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MIXED CONVECTIVE FLOW OF UNSTEADY HYDROMAGNETIC COUPLE STRESS FLUID THROUGH A VERTICAL CHANNEL FILLED WITH POROUS MEDIUM

C.R. MAKHALEMELE and L. RUNDORA*
Department of Mathematics and Applied Mathematics
University of Limpopo
Private Bag X1106, Sovenga 0727, SOUTH AFRICA
E-mail: lazarus.rundora@ul.ac.za

S.O. ADESANYA
Department of Mathematical Sciences, Redeemer's University
Ede, NIGERIA

In this paper, the mixed convective flow of an electrically conducting, viscous incompressible couple stress fluid through a vertical channel filled with a saturated porous medium has been investigated. The fluid is assumed to be driven by both buoyancy force and oscillatory pressure gradient parallel to the channel plates. A uniform magnetic field of strength B_0 is imposed transverse to the channel boundaries. The temperature of the right channel plate is assumed to vary periodically, and the temperature difference between the plates is high enough to induce radiative heat transfer. Under these assumptions, the equations governing the two-dimensional couple stress fluid flow are formulated and exact solutions of the velocity and the temperature fields are obtained. The effects of radiation, Hall current, porous medium permeability and other various flow parameters on the flow and heat transfer are presented graphically and discussed extensively.

Keywords: couple stress fluid, mixed convection, magnetohydrodynamics, oscillatory flow.

1. Introduction

Fluid flow and heat transfer in varied mechanical configurations filled with porous media continues to be an area of research interest for both Newtonian and non-Newtonian fluids. Engineers, scientists and technocrats have over time exploited the ubiquitous nature of porous media to develop machinery, equipment and industrial processes that has seen transformation of the world through stages from the first to the fourth industrial revolution. It cannot be an exaggeration to postulate that modernity has to a larger extent depended on exploitation and manipulation of fluid flow systems through porous materials. Khaled and Vafai [1] pointed out that transport theories in porous media have played a defining role in the advancement of a plethora of applications, examples of which are geology, chemical reactors, drying and liquid composite moulding, combustion and biological applications. The Handbook of Porous Media, Vafai [2], provides a succinct overview of the latest theories on flow, transport, and heat exchange processes in porous media. Purusothaman and Chamkha [3] and Rundora and Makinde [4] are some examples of recent studies on flow and heat transfer in porous media.

Liquid foams, geological materials, emulsions, hydrocarbon oils, polymeric fluids, etc, are examples of fluids belonging to a class broadly described by the generic term non-Newtonian. Industrial applications are dominated by fluids falling into this class and their complex hydrodynamic characteristics necessitated the emergence of a robust and complex non-Newtonian rheology. Models encountered in literature include

*To whom correspondence should be addressed the Power law model, fluids of the differential type, visco-elastic fluid models like the Johnson-Seagalman model, the Oldroyd model, the Casson fluid model, the couple stress model and many others (see for instance [5 - 7]). The couple stress fluid model is a generalisation of the classical Newtonian constitutive model for viscous fluids that account for the inclusion of couple stresses, body couples and non-symmetric tensors in the fluid medium [8]. Fluids such as lubricants, synthetic oils, paints, and blood, which contain tiny microstructures can be modelled efficiently by the couple stress fluid model. Couple stress fluids find applications as surfactants, coolants, lubricating fluids, toothpaste and gels, pharmaceutical mixtures, ferrofluids used in shock absorbers and many more [9].

Complexities arising from solving various couple stress fluid models and the wide application horizon has motivated many scholars to continue with active research in the couple stress fluid model. Hassan [10] used modified Adomian decomposition method to analyse a reactive hydromagnetic couple stress fluid flow through a channel filled with saturated porous media. Makinde and Eegunjobi [11] investigated the inherent irreversibility in a steady flow of a couple stress fluid through a vertical channel packed with saturated porous substances. Murthy *et al.* [12] studied the entropy generation in a steady flow of two immiscible couple stress fluids in a horizontal channel bounded by two porous beds at the bottom and top. Hassan and Fenuga [13] investigated the effects of thermal radiation on the flow of a reactive hydromagnetic heat generating couple stress fluid through a porous channel. When an external magnetic field is imposed onto a moving electrically conducting fluid, current is induced into the fluid which in turn polarises the fluid and a drag-like force (Lorentz force) is formed. This magnetohydrodynamic (MHD) phenomenon has pertinent applications in thermo-electrical systems like heat exchangers, cooling of electronic devices, electromagnetic processing of materials, metal purification and astrophysical applications [14].

Meanwhile studies of oscillatory MHD convection fluid flow in porous media have gained increased attention due to wide applications in physiology, physics and engineering. Examples in engineering include MHD generators, food processing industry, chemical process industry, centrifugation and filtration processes and rotating machinery (see [15 - 16]). In physiology, peristaltic motion is the major mechanism through which heat and fluid get transported through biological systems like the human body, and MHD principles are applied to accelerate the flow of blood which is useful in the treatment of some disorders [17]. Nayak and Dash [18] studied transient hydromagnetic flow of an electrically conducting couple stress fluid in a rotating frame of reference through a saturated porous channel under the influence of pulsatile pressure gradient. Sankad and Nagathan [19] examined the effects of MHD couple stress fluid in peristaltic flow with porous medium under the impact of slip, heat transfer and wall properties. MHD effects on peristaltic flow of a couple stress fluid in a channel with permeable walls was studied by Ramachandraiah *et al*. [20]. The novelty of the work of these researchers is that their work provides a guideline for some biomedical instruments like blood pumps in dialysis and heart lung machine. The problem of oscillatory MHD flow of blood in a porous arteriole in the presence of chemical reaction was investigated by Misra and Adhikary [21]. The work provides useful insights to biophysicists, physiologists and clinicians. Adesanya and Makinde [22] investigated the effect of slip on the hydromagnetic pulsatile flow through a channel filled with a saturated porous medium with time dependent boundary condition on the heated wall. Other related recent studies are found in [23 - 25].

Most of the studies cited above concentrated on one dimensional flow, overlooking the fact that the applied magnetic field induces a secondary flow in the direction parallel to it. Veera Krishna and Chand Basha [26] investigated the effects of radiation and Hall current on MHD oscillatory convection of a two dimensional flow of a viscous fluid in a vertical channel filled with a porous medium, and VeeraKrishna*et al*. [27] investigated the same type of flow but of a second grade fluid. Motivated by these papers and the need to contribute to the ongoing studies, this article studies mixed convective two dimensional flow of unsteady MHD couple stress fluid through a vertical channel filled with a porous medium. All the previous studies on oscillatory fluid flow neglect the effect of couple stresses and this explains why the current study is worthwhile.

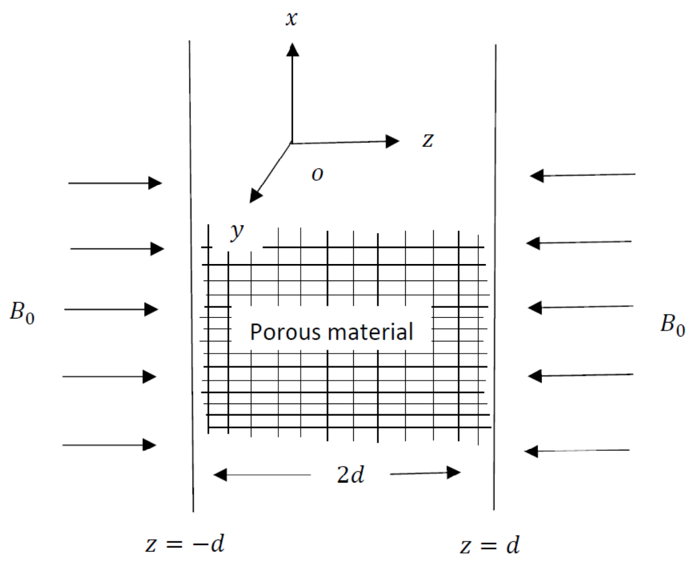

Fig.1. Physical model and coordinate system of the problem.

2. Mathematical formulation and solution of the problem

The schematic diagram of an unsteady mixed convection flow of an electrically conducting, viscous, incompressible couple stress fluid between two infinite vertical plates in the presence of Hall current and thermal radiation is illustrated in Fig.1. The fluid is driven by both buoyancy force and an oscillating pressure gradient parallel to the channel plates. The channel plates are at a distance $2d$ apart and the channel is filled with a homogeneous and isotropic porous medium. A Cartesian coordinate system $0(x,y,z)$ is chosen such that the x- axis lies along the centre of the channel in a vertical upward direction and the z-axis is oriented perpendicular to the planes of the plates. In this way, the boundary plates at $z=-d$ and $z=d$ are parallel to the xy- plane and the magnetic field of strength B_0 is applied in the transverse xz-plane as shown in the figure. The magnetic field induces a secondary flow in the z- direction. Following [27 - 28], the equations governing the flow under the influence of the imposed magnetic field are

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2}-\frac{\sigma B_0{}^2}{\rho}u-\frac{v}{k}u+g\beta T-\frac{\eta}{\rho}\frac{\partial^4 u}{\partial z^4}, \tag{2.1}$$

$$\frac{\partial w}{\partial t}=\nu\frac{\partial^2 w}{\partial z^2}-\frac{\sigma B_0{}^2}{\rho}w-\frac{v}{k}w-\frac{\eta}{\rho}\frac{\partial^4 w}{\partial z^4}, \tag{2.2}$$

$$\rho C_p\frac{\partial T}{\partial t}=\mathrm{K}\frac{\partial^2 T}{\partial z^2}-\frac{\partial q_r}{\partial z}. \tag{2.3}$$

The boundary conditions for the problem are given as

$$u''=u=w=T=0,\quad z=-d,$$

$$u''=u=w=0,\qquad T=T_w\cos\omega t,\qquad z=d \tag{2.4}$$

where T_w is the mean temperature of the plate $z = d$ and ω is the frequency of the oscillations.

Assuming that the fluid is optically thin with a relatively low density, Cogley *et al*. [29], the radiative heat flux $\frac{\partial q_r}{\partial z}$ in Eq.(2.3) is given by

$$\frac{\partial q_r}{\partial z} = 4\alpha_2^2 \left(T - T_0\right), \tag{2.5}$$

where α_2 is the mean radiation absorption coefficient. Taking the reference temperature at the left channel plate T_0 to be equal to 0 reduces Eq.(2.5) to

$$\frac{\partial q_r}{\partial z} = 4\alpha_2^2 T . \tag{2.6}$$

Introducing the dimensionless variables

$$\begin{gathered} z^* = \frac{z}{d}, \quad x^* = \frac{x}{d}, \quad u^* = \frac{u}{U}, \quad v^* = \frac{v}{U}, \quad q^* = \frac{q}{U}, \quad t^* = \frac{tU}{d}, \quad \omega^* = \frac{\omega d}{U}, \\ p^* = \frac{p}{\rho U^2}, \quad T^* = \frac{T}{T_w} , \end{gathered} \tag{2.7}$$

transforms the governing Eqs (2.1) – (2.3) to, after dropping the asterisks, the non-dimensional form

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{R}}\frac{\partial^2 u}{\partial z^2} - \frac{\mathrm{Ha}^2}{\mathrm{R}}u - \frac{D^{-1}}{\mathrm{R}}u + \frac{\mathrm{Gr}}{\mathrm{R}}T - \frac{1}{\kappa^2 \mathrm{R}}\frac{\partial^4 u}{\partial z^4} , \tag{2.8}$$

$$\frac{\partial w}{\partial t} = \frac{1}{\mathrm{R}}\frac{\partial^2 w}{\partial z^2} - \frac{\mathrm{Ha}^2}{\mathrm{R}}w - \frac{D^{-1}}{\mathrm{R}}w - \frac{1}{\kappa^2 \mathrm{R}}\frac{\partial^4 w}{\partial z^4} , \tag{2.9}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\mathrm{Pe}}\frac{\partial^2 T}{\partial z^2} - \frac{\delta^2}{\mathrm{Pe}}T \tag{2.10}$$

where $\mathrm{R} = \frac{Ud}{v}$ is the Reynolds number, $D = \frac{k}{d^2}$ is the Darcy parameter, $\mathrm{Gr} = \frac{g\beta d^2 T_w}{vU}$ is the thermal Grashof number, $\mathrm{Pe} = \frac{\rho C_p dU}{\mathrm{K}}$ is the Peclet number, $\delta = \frac{2\alpha_2 d}{\sqrt{\mathrm{K}}}$ is the thermal radiation parameter, $\mathrm{Ha}^2 = \frac{\sigma B_0^2 d^2}{\rho \nu}$ is the Hartmann number and $\kappa^2 = \frac{d^2 \nu}{\eta}$ is the couple stress parameter.

The transformed boundary conditions are

$$\begin{gathered} u'' = u = w = T = 0, \quad z = -1 , \\ u'' = u = w = 0, \quad T = \cos\omega t, \quad z = 1 . \end{gathered} \tag{2.11}$$

Assuming a complex solution of the form

$$q = u + iw, \tag{2.12}$$

reduces Eqs (2.8) and (2.9) to a single equation of the form

$$\frac{\partial q}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{R}}\frac{\partial^2 q}{\partial z^2} - \left(\frac{\mathrm{Ha}^2}{\mathrm{R}} + \frac{1}{D\mathrm{R}}\right)q - \frac{1}{\kappa^2\mathrm{R}}\frac{\partial^4 q}{\partial z^4} + \frac{\mathrm{Gr}}{\mathrm{R}}T\,. \tag{2.13}$$

The system of equations to be solved now reduces to a system of two equations, namely Eq.(2.10) and Eq.(2.13).

The boundary conditions in complex form are

$$\begin{aligned} &q = T = 0, \quad z = -1, \\ &q = 0, \quad T = e^{i\omega t}, \quad z = 1\,. \end{aligned} \tag{2.14}$$

For a purely oscillatory flow, we assume that

$$-\frac{\partial p}{\partial x} = \lambda e^{i\omega t}, \quad q(t,z) = \phi(z)e^{i\omega t} \quad \text{and} \quad T(t,z) = \theta(z)e^{i\omega t} \tag{2.15}$$

where λ is any positive constant and ω is the frequency of oscillation. Substituting Eq.(2.15) into Eq.(2.13) gives

$$\begin{aligned} &i\omega\phi(z)e^{i\omega t} = \lambda e^{i\omega t} + \frac{1}{\mathrm{R}}\phi''(z)e^{i\omega t} - \frac{1}{\mathrm{R}}\left(\mathrm{Ha}^2 + S^2\right)\phi(z)e^{i\omega t} + \\ &-\frac{1}{\kappa^2\mathrm{R}}\phi''''(z)e^{i\omega t} + \frac{\mathrm{Gr}}{\mathrm{R}}\theta(z)e^{i\omega t} \end{aligned} \tag{2.16}$$

where $S^2 = \frac{1}{D}$ is the porous medium parameter. Equivalently, since $e^{i\omega t}$ is common, we get

$$i\omega\phi(z) = \lambda + \frac{1}{\mathrm{R}}\frac{d^2\phi}{dz^2} - \frac{1}{\mathrm{R}}\left(\mathrm{Ha}^2 + S^2\right)\phi(z) - \frac{1}{\kappa^2\mathrm{R}}\frac{d^4\phi}{dz^4} + \frac{\mathrm{Gr}}{\mathrm{R}}\theta(z), \tag{2.17}$$

which simplifies to

$$\frac{d^4\phi}{dz^4} = \kappa^2\left[\mathrm{R}\lambda + \frac{d^2\phi}{dz^2} - \left(\mathrm{Ha}^2 + S^2 + \mathrm{R}i\omega\right)\phi + \mathrm{Gr}\theta\right], \tag{2.18}$$

along with boundary conditions

$$\phi(\pm 1) = 0 = \phi''(\pm 1)\,. \tag{2.19}$$

For the temperature field, after substituting Eq.(2.15) into Eq.(2.10), we get

$$\theta \mathrm{Pe} iwe^{iwt} = \frac{d^2\theta}{dz^2} e^{iwt} - \delta^2 \theta e^{iwt}, \tag{2.20}$$

which becomes

$$\frac{d^2\phi}{dz^2} - \gamma^2\theta = 0, \qquad \gamma^2 = \delta^2 + \mathrm{Pe} i\omega, \tag{2.21}$$

with the boundary conditions

$$\theta(1) = 1, \qquad \theta(-1) = 0. \tag{2.22}$$

Solving the ordinary differential Eq.(2.21) under boundary conditions given by Eq.(2.22) gives

$$\theta(z) = \frac{\sinh\gamma(z+1)}{\sinh 2\gamma}. \tag{2.23}$$

In exponential form, $\theta(z)$ is equivalently written as

$$\theta(z) = \frac{e^{\sqrt{\delta^2+\mathrm{Pe}i\omega} - z\sqrt{\delta^2+\mathrm{Pe}i\omega}\left(1+e^{2z\sqrt{\delta^2+\mathrm{Pe}i\omega}}\right)}}{1+e^{2\sqrt{\delta^2+\mathrm{Pe}i\omega}}}. \tag{2.24}$$

In this way, the exact solution for the temperature field is obtained as

$$T(z,t) = \frac{e^{\sqrt{\delta^2+\mathrm{Pe}i\omega} - z\sqrt{\delta^2+\mathrm{Pe}i\omega}\left(1+e^{2z\sqrt{\delta^2+\mathrm{Pe}i\omega}}\right)}}{1+e^{2\sqrt{\delta^2+\mathrm{Pe}i\omega}}} e^{i\omega t}. \tag{2.25}$$

Equation (2.18) becomes, after substituting Eq.(2.24),

$$\frac{d^4\phi}{dz^4} = \kappa^2 \left[R\lambda + \frac{d^2\phi}{dz^2} - \left(\mathrm{Ha}^2 + S^2 + \mathrm{R}i\omega\right)\phi + \mathrm{Gr}\frac{e^{\sqrt{\delta^2+\mathrm{Pe}i\omega} - z\sqrt{\delta^2+\mathrm{Pe}i\omega}\left(1+e^{2z\sqrt{\delta^2+\mathrm{Pe}i\omega}}\right)}}{1+e^{2\sqrt{\delta^2+\mathrm{Pe}i\omega}}} \right], \tag{2.26}$$

along with boundary conditions in Eq.(2.19). Due to the massive output of the symbolic solution for $\phi(z)$ only the graphical solution will be presented in the following section.

For the type of flow investigated herein, the two quantities of engineering importance are the rate of heat transfer Nu (Nusselt number) at the channel walls and the wall shear stress C_f (skin friction). From the temperature field, Eq.(2.24), we can obtain the rate of heat transfer at the left channel wall $z=-1$ which is given by

$$\mathrm{Nu}=-\left(\frac{\partial\theta}{\partial z}\right)_{z=-1}=-\frac{\sqrt{\delta^2+\mathrm{Pe}i\omega}\left(1-e^{2\sqrt{\delta^2+\mathrm{Pe}i\omega}}\right)}{1+e^{2\sqrt{\delta^2+\mathrm{Pe}i\omega}}}. \tag{2.27}$$

Similarly, from the velocity field we obtain the skin friction at the left channel wall as

$$C_f=\left(\frac{\partial\phi}{\partial z}-\frac{1}{\kappa^2}\frac{\partial^3\phi}{\partial z^3}\right)_{z=-1}. \tag{2.28}$$

Table 1. Skin friction (C_f) at the left wall plate $z=-1$.

κ	**δ**	**Ha**	***S***	**R**	**Pe**	**ω**	**Gr**	**λ**	C_f
1	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1.44055*
								2	*2.28076*
								3	*3.12096*
							1	*1*	*1.44055*
							2		*2.04090*
							3		*2.64125*
						1	*1*		*1.44055*
						2			*1.30865*
						3			*1.16637*
					0.71	*1*			*1.46063*
					1				*1.44055*
					2				*1.35264*
				1	*1*				*1.44055*
				2					*2.22470*
				3					*2.94170*
			1	*1*					*1.44055*
			2						*1.20823*
			3						*0.98416*
		1	*1*						*1.44055*
		2							*1.20823*
		3							*0.98416*
	1	*1*							*1.44055*
	2								*1.25160*
	3								*1.13504*
0.1	*1*								*1.71249*
0.2									*1.69894*
0.3									*1.67749*

Table 2. Rate of heat transfer (Nu).

Pe	δ	ω	Nu
0.71	*0.1*	*1*	*0.163580*
1			*0.293853*
2			*0.815061*
1	*0.1*		*0.293853*
	0.2		*0.314346*
	0.3		*0.348063*
	1	*1*	*0.875163*
		2	*1.133940*
		3	*1.408950*

3. Results and discussion

Equations (2.24) and (2.26) are coded into a computer symbolic package, MATHEMATICA, for successful computation of the graphical solutions. A qualitative as well as quantitative analysis of the effects of the underlying parameters on the velocity and fluid temperature profiles is carried out with the aid of simulated graphs. The computational results for the velocity profiles are presented in Figs 2 – 9 while results for the fluid temperature profiles are displayed in Figs 10 – 12.

In Fig.2, an increase in the couple stress parameter κ is seen to increase the fluid velocity profiles, signifying the thinning of the fluid. A reverse trend is seen with decreasing values of κ that shows the thickening of the fluid together with the decreasing flow velocity. The influence of the magnetic field on the flow field is modelled by the Hartmann number Ha. Increasing the Hartmann number means an increase in the intensity of the magnetic field. In Fig.3, consistent with expectation, the magnitudes of the velocity components are retarded by increasing magnetic field intensity. The transversely applied magnetic field B_0 gives rise to a drag-like force, called the Lorentz force, whose effect on the electrically conducting fluid is to damp the motion. Figure 4 illustrates the variation of the fluid velocity components with the porous medium shape parameter S. The effect of the porous medium shape parameter mirrors that of the Hartmann number. As the parameter S increases, the tortuosity of the porous matrix increases resulting in damping of the flow. The effect of the Reynolds number R on the velocity components is depicted in Fig.5, and the magnitude of either velocity component is enhanced with increasing Reynolds number. This is consistent with expectation. The influence of the Peclet number Pe on the velocity profiles is shown in Fig.6. The Peclet number is seen to retard the magnitude of the component u and enhance the component w. Similar trends are observed in Fig.7 where the effects of the frequency of oscillations on the velocity components are displayed. Figure 8 shows the magnitude of both velocity components increasing with increasing Grashof number. Radiative heat transferred from the hot plate at $z = 1$ into the fluid inevitably raises the temperature of the fluid and in the process the viscosity of the fluid is reduced, resulting in increased flow rate. This phenomenon is explained by the fact that when heat is transferred from the heated right plate into the fluid, the increased buoyancy forces enhance the velocity of the bulk of the fluid. The influence of the pressure gradient on the fluid flow is represented by the pressure gradient parameter λ. Figure 9 shows both velocity components increasing with an increase in this parameter. This is to be expected since the pressure gradient is one of the driving forces of motion in this study.

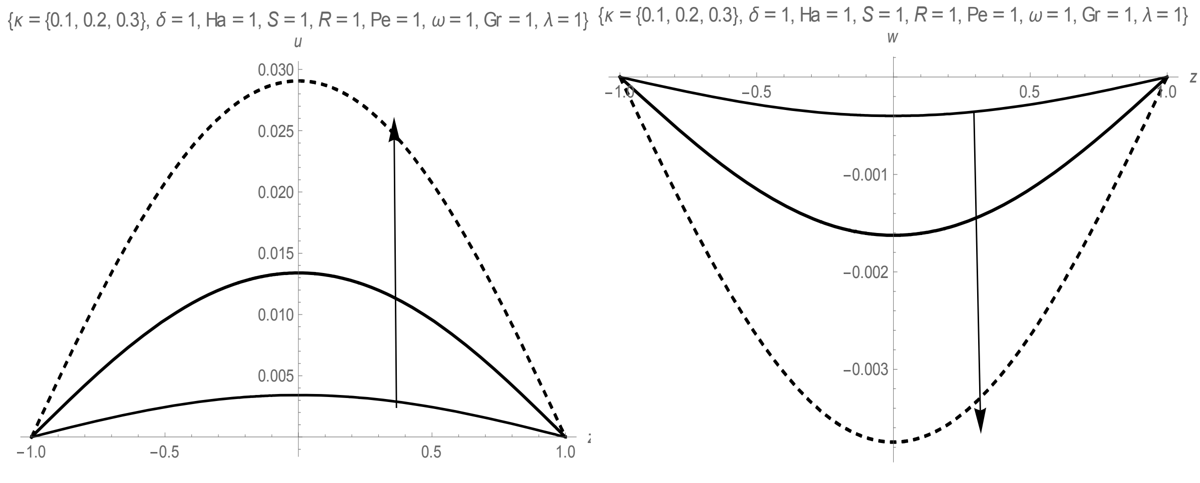

Fig.2. Effects of the couple stress parameter κ on fluid velocity profiles *u* and *w*.

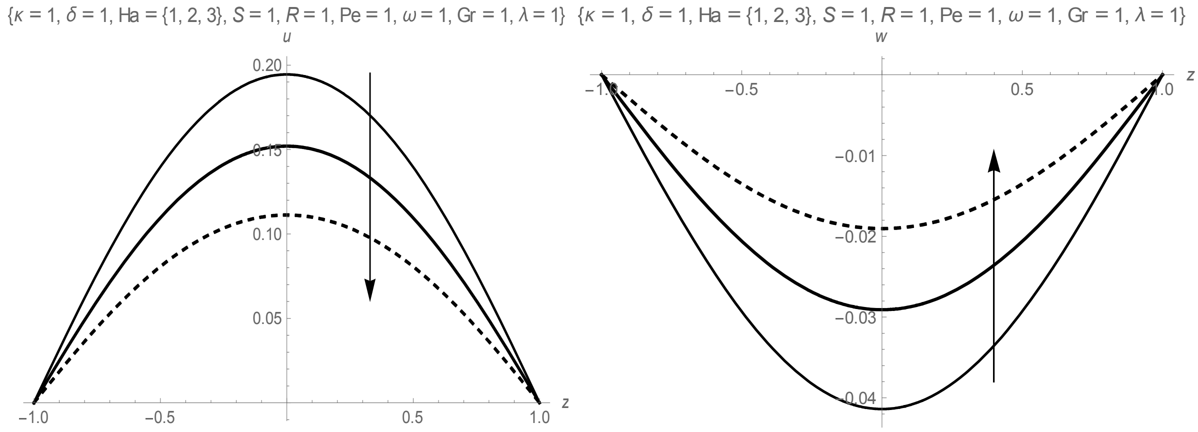

Fig.3. Effects of the Hartmann number Ha on fluid velocity profiles *u* and *w*.

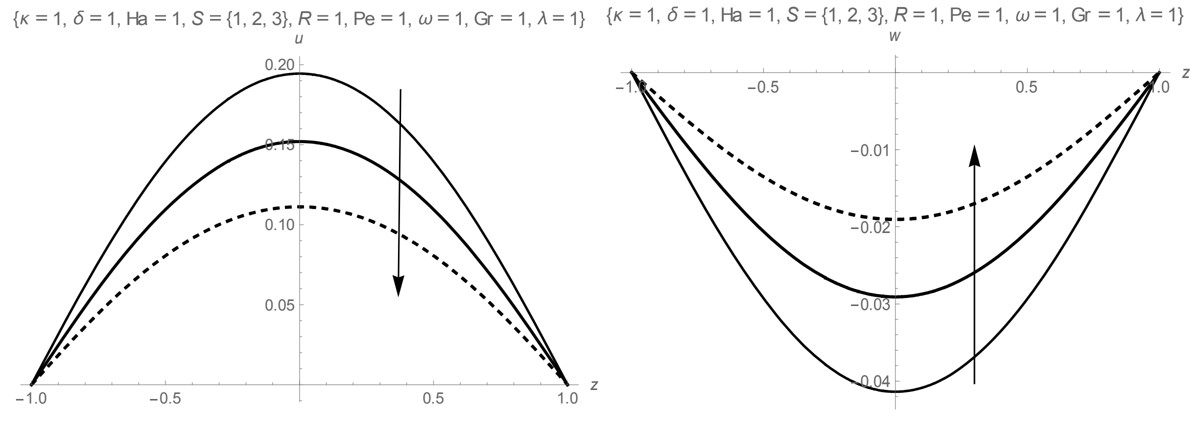

Fig.4. Effects of the porous medium parameter *S* on fluid velocity profiles *u* and *w*.

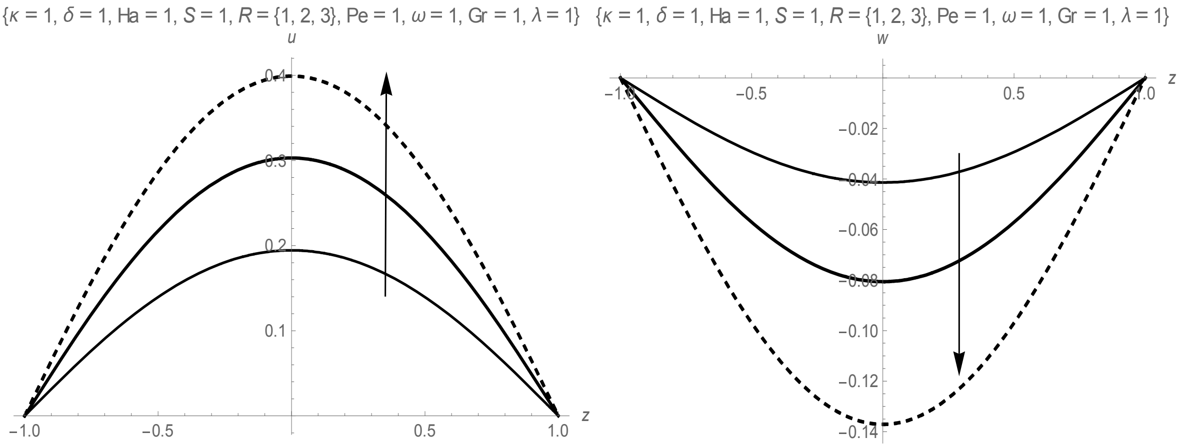

Fig.5. Effects of the Reynolds number R on fluid velocity profiles u and w.

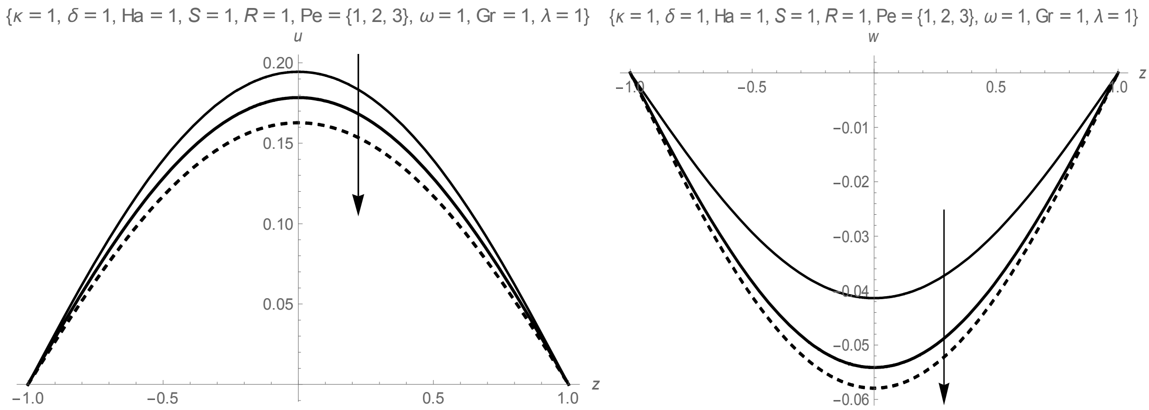

Fig.6. Effects of Peclet number Pe on fluid velocity profiles u and w.

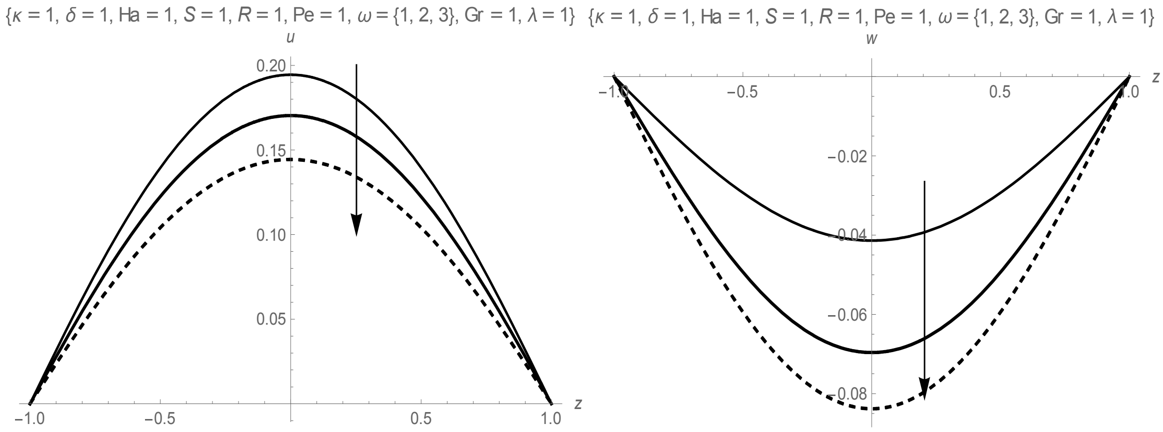

Fig.7. Effects of frequency of oscillations ω on fluid velocity profiles u and w.

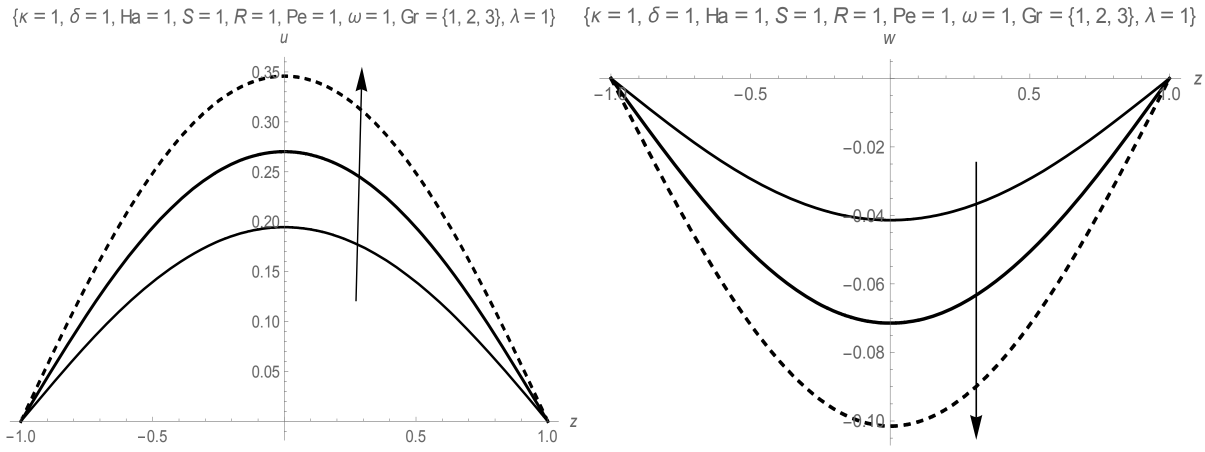

Fig.8. Effects of the Grashof number Gr on fluid velocity profiles u and w.

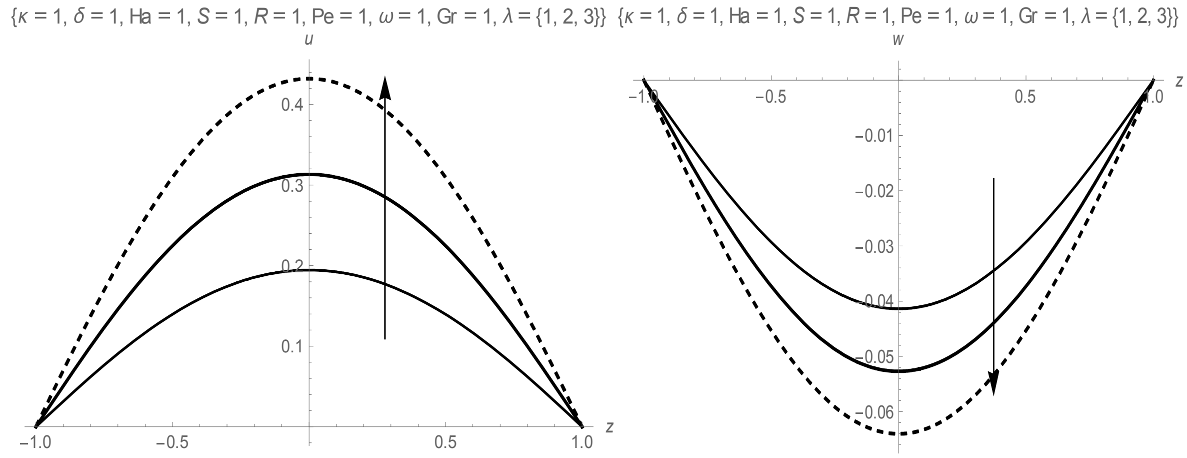

Fig.9. Effects of the pressure gradient parameter λ on fluid velocity profiles u and w.

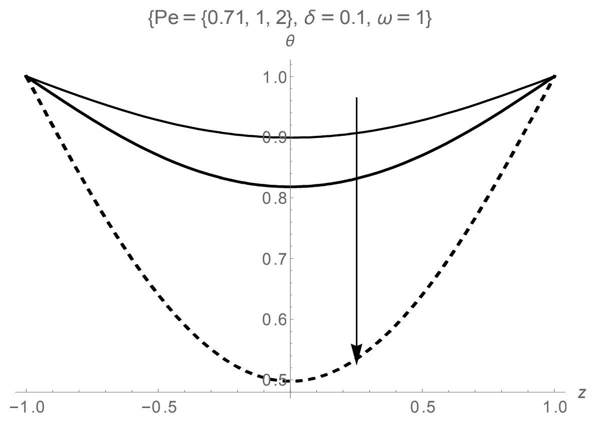

Fig.10. Effects of the Peclet number Pe on fluid temperature.

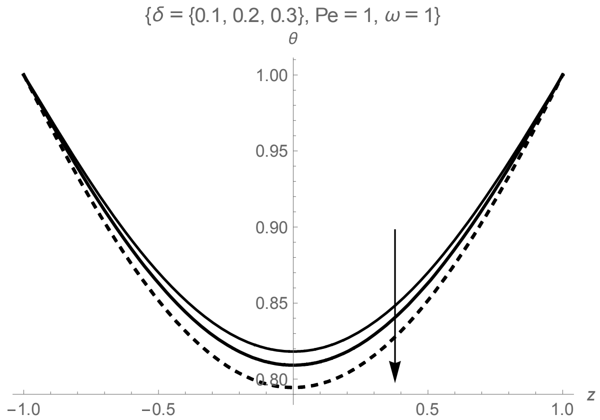

Fig.11. Effects of the thermal radiation parameter δ on fluid temperature.

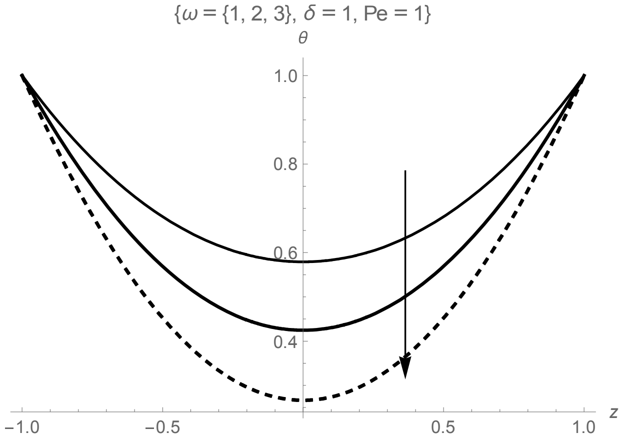

Fig.12. Effects of frequency of oscillations ω on fluid temperature.

Figure 10 shows the fluid temperature diminishing with increasing Peclet number. This phenomenon shows that within the flow, there is dominance of thermal diffusivity over momentum diffusivity. More heat is transferred from the fluid to the cooler channel plate at $z=-1$ resulting in the lowering of the fluid temperature. In Fig.11, an increase in the thermal radiation parameter leads to a decrease of the fluid temperature. The same explanation for the phenomenon in Fig.10 also applies in this case. Increasing the radiation parameter results in more radiative heat being transferred from the fluid to the cooler channel plate at $z=1$ and this, of course, lowers the temperature in the bulk of the fluid. Figure 12 shows that an increase in the frequency of oscillations reduces the fluid temperature as well. The increased rate of oscillations inevitably dissipates the heat out of the fluid into the ambient.

Tables 1 and 2 show the influence of the thermophysical parameters on the wall shear stress (skin friction) and the heat transfer rate (Nusselt number), respectively. Two interesting trends clearly stand out. Firstly, except for the couple stress parameter, parameters that increase the rate of flow also increase the skin friction and those that decrease the flow rate also decrease the skin friction. The couple stresses increase the flow rate while they marginally decrease the skin friction. Secondly, parameters that decrease the fluid temperature also decrease the heat transfer rate.

4. Conclusion

In this study, mixed convective flow of an electrically conducting, viscous incompressible couple stress fluid through a vertical channel filled with a saturated porous medium under the influence of an externally applied magnetic field has been investigated. It is observed that the velocity component for the primary flow is enhanced with an increase in the couple stress parameter, the Reynolds number, the Grashof number and the pressure gradient parameter while it is retarded with an increase in the magnetic field, the porous medium parameter, the Peclet number and the frequency of oscillations. The velocity component for the secondary flow is increased with an increase in all the parameters except the magnetic field and the porous medium parameter which retard it. It is further observed that the thermal radiation parameter, the Peclet number and the frequency of oscillations have a retardation effect on the fluid temperature. The investigation also concludes that, except for the couple stresses, parameters that increase (decrease) the fluid velocity also increase (decrease) the wall shear stress and parameters that decrease the fluid temperature also decrease the heat transfer rate. The couple stresses are observed to marginally decrease the skin friction.

Nomenclature

B_0 – applied magnetic field (Wb)
C_p – specific heat at constant pressure ($J\,kg^{-1}K$)
g – acceleration due to gravity ($m\,s^2$)
K – thermal conductivity ($W\,m^{-1}K$)
k – permeability of the porous medium ($Darcy$)
p – pressure ($N\,m^{-2}$)
q_r – radiative heat ($J\,K$)
T – temperature (K)
t – time (s)
U – mean axial velocity ($m\,s^{-1}$)
β – coefficient of volume expansion (K^{-1})
η – couple stress parameter ($N\,m^{-2}$)
ρ – density of the fluid ($Kg\,m^{-3}$)
ν – coefficient of kinematic viscosity ($NS\,m^{-2}$)
σ – electrical conductivity ($s\,m^{-1}$)

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