\right)$, $\alpha_{ij}^*\left(= \alpha_{ji}^*\right)$ are, respectively, the coefficients of thermal conductivity and diffusion tensor, β_{ij}, γ_{ij} are, respectively, the tensors of thermal and diffusion moduli, $A_{ij,} B_{ij}, \omega_0, \xi, b_1^*, b_2^*$ are the constitutive coefficients, T is the temperature distribution from the reference temperature T_0, ρ is the density, χ is the equilibrated inertia, φ is the volume fraction field, $e_{ij} = \dfrac{u_{i,j} + u_{j,i}}{2}$ are the components of the strain tensor, u_i are components of the displacement vector, a, d are, respectively, the coefficient describing the measure of thermodiffusion and mass diffusion effects, C is the concentration of diffusive material in the elastic body, C^* is the specific heat at constant strain and the above coefficient have the following symmetries. The symbol (",") followed by a suffix denotes differentiation with respect to the spatial coordinate and a superposed dot (".") denotes the derivative with respect to time.

3. Formulation of the problem

We consider a homogenous, transversely isotropic thermoelastic diffusion medium. Let us take *Oxyz* as the frame of reference in Cartesian coordinates.

For a two-dimensional static problem, we assume the displacement vector, temperature change and mass concentration, volume fraction field, respectively, of the form

$$\boldsymbol{u} = (u, 0, w), \qquad T(x,z,t), \qquad C(x,z,t), \qquad \varphi(x,z,t). \tag{3.1}$$

Equations (2.1)- (2.5) for a transversely thermoelastic material with diffusion and voids, with the aid ofEqs (3.1), can be written as

$$\left[c_{11}\frac{\partial^2}{\partial x^2} + c_{66}\frac{\partial^2}{\partial z^2}\right]u + (c_{13} + c_{44})\frac{\partial^2 w}{\partial x \partial z} + B_1\frac{\partial\varphi}{\partial x} - \beta_1\frac{\partial T}{\partial x} - \gamma_1\frac{\partial C}{\partial x} = 0\,, \tag{3.2}$$

$$(c_{13} + c_{44})\frac{\partial^2 u}{\partial x \partial z} + \left[c_{44}\frac{\partial^2}{\partial x^2} + c_{33}\frac{\partial^2}{\partial z^2}\right]w + B_3\frac{\partial\varphi}{\partial z} - \beta_3\frac{\partial T}{\partial z} - \gamma_3\frac{\partial C}{\partial z} = 0\,, \tag{3.3}$$

$$-B_1\frac{\partial u}{\partial x}-B_3\frac{\partial w}{\partial z}+\left[A_1\frac{\partial^2}{\partial x^2}+A_3\frac{\partial^2}{\partial z^2}-\xi\right]\varphi+b_1^*T+b_2^*C=0\,, \tag{3.4}$$

$$\begin{gathered}\frac{\partial}{\partial x}\left[\gamma_1\left(\alpha_1^*\frac{\partial^2}{\partial x^2}+\alpha_3^*\frac{\partial^2}{\partial z^2}\right)\right]u+\frac{\partial}{\partial z}\left[\gamma_3\left(\alpha_1^*\frac{\partial^2}{\partial x^2}+\alpha_3^*\frac{\partial^2}{\partial z^2}\right)\right]w+\left[b_2^*\left(\alpha_1^*\frac{\partial^2}{\partial x^2}+\alpha_3^*\frac{\partial^2}{\partial z^2}\right)\right]\varphi+\\ +\left[a\left(\alpha_1^*\frac{\partial^2}{\partial x^2}+\alpha_3^*\frac{\partial^2}{\partial z^2}\right)\right]T-\left[d\left(\alpha_1^*\frac{\partial^2}{\partial x^2}+\alpha_3^*\frac{\partial^2}{\partial z^2}\right)\right]C=0.\end{gathered} \tag{3.5}$$

Equations (3.2)-(3.5) can be written as

$$D\{u,w,\varphi,T\}^{tr}=0 \tag{3.6}$$

where D is the differential operator matrix given by

$$\begin{bmatrix} c_{11}\frac{\partial^2}{\partial x^2}+c_{66}\frac{\partial^2}{\partial z^2} & (c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z} & B_1\frac{\partial}{\partial x} & -\gamma_1\frac{\partial}{\partial x} & -\beta_1\frac{\partial}{\partial x} \\ (c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z} & c_{11}\frac{\partial^2}{\partial x^2}+c_{66}\frac{\partial^2}{\partial z^2} & B_3\frac{\partial}{\partial z} & -\gamma_3\frac{\partial}{\partial z} & -\beta_3\frac{\partial}{\partial x} \\ -B_1\frac{\partial}{\partial x} & -B_3\frac{\partial}{\partial z} & \left(A_1\frac{\partial^2}{\partial x^2}+A_3\frac{\partial^2}{\partial x^2}-\xi\right) & b_2^* & b_1^* \\ \frac{\partial}{\partial x}\left[\gamma_1\left(\alpha_1^*\frac{\partial^2}{\partial x^2}+\alpha_3^*\frac{\partial^2}{\partial z^2}\right)\right] & \frac{\partial}{\partial z}\left[\gamma_3\left(\alpha_1^*\frac{\partial^2}{\partial x^2}+\alpha_3^*\frac{\partial^2}{\partial z^2}\right)\right] & \left[b_2^*\left(\alpha_1^*\frac{\partial^2}{\partial x^2}+\alpha_3^*\frac{\partial^2}{\partial z^2}\right)\right] & \left[a\left(\alpha_1^*\frac{\partial^2}{\partial x^2}+\alpha_3^*\frac{\partial^2}{\partial z^2}\right)\right]T-\left[d\left(\alpha_1^*\frac{\partial^2}{\partial x^2}+\alpha_3^*\frac{\partial^2}{\partial z^2}\right)\right] & \\ 0 & 0 & 0 & 0 & \left(K_1\frac{\partial^2}{\partial x^2}+K_3\frac{\partial^2}{\partial z^2}\right) \end{bmatrix}. \tag{3.7}$$

Equation (3.6) is a homogeneous set of differential equations in u,w,φ,C,T. The general solution by the operator theory is as follows

$$\begin{gathered}u=A_{i1}F+\overline{A}_{i1}G,\quad w=A_{i2}F+\overline{A}_{i2}G,\quad \varphi=\overline{A}_{i3}G,\quad C=A_{i4}F+\overline{A}_{i4}G,\\ T=A_{i5}F+\overline{A}_{i5}G,\qquad (i=1,2,3,4,5)\end{gathered} \tag{3.8}$$

where A_{ij} are algebraic cofactors of the matrix D, of which the determinant is

$$\left|D\right| = \left(a^* \frac{\partial^8}{\partial z^8} + b^* \frac{\partial^8}{\partial x^2 \partial z^6} + c^* \frac{\partial^8}{\partial x^4 \partial z^4} + d^* \frac{\partial^8}{\partial x^6 \partial z^2} + e^* \frac{\partial^8}{\partial x^8} \right) \times \left(\frac{\partial^2}{\partial x^2} + \varepsilon_3 \frac{\partial^2}{\partial z^2} \right) + \\ + \left(\bar{a} \frac{\partial^6}{\partial z^6} + \bar{b} \frac{\partial^6}{\partial x^2 \partial z^4} + \bar{c} \frac{\partial^6}{\partial x^4 \partial z^2} + \bar{d} \frac{\partial^6}{\partial z^6} \right) \times \left(\frac{\partial^2}{\partial x^2} + \varepsilon_3 \frac{\partial^2}{\partial z^2} \right), \tag{3.9}$$

where a^*, b^*, c^*, d^*, e^* and $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$ are given in Appendix A.

The functions F and G in Eq.(3.8) satisfy the following homogeneous equation

$$\left|D\right| F = 0 \qquad \text{and} \qquad \left|D\right| G = 0. \tag{3.10}$$

It can be seen that if $i = 1,2,3,4$ are taken in Eqs (3.8), four general solutions are obtained in which $T = 0$. These solutions are identical to those without thermal fact and are not discussed here. Therefore if $i = 5$ should be taken in Eqs (3.8), the following solution is obtained

$$u = \left(p_1 \frac{\partial^6}{\partial x^6} + q_1 \frac{\partial^6}{\partial x^4 \partial z^2} + r_1 \frac{\partial^6}{\partial x^2 \partial z^4} + v_1 \frac{\partial^6}{\partial z^6} \right) \frac{\partial F}{\partial x} + \\ + \left(\bar{p}_1 \frac{\partial^4}{\partial x^4} + \bar{q}_1 \frac{\partial^4}{\partial x^2 \partial z^2} + \bar{r}_1 \frac{\partial^4}{\partial z^4} \right) \frac{\partial G}{\partial x}, \tag{3.11a}$$

$$w = \left(p_2 \frac{\partial^6}{\partial x^6} + q_2 \frac{\partial^6}{\partial x^4 \partial z^2} + r_2 \frac{\partial^6}{\partial x^2 \partial z^4} + v_2 \frac{\partial^6}{\partial z^6} \right) \frac{\partial F}{\partial z} + \\ + \left(\bar{p}_2 \frac{\partial^4}{\partial x^4} + \bar{q}_2 \frac{\partial^4}{\partial x^2 \partial z^2} + \bar{r}_2 \frac{\partial^4}{\partial z^4} \right) \frac{\partial G}{\partial z}, \tag{3.11b}$$

$$\varphi = \left(\bar{p}_3 \frac{\partial^6}{\partial x^6} + \bar{q}_3 \frac{\partial^6}{\partial x^4 \partial z^2} + \bar{r}_3 \frac{\partial^6}{\partial x^2 \partial z^4} + \bar{v}_3 \frac{\partial^6}{\partial z^6} \right) G, \tag{3.11c}$$

$$C = \left(p_4 \frac{\partial^8}{\partial x^8} + q_4 \frac{\partial^8}{\partial x^6 \partial z^2} + r_4 \frac{\partial^8}{\partial x^4 \partial z^4} + v_4 \frac{\partial^8}{\partial x^2 \partial z^6} + w_4 \frac{\partial^8}{\partial z^8} \right) F + \\ + \left(\bar{p}_4 \frac{\partial^6}{\partial x^6} + \bar{q}_4 \frac{\partial^6}{\partial x^4 \partial z^2} + \bar{r}_4 \frac{\partial^6}{\partial x^2 \partial z^4} + \bar{v}_4 \frac{\partial^6}{\partial x^6} \right) G, \tag{3.11d}$$

$$T = \left(a^* \frac{\partial^8}{\partial z^8} + b^* \frac{\partial^8}{\partial x^2 \partial z^6} + c^* \frac{\partial^8}{\partial x^4 \partial z^4} + d^* \frac{\partial^8}{\partial x^6 \partial z^2} + e^* \frac{\partial^8}{\partial x^8} \right) F + \\ + \left(\bar{a} \frac{\partial^6}{\partial z^6} + \bar{b} \frac{\partial^6}{\partial z^4 \partial x^2} + \bar{c} \frac{\partial^6}{\partial z^2 \partial x^4} + \bar{d} \frac{\partial^6}{\partial x^6} \right) G \tag{3.11e}$$

where a^*, b^*, c^*, d^*, e^* and $\overline{a}, \overline{b}, \overline{c}$ and $\overline{d}$ are given in Appendix A.

Equation (3.10) can be rewritten as

$$\prod_{j=1}^{5}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z_j^2}\right)F=0, \tag{3.12}$$

$$\prod_{j=1}^{4}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z_j^2}\right)G=0 \tag{3.13}$$

where

$z_j = s_j z,\ s_5 = \sqrt{\dfrac{K_1}{K_3}}$ and $s_j\,(j=1,2,3,4)$ are four roots (with positive real part) of the following algebraic equation

$$a^* s^8 - b^* s^6 + c^* s^4 - d^* s^2 + e^* = 0. \tag{3.14}$$

and

$\overline{z}_j = s_j z,\ \overline{s}_4 = \sqrt{\dfrac{K_1}{K_3}}$ and $s_j\,(j=1,2,3)$ are three roots (with positive real part) of the following algebraic equation

$$\overline{a}s^6 - \overline{b}s^4 + \overline{c}s^2 - \overline{d} = 0. \tag{3.15}$$

As known from the generalized Almansi (proved by Ding *et al.* [1]) theorem, the function F and G can be expressed, respectively, in terms of five and four harmonic functions

(i) $F = F_1 + F_2 + F_3 + F_4 + F_5$ for distinct $s_j\,(j=1,2,3,4,5)$,

$G = G_1 + G_2 + G_3 + G_4$ for distinct $\overline{s}_j\,(j=1,2,3,4)$, (3.16a)

(ii) $F = F_1 + F_2 + F_3 + F_4 + zF_5$ for $s_1 \neq s_2 \neq s_3 \neq s_4 = s_5$,

$G = G_1 + G_2 + G_3 + zG_4$ for $\overline{s}_1 \neq \overline{s}_2 \neq s_3 = s_4$, (3.16b)

(iii) $F = F_1 + F_2 + F_3 + zF_4 + z^2 F_5$ for $s_1 \neq s_2 \neq s_3 = s_4 = s_5$,

$G = G_1 + G_2 + zG_3 + z^2 G_4$ for $\overline{s}_1 \neq \overline{s}_2 = s_3 = s_4$, (3.16c)

(iv) $F = F_1 + F_2 + zF_3 + z^2 F_4 + z^3 F_5$ for $s_1 \neq s_2 = s_3 = s_4 = s_5$, (3.16d)

(v) $F = F_1 + zF_2 + z^2 F_3 + z^3 F_4 + z^4 F_5$ for $s_1 = s_2 = s_3 = s_4 = s_5$,

$G = G_1 + zG_2 + z^2 G_3 + z^3 G_4$ $\overline{s}_1 = \overline{s}_2 = \overline{s}_3 = \overline{s}_4$ (3.16e) where $F_j(j=1,2,3,4,5)$ and $G_j(j=1,2,3,4)$ satisfies the following harmonic equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z_j^2}\right)F_j=0, \qquad (j=1,2,3,4)\,, \tag{3.17a}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z_j^2}\right)G_j=0, \qquad (j=1,2,3). \tag{3.17b}$$

The general solution for the case of distinct roots, can be derived as follows

$$u=\sum_{j=1}^{5}p_{1j}\frac{\partial^7 F_j}{\partial x\partial z_j^6}+\sum_{j=1}^{4}\overline{p}_{1j}\frac{\partial^5 G_j}{\partial x\partial z_j^4}, \qquad w=\sum_{j=1}^{5}s_j p_{2j}\frac{\partial^7 F_j}{\partial z_j^7}+\sum_{j=1}^{4}s_j\overline{p}_{2j}\frac{\partial^5 G_j}{\partial z_j^5};$$

$$\varphi=\sum_{j=1}^{4}\overline{p}_{3j}\frac{\partial^6 G_j}{\partial z_j^6}, \quad C=\sum_{j=1}^{5}p_{4j}\frac{\partial^8 F_j}{\partial z_j^8}+\sum_{j=1}^{4}\overline{p}_{4j}\frac{\partial^6 G_j}{\partial z_j^6}, \quad T=p_{55}\frac{\partial^8 F_5}{\partial z_5^8}+\overline{p}_{54}\frac{\partial^6 G_4}{\partial z_4^6}, \tag{3.18}$$

$$p_{kj=}-p_k+q_k s_j^2-r_k s_j^4+v_k s_j^6, \ (k=1,2,3\,\&\, j=1,2,3,4,5);$$

$$p_{4j}=p_4-q_4 s_j^2+r_4 s_j^4-v_4 s_j^6+w_4 s_j^8,$$

$$p_{55}=a^* s_5^8-b^* s_5^6+c^* s_5^4-d^* s_5^2-e^*,$$

$$\overline{p}_{kj}=\overline{p}_k-\overline{q}_k\overline{s}_j^2+\overline{r}_k\overline{s}_j^4, \qquad (k=1,2\,\&\, j=1,2,3,4),$$

$$\overline{p}_{kj}=-\overline{p}_k+\overline{q}_k s_j^2-\overline{r}_k s_j^4+\overline{v}_k s_j^6, \qquad (k=3,4\ \&\ j=1,2,3,4),$$

$$\overline{p}_{54}=\overline{a}\ \overline{s}_4^6-\overline{bs}_4^4+\overline{cs}_4^2-\overline{d}\ .$$

In a similar way, the general solution for the four three cases can be derived. Equation (3.18) can be further simplified by taking

$$p_{1j}\frac{\partial^6 F_j}{\partial z_j^6}=\Psi_j, \tag{3.19a}$$

and

$$\overline{p}_{1j}\frac{\partial^4 G_j}{\partial \overline{z}_j^4}=\overline{\Psi}_j. \tag{3.19b}$$

$$u=\sum_{j=1}^{5}\frac{\partial\psi_j}{\partial x}+\sum_{j=1}^{4}\frac{\partial\overline{\psi}_j}{\partial x},\qquad w=\sum_{j=1}^{5}s_jP_{1j}\frac{\partial\psi_j}{\partial z_j}+\sum_{j=1}^{4}\overline{s}_j\overline{P}_{1j}\frac{\partial\overline{\psi}_j}{\partial\overline{z}_j},$$

$$\varphi=\sum_{j=1}^{4}\overline{P}_{2j}\frac{\partial^2\overline{\psi}_j}{\partial\overline{z}_j^2},\qquad C=\sum_{j=1}^{5}P_{3j}\frac{\partial^4\psi_j}{\partial z_j^4}+\sum_{j=1}^{4}\overline{P}_{3j}\frac{\partial^2\overline{\psi}_j}{\partial\overline{z}_j^2},\qquad T=P_{45}\frac{\partial^4\psi_5}{\partial z_5^2}+\overline{p}_{44}\frac{\partial^2\overline{\psi}_4}{\partial\overline{z}_4^2},\qquad (3.20)$$

where

$$P_{1j}=p_{2j}/p_{1j},\qquad P_{2j}=p_{3j}/p_{1j},\qquad P_{3j}=p_{4j}/p_{1j},\qquad P_{45}=p_{55}/p_{15}.$$

$$\overline{P}_{1j}=\overline{p}_{2j}/\overline{p}_{1j},\qquad \overline{P}_{23}=\overline{p}_{33}/\overline{p}_{13},\ \ \overline{P}_{34}=\overline{p}_{44}/\overline{p}_{14}.$$

The functions $\psi_j\ (j=1,2,3,4,5)$ and $\overline{\psi}_j\ (j=1,2,3,4)$ satisfy the harmonic equations

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z_j^2}\right)\psi_j=0,\qquad j=1,2,3,4,5\ , \qquad (3.21a)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial \overline{z}_j^2}\right)\overline{\psi}_j=0\qquad j=1,2,3,4, \qquad (3.21b)$$

$$\begin{aligned}\sigma_{xx}&=\sum_{j=1}^{5}\left(-c_{11}+c_{13}s_j^2P_{1j}-b_1P_{3j}-a_1P_{4j}\right)\frac{\partial^2\psi_j}{\partial z_j^2}+\\&+\sum_{j=1}^{4}\left(-c_{11}+c_{13}\overline{s}_j^2\overline{P}_{1j}+B_1\overline{P}_{2j}-b_1\overline{P}_{3j}-a_1\overline{P}_{4j}\right)\frac{\partial^2\overline{\psi}_j}{\partial\overline{z}_j^2},\end{aligned} \qquad (3.22\ a)$$

$$\begin{aligned}\sigma_{zz}&=\sum_{j=1}^{5}\left(-c_{13}+c_{33}s_j^2P_{1j}-b_3P_{3j}-a_3P_{4j}\right)\frac{\partial^2\psi_j}{\partial z_j^2}+\\&+\sum_{j=1}^{4}\left(-c_{13}+c_{33}s_j^2\overline{P}_{1j}+B_3\overline{P}_{2j}-b_3\overline{P}_{3j}-a_3\overline{P}_{4j}\right)\frac{\partial^2\overline{\psi}_j}{\partial\overline{z}_j^2},\end{aligned} \qquad (3.22\ b)$$

$$\sigma_{zx}=\sum_{j=1}^{5}c_{44}(1+P_{1j})s_j\frac{\partial^2\psi_j}{\partial x\partial z_j}+\sum_{j=1}^{4}c_{44}(1+\overline{P}_{1j})\overline{s}_j\frac{\partial^2\overline{\psi}_j}{\partial x\partial\overline{z}_j}, \qquad (3.22c)$$

$$c_{11}-c_{13}s_j^2P_{1j}+b_1P_{3j}+a_1P_{4j}=c_{44}(1+P_{1j})s_j^2\ , \qquad (3.23a)$$

$$c_{11}-c_{13}\overline{s}_j^2\overline{P}_{1j}-B_1\overline{P}_{2j}+b_1\overline{P}_{3j}+a_1\overline{P}_{4j}=c_{44}(1+\overline{P}_{1j})\overline{s}_j^2\ , \qquad (3.23b)$$

$$-c_{13}+c_{33}s_j^2P_{1j}-b_3P_{3j}-a_3P_{4j}=c_{44}(1+P_{1j})\ , \qquad (3.24a)$$

$$-c_{13}+c_{33}s_j^2\overline{P}_{1j}+B_3\overline{P}_{2j}-b_3\overline{P}_{3j}-a_3\overline{P}_{4j}=c_{44}(1+\overline{P}_{1j})\,. \tag{3.24b}$$

The general solution Eqs (3.22a)-(3.22c) with the help of Eqs (3.23a, b) and (3.24a, b) can be simplified as

$$\sigma_{xx}=-\sum_{j=1}^{5}s_j^2 w_{1j}\frac{\partial^2\psi_j}{\partial z_j^2}-\sum_{j=1}^{4}\overline{s}_j^2\overline{w}_{1j}\frac{\partial^2\overline{\psi}_j}{\partial \overline{z}_j^2},\qquad \sigma_{zz}=\sum_{j=1}^{5}w_{1j}\frac{\partial^2\psi_j}{\partial z_j^2}+\sum_{j=1}^{4}\overline{w}_{1j}\frac{\partial^2\overline{\psi}_j}{\partial \overline{z}_j^2},$$

$$\sigma_{zx}=\sum_{j=1}^{5}s_j w_{1j}\frac{\partial^2\psi_j}{\partial x\partial z_j}, \tag{3.25}$$

where

$$w_{1j}=\frac{c_{11}-c_{13}s_j^2P_{1j}+b_1P_{3j}+a_1P_{4j}}{s_j^2}=c_{44}(1+P_{1j})=-c_{13}+c_{33}s_j^2P_{1j}-b_3P_{3j}-a_3P_{4j}. \tag{3.26}$$

$$\overline{w}_{1j}=\frac{c_{11}-c_{13}\overline{s}_j^2\overline{P}_{1j}-B_1\overline{P}_{2j}+b_1\overline{P}_{3j}+a_1\overline{P}_{4j}}{\overline{s}_j^2}=c_{44}(1+\overline{P}_{1j})=$$

$$=-c_{13}+c_{33}s_j^2\overline{P}_{1j}+B_3\overline{P}_{2j}-b_3\overline{P}_{3j}-a_3\overline{P}_{4j}. \tag{3.27}$$

4. Fundamental solution for a point heat source in a semi-infinite orthotropic thermoelastic material with voids

We consider a semi-infinite orthotropic thermoelastic material with diffusion and voids $z\geq 0$. A point heat source H is applied at the origin and the surface $z=0$ is free, equilibrated thermally insulated. The complete geometry of the problem is shown in Fig.1. The general solution given by Eqs (3.20) and (3.25) is derived in this section.

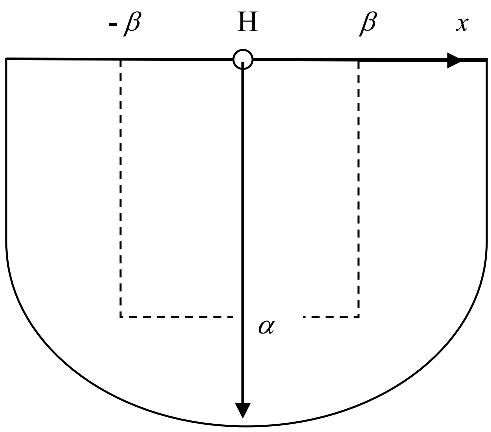

Fig.1. Geometry of the problem.

Introduce the harmonic functions as

$$\psi_j = A_j\left[\frac{1}{2}(z_j^2 - x^2)\left(\log r_j - \frac{3}{2}\right) - xz_j \tan^{-1}\frac{x}{z_j}\right] \quad j = 1,2,3,4,5 \tag{4.1}$$

where A_j ($j=1, 2, 3, 4, 5$) are arbitrary constants to be determined and

$$r_j = \sqrt{x^2 + z_j^2}, \tag{4.2}$$

and

$$\overline{\psi}_j = \overline{A}_j\left[\frac{1}{2}\left(\overline{z}_j^2 - x^2\right)\left(\log\overline{r}_j - \frac{3}{2}\right) - x\overline{z}_j \tan^{-1}\frac{x}{\overline{z}_j}\right], \qquad j = 1,2,3,4 \tag{4.3}$$

where $\overline{A}_j$ $(j = 1,2,3,4)$ are arbitrary constants to be determined and

$$\overline{r}_j = \sqrt{x^2 + \overline{z}_j^2}. \tag{4.4a}$$

Here, $\overline{A}_4$ can be written as a linear combination of A_5 i.e. $\overline{A}_4 = \eta A_3$ (4.4b)
where η is some arbitrary constant.

The boundary conditions on the surface $z = 0$ are

$$\sigma_{zz} = \sigma_{zx} = 0, \qquad \frac{\partial T}{\partial z} = 0, \qquad \frac{\partial \varphi}{\partial z} = 0, \qquad \frac{\partial C}{\partial z} = 0. \tag{4.5}$$

When the volume fraction field, concentration and thermal condition for a rectangle of $0 \le z \le \alpha$ and $-\beta \le x \le \beta$ $(b > 0)$ are considered [Fig.1], the following equations can be obtained

$$\int_{-\beta}^{\beta} \sigma_{zz}(x,\alpha)dx + \int_0^{\alpha}\left[\sigma_{zx}(\beta,z) - \sigma_{zx}(-\beta,z)\right]dz = 0, \tag{4.6a}$$

$$\int_{-\beta}^{\beta} \frac{\partial\varphi}{\partial z}(x,\alpha)dx + \int_0^{\alpha}\left[\frac{\partial\varphi}{\partial x}(\beta,z) - \frac{\partial\varphi}{\partial x}(-\beta,z)\right]dz = 0, \tag{4.6b}$$

$$\int_{-\beta}^{\beta}\left[\frac{\partial C}{\partial z}(x,\alpha)\right]dx - \int_0^{\alpha}\left[\frac{\partial C}{\partial x}(\beta,z) - \frac{\partial C}{\partial x}(-\beta,z)\right]dz = 0. \tag{4.6c}$$

$$-a_3\int_{-\beta}^{\beta}\left[\frac{\partial T}{\partial z}(x,\alpha)\right]dx - a_1\int_0^{\alpha}\left[\frac{\partial T}{\partial x}(\beta,z) - \frac{\partial T}{\partial x}(-\beta,z)\right]dz = H. \tag{4.6d}$$

Substituting the values of ψ_j and $\overline{\psi}_j$ from Eqs (4.1) and (4.3) in Eqs (3.20) and (3.25), we obtain the expressions for components of displacement, temperature change, volume fraction field and stress components as follows

$$u = -\sum_{j=1}^{5} A_j \left[x(\log r_j - 1) + z_j \tan^{-1} \frac{x}{z_j} \right] - \sum_{j=1}^{4} \overline{A}_j \left[x(\log \overline{r}_j - 1) + \overline{z}_j \tan^{-1} \frac{x}{\overline{z}_j} \right], \tag{4.7a}$$

$$w = \sum_{j=1}^{5} s_j P_{1j} A_j \left[z_j(\log r_j - 1) - x \tan^{-1} \frac{x}{z_j} \right] + \sum_{j=1}^{4} \overline{s}_j \overline{P}_{1j} \overline{A}_j \left[\overline{z}_j(\log \overline{r}_j - 1) - x \tan^{-1} \frac{x}{\overline{z}_j} \right], \tag{4.7b}$$

$$\varphi = \sum_{j=1}^{4} \overline{A}_j \overline{P}_{2j} \log \overline{r}_j, \tag{4.7c}$$

$$C = \sum_{j=1}^{5} A_j P_{2j} \log r_j + \sum_{j=1}^{4} \overline{A}_j \overline{P}_{2j} \log \overline{r}_j, \tag{4.7d}$$

$$T = A_5 P_{45} \log r_5 + \overline{A}_4 \overline{P}_{44} \log \overline{r}_4, \tag{4.7e}$$

$$\sigma_{xx} = -\sum_{j=1}^{5} s_j^2 w_{1j} A_j \log r_j - \sum_{j=1}^{4} \overline{s}_j^2 \overline{w}_{1j} \overline{A}_j \log \overline{r}_j, \tag{4.7f}$$

$$\sigma_{zz} = \sum_{j=1}^{5} w_{1j} A_j \log r_j + \sum_{j=1}^{4} \overline{w}_{1j} \overline{A}_j \log \overline{r}_j, \tag{4.7g}$$

$$\sigma_{zx} = -\sum_{j=1}^{5} s_j w_{1j} A_j \tan^{-1} \frac{x}{z_j} - \sum_{j=1}^{4} \overline{s}_j \overline{w}_{1j} \overline{A}_j \tan^{-1} \frac{x}{\overline{z}_j}. \tag{4.7h}$$

Making use the values of $\sigma_{zz}, \sigma_{zx}, C, \varphi$ and T from Eqs (4.7 c, d, e, g, h) in Eq.(4.5), we obtain

$$\sum_{j=1}^{5} w_{1j} A_j = 0, \tag{4.8a}$$

$$\sum_{j=1}^{5} \overline{w}_{1j} \overline{A}_j = 0, \tag{4.8b}$$

$$\sum_{j=1}^{4} s_j w_{1j} A_j = 0, \tag{4.8c}$$

$$\sum_{j=1}^{4} \overline{s}_j \overline{w}_{1j} \overline{A}_j = 0, \tag{4.8 d}$$

$\frac{\partial C}{\partial z}, \frac{\partial T}{\partial z}$ and $\frac{\partial \varphi}{\partial z}$ are automatically satisfied at the surface $z = 0$.

Making use of the values of σ_{zz} and σ_{zx} from Eqs (4.7 f, g) in Eq.(4.6a), we obtain

$$\sum_{j=1}^{5} w_{1j} A_j I_3 + \sum_{j=1}^{4} \overline{w}_{1j} \overline{A}_j I_4 = 0, \tag{4.9}$$

where

$$I_3 = \left[x\left(\log\sqrt{x^2 + s_j^2\alpha^2} - 1\right) + s_j\alpha \tan^{-1}\frac{x}{s_j\alpha} \right]_{x=-\beta}^{x=\beta} +$$

$$-2\left[z_j \tan^{-1}\frac{\beta}{s_j z} + b\log\sqrt{\beta^2 + s_j^2 z^2} \right]_{z=0}^{z=\alpha} = 2\beta(\log\beta - 1), \tag{4.10a}$$

and

$$I_4 = \left[x\left(\log\sqrt{x^2 + \overline{s}_j^2\alpha^2} - 1\right) + \overline{s}_j\alpha \tan^{-1}\frac{x}{\overline{s}_j\alpha} \right]_{x=-\beta}^{x=\beta} +$$

$$-2\left[\overline{z}_j \tan^{-1}\frac{\beta}{\overline{s}_j z} + \beta\log\sqrt{\beta^2 + \overline{s}_j^2 z^2} \right]_{z=0}^{z=\alpha} = 2\beta(\log\beta - 1). \tag{4.10b}$$

By virtue of Eqs (4.10 a, b), Eq.(4.9) degenerate to Eqs (4.8 a, b) i.e., Eqs (3.6a) and (4.9) are satisfied automatically.

Some useful integrals are given as follows

$$\int \frac{\partial \varphi}{\partial z} = \sum_{j=1}^{4} \overline{A}_j \overline{s}_j^2 \overline{P}_{2j} \int \frac{\overline{z}}{x^2 + \overline{s}_j^2 \overline{z}^2} dx = \sum_{j=1}^{5} \overline{A}_j \overline{s}_j \overline{P}_{2j} \tan^{-1}\frac{x}{\overline{z}_j}, \tag{4.11a}$$

$$\int \frac{\partial \varphi}{\partial x} dz = \sum_{j=1}^{4} \overline{A}_j \overline{P}_{2j} \int \frac{x}{x^2 + \overline{s}_j^2 \overline{z}^2} dz = -\sum_{j=1}^{4} \frac{\overline{A}_j}{\overline{s}_j} \overline{P}_{2j} \tan^{-1}\frac{x}{\overline{z}_j}, \tag{4.11b}$$

$$\begin{aligned} \int \frac{\partial C}{\partial z} dx &= \sum_{j=1}^{5} A_j s_j^2 P_{2j} \int \frac{z}{x^2 + s_j^2 z^2} dx + \sum_{j=1}^{4} \overline{A}_j \overline{s}_j^2 \overline{P}_{2j} \int \frac{\overline{z}}{x^2 + \overline{s}_j^2 \overline{z}^2} dx = \\ &= \sum_{j=1}^{5} A_j s_j \overline{P}_{2j} \tan^{-1}\frac{x}{z_j} + \sum_{j=1}^{4} \overline{A}_j \overline{s}_j \overline{P}_{2j} \tan^{-1}\frac{x}{\overline{z}_j}, \end{aligned} \tag{4.11c}$$

$$\begin{aligned} \int \frac{\partial C}{\partial x} dz &= \sum_{j=1}^{5} A_j P_{2j} \int \frac{x}{x^2 + s_j^2 z^2} dz + \sum_{j=1}^{4} \overline{A}_j \overline{P}_{2j} \int \frac{x}{x^2 + \overline{s}_j^2 \overline{z}^2} dz = \\ &= -\sum_{j=1}^{5} \frac{A_j}{s_j} P_{2j} \tan^{-1}\frac{x}{z_j} - \sum_{j=1}^{4} \frac{\overline{A}_j}{\overline{s}_j} \overline{P}_{2j} \tan^{-1}\frac{x}{\overline{z}_j}, \end{aligned} \tag{4.11d}$$

$$\int \frac{\partial T}{\partial z} dx = s_5 P_{45} A_5 \int \frac{z_5}{r_5^2} dx + \bar{s}_4 \bar{P}_{44} \bar{A}_4 \int \frac{\bar{z}_4}{\bar{r}_4^2} dx = s_5 P_{45} A_5 \tan^{-1} \frac{x}{z_5} + \bar{s}_4 \bar{P}_{44} \bar{A}_4 \tan^{-1} \frac{x}{\bar{z}_4}, \tag{4.11 e}$$

$$\int \frac{\partial T}{\partial x} dz = P_{45} A_5 \int \frac{x}{r_5^2} dz + \bar{P}_{44} \bar{A}_4 \int \frac{x}{\bar{r}_4^2} dz = -\frac{P_{45}}{s_5} A_5 \tan^{-1} \frac{x}{z_5} - \frac{\bar{P}_{44}}{\bar{s}_4} \bar{A}_4 \tan^{-1} \frac{x}{\bar{z}_4}, \tag{4.11 f}$$

Making use of Eq.(4.7e) in Eq.(4.6d), with the aid of $s_5 = \sqrt{\frac{K_1}{K_3}} = \bar{s}_4$ and the integrals (4.11 e, f), we obtain

$$P_{45} A_5 I_5 + \bar{P}_{44} \bar{A}_4 I_6 = \frac{H}{\sqrt{K_3/K_1}}, \tag{4.12}$$

$$I_5 = -\left[\tan^{-1}\left(\frac{x}{s_5 \alpha}\right)\right]_{x=-\beta}^{x=\beta} + \left[\tan^{-1}\left(\frac{\beta}{s_5 z}\right)\right]_{z=0}^{z=\alpha} = -\pi, \tag{4.13a}$$

$$I_6 = -\left[\tan^{-1}\left(\frac{x}{\bar{s}_4 \alpha}\right)\right]_{x=-\beta}^{x=\beta} + \left[\tan^{-1}\left(\frac{\beta}{\bar{s}_4 z}\right)\right]_{z=0}^{z=\alpha} = -\pi. \tag{4.13b}$$

A_5 can be determined from Eqs (4.12) and (4.13 a, b), as follows

$$A_5 = -\frac{H}{\pi (P_{45} + \alpha \bar{P}_{44}) \sqrt{K_3/K_1}}. \tag{4.14}$$

Substituting the value of φ from Eq.(4.7c) in Eq.(4.6b) and with the aid of the integrals (4.11 a, b), we obtain

$$\sum_{j=1}^{4} \bar{r}_j \bar{P}_{2j} \bar{A}_j = 0, \tag{4.15}$$

where

$$r_j = \left[\bar{s}_j^2 \tan^{-1}\left(\frac{x}{\bar{s}_j \alpha}\right)\right]_{x=-\beta}^{x=\beta} - \left[2\tan^{-1}\left(\frac{\beta}{\bar{s}_j z}\right)\right]_{z=0}^{z=\alpha}. \tag{4.16}$$

On simplifying, we obtain

$$r_j = 2\left(\bar{s}_j^2 - 1\right)\tan^{-1}\left(\frac{\beta}{\bar{s}_j \alpha}\right) + \pi.$$

Substituting the value of C from Eq.(4.7d) in Eq.(4.6c) and with the aid of the integrals (4.11 c, d), we obtain

$$\sum_{j=1}^{5} q_j P_{2j} A_j = 0, \tag{4.17}$$

$$\sum_{j=1}^{4} \overline{q}_j \overline{P}_{2j} \overline{A}_j = 0 \tag{4.18}$$

where

$$q_j = 2\left(s_j^2 - 1\right)\tan^{-1}\left(\frac{\beta}{s_j \alpha}\right) + \pi.$$

$$\overline{q}_j = 2\left(\overline{s}_j^2 - 1\right)\tan^{-1}\left(\frac{\beta}{\overline{s}_j \alpha}\right) + \pi.$$

Thus the nine constants $A_j\ (j = 1,2,3,4,5)$, $\overline{A}_j (j = 1,2,3,4)$ can be determined by nine equations including Eqs (4.8a) - (4.8d), (4.14) and (4.15), (4.17) and (4.18) and by the relation given in Eq. (4.4b).

5. Special cases

Case I: In the absence of diffusion effect

In the absence of voids effect Eqs (4.7a)-(4.7h) reduce to

$$u = -\sum_{j=1}^{4} A_j \left[x(\log r_j - 1) + z_j \tan^{-1}\frac{x}{z_j} \right] - \sum_{j=1}^{3} \overline{A}_j \left[x(\log \overline{r}_j - 1) + \overline{z}_j \tan^{-1}\frac{x}{\overline{z}_j} \right], \tag{5.1 a}$$

$$w = \sum_{j=1}^{4} s_j P_{1j} A_j \left[z_j (\log r_j - 1) - x \tan^{-1}\frac{x}{z_j} \right] + \sum_{j=1}^{3} \overline{s}_j \overline{P}_{1j} \overline{A}_j \left[\overline{z}_j (\log \overline{r}_j - 1) - x \tan^{-1}\frac{x}{\overline{z}_j} \right], \tag{5.1 b}$$

$$\varphi = \sum_{j=1}^{3} \overline{A}_j \overline{P}_{2j} \log \overline{r}_j, \tag{5.1 c}$$

$$T = A_4 P_{34} \log r_4 + \overline{A}_3 \overline{P}_{33} \log \overline{r}_3, \tag{5.1 d}$$

$$\sigma_{xx} = -\sum_{j=1}^{4} s_j^2 w_{1j} A_j \log r_j - \sum_{j=1}^{3} \overline{s}_j^2 \overline{w}_{1j} \overline{A}_j \log \overline{r}_j, \tag{5.1 e}$$

$$\sigma_{zz} = \sum_{j=1}^{4} w_{1j} A_j \log r_j + \sum_{j=1}^{3} \overline{w}_{1j} \overline{A}_j \log \overline{r}_j, \tag{5.1f}$$

$$\sigma_{zx} = -\sum_{j=1}^{4} s_j w_{1j} A_j \tan^{-1}\frac{x}{z_j} - \sum_{j=1}^{3} \bar{s}_j \bar{w}_{1j} \bar{A}_j \tan^{-1}\frac{x}{\bar{z}_j}. \tag{5.1g}$$

The above results are similar to those obtained by Kumar and Chawla [14].

Case II: In the absence of voids and diffusion effects

In the absence of voids and diffusion effects Eqs (4.7a)-(4.7h) reduce to

$$u = -\sum_{j=1}^{3} A_j \left[x(\log r_j - 1) + z_j \tan^{-1}\frac{x}{z_j} \right], \tag{5.2a}$$

$$w = \sum_{j=1}^{3} s_j P_{1j} A_j \left[z_j (\log r_j - 1) - x \tan^{-1}\frac{x}{z_j} \right], \tag{5.2b}$$

$$T = A_3 P_{23} \log r_3, \tag{5.2c}$$

$$\sigma_{xx} = -\sum_{j=1}^{3} s_j^2 w_{1j} A_j \log r_j, \tag{5.2d}$$

$$\sigma_{zz} = \sum_{j=1}^{3} w_{1j} A_j \log r_j, \tag{5.2e}$$

$$\sigma_{zx} = -\sum_{j=1}^{3} s_j w_{1j} A_j \tan^{-1}\frac{x}{z_j}. \tag{5.2f}$$

The above results are similar to those obtained by Hou *et al*. [12].

Case III: In the absence of voids, thermal and diffusion effects

In the absence of voids, thermal and diffusion effects, we obtain the corresponding results for a transversely isotropic elastic medium.

6. Conclusion

The general solution and fundamental solution for a two-dimensional problem in transversely isotropic thermoelastic media with mass diffusion and voids have been constructed. The two-dimensional general solution in transversely isotropic thermoelastic media with mass diffusion and voids is derived first by using the operator theory. On the basis of the obtained general solution, the fundamental solution for a steady point heat source on the surface of a semi-infinite transversely isotropic thermoelastic material with mass diffusion and voids is derived by nine new introduced harmonic functions. The components of displacement, stress, temperature change, mass concentration and voids are expressed in terms of elementary functions, so it is convenient to use them. From the present investigation, some special cases of interest are also deduced and compared with the previous results.

Applications: fundamental solutions for two dimensional in anisotropic media are important for the solution of inclusion problems and of the boundary integral equations. This type of solution technique (which, has been used in this research paper) is very useful for finding the general solution and fundamental solution in anisotropic media for different theories, i.e. micropolar thermoelastic material with voids, micropolar thermoelastic material with mass diffusion and voids, microstretch thermoelastic material, microstretch thermoelastic material with mass diffusion, etc. This type of solution technique provides a wonderful platform for new researcher studies to construct the general solution in thermoelasticity with double porosity and triple porosity. Also, this type of solution technique will be very useful to construct fundamental solution for three dimensional problems and Green's function in different symmetries which will be very useful for solving boundary value problems as well as for the study of cracks, defects and inclusions.

Appendix A

$$a^* = c_{66}\delta_1,\ b^* = c_{11}\delta_1 + c_{66}[\,\gamma_3^2\delta_2 - d\alpha_3^* A_3 c_{44} + dc_{33}\delta_2] +$$
$$+\alpha_3^* A_3\delta_3(d\delta_3 - \gamma_1 A_3) + \gamma_1\alpha_3^*(\gamma_3 A_1\delta_3 - \gamma_1 A_3 c_{33}),$$

$$c^* = c_{11}[\gamma_3^2\delta_2 - d(\alpha_3^* A_3 c_{44} - \delta_2 c_{33})] + \alpha_1^* A_1(\gamma_3^2 - dc_{33}) + d\delta_2 +$$
$$-\delta_3\delta_2(d\delta_3 - 2\gamma_1\gamma_3) - \gamma_1^2(\alpha_3^* A_3 c_{44} + c_{33}\,\delta_2),$$

$$d^* = c_{11}[\,\alpha_1^* A_1(\gamma_3^2\delta_2 - dc_{33}) + d\delta_2] + \alpha_1^* A_1[\delta_3(d\delta_3 - \gamma_1\gamma_3) - c_{44}c_{66}d] +$$
$$+\gamma_1^2\ (c_{44}\delta_2 - \alpha_1^* A_1 c_{33}) + \gamma_1\gamma_3\alpha_1^* A_1\delta_3,$$

$$e^* = -\alpha_1^* A_1 c_{44}(dc_{11} + \gamma_1^2)\,, \qquad \overline{a} = c_{66}\alpha_3^*[B_3\delta_4 + \gamma_2\delta_5 - c_{33}\delta_6],$$

$$\overline{b} = c_{66}\alpha_1^*[c_{33}\delta_6 + B_3\delta_4 + \gamma_3\delta_5] - c_{66}\alpha_3^*\delta_6 + c_{11}\alpha_3^*[B_3\delta_4 + \gamma_3\delta_5 - c_{33}\delta_6] +$$
$$-\delta_3\alpha_3^*[\gamma_3(B_1 b_2^* - \xi\gamma_1) + \delta_3\delta_6 + B_3(b_2^*\gamma_1 - B_1 d)] +$$
$$+\alpha_3^*[\delta_3 B_3 d - b_2^*\gamma_3 - B_1 dc_{33} - b_2^*\gamma_1 c_{33} + B_1\gamma_3^2 - B_1\gamma_1\gamma_3 + \gamma_1\delta_3(B_3 b_2^* - \xi\gamma_3) +$$
$$+(\xi\gamma_1 - b_2^* B_1)c_{33} + B_3\delta_{12}], \qquad \overline{d} = \alpha_1^*[c_{44}(c_{11}\delta_6 + B_1\delta_7) + \gamma_1(\xi\gamma_1 - B_1 b_2^*)],$$

$$\overline{c} = \delta_6(c_{33}\alpha_1^* - \alpha_3^*) + \alpha_1^*(B_3\delta_4 + \gamma_3\delta_5 + c_{44}c_{66}\delta_6) +$$
$$-\delta_3\alpha_1^*[B_1\gamma_1 b_2^* + \delta_3\delta_6 + B_3\delta_7] + B_1[-\alpha_1^*\delta_3\delta_4 + \delta_7(c_{44}\alpha_3^* + c_{33}\alpha_1^*) + \alpha_1^*\gamma_3\delta_{12}] +$$
$$+\alpha_1^*\gamma_1[\delta_3(b_2^* B_3 + \xi\gamma_3) + B_3\delta_{12}] + (\xi\gamma_1 - b_2^* B_1)(c_{44}\alpha_3^*\gamma_1 + c_{33}\alpha_1^*).$$

Appendix B

$$p_1 = -\gamma_1 a\alpha_1^* c_{44},$$

$$q_1 = A_1\alpha_1^*\delta_3(a\gamma_3 + d\beta_3) + \gamma_1[ac_{44}\delta_2 + A_1\alpha_1^*(ac_{33} + \gamma_3\beta_3)] +$$
$$-dB_1(\alpha_3^* + \alpha_1^*)(A_1 c_{33} + A_3 c_{44}),$$

$$r_1 = -\delta_2[(a\gamma_3 + d\beta_3)\delta_3 + \gamma_1(\gamma_3 B_3 - ac_{33} + \gamma_3^2)] - ac_{44}\gamma_1 A_3\alpha_3^*$$

$$v_1 = \delta_3 A_3 \alpha_3^*(a\gamma_3 + d\beta_3) - \gamma_1 A_3(\gamma_3 B_3 + a\alpha_3^* c_{33}) - B_1 A_3 \alpha_3^*(dc_{33} + \gamma_3^2),$$

$$\overline{p}_1 = \gamma_1 \alpha_1^* c_{44}(a\xi + b_1^* b_2^*) + \xi d\alpha_1^* c_{44} B_1$$

$$\overline{q}_1 = \alpha_1^* \delta_3[B_3\delta_9 - \gamma_3\delta_8 - \beta_3\delta_6] +$$
$$+\alpha_1^* B_1\left[\gamma_3\left\{(aB_3 + \gamma_3 b_1^*) - B_3(\gamma_3 b_2^* - dB_3) - c_{33}\delta_9 - \delta_4\right\} - \alpha_3^* c_{33}\delta_9\right] +$$
$$-\gamma_1\left[\alpha_1^*\left\{B_3(aB_3 + b_1^*\gamma_3) - c_{33}(a\xi + b_1^* b_2^*) + B_3(B_3 b_2^* - \xi\gamma_3)\right\} - \alpha_3^* c_{44}\delta_8\right] +$$
$$+\xi dB_1(\alpha_1^* c_{33} + \alpha_3^* c_{44}) + B_1 B_3 \alpha_1^* \delta_4 - \alpha_1^* \gamma_1(\xi\gamma_3 - b_2^*),$$

$$\overline{r}_1 = \alpha_3^* \delta_3[B_3\delta_9 - \gamma_3\delta_8 - \beta_3\delta_6] + \alpha_3^* B_1[\gamma_3(aB_3 + \gamma_3 b_1^*) - B_3\delta_4 - c_{33}\delta_9] +$$
$$-\gamma_1[\alpha_3^* B_3(aB_3 + \gamma_3 b_1^*) - \alpha_3^* \delta_8] + \xi c_{33} d\alpha_3^* B_1 + B_1 B_3 \alpha_3^* \delta_4 + \gamma_3 \alpha_3^* \delta_5,$$

$$\overline{p}_3 = \alpha_1^* c_{44}(\delta_9 c_{11} + \delta_7 \beta_1),$$

$$\overline{q}_3 = c_{11}[\delta_9(\alpha_3^* c_{44} + \alpha_1^*) - \gamma_3 \alpha_1^* \delta_{11} + \alpha_1^* \delta_4] - \alpha_1^* \delta_3[\delta_9\delta_3 - \gamma_3\delta_{10} + \beta_3\delta_7] +$$
$$+\alpha_1^* \delta_9 c_{44} c_{66} + \gamma_1 \alpha_3^* c_{44} \delta_{10} - \gamma_1 \alpha_1^* \delta_3 + \beta_3 \gamma_1 \alpha_1^* \delta_{12} - \beta_1 \delta_3 \alpha_1^* \delta_4 + \delta_7(c_{33}\beta_1 + c_{44}\alpha_3^* \beta_1) +$$
$$-\beta_1 \gamma_3 \alpha_1^* \delta_{12},$$

$$\overline{r}_3 = [\alpha_3^* c_{11}\left\{\delta_9 c_{44} - \beta_3\delta_4\right\} + c_{66}\left\{\delta_9(\alpha_3^* c_{44} + \alpha_1^* c_{33}) + \alpha_1^*(\beta_3\delta_4 - \gamma_3\delta_{11})\right\} +$$
$$-\delta_3\alpha_3^*\left\{\delta_3\delta_9 - \delta_{10} + \delta_7\right\} + \alpha_3^*(\gamma_1\delta_3\delta_{11} + \beta_3\delta_{12}) - \beta_1\alpha_3^*(\delta_3\delta_4 + c_{33}\delta_7 - \gamma_3\delta_{12}),$$

$$\overline{v}_3 = \alpha_3^* c_{66}[\delta_9 c_{44} d + \beta_3\delta_4 - \gamma_3\delta_{11}],$$

$$p_4 = \alpha_1^* c_{44} A_1(\gamma_1 B_1 - ac_{11}),\ w_4 = -c_{66} A_3 \gamma_3 \beta_3 \alpha_3^*,$$

$$q_4 = c_{11}\left\{ac_{44}\delta_2 + A_1\alpha_1^*(ac_{33} - \gamma_3\beta_3)\right\} + \delta_3\left\{A_1\alpha_1^*(\delta_3 a + \gamma_1\beta_3)\right\} +$$
$$+A_1\alpha_1^* B_1(\gamma_1 c_{33} - \gamma_3\delta_3) + c_{44} B_1 \gamma_1 \delta_2,$$

$$r_4 = c_{66}\left\{ac_{44}\delta_2 - A_1\alpha_1^*(ac_{33} - \gamma_3\beta_3)\right\} + c_{11}\left\{ac_{44}A_1\alpha_3^* - \delta_2(a + \gamma_3\beta_3)\right\} +$$
$$+\delta_3\left\{\delta_2(\gamma_1\beta_3 - a\delta_3 A_1\alpha_3^*)\right\} + B_1\delta_3(\delta_3\gamma_3 + c_{33}\gamma_1),$$

$$v_4 = c_{66}\left\{ac_{44}\delta_2 - A_1\alpha_1^*(ac_{33} - \gamma_3\beta_3)\right\} + c_{11}\gamma_3\alpha_3^* A_3\beta_3 +$$
$$+\delta_2\alpha_3^* A_3(a\delta_3 + \gamma_1\beta_3) - \alpha_3^* B_1(\delta_3\gamma_3 A_1 + c_{33}\gamma_1\beta_3),$$

$$\overline{p}_4 = \alpha_1^* c_{44}[\delta_8 c_{11} - B_1\delta_{10} + B_1\delta_{13}], \qquad \overline{v}_4 = c_{44}\alpha_1^*\{c_{11}\delta_8 - B_1\delta_{10} + B_1\delta_{13}\},$$

$$\overline{r}_4 = c_{66}\left\{\delta_8(c_{44}\alpha_3^* + c_{33}\alpha_1^*) - \beta_3\alpha_1^*\delta_{11} + \alpha_1^*\delta_5\right\} + c_{11}\alpha_3^*(c_{33}\delta_8 - B_3\delta_{11}) +$$
$$-\delta_3\alpha_3^*(\delta_3\delta_8 + B_3\delta_{10} + \beta_3\delta_{13}) + B_1\alpha_3^*(\delta_{11}\delta_3 - c_{33}\delta_{10} + \beta_3\delta_{12} - \delta_3\delta_5 + B_3\delta_{12} + c_{33}\delta_{14}),$$

$$\overline{q}_4 = c_{44}c_{66}\alpha_1^*\delta_8 + c_{11}\left\{\delta_8(c_{44}\alpha_3^* + c_{33}\alpha_1^*) - \alpha_1^* + A_1\alpha_1^*(ac_{33} - \gamma_3\beta_3)\right\} +$$
$$+\delta_3\left\{A_1\alpha_1^*(\delta_3 a + \gamma_1\beta_3) + \alpha_1^*\delta_5\right\} + \delta_3\alpha_1^*(\beta_3\delta_{13} - \delta_3 + B_3\delta_{10}) +$$
$$+B_1\alpha_1^*(\delta_3\delta_{11} - c_{33}\delta_{10}) - B_1\left\{\beta_3(B_1\gamma_3 - \gamma_1\alpha_1^*) - c_{44}\alpha_3^*\delta_{10}\right\} + \delta_3 B_1\alpha_1^*(\xi\gamma_3 - \alpha_1^* b_2^*) +$$
$$+c_{33}B_1\alpha_1^*\delta_{13} + c_{44}B_1\alpha_3^*(\xi\gamma_1 - \gamma\beta_3) + B_1B_3\alpha_1^*\delta_{13},$$

$$\delta_1 = \alpha_3^*(\gamma_3^2 - dc_{33}), \qquad \delta_2 = \alpha_1^* A_3 - \alpha_3^* A_1, \qquad \delta_3 = c_{13} + c_{44},$$

$$\delta_4 = b_2^*\gamma_3 - B_3 d, \qquad \delta_5 = b_2^* B_3 - \xi\gamma_3, \qquad \delta_6 = \xi d - b_2^{*2},$$

$$\delta_7 = b_2^*\gamma_1 - B_1 d, \qquad \delta_8 = a\xi + b_1^* b_2^*, \qquad \delta_9 = ab_2^* + db_1^*, \qquad \delta_{10} = aB_1 + \gamma_1 b_1^*,$$

$$\delta_{11} = aB_3 + \gamma_3 b_1^*, \qquad \delta_{12} = \gamma_1 B_3 - \gamma_3 B_1, \qquad \delta_{13} = b_2^* B_1 - \gamma_1\xi, \qquad \delta_{14} = \gamma_1\xi - \gamma_1 B_3.$$

Nomenclature

a, d	– are, respectively, coefficients describing the measure of thermodiffusion and mass diffusion effects
C	– concentration of diffusive material in the elastic body
C^*	– specific heat at constant strain
$c_{ijkm}(= c_{kmij} = c_{jikm})$	– tensor of elastic tensor
$e_{ij} = \frac{u_{i,j} + u_{j,i}}{2}$	– components of the strain tensor
$k_{ij}(= k_{ji})$	– coefficients of thermal conductivity
T	– temperature distribution from the reference temperature T_0
u_i	– components of displacement vector
$\alpha_{ij}^*(= \alpha_{ji}^*)$	– coefficients of diffusion tensor
β_{ij}	– tensors of thermal moduli
γ_{ij}	– tensors of diffusion moduli
ρ	– density
χ	– equilibrated inertia
φ	– volume fraction field

The symbol (“,”) followed by a suffix denotes differentiation with respect to the spatial coordinate and a superposed dot (“.”) denotes the derivative with respect to time.

References

[1] Ding H.J., Chen B. and Liang J. (1996): *General solutions for coupled equations in piezoelectric media*. – Int. J. Solids Struct., vol.33, pp.2283-2298.

[2] Dunn M.L. and Wienecke H.A. (1999): *Half space Green's functions for transversely isotropic piezoelectric solids.* – Journal of Applied Mechanics, vol.66, pp.675-699.

[3] Pan E. and Tanon F. (2000): *Three dimensional Green's functions in anisotropic piezoelectric solids.* – Int. J. Solids Struct., vol.37, pp. 943-958.

[4] Chen W.Q. (2000): *On the general solution for piezothermoelasticity for transverse isotropy with applications*. – ASME J. Appl. Mech., vol.67, pp.705-711.

[5] Chen W.Q., Lim C.W. and Ding H.J. (2005): *Point temperature solution for a penny- shaped crack in an infinite transversely isotropic thermo-piezo-elastic medium*. − Eng. Anal. with Bound. Elem., vol29, pp524-532.

[6] Sharma B. (1958): *Thermal stresses in transversely isotropic semi-infinite elastic solids*. − ASME J. Appl. Mech., vol.23, pp.86-88.

[7] Ciarletta M., Scalia A. and Svanadze M. (2007): *Fundamental solution in the theory of micropolar thermoelastic for materials with voids*. − J. Therm. Stress, vol.30, pp.213-229.

[8] Hou P.F., Leung A.Y.T. and He Y.J. (2008): *Three-dimensional Green's functions for transversely isotropic thermoelastic biomaterials*. − Int. J. Solids Struct., vol.45, pp.6100-6113.

[9] Hou P.F., Wang L. and Yi T. (2009): *2D Green's functions for semi-infinite orthotropic thermoelastic plane*. − Appl. Math. Model., vol.33, pp.1674-1682.

[10] Xiong S.M., Hou P.F. and Yang S.Y. (2010): *2D Green's functions for semi-infinite orthotropic piezothermoelastic plane*. − IEEE Trans. Ultrason. Ferroelectr. Freq. Control, vol.57, pp.1003-1010.

[11] Aouadi M. (2010): *A theory of thermoelastic diffusion materials with voids*. − Z. Angew. Math. Phys., vol.61, pp.357-379.

[12] Hou P.F., Sha H. and Chen C.P. (2011): *2D general solution and fundamental solution for orthotropic thermoelastic materials.* − Engineering Analysis with Boundary Elements, vol.35, pp.56-60,

[13] Seremet V. (2011): *Deriving exact Green's functions and integral formulas for a thermoelastic wedge*. − Engng. Anal. with Bound. Elements., vol.35, pp.527-532.

[14] Seremet V. (2012): *New closed form Green's function and integral formula for a thermoelastic quadrant*. − Appl. Math. Model., vol.36, pp.799-812.

[15] Kumar R. and Kansal T. (2012): *Plane waves and fundamental solution in the generalized theories of thermoelastic diffusion.* − Int. J. Appl. Math. Mech., vol.8, pp1-20.

[16] Kumar R. and Chawla V. (2011): *A study of fundamental solution in orthotropic thermodiffusive elastic media*. − International Communication in Heat and Mass Transfer, vol.27, pp.456-462.

[17] Kumar R. and Chawla V. (2012*): Green's functions in orthotropic thermoelstic diffusion media*. − Engineering Analysis with Boundary Elements, vol.36, pp.1272-1277.

[18] Kumar R. and Chawla V. (2012): *General steady-state solution and green's function in orthotropic piezothermoelastic diffusion medium.* − Archives of Mechanics, vol.64, pp.555-579.

[19] Kumar R. and Chawla V. (2013): *Fundamental solution for two-dimensional problem in orthotropic piezothermoelastic diffusion media.* − Material Physics and Mechanics, vol.16, pp.159-174.

[20] Kumar R. and Chawla V. (2013): *Reflection and refraction of plane wave at the interface between elastic and thermoelastic media with three-phase-lag*. − International Communication in Heat and Mass Transfer, vol48, pp.53-60.

[21] Kumar R. and Gupta V. (2014): *Green's function for transversely isotropic thermoelastic diffusion bimaterials*. − Journal of Thermal Stresses, vol.37, pp.1201-1229.

[22] Kumar R. and Chawla V. (2015): *General solution and fundamental solution for two-dimensional problem in orthotropic thermoelastic media with voids*. − Journal of Advanced Mathematics and Applications, American Scientific Publishers, vol.3, pp.1-8.

[23] Şeremet V. (2016): *A method to derive thermoelastic Green's functions for bounded domains (on examples of two-dimensional problems for parallelepipeds).* − Acta Mechanica, vol.227, pp.3603-3620.

[24] Pan L.H, Hou P.F and Chen J.Y. (2016): *2D steady-state general solution and fundamental solution for fluid-saturated*. − Z. Angew. Math. Phys ZAMP, pp.67-84.

[25] Chawla V., Ahuja S. and Rani V. (2017): *Fundamental solution for a two-dimensional problem in transversely isotropic micropolar thermoelastic media.* − Multidiscipline Modeling in Materials and Structures, vol.13, pp.409-423.

[26] Dang H.Y., Zhao M.H., Fan C.Y. and Chen Z.T. (2018): *Analysis of arbitrarily shaped planar cracks in three-dimensional isotropic hygrothermoelastic media*. − J. Therm. Stress.,vol.6, pp.1-28.

[27] Zhao M.H., Dang H.Y., Fan C.Y. and Chen Z.T. (2018): *Three dimensional steady-state general solution for isotropic hygrothermoelastic media*. − Journal of Thermal Stresses, vol.41, pp.951-972.

[28] Tomar T., Goyal N. and Szekeres A. (2019): *Plane waves in thermo-viscoelastic material with voids under different theories of thermoelasticity.* − Int. J. of Applied Mechanics and Engineering, vol.24, pp.691-708.

[29] Biswas S. (2020): *Fundamental solution of steady oscillations equations in nonlocal thermoelastic medium with voids.* − Journal of Thermal Stresses, vol.43, pp.284-304.

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